Chiral-symmetry-breaking corrections to the Goldberger-Treiman relation

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The problem of accounting for the corrections to the Goldberger-Treiman relation (GTR) in neutron β decay, Δ_π, is reexamined. It is argued that the failure of the traditional approach, based on dispersion relations or Reggeized one-pion-exchange fits to high-energy hadronic reactions, is probably due to the fact that the radial excitations of the pion have not been explicitly incorporated into these calculations. Although there is now experimental confirmation of the existence of a three-pion resonance, more data is needed in order to update these analyses. A different approach to the calculation of Δ_π is proposed here. Starting from the chiral-SU(2)$\times$SU(2)-symmetry limit, where the GTR is exact, one interprets all the GTR parameters as referring strictly to that limit. By calculating the renormalizations induced by symmetry breaking in *all* four parameters, one can then predict Δ_π. As a result of this interpretation a new relation between Δ_π and the πN σ commutator $\sigma_{\pi N}$ is obtained. Using the most recent theoretical value of $\sigma_{\pi N}$ (35$\pm$10 MeV), one predicts $\Delta_\pi=0.06\pm0.02$, in good agreement with experiment ($\Delta_\pi^{\exp}=0.06\pm0.01$). On the other hand, a large σ term such as $\sigma_{\pi N}\simeq 60$ MeV, which in itself is very hard to understand in the framework of QCD and SU(3), would lead to $\Delta_\pi\simeq-0.30$, in obvious conflict with experiment.

I. INTRODUCTION

The Goldberger-Treiman relation (GTR) in neutron β decay is currently understood as a consequence of exact chiral SU(2)$\times$SU(2) symmetry realized in the Nambu-Goldstone fashion,[1] a view that finds a natural place in the theory of quantum chromodynamics[2] (QCD). When SU(2)$\times$SU(2) is broken by the up- and down-quark masses in the QCD Lagrangian, the GTR is no longer exact and the quantity

$$\Delta_\pi=1-\frac{Mg_A(0)}{f_\pi g_{\pi NN}(\mu_\pi{}^2)} \tag{1}$$

measures the departure from the chiral-symmetry limit. In Eq. (1) M is the nucleon mass, $g_A(0)$ is the axial-vector coupling at zero momentum transfer, f_π is the pion decay constant, and $g_{\pi NN}(\mu_\pi{}^2)$ is the on-mass-shell pion-nucleon coupling. Using current experimental values[3] [$f_\pi=93.24\pm0.09$ MeV, $g_A(0)=1.254\pm0.007$, and $g_{\pi NN}(\mu_\pi{}^2)=13.4\pm0.01$] one finds

$$\Delta_\pi^{\exp}=0.06\pm0.01 \; . \tag{2}$$

The smallness of Δ_π provides strong support to the view that SU(2)$\times$SU(2) is the most accurate of the broken hadronic symmetries after isospin.

A very difficult problem arises when one tries to predict the actual value of Δ_π. In fact, despite all the attempts made over the years,[4–7] theoretical predictions disagree with Eq. (2) at least by a factor of two.

The standard approach[1] has been to rederive the GTR in broken SU(2)$\times$SU(2), assuming partial conservation of the axial-vector current (PCAC), in which case the discrepancy is due entirely to the change in the πNN coupling from $q^2=0$ (off-mass-shell) to $q^2=\mu_\pi{}^2$ (on-mass-shell), i.e.,

$$\Delta_\pi=1-\frac{g_{\pi NN}(0)}{g_{\pi NN}(\mu_\pi{}^2)}=1-F_{\pi NN}(0) \; , \tag{3}$$

where $F_{\pi NN}(q^2)$ is the πNN hadronic form factor and $F_{\pi NN}(\mu_\pi{}^2)=1$. In other words, PCAC implies that the GTR in broken SU(2)$\times$SU(2) is still exact at $q^2=0$ (but not necessarily at $q^2=\mu_\pi{}^2$). The value of the πNN form factor at zero momentum transfer, $F_{\pi NN}(0)$, has been estimated in a variety of ways such as (a) by means of dispersion relations,[4] (b) invoking specific dynamical models such as the σ model[5] or the dual model,[6] (c) performing Reggeized one-pion-exchange (OPE) analyses of hadronic and electromagnetic reactions at high energy and small momentum transfers.[7] All these estimates are in fairly good agreement with each other but lead to $\Delta_\pi \sim 0.01-0.03$.

In this paper I wish to discuss a possible model-independent solution to this old puzzle based on the following two premises: (i) abandon the assumption that the GTR is exact at $q^2=0$ when $SU(2)\times SU(2)$ is broken, and (ii) interpret *all* the parameters that appear in the GTR as referring strictly to the chiral-symmetry limit. By calculating the renormalizations induced by symmetry breaking one can then predict Δ_π.

Since an exact GTR at $q^2=0$, when $\mu_\pi{}^2 \neq 0$, is a consequence of PCAC, it may appear that requirement (i) above could spoil the success of the current-algebra—PCAC low-energy theorems. It will be argued in Sec. II that this is not the case and, moreover, that a breakdown of the GTR at $q^2=0$ ($\mu_\pi{}^2 \neq 0$) is to be expected naturally from a pseudoscalar meson mass spectrum with radial excitations. These states were first predicted in the framework of the dual model,[6] where they arise as Regge recurrences (daughters) of the pion trajectory, and later in QCD bag models, where they correspond to radial excitations of the quark-antiquark bound state.[8] Also, Pagels[9] had suggested earlier that a heavy pion might account for the GTR discrepancy through an enhancement in the three-pion contribution to the dispersion relation for the πNN form factor. These states eluded experimental detection for many years until very recently when the first radial excitation of the pion was found in two independent analyses[10,11] of the reaction $\pi N \to (3\pi)N$, with a mass $\mu_{\pi'} \sim 1.3$ GeV and a broad width $\Gamma_{\pi'} \sim 0.6$ GeV.

With this in mind, the GTR becomes an exact statement valid strictly in the chiral-symmetry limit and the discrepancy Δ_π is the result of the renormalizations induced by $SU(2)\times SU(2)$ breaking in *all* four GTR parameters. An interesting consequence of this interpretation is that Δ_π becomes strongly correlated to the πN σ commutator.[12] This is discussed in Sec. III together with the calculation of the shifts in the GTR parameters that leads to a correct prediction of Δ_π.

II. THE GOLDBERGER-TREIMAN RELATION

Let us consider the matrix element of the axial-vector current between nucleons,

$$\langle N(p')\,|\,A_\mu\,|\,N(p)\rangle = \bar{u}(p')[\gamma_\mu\gamma_5 g_A(q^2)+q_\mu\gamma_5 g_P(q^2)]u(p)\ , \tag{4}$$

where $q^2=(p'-p)^2$ and isospin indices have been omitted for simplicity. Taking the divergence of Eq. (4) in the $SU(2)\times SU(2)$ limit one finds

$$2\tilde{M}\tilde{g}_A(q^2)+q^2\tilde{g}_P(q^2)=0\ , \tag{5}$$

where all quantities with a tilde refer to the symmetry limit, i.e., $\tilde{M}=M(\mu_\pi{}^2=0)$, $\tilde{g}_A(q^2)=g_A(q^2,\mu_\pi{}^2=0)$, etc. As $q^2\to 0$, the induced pseudoscalar form factor $\tilde{g}_P(q^2)$, exhibits the Goldstone-boson pole and one obtains the GTR

$$\frac{\tilde{M}\tilde{g}_A(0)}{\tilde{f}_\pi \tilde{g}_{\pi NN}(0)}=1\ , \tag{6}$$

where $\tilde{g}_{\pi NN}(0)$ is to be understood as the on-mass-shell "Goldstone-pion"—nucleon coupling.

In the traditional approach[1] one rederives the GTR in broken $SU(2)\times SU(2)$ by assuming PCAC,

$$\partial^\mu A_\mu = \mu_\pi{}^2 f_\pi \phi_\pi \tag{7}$$

with

$$\langle N(p')\,|\,\phi_\pi\,|\,N(p)\rangle = \frac{\langle N(p')\,|\,j_\pi\,|\,N(p)\rangle}{\mu_\pi{}^2-q^2}\ , \tag{8}$$

where $j_\pi=(\Box+\mu_\pi{}^2)\phi_\pi$ and the following definition of the πNN form factor:

$$\langle N(p')\,|\,j_\pi\,|\,N(p)\rangle = -i\bar{u}(p')\gamma_5\sqrt{2}g_{\pi NN}(q^2)u(p)\ . \tag{9}$$

In fact, taking the divergence of Eq. (4) and using Eqs. (7)—(9) one finds in the limit $q^2\to 0$

$$\frac{Mg_A(0)}{f_\pi g_{\pi NN}(0)}=1\ , \tag{10}$$

where $g_{\pi NN}(0)$ is now the "physical-pion"—nucleon coupling evaluated off the mass shell ($q^2=0$), and $g_p(0)$ is no longer singular, i.e., $q^2 g_p(q^2)=0$ as $q^2\to 0$. But Eq. (10) leads inevitably to Eq. (3) and thus to an incorrect prediction of Δ_π.

It has been shown some time ago[13] that the incorporation of heavy pions into Eq. (7), i.e., extended PCAC (EPCAC), leads to a model-independent and unified technique for calculating the leading analytic chiral-symmetry-breaking corrections to current-algebra low-energy theorems. This can be done without invoking any specific model for the mass spectrum and decay constants of the heavy pions. Moreover, these corrections always act in the right direction bringing the predictions of the low-energy theorems into agreement with experiment at the one-standard-deviation level. The recent discovery of the first radial excitation of the pion lends even more support to EPCAC and it is only natural to reexamine in this light the above derivation of the GTR in broken SU(2)×SU(2).

In this case, Eq. (7) now reads

$$\partial^\mu A_\mu = \sum_{i=0}^{N} \mu_{\pi_i}^2 f_{\pi_i} \phi_{\pi_i} , \tag{11}$$

where $N \geq 1$. The only requirement one needs to impose on the heavy pions is that they do not become Goldstone bosons in the symmetry limit, i.e., $f_{\pi_i} = O(\mu_\pi^2)$ for $i \geq 1$. This guarantees that the chiral symmetry is still SU(2)×SU(2) and that the corrections to the low-energy theorems are of $O(\mu_\pi^2)$, as opposed to $O(1)$ otherwise.

It is straightforward to show that Eq. (10) now becomes

$$Mg_A(0) = f_\pi g_{\pi NN}(0) \left[1 + \sum_{i=1}^{N} \frac{f_{\pi_i}}{f_\pi} \frac{g_{\pi_i NN}(0)}{g_{\pi NN}(0)} \right] , \tag{12}$$

where $g_{\pi NN}(0) = g_{\pi NN}(\mu_\pi^2) F_{\pi NN}(0)$ has the *same* meaning as before [see Eq. (9)]. Using Eqs. (12) and (1) one finds

$$\Delta_\pi = 1 - F_{\pi NN}(0) \left[1 + \sum_{i=1}^{N} \frac{f_{\pi_i}}{f_\pi} \frac{g_{\pi_i NN}(0)}{g_{\pi NN}(0)} \right] . \tag{13}$$

It is obvious that by redefining the πNN form factor one could still arrive formally at Eq. (10) and thus to Eq. (3). This can also be accomplished by defining a new pion field in such a way that Eq. (11) looks formally like Eq. (7). However, in this case the numerical value of the new form factor at $q^2 = 0$ would be different from that extracted from Reggeized OPE analyses.[7] It should be recalled that these analyses probe the small- and spacelike-momentum-transfer region of high-energy hadronic reactions and thus are sensitive only to the pion-pole contribution. In other words, no *explicit* heavy-pion pole piece has ever been included in these analyses. This is also true of the dispersion relation calculation of the form factor,[4] except that since in this case one is probing the timelike region, one would expect a stronger sensitivity to the heavy-pion contributions.

In summary, the radial excitations of the pion generate an additional contribution of $O(\mu_\pi^2)$ to Δ_π no matter what definition one chooses for the πNN form factor or the pion field. This extra piece has not been included in the dispersion relation[4] or Reggeized OPE extractions[7] and, therefore, the conclusion that $\Delta_\pi \sim 0.01 - 0.03$ does not necessarily follow. At the present time only the mass and width of the first heavy pion are known experimentally[10,11] and so, Eq. (13) cannot be used to predict Δ_π in a model-independent fashion. A different approach to the problem will be discussed in the next section.

III. CHIRAL-SYMMETRY-BREAKING CORRECTIONS

Adopting the point of view that the GTR is an exact statement valid strictly in the chiral-symmetry limit and using Eqs. (1) and (6), one finds

$$\Delta_\pi = 1 - F_{\pi NN}(0) \frac{(\tilde{f}_\pi / f_\pi)[\tilde{g}_{\pi NN}(0)/g_{\pi NN}(0)]}{(\tilde{M}/M)[\tilde{g}_A(0)/g_A(0)]} , \tag{14}$$

where $g_{\pi NN}(0) = g_{\pi NN}(\mu_\pi^2) F_{\pi NN}(0)$ is defined in the usual way by Eq. (9) and, therefore, $F_{\pi NN}$ is the form factor extracted from Reggeized OPE analyses,[7] i.e.,

$$F_{\pi NN}(0) = 0.976 \pm 0.006 . \tag{15}$$

According to this approach the actual value of Δ_π is to emerge as the result of a very delicate balance between the renormalizations induced by SU(2) ×SU(2) breaking in *all* four GTR parameters. At the same time, since the σ commutator[12] $\sigma_{\pi N}$ is a measure of the shift in the nucleon mass, Eq. (14) will lead to a relation between Δ_π and $\sigma_{\pi N}$.

The shift in f_π induced by chiral SU(2)×SU(2) symmetry breaking has been calculated by Langacker and Pagels[14] in chiral perturbation theory. Due to the nonanalytic approach to the symmetry limit in this application, the leading nonanalytic correction to $\tilde{f}_\pi / f_\pi$ can be computed exactly up to a cutoff which has very little effect on the numerical prediction. The result of this calculation is[14]

$$\frac{\tilde{f}_\pi}{f_\pi} = \left[1 - \frac{\mu_\pi^2}{16\pi^2 f_\pi^2} \ln \left[\frac{4\mu_\pi^2}{\Lambda} \right] \right]^{-1} , \tag{16}$$

which, for $M_\rho^2 \leq \Lambda \leq 4M^2$, gives

$$\frac{\tilde{f}_\pi}{f_\pi} = 0.96 \pm 0.01 . \tag{17}$$

The calculation of the shifts in $g_{\pi NN}$ and in g_A

will be done here by studying the current-algebra constraints on the isospin-even and -odd πN amplitudes, i.e., the Adler and Adler-Weisberger relations.[15] Let us consider the process

$$\pi(q)+N(p)\to\pi(q')+N(p')$$

and define

$$\nu=\frac{s-u}{4M},\qquad \nu_B=\frac{-q\cdot q'}{2M}=\frac{t-q^2-q'^2}{4M}, \tag{18}$$

where s, t, and u are the usual Mandelstam variables. In the chiral-symmetry limit the isopin-even πN amplitude $A^{(+)}(\nu,\nu_B,q^2,q'^2)$ is given by

$$\tilde{A}^{(+)}(0,0,0,0)=\frac{\tilde{g}_{\pi NN}^2(0)}{\tilde{M}}, \tag{19}$$

where $g_{\pi NN}(0)$ is the "Goldstone-pion"—nucleon coupling evaluated at $\nu=\nu_B=0$. Breaking the symmetry and evaluting $A^{(+)}$ at the same kinematical point (now an off-mass-shell point) gives

$$A^{(+)}(0,0,0,0)=\frac{g_{\pi NN}^2(0)}{M}-\frac{\sigma_{\pi N}(0)}{f_\pi^2}+O(\mu_\pi^4), \tag{20}$$

where $\sigma_{\pi N}$ is the σ commutator of $O(\mu_\pi^2)$. Once the leading-order chiral-symmetry-breaking terms are included, the chiral amplitude extrapolates smoothly into the off-mass-shell amplitude[14] and one obtains

$$\tilde{g}_{\pi NN}^2(0)=\frac{\tilde{M}}{M}\left[g_{\pi NN}^2(0)-\frac{M\sigma_{\pi N}(0)}{f_\pi^2}\right]+O(\mu_\pi^4). \tag{21}$$

It must be pointed out that although $A^{(+)}$ changes rapidly as one approaches the on-mass-shell point,[16] i.e.,

$$T^{(+)}(0,0,\mu_\pi^2,\mu_\pi^2)=-T^{(+)}(0,0,0,0)+O(\mu_\pi^4),$$

this does not affect the above derivation as it is based on a comparison of amplitudes at the fixed point $q^2=q'^2=0$ ($T^{(+)}$ is essentially equal to $A^{(+)}$ minus the Born term).

Although the σ term appears explicitly in Eq. (21), it has little effect on the numerical value of $\tilde{g}_{\pi NN}(0)$. In fact, allowing a broad range of values $\sigma_{\pi N}(0)\sim 25-65$ MeV, one finds that $M\sigma_{\pi N}(0)/f_\pi^2\simeq 3-7$, to be compared with $g_{\pi NN}^2(0)\simeq 170$.

The strong relation between Δ_π and $\sigma_{\pi N}$ will emerge through the implicit dependence of $\tilde{M}$ on $\sigma_{\pi N}$ as discussed later.

Turning now to the isospin-odd πN scattering amplitude $T^{(-)}(\nu,\nu_B,q^2,q'^2)$, one finds in the chiral-symmetry limit

$$\lim_{\nu\to 0}\frac{\tilde{T}^{(-)}(\nu,0,0,0)}{\nu}=\frac{1-\tilde{g}_A^2(0)}{\tilde{f}_\pi^2}. \tag{22}$$

The leading-order chiral-symmetry-breaking corrections to Eq. (22) are known to be analytic[1] and of $O(\mu_\pi^2)$ and, therefore, they should be taken into account since Δ_π is of the same order. Due to the analytic nature of these corrections they have to be calculated using EPCAC rather than chiral perturbation theory. The result is[13]

$$\lim_{\nu\to 0}\frac{T^{(-)}(\nu,0,0,0)}{\nu}=\frac{1-g_A^2(0)}{f_\pi^2(1-\Delta_\pi)^2}+O(\mu_\pi^4), \tag{23}$$

where Δ_π is not predicted by EPCAC. Comparing Eqs. (22) and (23) one finds

$$\tilde{g}_A^2(0)=1-\left[\frac{\tilde{f}_\pi}{f_\pi}\right]^2\frac{[1-g_A^2(0)]}{(1-\Delta_\pi)^2}+O(\mu_\pi^4). \tag{24}$$

Clearly, Eq. (24) should not be used to predict the numerical value of $\tilde{g}_A(0)$ but, instead, it should be substituted in full into Eq. (14) in order to predict Δ_π.

When Eqs. (15), (17), (21), and (24) are substituted in Eq. (14) one finds

$$\Delta_\pi=1-(0.96\pm0.01)\times[(1.45\pm0.03)(M/\tilde{M})-(0.57\pm0.02)]^{1/2}. \tag{25}$$

In the absence of any knowledge about $\tilde{M}$, if one would assume that this renormalization is of the same magnitude as the other renormalizations, i.e., $\tilde{M}/M\simeq 0.96$, then one would obtain $\Delta_\pi\simeq 0.07\pm0.02$. But this argument, though plausible, is by no means compelling.

The problem with the calculation of $\tilde{M}$ is that chiral perturbation theory fails in this application, i.e., the leading nonanalytic corrections are huge, as argued recently by Gasser.[17] The best one can do at the present time is to relate $\tilde{M}$ to other QCD quantities such as quark masses, matrix elements of bilinear quark operators between nucleons, the σ term, etc. In this way Gasser[17] has obtained a relation between $\tilde{M}$ and $\sigma_{\pi N}(0)$ which when used in conjunction with Eq. (25) leads to the values of Δ_π listed in Table I. The reason why a given value of

TABLE I. The chiral mass of the nucleon $\tilde{M}$ is related to the σ term $\sigma_{\pi N}(0)$ through the Gasser analysis (Ref. 17) and to Δ_π through Eq. (25).

$\tilde{M}$ (MeV)	$\tilde{M}/M$	$\sigma_{\pi N}(0)$ (MeV)	Δ_π
M	1	21−40	0.10±0.02
900	0.96	23−43	0.07±0.02
800	0.85	28−50	−0.02±0.02
700	0.75	31−56	−0.12±0.02
600	0.64	34−60	−0.25±0.02

$\tilde{M}$ does not give a unique value of $\sigma_{\pi N}(0)$ is because of the uncertainties in the rest of the QCD quantities involved in the analysis.[17]

It has been argued recently[12] that the theoretical value of the σ commutator is expected to lie in the range $\sigma_{\pi N}(0)\simeq 25-45$ MeV and also that these values are in no obvious conflict with the available experimental information. This should be contrasted with previous extractions[18] that claim $\sigma_{\pi N}^{\text{expt}}=65\pm5$ MeV, but which underestimate considerably the uncertainties. It should be clear from Table I that a σ term $\sigma_{\pi N}(0)\simeq 65$ MeV, which in itself is very hard to understand in the framework of QCD and SU(3), is in obvious conflict with the corrections to the GTR.

Using the recent value[12] $\sigma_{\pi N}(0)=35\pm10$ MeV in the Gasser analysis, one finds $\tilde{M}=885\pm15$ MeV, and using Eq. (19) one predicts

$$\Delta_\pi=0.06\pm0.02\ . \qquad (26)$$

This result is consistent with the idea that the renormalization of the nucleon mass is of the same magnitude as that of the other GTR parameters.

IV. SUMMARY

The deviation from the GTR in neutron β decay has been traditionally interpreted as due to the change in the πNN coupling from $q^2=0$ to $q^2=\mu_\pi^2$ [see Eq. (3)]. But the extraction of the πNN form factor through dispersion relations[4] or Reggeized OPE analyses[7] leads inevitably to a discrepancy which is too small. It has been argued here that this failure is likely due to the fact that the radial excitations of the pion have not been included in those analyses. These radial excitations were predicted long ago[6,8,9] and the first experimental confirmations of a heavy pion have been reported recently.[10,11] Whether or not new dispersion-relation extractions of $F_{\pi NN}$ will succeed in accounting for the value of Δ_π remains to be seen. More information on these states is clearly needed for this type of calculation.

In this paper a different approach to the calculation of Δ_π is being proposed. Starting from the GTR in the chiral SU(2)×SU(2) limit, where it is exact, one interprets all the GTR parameters as referring strictly to that limit. By calculating the renormalizations induced by SU(2)×SU(2) breaking in *all* four parameters, one can then predict Δ_π.

The only quantity which cannot be calculated independently at the present time is the chiral mass of the nucleon. However, using the recent analysis of Gasser[17] one can relate $\tilde{M}$ to the σ commutator and obtain a new relation between Δ_π and $\sigma_{\pi N}(0)$. When the most recent value of $\sigma_{\pi N}(0)$ is inserted into this relation, the prediction of Δ_π is in good agreement with experiment.

In closing, a few comments about Eq. (23) are perhaps in order. Equation (23) follows from the EPCAC version of the current-algebra soft-pion theorem which in this case is[13]

$$\lim_{\substack{q\to0\\ q'\to0}} T^{\alpha\beta}=f_\pi^{\,2}(1-\Delta_\pi)^2A^{\alpha\beta}\ , \qquad (27)$$

where $A^{\alpha\beta}$ is the reduced amplitude (with poles removed) and α,β are isospin indices. The term $(1-\Delta_\pi)$ raised to a power equal to the number of pions taken soft, and which always multiplies f_π, represents the chiral-symmetry-breaking correction to the PCAC soft-pion theorem and reflects the contribution from all possible heavy pions to the divergence of the axial current. Although Δ_π is not predicted by EPCAC it is assumed that it should be accounted for by the heavy-pion contributions.

If one would use the experimental value of Δ_π, Eq. (2), then the right-hand side of (23) is equal to $-1.45\pm0.07\ \mu_\pi^{-2}$ while for $\Delta_\pi=0$ (PCAC) it is

$-1.28\pm0.05\ \mu_\pi^{-2}$. Before comparing these numbers with those extracted from πN and $\pi\pi$ data analyses[18] one should bear in mind that the latter deal with on-shell quantities, while Eq. (23) is an off-shell prediction and although the difference is expected to be small this does introduce some uncertainty. A much larger and mostly unknown uncertainty, however, comes from the data analysis itself because it involves extrapolations to the unphysical point $\nu=0$ at subthreshold. Some of these problems have been discussed recently[12] in connection with the even amplitude, $T^{(+)}$ and it has been argued that the extracted data points are likely to have errors several times bigger than what a measure of their dispersion around the fits would tend to indicate. According to the Karlsruhe analysis[18] the right-hand side of Eq. (23) for on-mass-shell pions is $-1.22\ \mu_\pi^{-2}$. The uncertainty estimated just from the dispersion of data points is at the $4-5\,\%$ level, but the (unknown) true uncertainty is probably much larger. On the other hand, it must be mentioned that the kinematic point for Eq. (23) lies to the left of the ρ resonance in a region mostly affected by background. Model predictions[19] for the $\pi\pi\to N\bar{N}$ P-wave amplitude f_+^1, which essentially determines the left-hand side of Eq. (23), agree very well with the Karlsruhe results at and beyond resonance, but there is a large systematic disagreement below the ρ peak and especially in the region relevant to Eq. (23).

In view of all these uncertainties a direct test of Eq. (23) does not appear feasible. A more meaningful test, though, is provided by the Weinberg-Tomozawa relation[15], which follows from a calculation of $T^{(-)}$ at threshold. Translated into the difference between the $I=\frac{1}{2}$ and $I=\frac{3}{2}$ πN scattering lengths, which can be extracted directly from the data, one has in EPCAC (Ref. 12)

$$a_{1/2}-a_{3/2}=\frac{3\mu_\pi M}{8\pi(\mu_\pi+M)}\frac{1}{f_\pi^{\,2}(1-\Delta_\pi)^2}\,, \tag{28}$$

while for $\Delta_\pi=0$ one recovers the PCAC prediction.[15] Using Eq. (2) in Eq. (28), one finds the following numerical results:

$$a_{1/2}-a_{3/2}=\begin{cases}0.233 & \mu_\pi^{-1}, \quad \text{PCAC } (\Delta_\pi=0)\\ 0.265\pm0.005 & \mu_\pi^{-1}, \quad \text{EPCAC}\\ 0.276\pm0.005 & \mu_\pi^{-1}, \quad \text{experiment (Ref. 18)}\end{cases}$$

which show that EPCAC improves considerably the agreement with experiment.

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