PHYSICAL REVIEW D VOLUME 25, NUMBER 7 1 APRIL 1982

Pionic decays and saturation of current-algebra sum rules in a nonrelativistic expansion of the quark shell model

M. B. Gavela*
Centre de Physique Théorique de l'Ecole Polytechnique, 91120 Palaiseau, France

A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal
Laboratoire de Physique Théorique et Hautes Energies,† Orsay, France
(Received 18 February 1981)

Pionic decays of hadrons are calculated using a PCAC (partial conservation of axial-vector current) prescription and a quark shell model with quarks bound by a central potential, described by the Dirac equation. The Dirac Hamiltonian and operators are expanded in v/c, the internal quark velocity. Then, one finds an exact saturation of the current-algebra sum rules as defined in the SU(2) ⊗ SU(2) symmetry of Gilman-Harari and Weinberg up to order v^4/c^4. The saturation is obtained without need of exotics, with the usual excitations of the ground state. The relation with the $P=\infty$ approach is clarified. The corrections found with respect to previous quark models in $L=2$ decays are discussed. They do not solve the problem of $SU(6)_W$ coupling signs. Finally, the whole Weinberg scheme of linear SU(2) ⊗ SU(2) symmetry is completed by the expression of the chiral-breaking part of the mass operator $m_4{}^2$.

I. INTRODUCTION

Although the quark model has enjoyed fair empirical success in describing pionic decays, the situation is not completely clear both on the empirical side and with respect to general principles.

Starting from the old $\vec{\sigma}\cdot\vec{k}_\pi$ coupling,[1] the naive quark model has evolved towards more refined treatments: the Mitra and Ross recoil term[2] or the somewhat equivalent Feynman-Kislinger-Ravndal (FKR) model,[3] the quark-pair-creation model.[4] These refinements have been stimulated both by general requirements like covariance and compositeness of the emitted pion, and by the empirical fact that the naive $SU(6)_W$ vertex symmetry is badly broken, a fact strongly emphasized by Rosner and co-workers with their l-broken $SU(6)_W$ symmetry.[5] But explicit quark models are still unsatisfactory in two respects:

(i) Coupling signs opposite to those of $SU(6)_W$ are unambiguously predicted for any resonance decay, in apparent contradiction with experiment for the case of positive-parity excited baryons.[6]

(ii) The current-algebra principles are not explicitly included, and the eventual role of a PCAC (partial conservation of axial-vector current) prescription (as used for instance by FKR) is unclear. In particular, one would like to know if the Adler-Weisberger sum rule is satisfied. This requirement seems important because current algebra has played a very large role in the discussion of pion emission.

On the other hand, in the $P=\infty$ approach, the general principles of PCAC and current algebra have been included from the start[7,8] and seem to play a central role. Moreover, evolving from purely phenomenological SU(3) ⊗ SU(3) mixing schemes, one generally believes that the Melosh transformation allows a fully relativistic calculation of pion decays.[9] And there has even been the claim that the $SU(6)_W$-like coupling signs of the $L=2$ band of baryons could then be naturally understood.[10] However, a general drawback of this approach is that matrix elements are left unspecified for lack of a $P=\infty$ model of bound states sufficiently easy to handle.[11]

Therefore, one would like to combine the good features of naive quark models and $P=\infty$ approaches. At the same time, one would like to understand in a precise manner the connection between $P=0$ and $P=\infty$. One further hope, if such an understanding is obtained, is to reconcile naive quark models with $SU(6)_W$-like signs for $L=2$ decays.

We want to show that it is possible to include PCAC and the Adler-Weisberger sum rule in the $P=0$ approach of the naive quark model. In fact, by a suitable and very natural definition of the pion coupling, the Adler-Weisberger (AW) sum rule, as well as the whole Weinberg scheme of SU(2) ⊗ SU(2) linear symmetry[12] are satisfied in a quark shell model with a fixed number of constituents, order by order in a nonrelativistic expansion. In fact, this saturation of the AW sum rule was suggested in a very interesting paper by Donoghue and Wyler.[13] But they could not give a precise demonstration, for lack of a systematic and coherent v/c expansion. This is a crucial point. Let us recall that Brodsky and Primack[14] and Close and Osborne[15] were able, many years ago, to demonstrate the saturation of much more difficult sum rules for Compton scattering by such a v/c expansion. However, the relevance of the method to study the chiral-algebra sum rules was not noticed. This may be due partly to the fact that the solution for Compton sum rules was rather complicated and partly to the fact that chiral symmetry was believed to involve uneasy features, especially the phenomenon of spontaneous breakdown. In fact, we are able to show that the demonstration of the AW rule does not really imply the knowledge of a dynamical mechanism for the symmetry breakdown. Also, the choice of a quark shell model (instead of two-body forces) enormously simplifies the v/c expansion, giving a very transparent discussion at least to the order v^2/c^2. We are also able to give the expressions to order v^4/c^4 and to demonstrate the saturation of this order. The general demonstration, which implies another method because the explicit expressions become untractable, will be given in a forthcoming paper. As a by-product of this work, the connection with the parallel work at $P=\infty$ (Ref. 16) becomes clear.

Of course, the pion decay amplitudes which are found are somewhat different from the previous expressions of naive quark models. However, we find that the unpleasant prediction of signs opposite to those of $SU(6)_W$ for $L=2$ remains unaffected by the additional terms which appear in the new expressions to order v^2/c^2. The apparently opposite conclusion of the $P=\infty$ approach is not confirmed by a more careful discussion, which shows agreement with the $P=0$ framework.

II. THE MODEL AND METHOD

Let us first remark that there is no difficulty in principle in including a PCAC prescription for pion transitions in a potential approach.[17] As emphasized by Weinberg,[12] a quite general expression is

$$M(\pi+\alpha\to\beta)=\frac{1}{f_\pi}\langle\beta|n^\mu A_\mu|\alpha\rangle\omega_\pi\,, \qquad (2.1)$$

where A_μ is the axial-vector current and n^μ is a unit null vector ${n_0}^2=\vec{n}^2=1$ along the pion momentum, which we choose to be in the direction of $0z$: $n_x=n_y=0$. The usual $P=\infty$ expression corresponds to taking the axial-vector charge. In the other frames, the operator will not reduce to a charge. Its matrix elements are, however, perfectly calculable. But first let us define A_μ and the hadron state vector α, β in a potential model.

One could believe that we have to know the whole mechanism of $SU(2)\otimes SU(2)$ symmetry breakdown, or at least some phenomenological chiral quark-pion Lagrangian. However, it is generally recognized that the two main effects of symmetry breakdown are the appearance of (i) a quark mass m, and (ii) the pion as a Goldstone boson. Here m is of course the medium size "constituent quark mass" and not the small "current" quark mass. Anyway, we do not treat the question of explicit chiral symmetry breaking and set $m_\pi=0$ in accordance with Weinberg's developments. If we adopt this Nambu–Jona Lasino[18] picture, an effective pure quark Lagrangian such as

$$L_{\text{eff}}=\bar{q}i\partial\!\!\!/ q-m\bar{q}q-\sum_i V_i\bar{q}O_iq \qquad (2.2)$$

(where V_iO_i are various types of central potentials) seems quite reasonable to describe the hadron states, inasmuch as the Goldstone pion is not concerned. This is precisely the case for pionic decays: the pion will enter only through the formula (2.1), where A_μ will operate only on "normal" composite hadron states. In fact, A_μ will not be the full axial current, but the one obtained by removing all the pion pole contributions. Therefore, we do not expect a conservation law $\partial_\mu\vec{A}_\mu=0$, which should hold only for the full current.[18] It is precisely necessary to have $\partial_\mu\vec{A}^\mu\neq0$ in order to get the nonzero pion transition amplitude from (2.1). In fact, the canonical procedure gives

$$\vec{A}_\mu=\bar{q}\gamma_\mu\gamma_5\frac{\vec{\tau}}{2}q \qquad (2.3)$$

for the infinitesimal chiral transformation defined as

$$q\to q+i\vec{\alpha}\cdot\frac{\vec{\tau}}{2}\gamma_5q\,. \qquad (2.4)$$

In the Lagrangian (2.2), we have totally lost the $SU(2)\otimes SU(2)$ symmetry, since the Goldstone boson has been dropped out. However, we still have the $SU(2)\otimes SU(2)$ current commutation relations, such as

$$[A_0^a,A_0^b]=i\epsilon_{abc}A_0^c\delta^3(\vec{x}-\vec{y}) \qquad (2.5)$$

which come only from the canonical commutations of fields and of the τ's algebra. Therefore, following the old argument of Gell-Mann, we may expect to get the AW relation considered as a sum rule. On the contrary, we do not claim to calculate pion scattering amplitudes or to get Tomazawa-Weinberg soft-pion theorems; they will come only indirectly from the use of dispersion relations saturated by quark model resonances. There are still limitations which we must recognize: We are not able to include internal pion loop corrections with the simple Lagrangian (2.2), while they may be practically important; there is ambiguity in treating pion targets, although it is not always unreasonable to treat the pion as a normal bound state in SU(6) quark models; finally there are the limitations coming from the use of a central potential, i.e., a quark shell model.

Let us then discuss the phenomenological relevance of the Lagrangian (2.2). A quark shell model with a confining static potential of vector or scalar type ($O_i = 1$ or γ^0) is certainly relevant to spectroscopy: in the nonrelativistic limit, it gives the usual SU(6) spectroscopy. Moreover, the use of the Dirac equation, initiated by Bogolioubov,[19] has certain good features.[20] Of course, in such a static model, we completely neglect the motion of the recoiling hadron, while we treat relativistically the quark internal velocities. It may seem reasonable for baryon decays $N^* \to N\pi$, which are phenomenologically the most interesting (first and second levels of the harmonic oscillator): the internal velocities are known to be very large,[16] while the recoil energy is suppressed by the mass of the baryon. However, we have at least to consider high-order corrections (beyond v^2/c^2) as being not very significant. Much more serious is the lack of two-body forces. First, the quark shell model by itself generates spurious states such as a $(56, 1^-)$ which have to be included in the closure relation. Second, we can not describe the saturation of color forces between color singlets as was done, e.g., in our model[21]; however, multiquark states do not appear in our closure relations, as will be seen. Finally, we cannot account for the spin-spin forces which are recognized to be the dominant spin-dependent forces.[22]

Let us now proceed to the v/c expansion. From now on, we abandon the quantum field framework and turn to the ordinary Dirac equation formalism. The matrix elements of (2.1) will then be simply the matrix elements of

$$\hat{X} = \sum_i \vec{\tau}(i) n^\mu \gamma_\mu(i) \gamma_5(i) e^{i\vec{k}\cdot\vec{r}(i)} \tag{2.6}$$

between tensor products of solutions of the Dirac equation in a central potential, where $\vec{k}$ is the pion momentum ($+i\vec{k}\cdot\vec{r}$ for absorption). The sum runs on quarks. We consider only hadrons with the same number of constituents (in general $3q$ states). The v/c expansion has then several converging motivations:

(i) The natural starting basis of states is the well-known SU(6) ⊗ O(3) basis. It is obtained in the lowest nontrivial v/c order approximation of the Hamiltonian of Dirac. We would like to know systematically how the states depart from this simple basis in the higher approximations (configuration mixing). At the same time, the lowest-order approximation to the operator $\hat{X}$,

$$-\sum_i \vec{\tau}(i)\sigma_z(i)\,, \tag{2.7}$$

has a very simple $SU(6)_W \otimes O(3)$ structure. However, it is not coherent to use

$$-\sum_i \vec{\tau}(i)\sigma_z(i) e^{ikz} \tag{2.8}$$

for the calculations of transitions between the ground state and excited states, since e^{ikz} includes corrections of order v/c and higher: $k = E_f - E_i = mO(v^2/c^2)$, while one neglects other terms of the same order. Therefore, one would like to know the various terms to be added, at each order of expansion, to (2.7) and which introduce new $SU(6)_W \otimes O(3)$ structures.

(ii) One wants to demonstrate sum rules, more precisely sums over states of a fixed number of constituents, avoiding exotics (like $3q + q\bar{q}$). In the expression (2.6), as well as in the Hamiltonian [which comes into the second Weinberg sum rule for linear SU(2) ⊗ SU(2) symmetry], the space of states includes the negative energies, so that we would have to include them in the closure relations; this is equivalent to allowing for additional pairs in the sum over states. To avoid this, one has to separate out negative energy states, which can be done by performing a Foldy-Wouthuysen (FW) transformation order in v/c. Moreover, the operator $\hat{X}$ is depending on the states $k = E_f - E_i = E_\beta - E_\alpha$. To use the closure, one must eliminate this dependence. It may be done by expanding $e^{ikz} = \sum_m i^m k^m z^m / m!$ and replacing the k^m factors by commutators with H:

$$\langle\beta|k^m O|\alpha\rangle = \langle\beta|[H,[H,[H,\ldots[H,O]\ldots]]]|\alpha\rangle\,. \tag{2.9}$$

But the expansion of $\exp(ikz)$ is once more a v/c expansion: kz is, in general, of order v/c.

We will perform the nonrelativistic expansion in the following way (from now on we drop the isospin operators and indices, as well as the sum over quarks, which have trivial effects). First, to get a nonrelativistic expansion of the Hamiltonian ($n = 2\nu$)

$$\hat{H} = \vec{\alpha}\vec{p} + m\beta + V\,, \tag{2.10}$$

$$V = \beta V_S + V_V\,, \tag{2.11}$$

we perform $\nu + 1$ successive FW transformations,

$$e^S = e^{F_{\nu+1}} \ldots e^{F_1}\,. \tag{2.12}$$

Let us recall that in this simple case F_m is of the order $2(m-1)+1$ (F_1 first order, F_2 third order, . . .), considering the potential of order $O(v^2/c^2)$. $\exp S\,\hat{H}\exp(-S)$ is then without odd terms up to order $m(v/c)^{n+2}$. We call this Hamiltonian H_{n+2}, its eigenfunctions φ_n, and we denote by $H^{(i)}$, $\varphi^{(i)}$ each term of their expansion:

$$H_{n+2}=\sum_0^{n+2} H^{(i)}\,, \tag{2.13}$$

$$\varphi_n=\sum_0^{n} \varphi^{(i)}\,. \tag{2.14}$$

$H^{(0)}$ is in fact m, and $\varphi^{(0)}$ is the eigenfunction of

$$H^{(2)}=p^2/2m+V\,. \tag{2.15}$$

$H^{(4)}$ contains the well-known spin-orbit and Darwin terms, as well as a correction to the kinetic energy[23]:

$$H^{(4)}=-\frac{p^4}{8m^3}+\frac{1}{8m^2}\vec\nabla^2 V_V-\frac{1}{8m^2}[\vec p,[\vec p, V_S]_+]_+ + \frac{1}{4m^2}[\vec\nabla(V_V-V_S)\times\vec p]\,. \tag{2.16}$$

In the new basis, the exact wave function is

$$\Psi=\varphi_n+O((v/c)^{n+1})\,. \tag{2.17}$$

In particular, the small components are of order $(v/c)^{n+1}$. This simplifies greatly the calculation of X. To obtain the expansion of its transformed expression $X=X\exp S\,\hat X\exp(-S)$, we have only to expand $\exp(\pm S)$. In principle, we must stop at order $(v/c)^n$, because the wave functions are known to this order. But, moreover, we can drop all the odd operators found in X, since the small components are then of order $(v/c)^{n+1}$ at least. It will remain to apply (2.9). Let us denote the expansion finally obtained:

$$X_n=\sum_0^n X^{(i)}\,. \tag{2.18}$$

In reality, the knowledge of X_n does not imply more than ν FW transformations because $F_{\nu+1}$ is of order $n+1$ and moreover, in applying (2.9) to the expansion of $\exp(ikz)$, we have to retain only $H=H_n$, because $H_{n+2}z$ is of order $n+1$. For instance, X_2 will be given by the first FW transformation, $F_1=\beta\vec\alpha\vec p/2m$, which is exactly the same as for free quarks. The potential appears only through the expansion of $\exp(ikz)$ and the use of (2.9). Since sum rules are demonstrated in the operator form, we shall have to deal only with X_n as far as we look for sum rules.

On the contrary, it would seem that the knowledge of matrix elements requires $\varphi^{(n)}$ and therefore $\nu+1$ FW transformations. However, $\varphi^{(n)}$ is needed only for the zero-order contributions to $\langle\beta|X_n|\alpha\rangle$: $\langle\beta|x^{(0)}|\alpha\rangle$, where $X^{(0)}=-\sigma_3$. Otherwise, one needs only $\varphi^{(n-1)}$. But if α or β is the ground state $l=0$, $j=\frac{1}{2}$, $\langle\beta|\sigma_3|\alpha\rangle$ has a very simple expression. In fact H_{n+2} has always its solutions of the factorized form (only spin-orbit coupling):

$$\psi=\sum \varphi_l\chi_S \tag{2.19}$$

and the ground state $|0\rangle$ is $l=0$, $j_z=s_z$. Therefore $\sigma_3|0\rangle=\pm|0\rangle$ and $\langle\beta|\sigma_3|\alpha\rangle=1$ or 0. And of course, the most usual cases correspond to at least one of the states being the ground state.

Our final conclusion will be that if we are looking for the expression of axial-vector currents and their matrix elements up to order v^2/c^2, we need only the expression of the first FW transformation $F_1=\beta\vec\alpha\cdot\vec p/2m$, the usual nonrelativistic Hamiltonian $H=p^2/2m+V$, and its unperturbed SU(6) wave functions. The order v^4/c^4 is still easy to handle.

III. PIONIC TRANSITIONS AND CHIRAL SUM RULES UP TO ORDER v^2/c^2

A. Chiral sum rules

Using the first FW transformation $\exp F_1$, it is trivial to get the pion absorption operator:

$$X=-e^{ikz}\sigma_3+\frac{1}{2m}[\vec\sigma\cdot\vec p, e^{ikz}]_+ +\frac{1}{4m^2}(\vec p^2\sigma_3-\vec\sigma\cdot\vec p\sigma_3\vec\sigma\cdot\vec p) \tag{3.1}$$

whence, by expanding and reducing e^{ikz},

$$X^{(0)}=-\sigma_3\,, \tag{3.2}$$

$$X^{(1)}=\frac{\vec\sigma_T\cdot\vec p_T}{m}\,, \tag{3.3}$$

$$X^{(2)}=\frac{\vec p_T^{\,2}\sigma_3}{2m^2}+\frac{p_z\vec\sigma_T\cdot\vec p_T}{2m^2}-\frac{1}{m}z\vec\sigma_T\cdot\vec\nabla_T V\,. \tag{3.4}$$

The AW sum rule can be written

$$g_A{}^2-f_\pi{}^2\frac{1}{\pi}\int\frac{ds}{s-m_N{}^2}[\sigma(\pi^+p)-\sigma(\pi^-p)]=1\,. \tag{3.5}$$

If one saturates the continuum integral by a set of states, it is easy to rewrite it in the Weinberg form:

$$[X^a, X^b]=i\epsilon_{abc}I^c\,. \tag{3.6}$$

Here, a, b, and c, are isospin indices, X^a denotes the matrix of the axial current $\eta^\mu A_\mu{}^a$ between the states. The usual AW sum rule corresponds to $a=+$, $b=-$, $c=0$. I is the isospin operator. Of course, (3.6) is finally completely independent of the particular basis of states. We may therefore identify

$$X^a=XI^a \tag{3.7}$$

or in the case of n quarks, by additivity

$$X^a=\sum_i X(i)I^a(i)\,. \tag{3.8}$$

In fact, any commutator of additive quantities will be itself additive, since the cross terms are automatically eliminated. Therefore (3.6) can be written in terms of (3.8),

$$\sum_i X^2(i)[I^a(i)I^b(i) - I^b(i)I^a(i)] = i\epsilon_{abc}\sum_i I^c(i)\,, \qquad (3.9)$$

and the AW rule has finally the transparent form

$$X^2 = 1\,. \qquad (3.10)$$

One must emphasize that the X^a's are additive the $\hat{X}$'s are so (2.6) and because the FW transformations on each quark are independent. This is a specific property of the central potential (shell model) and will not be true for two-body forces.[14,15]

Let us now express X^2 in terms of our $X^{(i)}$'s:

$$X^2 = [X^{(0)}]^2 + [X^{(0)}, X^{(2)}]_+ + [X^{(1)}]^2$$

$$= 1 - \mathrm{p}_T{}^2/m^2 + \mathrm{p}_T{}^2/m^2 = 1\,. \qquad (3.11)$$

The generalized AW sum rule is identically satisfied. Let us comment in more detail on this saturation in the case of the ground-state matrix element of (3.11). The first two terms represent $g_A{}^2$:

$$-g_A = \langle 0|X^{(0)}|0\rangle + \langle 0|X^{(2)}|0\rangle\,. \qquad (3.12)$$

The second term comes from the v^2/c^2 corrections to g_A:

$$-g_A = -1 + \langle \vec{\mathrm{p}}_T{}^2\rangle/2m^2 \qquad (3.13)$$

which is exactly the expression found by Gell-Mann.[7] The corrections to $g_A{}^2$ are canceled in (3.11) by the negative-parity state's contribution, which is the third term:

$$\langle 0|[X^{(1)}]^2|0\rangle = \sum_\gamma |\langle 0|X^{(1)}|\gamma\rangle|^2\,. \qquad (3.14)$$

The γ's are necessarily of negative parity. Moreover, in the case of an harmonic-oscillator model, $X^{(1)}$ can only excite the first excited states $(56, 1^-)(70, 1^-)$ [let us recall that spurious states are necessarily present in a shell model; the $(56, 1^-)$, as commented in Ref. 24, represents spurious motions of the center of mass]. This demonstration confirms the qualitative suggestions of Donoghue and Wyler.[13,25] It must be emphasized that the saturation is obtained with ordinary excitations of the ground state—there are no exotics.

The second Weinberg sum rule

$$[[X^a, [X^b, m^2]]_{I=2} = 0\,. \qquad (3.15)$$

is trivially satisfied because of the additivity of commutators and because the one-quark contribution to the left-hand side of (3.15) cannot of course have an $I=2$ part: $\pi^a q \to \pi^b q$ has $I_t = 0$ or 1. The saturation of (3.15) is also very simple. One can rewrite it as

$$\sum_\gamma (m_\gamma{}^2 - m_\alpha{}^2 - m_\beta{}^2) X^+_{\alpha\gamma} X^+_{\gamma\beta} = 0\,, \qquad (3.16)$$

where one has set $a = b = +$ to select the $I=2$ part. Taking α and β to be the ground state, this simplifies to

$$\sum_\gamma (m_\gamma{}^2 - m_0{}^2) X^+_{0\gamma} X^+_{\gamma 0} = 0\,. \qquad (3.17)$$

The saturation at the present order of approximation will be only by negative-parity states: $\gamma = 0$ does not contribute and, for excited positive-parity states, $X^+_{0\gamma}X^+_{\gamma 0}$ will be order v^4/c^4. Furthermore, in the harmonic-oscillator model, only $L^P = 1^-$ states will be excited by $X^{(1)}$ and, since they are degenerate, one can factorize $m_\gamma{}^2 - m_0{}^2$, whence

$$\sum_{\gamma\in 1^-} X^+_{0\gamma} X^+_{\gamma 0} = 0\,. \qquad (3.18)$$

This result sheds no light on the abundantly discussed saturation of the $\pi\rho$ sum rule for mesons. For meson systems, one must replace 1^- by 1^+ states. Equation (3.18) is satisfied by a cancellation between the $A_{1,2}$ and spurious $I=0$, $G=-1$ states [these are once again the spurious states of motion of the center of mass, with a "wrong" G parity $\neq (-1)^{L+S+I}$]. Therefore, we do not get a physically significant saturation scheme, and we cannot solve the difficulty encountered by Gilman and Harari.[8,26] More interesting is the expression found for $m_4{}^2$,[27]

$$m_4{}^2 = 2m\left\{ \frac{(\vec{\sigma}_T \times \vec{\nabla}_T V)_z}{2m} + \frac{\vec{\sigma}_T \cdot [\vec{\nabla}(V_V - V_S)\times \vec{\mathrm{p}}]_T}{4m^2} + \frac{\{-\nabla_T{}^2 V + 3\nabla_z(\vec{\sigma}_T \times \vec{\nabla}_T V)_z + 3i(\vec{\sigma}_T \times \vec{\nabla}_T V)_z p_z + i\nabla_z V(\vec{\sigma}_T \times \vec{\mathrm{p}}_T)_z - \sigma_z(\vec{\nabla}_T V \times \vec{\mathrm{p}}_T)_z + z[(\vec{\sigma}_T \times \vec{\nabla}_T V)_z, \vec{\mathrm{p}}^2]\}}{4m^2} \right\}. \qquad (3.19)$$

One notes a strong spin dependence. Remember that Weinberg has assumed that $m_4{}^2$ is in fact helicity independent. The argument was based on the assumption of helicity independence of forward inelastic $I_t = 0$ scattering. However, one does not expect this assumption to hold in an additive quark model.[28] One also notices that $m_4{}^2$ is nonzero because of the presence of the potential. In a free-quark model, H would be chiral invariant [in the sense of linear $\mathrm{SU}(2)\otimes\mathrm{SU}(2)$ symmetry].

But one sees also that $m_4{}^2$ is of order v^3/c^3. Therefore,

$$H^{(2)} = p^2/2m + V \quad (3.20)$$

is still chiral invariant to order v^2/c^2. This could seem paradoxical, because chiral invariance implies the absence of inelastic transitions, and we have found on the other hand inelastic transitions at order v/c and v^2/c^2. The paradox is solved by considering more carefully the orders of magnitude in the equation

$$[X, H]_{\alpha\beta} = X_{\alpha\beta}(E_\alpha - E_\beta)\,. \quad (3.21)$$

If the left-hand side was exactly 0, one should have $X_{\alpha\beta}=0$ for $E_\alpha \neq E_\beta$. But (3.21) is still satisfied with the left-hand side being of order v^3/c^3, $X_{\alpha\beta} = O(v/c)$, $E_\alpha - E_\beta = mO(v^2/c^2)$. This is precisely the situation we are finding.

As a conclusion, one must emphasize that SU(2) ⊗SU(2) linear symmetry is not so good as SU(6) symmetry. The Hamiltonian chiral symmetry is broken by $mO(v^3/c^3)$ terms, while SU(6) is broken by $mO(v^4/c^4)$. There is a SU(2)⊗SU(2) mixing of states of order v/c, while SU(6) representations are mixed only by v^2/c^2 terms. This is a consequence of a different behavior under parity: SU(2)⊗SU(2) mixes parity, while SU(6) conserves it. Of course the conclusion is physically significant only if the parameter v/c is really small (Ref. 29).

B. Comparison with the $P_z = \infty$ approach

The $P=\infty$ approach is usually considered as the best suited to study SU(2)⊗SU(2) linear symmetry. The main reasons seem (i) the quark-pair contributions are automatically eliminated, and (ii) the pionic transition operator takes the especially simple form of a charge. As to the quark-pair contribution, the advantage is that one quite generally demonstrates its disappearance. It is especially transparent in the null plane formalism.[30] The pair contributions to charges are affected by a $\delta(\eta+\eta')$ for longitudinal-momentum conservation; but this is necessarily zero because $\eta, \eta' > 0$. However, this quantum-field-theoretic argument does not apply to our problem of quarks bound by a potential. In fact, the null-plane formalism adapted to our problem has been defined by Bell and Ruegg.[31] In this formalism, the positivity condition no longer holds: we have no kinematical bounds. Indeed, as will be demonstrated in a forthcoming paper, the contribution of bound quark pairs disappears only in the nonrelativistic expansion: they give a contribution which is nonanalytic in v/c.

As to the simplicity of the operator $\hat{X}$, it is seen in its expression

$$X = -\sum_i \tau(i)\sigma_3(i) \quad (3.22)$$

which we obtain using the definitions of Bell and Ruegg. We have an operator acting within a two-component spin space with the very simple SU(6) structure of the lowest-order approximation to the $P=0$ case (2.7). Moreover, there is no longer the e^{ikz} factor which led to much complication. However, what is gained for the operator is lost on the side of the wave functions; first, the wave function of a state (of definite spin) does not have a simple SU(2)⊗SU(2) content. It does not have the simple factorized SU(6)⊗O(3) structure of the $P=0$ wave functions. We have shown this by a direct boost from $P=0$ to $P=\infty$.[16] It is also possible to remain within the $P=\infty$ framework. Then, the crucial thing to satisfy is the angular momentum condition.[32] The explicit solution in the free quark model has been given by Melosh[33] and has been further clarified.[34-36] It has also been discussed for interacting fields.[37,38] Bell and Ruegg give the discussion for our case of a central potential.[31] The conclusion is that in order to recover a simple $SU(6)_W$ behavior of states, one must pass to a new basis, the so-called "constituent basis," where of course X takes a more complicated form; one must perform a v/c expansion and one finds to order v^2/c^2:

$$X_M = \sigma_3 + \vec{\sigma}_T\cdot\vec{p}_T/m - p_T{}^2\sigma_3/2m^2 - p_z\vec{\sigma}_T\cdot\vec{p}_T/2m^2\,. \quad (3.23)$$

It is to be emphasized, however, that the constituent basis is not yet identical to the usual SU(6) basis of quark modelists. This is seen from the difference between (3.23) and our expression (3.4). We have a further potential dependent term. One can show that our SU(6) basis and the Melosh transformed basis of Bell and Ruegg are related to order v^2/c^2 by

$$\varphi_M = e^{iEz}(1 + p_z/2m - p_z{}^2/8m)\varphi_{\rm FW}\,. \quad (3.24)$$

This expression shows in particular that the e^{ikz} factor has been transferred to the wave functions in the $P=\infty$ frame. But there is also the nontrivial factor involving the longitudinal momenta, which is crucial for calculating inelastic transitions.

It is to be noted that in all these discussions, the v/c expansion appears once more as a natural tool, apart from its general physical interest which would come from an actual smallness of v/c.[39] The departure of states, in the constituent basis, from a simple SU(6) structure, is controlled by the successive v/c orders. And so is it for the transition operator. And, finally, one needs the v/c expansion to separate out quark pair contributions. These motivations are fully parallel to the motivation of our FW approach.

As to the respective merits of the two approaches, one can conclude that the closure is more easily performed in the $P=\infty$ approach, because of the absence of the e^{ikz} factor which introduces a dependence on states at $P=0$. On the other hand, the calculation of actual matrix elements is more easily done in our approach, where the wave functions have a simpler structure, as illustrated by the relation (3.24).

Last but not least, we have to formulate the following remark. There has been the claim[10] that the $P=\infty$ frame naturally yields signs opposite to $SU(6)_W$ for odd-L transitions to the ground state, and $SU(6)_W$-like signs for even-L states, as seems to be wanted by experiment. The claim was based on the fact that, apparently, one was getting, respectively, pure $\Delta L_z=\pm 1$ and $\Delta L_z=0$ behavior according to the parity of L, in the models.[40,39] The explicit quark-model expression (3.23) shows that it is not true already at order v^2/c^2: one has both $\Delta L_z=0$ and $\Delta L_z=\pm 1$ terms. And this is precisely the order responsible for $L=2$ to $L=0$ transitions. In fact, in terms of Ref. 40, the Melosh transformation is $\exp(iZ)$, but Z can be approximated by

$$Z=(\vec{\sigma}_T\times\vec{p}_T)_z/2m \tag{3.25}$$

only at first order. At second order in v/c, one must include the p_z dependence of the denominator of the Melosh expression.[33] The same conclusion is drawn from our $P=0$ expression (3.4).

C. Comparison with usual quark models and phenomenology

Our model for pion coupling is nothing but the pseudovector coupling, but with a consistent treatment of internal velocity relativistic corrections. According to (2.2), it is equivalent to the pseudoscalar coupling in the case of a vector binding potential, but not in the case of a scalar potential. We have now to compare with the standard quark model of pion emission as proposed long ago by Mitra and Ross.[2] The FKR model is not essentially different for our purpose, since it introduces only a relativistic treatment of the overall hadron motion, but still neglects the internal velocity relativistic corrections beyond v/c order. We write the Mitra and Ross operator (with an irrelevant symmetrization of the recoil term):

$$-X=e^{ikz}\sigma_3-\frac{1}{2m}[\vec{\sigma}\cdot\vec{p},e^{ikz}]_+ . \tag{3.26}$$

To make a more transparent comparison with (3.2)–(3.4), we retain only terms up to v^2/c^2 in e^{ikz}. We then get

$$-X=\sigma_3-\frac{\vec{\sigma}_T\cdot\vec{p}_T}{m}-p_z\frac{\vec{\sigma}_T\cdot\vec{p}_T}{m^2}+z\frac{1}{m}\vec{\sigma}_T\cdot\vec{\nabla}_T V . \tag{3.27}$$

We notice that X differs from (3.2)–(3.4) only in the second order $X^{(2)}$. Equation (3.27) lacks a $\Delta L_z=0$ contribution $-p_T^2/2m^2$ and the coefficient of $p_z p_T$ is not the same.

Equation (3.27) is pure $\Delta L_z=\pm 1$ save for the zero-order σ_3 and therefore signs opposite to those of $SU(6)_W$ automatically come out for $L=2$ to $L=0$ decays as well as for $L=1$ decays. One must emphasize that our additional $\Delta L_z=0$ contribution at second order is not sufficiently large to change the conclusion. We still find definite signs opposite to $SU(6)_W$ in $L=2$ decays. In fact the calculation in a harmonic-oscillator model shows that

$$X(L=0\to L=2)=(\vec{r}_T^{\,2}\sigma_z+3z\,\vec{r}_T\cdot\vec{\sigma}_T)/2m^2R^4 . \tag{3.28}$$

Calculating for instance the transition $P_{13}\to\Delta\pi$, we find a ratio $R=(\lambda=\frac{3}{2})/(\lambda=\frac{1}{2})=-18/26$, corresponding to $F/P=-96/84$ in the notations of Ref. 9. Therefore, although it is possible to fit the coefficients of the direct and recoil terms in a purely phenomenological approach,[41] and to get $SU(6)_W$-like signs in $L=2$ decays, no explicit quark-model calculation is able up to now to support such a fit. $F_{15}\to\Delta\pi$ remains a challenge to our understanding of pion decays. Of course, one cannot trust too much the model calculations, because there are so many uncontrollable approximations involved. On the other hand, experiment could be blamed since, in $N\pi$ decays, one finds contradictory indications.[42]

We recall that a similar negative conclusion was found in our quark-pair-creation (QPC) model.[43] It has not been possible, however, to include in a natural manner in this model the relativistic corrections considered in the present paper (Ref. 44). Anyway, the QPC model in its present form has a serious drawback: Although it yields a recoil term which is of a reasonable order of magnitude in all actual reactions, this recoil term, corresponding to an effective emission operator

$$\vec{\sigma}\cdot(\vec{k}_\pi-\vec{p}_i) , \tag{3.29}$$

does not include the k/m factor which is implied by the PCAC prescription. This drawback has been emphasized in a recent paper by Mitra and Sood.[45]

Returning now to the v/c term on which there is agreement between all models of elementary emission,

$$X^{(1)}=\frac{\vec{\sigma}_T\cdot\vec{p}_T}{m}\,, \qquad (3.30)$$

one must look for experimental confrontation. It induces the $L=1$ decays. For mesons, one must remember that the phenomenological discussion of Buccella *et al.*[46] gives good results. It is more difficult to draw a firm conclusion in the baryon sector, because of the large possible mixings. We address the reader to the discussion of Gilman, Kugler, and Meshkov[9] concerning the hypothesis of a pure $\Delta L_z=\pm 1$ transition.

It is fair to emphasize that if v/c is not actually small, the phenomenological relevance of the whole approach becomes weak. Higher-order corrections (v^3/c^3 and v^4/c^4) can be calculated, but are not very significant in view of all other possible corrections. The main interest of these higher-order calculations is rather theoretical: one can still demonstrate the saturation of the AW sum rule. We relegate the expressions and proof to an appendix because there is nothing essentially new to add to v^2/c^2 order.

IV. CONCLUSION

We have shown that the rather naive, but phenomenologically very significant quark shell model with a nonrelativistic expansion can realize the Weinberg scheme of linear SU(2)⊗SU(2) symmetry. In particular, we have obtained the AW sum rule up to order v^4/c^4 as well as the expression of $m_4{}^2$. This is done with standard methods of the $P=0$ frame, and shows that one can include in this frame the good features of the $P=\infty$ approach. It must be emphasized that all these results are obtained in a model of massive quarks bound by a potential and with approximate SU(6) symmetry: We do not have to deal with the fundamental Lagrangian and its massless quarks; it appears only very indirectly through the spontaneously generated quark mass and the PCAC prescription. The old exact SU(2)⊗SU(2) symmetry has completely disappeared. The new SU(2)⊗SU(2) symmetry is approximate in the same sense as the SU(6) symmetry: both are broken by higher-order terms of the nonrelativistic expansion. There is, however, a typical difference between these two approximate symmetries of a massive-quark model: They are not broken by terms of the same order in v/c. In this scheme, one must be aware that the "current" basis (the one where the X's are diagonal) is actually a basis of massive valence quarks: It is defined without reference to the current masses. Finally, another striking conclusion is that the sum rules of SU(2)⊗SU(2) symmetry do not severely constrain the potential, except for some very general conditions of differentiability.[46]

APPENDIX: ADLER-WEISBERGER SUM RULE AT ORDER v^4/c^4

The v^4/c^4 order is still easy to handle, with the expression of F_2:

$$F_2=\frac{1}{4m^2}[\vec{\alpha}\cdot\vec{p},V]-\frac{\beta}{6m^3}\vec{p}^2\vec{\alpha}\cdot\vec{p}\,. \qquad (A1)$$

There is no difficulty in demonstrating explicitly the AW sum rule saturation. It is lengthy because we have to reduce a large number of commutators coming from the application of the formula (2.9). A simple way of proceeding is to order the terms according to the powers of V. We have got the result, but it is not very enlightening to draw pages of equations. Instead, we shall try to convince the reader by the proof in a simple case, where (i) the model is a harmonic oscillator one of the vector type, and (ii) the target is the ground state.

What we have to demonstrate is the cancellation of terms of order four:

$$\langle 0|[-\sigma_3,X^{(4)}]_+ + [X^{(1)},X^{(3)}]_+ + (X^{(2)})^2|0\rangle=0\,. \quad (A2)$$

$|0\rangle$ is here the unperturbed ground state; since the operator itself is already of order v^4/c^4, we do not have to consider perturbations of the wave functions. $(X^{(2)})^2$ is very easily evaluated from the expression (3.4). We get

$$\langle 0|(X^{(2)})^2|0\rangle=\langle 0|\frac{\vec{p}^2\vec{p}_T{}^2}{4m^4}+\frac{1}{m^2}z^2(\vec{\nabla}_T V)^2 -\frac{1}{2m^3}[p_3\vec{p}_T,z\vec{\nabla}_T V]_+|0\rangle \qquad (A3)$$

without any particular hypothesis. The first term is also evaluated without additional hypotheses. In fact, as $\sigma_3|0\rangle=\pm|0\rangle$,

$$\langle 0|[-\sigma_3,X^{(4)}]_+|0\rangle=\mp 2\langle 0|X^{(4)}|0\rangle\,. \qquad (A4)$$

$X^{(4)}$ contains many commutators with $H^{(2)}$ coming from the transformation of e^{ikz} according to (2.9), as well as some commutators with $H^{(4)}$ having the same origin. However, let O be any operator:

$$\langle 0|[H^{(2)},O]|0\rangle=0\,, \qquad (A5)$$

because $|0\rangle$ is an eigenfunction of $H^{(2)}$. Therefore we can drop all the terms containing such commutators. We find also a simple cancellation between the terms containing commutators with $H^{(4)}$. Therefore, the final result is the one obtained by retaining only the first term in the expansion of $e^{ikz}\sim 1$. We get this straightforwardly, because the average on the ground state will then only involve the FW transformation of σ_3:

$$e^{F_2}e^{F_1}\sigma_3 e^{-F_1}e^{-F_2}\,. \qquad (A6)$$

Then,

$$\langle 0|X^{(4)}|0\rangle=\left\langle\frac{3\vec{p}^2\vec{p}_T{}^2}{8m^4}-\frac{\nabla_T{}^2V_V}{4m^3}\right\rangle \text{ (vector case)}. \tag{A7}$$

The real complication is in the second term, because there the host of $H^{(2)}$ commutators cannot be eliminated so easily. There, we have to make the additional hypothesis of a harmonic-oscillator potential. The exact expression of $X^{(3)}$ is

$$-X^{(3)}=\frac{\vec{p}^2}{2m^2}\frac{\vec{\sigma}\cdot\vec{p}}{m}+i[H^{(2)},-\vec{p}_T{}^2z\sigma_3+\vec{\sigma}_T\cdot\vec{p}_T(p_z,z)_+]/2m^2$$
$$+i[H^{(4)},z\sigma_3]-\frac{i^2}{2!}\left[H^{(2)},\left[H^{(2)},\left(\frac{\vec{\sigma}\cdot\vec{p}}{2m},z^2\right)_+\right]\right]$$
$$+\frac{i^3}{3!}[H^{(2)},[H^{(2)},[H^{(2)},z^3\sigma_3]]]. \tag{A8}$$

Let us then calculate $\langle 0|[X^{(1)},X^{(3)}]_+|0\rangle$. In a harmonic-oscillator model, we can saturate the products by the first excited states $|n\rangle$,

$$\langle 0|[X^{(1)},X^{(3)}]_+|0\rangle=2\sum_n\langle 0|X^{(1)}|n\rangle\langle n|X^{(3)}|0\rangle, \tag{A9}$$

since $X^{(1)}$ can only excite these states from the ground state. But in $\langle n|X^{(3)}|0\rangle$, we can substitute any $H^{(2)}$ commutator by a multiplication by $\omega(k=\omega)$,

$$\langle n|[H^{(2)},O]|0\rangle=\omega\langle n|O|0\rangle, \tag{A10}$$

and we return to a much simpler expression. Moreover, we can redo the closure on $|n\rangle$ and then since $X^{(1)}$ is odd in p_T, we have to retain only the odd powers of p_T in $X^{(3)}$ to get an even operator under $p_T\to -p_T$. The average of an odd operator will be zero. This makes it possible to drop a number of terms in (A8) and finally we get very straightforwardly

$$\langle 0|[X^{(1)},X^{(3)}]_+|0\rangle=-\langle\vec{p}^2\vec{p}_T{}^2/m^4\rangle. \tag{A11}$$

Combining (A11), (A7), and (A3) and calculating them in the harmonic-oscillator model (every term is reduced to a monomial of coordinates of order 4), one gets (A2).

---

*Now at Laboratoire de Physique des Particules, 74019 Annecy-le-Vieux cedex, France.

†Laboratoire associé au Centre National de la Recherche Scientifique. Postal address: Bât. 211, Université Paris—Sud, 91405 Orsay cedex, France.

[1]C. Becchi and G. Morpurgo, Phys. Rev. 149, 1284 (1966).

[2]A. Mitra and M. Ross, Phys. Rev. 158, 1630 (1967).

[3]R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D 3, 2706 (1971).

[4]A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal, Phys. Rev. D 8, 2223 (1973).

[5]D. Faiman and J. Rosner, Phys. Lett. 45B, 357 (1973); W. Petersen and J. Rosner, Phys. Rev. D 6, 820 (1972); 7, 747 (1973).

[6]See, e.g., R. S. Longacre, T. Lasinski, A. H. Rosenfeld, G. Smadja, R. J. Cashmore, and D. W. G. S. Leith, Phys. Rev. D 17, 1795 (1978).

[7]M. Gell-Mann, in *Strong and Weak Interactions: Present Problems*, proceedings of the 1966 International School of Physics "Ettore Majorana," edited by A. Zichichi (Academic, New York, 1967), p. 173; *Hadrons and Their Interactions: Current and Field Algebra, Soft Pions, Supermultiplets and Related Topics*, proceedings of the 1967 International School of Physics "Ettore Majorana," edited by A. Zichichi (Academic, New York, 1968).

[8]F. J. Gilman and H. Harari, Phys. Rev. 165, 1803 (1968).

[9]F. J. Gilman, M. Kugler, and S. Meshkov, Phys. Rev. D 9, 715 (1974).

[10]R. Carlitz and J. Weyers, Phys. Lett. 56B, 154 (1975).

[11]In fact, Gell-Mann (Ref. 7) bypasses this problem by defining a mass operator defined in the rest frame. One could of course alternatively introduce covariant wave equations. See the discussion in H. Leutwyler and J. Stern, Ann. Phys. (N.Y.) 112, 94 (1978).

[12]S. Weinberg, Phys. Rev. 177, 2604 (1969).

[13]J. F. Donoghue and D. W. Wyler, Phys. Rev. D 17, 280 (1978).

[14]S. J. Brodsky and J. R. Primack, Ann. Phys. (N.Y.) 52, 315 (1969).

[15]F. Close and L. A. Copley, Nucl. Phys. B19, 477 (1970).

[16]A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal [Phys. Rev. D 9, 2636 (1974); 11, 680 (1975)] already made this connection explicit.

[17]It amounts evidently to taking the pseudovector coupling for one-pion emission. The nontrivial problem is when two pions or more are present. We do not deal directly with this problem. The following discussion is then only to settle the place of our specific work in the general context of SU(2)⊗SU(2) symmetry.

[18]Y. Nambu and J. Jona Lasinio, Phys. Rev. D 122, 345 (1961). This point of view is very different from the usual bag model.

[19]P. N. Bogolioubov, Ann. Inst. Henri Poincaré 8, 163 (1968). For a systematic development of this point of view, see M. Bando, I. Kubo, and S. Tanaka, Progr. Theor. Phys. 53, 544 (1975) and further papers. However, in the description of pion emission, these authors drop the time component of the axial-vector current, which leads to very different results.

[20]For recent arguments in favor of a Dirac equation with scalar potential, see I. Cohen and H. J. Lipkin, Report No. FERMILAB PUB 80/33 THY (unpublished).

[21]M. B. Gavela, A. Le Yaouanc, L. Oliver, J. C. Raynal, and S. Sood, Phys. Lett. B82, 431 (1979).

[22]D. Gromes, Nucl. Phys. B130, 18 (1977); U. Ellwanger, Nucl. Phys. B139, 422 (1978); N. Isgur and G. Karl, Phys. Rev. D 18, 4187 (1978).

[23]J. L. Friar, Phys. Rev. C 11, 274 (1975). The same author gives a discussion of the pion absorption operator to order v^2/c^2 in Phys. Rev. C 10, 955 (1974).

[24]D. Faiman and A. Hendry, Phys. Rev. 173, 1720 (1968).

[25]On the other hand, we are not convinced by the statement of the authors concerning the cancellation between the effect of Δ-N mass difference and gluon radiative corrections to g_A. The Δ-N mass difference has an order $\alpha_s m O(v^3/c^3)$, while $g_A - 1 = \alpha_s O(1)$ and therefore would not vanish in the nonrelativistic limit.

[26]The difficulty is that one finds a zero transverse coupling in $A_1 \to \rho\pi$ in contradiction with hard pion calculations and quark models.

[27]R. Carlitz [Nuovo Cimento 12A, 1049 (1972)] has shown that ${m_4}^2$ is actually a divergent quantity. However, our v/c expansion truncates the sum over states [in a harmonic oscillator, the excitation of degree n is induced by the order $(v/c)^n$] and it is quite understandable that the result is then finite.

[28]A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal, Nucl. Phys. B37, 541 (1972).

[29]See, e.g., M. Ida, Prog. Theor. Phys. 54, 1775 (1975).

[30]J. Jersak and J. Stern, Nucl. Phys. B7, 413 (1968); Nuovo Cimento 59, 315 (1969).

[31]J. S. Bell and H. Ruegg, Nucl. Phys. B93, 12 (1975).

[32]R. Dashen and M. Gell Mann, Phys. Rev. Lett. 17, 340 (1966).

[33]H. J. Melosh, Phys. Rev. D 9, 1095 (1974).

[34]S. P. De Alwis and J. Stern, Nucl. Phys. B77, 509 (1974).

[35]F. Buccella, C. A. Savoy, and P. Sorba, Lett. Nuovo Cimento 10, 445 (1974).

[36]N. Marinescu and M. Kugler, Phys. Rev. D 12, 3315 (1975).

[37]N. H. Fuchs, Phys. Rev. D 13, 1309 (1976).

[38]R. Carlitz and W. K. Tung, Phys. Rev. D 13, 3446 (1976). Note however that the $1/m$ expansion of this paper is not exactly a v/c expansion.

[39]See Carlitz and Weyers, Ref. 10.

[40]F. Bucella, H. Kleinert, C. A. Savoy, E. Celeghini, and E. Sorace, Nuovo Cimento 69A, 133 (1970).

[41]R. Koniuk and N. Isgur, Phys. Rev. D 21, 1868 (1980).

[42]For indications of $SU(6)_W$ signs opposite to those of $SU(6)_W$ in $L=2$ decays, see J. Babcock and J. L. Rosner, Phys. Rev. D 18, 4027 (1978).

[43]A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal, Phys. Rev. D 11, 1272 (1975).

[44]For an attempt at a a relativistic formulation of the quark-pair-creation model, see Y. S. Kim and M. Noz, Phys. Rev. D 12, 129 (1975).

[45]A. N. Mitra and S. Sood, Fortschr. Phys. 25, 649 (1977), especially p. 704.

[46]This is in agreement with the final conclusion of Gell-Mann, Ref. 7.