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Permutation symmetries and the fermion mass matrix

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We consider $SU(2)\times U(1)$ gauge models with an additional global symmetry of S_n type. In particular, we study S_4 symmetry for a six-quark—six-lepton standard model. We find that the Kobayashi-Maskawa matrix is completely determined and the mass of the t quark is predicted. Some of the results persist in S_n applied to n families. Restrictions on neutrino masses and mixings are given. Implications of *CP* and flavor nonconservation that arise in the neutral-Higgs-boson sector are discussed.

I. INTRODUCTION

We present here an account of our work on imposing permutation symmetry on the Higgs-boson couplings in $SU(2)\times U(1)$ and the resulting constraints on fermion masses and mixing parameters.[1,2]

The general idea of this approach is to ask whether some pattern can be recognized in the fermion mass matrix already and to see how far one can go in understanding it at $SU(2)\times U(1)$ level, i.e., without invoking mass scales of order $M_X \sim 10^{15}$ GeV or even 10^5 GeV. Such scales, corresponding to SU(5) or $SU(2)_R$, are presumably relevant and would give further restrictions but we wish to study whether we can understand some masses here and now. The view taken is that perhaps these higher mass scales are needed to understand m_e, m_ν, etc., but not for all masses.

To the extent that fermion masses arise from Higgs-boson Yukawa couplings, patterns in fermion masses reflect those in Higgs-boson Yukawa couplings. Whether the Higgs bosons are true elementary scalars or dynamical manifestations need not concern us here.

The program to be outlined below is more modest than in some grand unified theories in that it is possible that (1) some masses such as m_e, etc., may not be understood and (2) masses are not "calculable." All our results are at tree level and for mass ratios. However, we are more ambitious in that we present very specific proposals wherein fermion masses and mixings are *predicted* and hence are vulnerable to experiment.

It is obvious that some horizontal pattern of the form $SU(2)\times U(1)\times G$ is required to constrain the fermion mass matrix. One possibility is that G is a gauge group. This has been discussed in the literature.[3] In general, in gauging G one needs to impose more *ad hoc* assumptions to make both the new gauge bosons and the new Higgs bosons sufficiently heavy. If one prefers to introduce as few new assumptions as necessary, then it seems preferable to let G be a global symmetry. If G is to be broken spontaneously (how else?), then to avoid zero-mass Nambu-Goldstone bosons G should be a discrete symmetry. So we are led to consider a discrete symmetry group for G to be chosen below.

It is evident that to get constraints by any of the above choices of G the Higgs-boson sector of $SU(2)\times U(1)$ has to be proliferated. This is the price to be paid: either one Higgs doublet with many free parameters or several Higgs doublets with few parameters (at least in the fermion sector). With many Higgs bosons, in general, there will be flavor-changing couplings of neutral Higgs bosons. This need not be alarming. Since one is hoping to *predict* fermion masses and mixings, flavor-changing couplings should be allowed to occur as dictated by the model. The real constraints are *experimental,* viz., K^0-$\bar{K}^0$ and D^0-$\bar{D}^0$ mixings, $K_L \to \mu\bar{\mu},\bar{\mu}e$ rates, etc. In this connection there are a number of theorems[4] which seem to suggest that it is not possible to predict fermion mixings. However, the proofs depend upon forbidding flavor-changing couplings of neutral Higgs bosons. Once they are allowed as just discussed, the theorems are irrelevant. There are cases where partial flavor conservation obtains, i.e., d-s and/or c-u coupling is absent in the neutral-Higgs-boson sector. This happens to be the case in the four-quark S_3 example.[1]

Here we consider models in which G is chosen to be a permutation symmetry group, and where the fermions but not the Higgs bosons can be regarded as objects of permutation. The kinds of results for fermion masses and mixings that arise are given in the next section. A more specific model based on S_4 for three families is analyzed in detail in Sec. III. The resulting mass formula for all fermions is given and the full prediction of the Kobayashi-Maskawa matrix is given. The possible restrictions on neutrino masses and mixings are discussed. The implications of the Higgs-boson (especially neutral) couplings which violate CP and flavor conservation are considered next. Finally we summarize the predictions of the model which can be easily tested in the near future.

II. THE S_n MODELS

Next we turn to the choice of the discrete global symmetry group G. If the fermion family spectrum is sequential (as it seems to be so far), i.e., families come as left-handed (LH) doublets and right-handed (RH) singlets of SU(2) then $\mathscr{L}_{\text{gauge}}$ in

$$\mathscr{L}_{\text{SU(2)}\times\text{U(1)}} = \mathscr{L}_{\text{gauge}} + \mathscr{L}_{\text{Higgs}}$$

is invariant under interchange/mixing of

$$\begin{bmatrix} u \\ d \end{bmatrix}_L \leftrightarrow \begin{bmatrix} c \\ s \end{bmatrix}_L \leftrightarrow \begin{bmatrix} t \\ b \end{bmatrix}_L, \quad \begin{bmatrix} \nu \\ e \end{bmatrix}_L \leftrightarrow \begin{bmatrix} \nu \\ \mu \end{bmatrix}_L \leftrightarrow \begin{bmatrix} \nu \\ \tau \end{bmatrix}_L ,$$

$u_R \leftrightarrow c_R \leftrightarrow t_R$, etc. In fact, $\mathscr{L}_{\text{gauge}}$ is invariant under SU(n) acting on quark LH doublets [also SU(n) on lepton doublets, SU(n) on $\frac{2}{3}$-charge RH singlets, etc.] for n families. The discrete symmetry which suggests itself is permutation symmetry leading to a choice of S_n for G. Then we insist that $\mathscr{L}_{\text{Higgs}}$ be also symmetric under S_n and look for spontaneous breakdown of S_n by $V(\phi)$. The resulting mass- mixing pattern is the prediction of the model.

The most straightforward and obvious implementation of the above ideas would be the following. Let ψ_L^i, ψ_R^i, and ϕ^i ($i=1$ to n for n families) be the objects being permuted under S_n. Here ψ_L and ψ_R are LH doublets and RH singlets, respectively (both quarks and leptons). So we have

$$\begin{bmatrix} \nu_i \\ l_i \end{bmatrix}_L, \quad \begin{bmatrix} u_i \\ d_i \end{bmatrix}_L, \quad \phi_i = \begin{bmatrix} \phi_i^+ \\ \phi_i^0 \end{bmatrix},$$

$$l_{iR},\ u_{iR},\ d_{iR} ,$$

and $i=1$ to n. The Higgs-boson Yukawa couplings are $\sum_{i,j,k} g_{ijk}(\overline{\nu_i,l_i})_L l_{jR}\phi_k$, etc. One writes the general Higgs potential invariant under $\text{SU(2)}\times\text{U(1)}\times S_n$ and finds the minimum, breaking S_n spontaneously with the corresponding vacuum expectation values for ϕ_i^0: $\langle\phi_i^0\rangle=\eta_i$. Calculate the resulting mass matrices M_l, M_u, and M_d. Find the diagonalizing matrices:

$$U_{uL}^\dagger M_u V_{uR} = \begin{bmatrix} m_u & & \\ & m_c & \\ & & \ddots \end{bmatrix}$$

$$U_{dL}^\dagger M_d V_{dR} = \begin{bmatrix} m_d & & \\ & m_s & \\ & & \ddots \end{bmatrix} \tag{2.1}$$

Then the Kobayashi-Maskawa (KM) matrix is just

$$U_{\text{KM}} = U_{uL}^\dagger U_{dL} . \tag{2.2}$$

In this class of models the form for U_{KM} turns out to be especially simple, viz., U_{KM} is block diagonal;[5] e.g., for $n=3$,

$$U_{\text{KM}} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

for $n=4$, (2.3)

$$U_{\text{KM}} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

etc.

Hence, in this class of models one quark doublet is always decoupled. This means b cannot decay by W exchange and it can also be shown that b cannot decay via nonleptonic modes. Such properties of b seem to be ruled out experimentally. Another unsatisfactory feature is that the mixing angles are not predictable.

Let us turn to a slightly different model which is in some sense more economical. Consider n sequential families. Now if all of ψ_L^i and ψ_R^i are objects of permutation under S_n, then they transform as $\underline{(n-1)} \oplus \underline{1}$ under S_n (where $\underline{n-1}$ and $\underline{1}$ refer to irreducible representations of S_n of dimensionality $n-1$ and 1, respectively). In the model just discussed, the Higgs bosons also reduce to $\underline{(n-1)} \oplus \underline{1}$. We now propose that there are only $n-1$ Higgs doublets transforming as $\underline{n-1}$ under S_n. Then in the Higgs-boson Yukawa couplings of each charge sector (up, down, and lepton) there are just three terms as follows:

$$\begin{array}{ccccc} L & & R & & \text{Higgs} \\ \hline \underline{(n-1)} & \otimes & \underline{(n-1)} & \otimes & \underline{(n-1)} \\ \underline{(n-1)} & \otimes & \underline{1} & \otimes & \underline{(n-1)} \\ \underline{1} & \otimes & \underline{(n-1)} & \otimes & \underline{(n-1)} \end{array} \tag{2.4}$$

Because $\underline{(n-1)} \otimes \underline{(n-1)} \otimes \underline{(n-1)}$ contains $\underline{1}$ exactly once each of the above couplings is described by one coupling constant. Notice that the fermion family classified as $\underline{1}$ gets no mass in the limit in which it is decoupled from the remaining $n-1$ families. It is attractive to suppose that $\underline{1}$ corresponds to (e,ν_e,u,d) and consider the limit where the masses and couplings of $\underline{1}$ are neglected compared to those of $\underline{(n-1)}$. Then for the $n-1$ families there is one unique Yukawa coupling. The only difference between each charge sector ($\frac{2}{3}$, $-\frac{1}{3}$, and -1) is the overall strength. Hence, the mass matrices are proportional, to wit,

$$M_d \propto M_l \propto M_u^* \; . \tag{2.5}$$

Also, the M's are symmetric. Since $M^\dagger M$'s are proportional, so are the mass-squared eigenvalues and so[2]

$$\begin{aligned} m_c/m_t/m_{t'} \cdots &= m_\mu/m_\tau/m_L \cdots \\ &= m_s/m_b/m_{b'} \cdots \; . \end{aligned} \tag{2.6}$$

This is satisfied if $m_s \sim 300$ MeV and predicts $m_t \sim 26$ GeV. Since $M_d \propto M_u^*$, the KM matrix is symmetric (before phases are absorbed): $U_{\rm KM} = U_{\rm KM}^T$ and hence $|U_{cb}| = |U_{ts}|$, etc. Couplings of neutral Higgs bosons change flavor as expected and processes such as $K_L \to e\mu$, $\Sigma \to p\mu e$ are expected at some level. So far we have not introduced either ν_R or $I=1$ Higgs bosons, hence leaving the neutrinos massless. If ν_R^i transform in the same way as the other fermions, i.e., as $\underline{(n-1)} \oplus \underline{1}$, then neutrinos get Dirac masses which also satisfy the same scaling law as Eq. (2.6), i.e.,

$$m_{\nu_\mu}/m_{\nu_\tau}/m_{\nu_L} \cdots = m_\mu/m_\tau/m_L \cdots \; . \tag{2.7}$$

The proportionality of masses found above predicts the fourth-generation quark masses if the fourth charged lepton mass is known:

$$\begin{aligned} m_{t'} &= m_t \frac{m_L}{m_\tau} \; , \\ m_{b'} &= m_b \frac{m_L}{m_\tau} \; , \end{aligned} \tag{2.8}$$

e.g., if m_L is near 30 GeV then $m_{b'} \sim 80$ to 100 GeV and $m_{t'} \sim 300$ to 400 GeV. These values have two interesting features: (a) the near degeneracy of b' with W and Z which makes its detection difficult and (b) the saturation of various bounds on fermion masses[6] in $\mathrm{SU}(2)\times\mathrm{U}(1)$ suggesting that this is the last sequential family. Why the choice of $m_L \sim 30$ GeV? We already know $m_L > 18$ GeV (Ref. 7) and perhaps L and t are nearly degenerate, similarly to τ and c. Or else there is geometric scaling in lepton masses as once speculated,[8] i.e., $m_L/m_\tau = m_\tau/m_\mu$. In addition, a quark flavor in this mass range is a very useful factory for W's, Z's, and Higgs bosons.

III. THE S_4 MODEL

For just three families a somewhat different assignment for fermions in S_4 is very interesting.[2] S_4 has irreducible representations of dimensions 3, 3′, 2, 1′, and 1. The Higgs doublets are assigned to $\underline{3}$. The fermions are assigned as follows:

$$\begin{array}{ccc} \begin{bmatrix} \nu_i \\ l_i \end{bmatrix}_L , & l_{1R}, & (l_{2R}, l_{3R}) \; , \\ \underline{3} & \underline{1} & \underline{2} \\ \begin{bmatrix} u_i \\ d_i \end{bmatrix}_L , & \begin{matrix} u_{1R} \\ d_{1R} \end{matrix} & \begin{matrix} (u_{2R}, u_{3R}) \\ (d_{2R}, d_{3R}) \end{matrix} \; . \\ \underline{3} & \underline{1} & \underline{2} \end{array} \tag{3.1}$$

The Higgs potential invariant under $S_4 \times \mathrm{SU}(2) \times \mathrm{U}(1)$ is

$$V=\mu_0{}^2(\bar\phi_0\phi_0+\bar\phi_1\phi_1+\bar\phi_2\phi_2)+\alpha(\bar\phi_0\phi_0+\bar\phi_1\phi_1+\bar\phi_2\phi_2)^2+\beta[\tfrac{1}{2}(\bar\phi_0\phi_1+\bar\phi_1\phi_0)^2+\tfrac{1}{6}(\bar\phi_0\phi_0+\bar\phi_1\phi_1-2\bar\phi_2\phi_2)^2]$$
$$+\gamma[\tfrac{1}{2}(\bar\phi_0\phi_2+\bar\phi_2\phi_0)^2+\tfrac{1}{2}(\bar\phi_1\phi_2+\bar\phi_2\phi_1)^2+\tfrac{1}{2}(\bar\phi_0\phi_0-\bar\phi_1\phi_1)^2]$$
$$+\delta\{\tfrac{1}{6}[\bar\phi_0\phi_1-\bar\phi_1\phi_0+\sqrt{2}(\bar\phi_0\phi_2-\bar\phi_2\phi_0)]^2+\tfrac{1}{2}(\bar\phi_1\phi_2-\bar\phi_2\phi_1)^2+\tfrac{1}{6}[\sqrt{2}(\bar\phi_0\phi_1-\bar\phi_1\phi_0)-(\bar\phi_0\phi_2-\bar\phi_2\phi_0)]^2\}\ . \tag{3.2}$$

Denoting the vacuum expectation values of neutral members of the three Higgs doublets by

$$\langle\phi_2^0\rangle=\xi_2,\ \ \langle\phi_0^0\rangle=\xi\cos\alpha\, e^{i\phi},\ \ \langle\phi_1^0\rangle=\xi\sin\alpha\, e^{i\psi}\ , \tag{3.3}$$

the potential is minimized when $\alpha=\pi/4$, $\phi+\psi=0$,

$$\cos2\phi=-\frac{\gamma+\delta}{\beta+\delta}\left[\frac{\xi_2}{\xi}\right]^2 . \tag{3.4}$$

The minimum is stable if

$$\gamma+\delta>\beta+\delta>0,\ \ |\xi_2|<|\xi|\ .$$

The Higgs-boson Yukawa coupling invariant under $S_4\times SU(2)\times U(1)$ is

$$\mathcal{L}_Y=F[(\overline{\nu_1,l_1})_L\phi_0+(\overline{\nu_2,l_2})_L\phi_1+(\overline{\nu_3,l_3})_L\phi_2]l_{1R}$$
$$+H\left[\frac{1}{\sqrt{2}}[(\overline{\nu_1,l_1})_L\phi_1+(\overline{\nu_2,l_2})_L\phi_0]l_{2R}+\frac{1}{\sqrt{6}}[(\overline{\nu_1,l_1})_L\phi_0+(\overline{\nu_2,l_2})_L\phi_1-2(\overline{\nu_3,l_3})\phi_2]l_{3R}\right]+\text{H.c.}$$
$$+\left[\text{similar terms for quarks with }\begin{bmatrix}F\\H\end{bmatrix}\to\begin{bmatrix}f_+,f_-\\h_+,h_-\end{bmatrix},\ l_{iL}\to q_{iL},\ l_{iR}\to(u_{iR},d_{iR}),\ \phi_i\to(\tilde\phi_i,\phi_i)\right], \tag{3.5}$$

where

$$\tilde\phi_i=\begin{bmatrix}\bar\phi_i^0\\-\phi_i^-\end{bmatrix}.$$

We expect ϵ_i (where $\epsilon_l=\sqrt{2}F/H$, $\epsilon_\pm=\sqrt{2}f_\pm/h_\pm$) to be small since ϵ_i vanish in the limit m_e m_n, $m_d\to0$. At tree level the mass matrices of quarks and leptons are

$$(\overline{l_1,l_2,l_3})_LM_l\begin{bmatrix}l_1\\l_2\\l_3\end{bmatrix}_R+(\overline{u_1,u_2,u_3})_LM_+\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix}_R+(\overline{d_1,d_2,d_3})_LM_-\begin{bmatrix}d_1\\d_2\\d_3\end{bmatrix}_R+\text{H.c.}\ , \tag{3.6}$$

where

$$M_l=\frac{\xi H}{2}\begin{bmatrix}\epsilon_le^{i\phi} & \bar e^{i\phi} & \frac{1}{\sqrt{3}}e^{i\phi}\\ \epsilon_l\bar e^{i\phi} & e^{i\phi} & \frac{1}{\sqrt{3}}\bar e^{i\phi}\\ \kappa\epsilon_l & 0 & \frac{-2}{\sqrt{3}}\kappa\end{bmatrix}. \tag{3.7}$$

$M_\mp$ have the same form as M_l with $h_\mp$ replacing H, $\epsilon_\mp$ replacing ϵ_l, and $\phi \to \pm\phi$. Here $\kappa = \sqrt{2}\xi_2/\xi$. Diagonalizing $M_l M_l^\dagger$ to $O(\epsilon_l^2)$ and eliminating the parameters ϵ_l^2, ϵ_+^2, ϵ_-^2, κ^2, and ϕ one relation[2,9] among the nine masses is found:

$$\begin{vmatrix} m_e^2 & m_\mu^2 & m_\tau^2 \\ m_u^2 & m_c^2 & m_t^2 \\ m_d^2 & m_s^2 & m_b^2 \end{vmatrix} = 0 . \tag{3.8}$$

Using the known values of other masses, this gives $m_t \sim 26$ GeV. The value of m_t is rather insensitive to the actual quark masses used. Also in the approximation where m_e, m_u, and m_d vanish one recovers Eq. (2.6):

$$m_t \approx m_c (m_\tau / m_\mu) .$$

To interpret the quark mass for a confined quark one may use the Georgi-Politzer[10] definition, viz., writing the quark propagator as $iS^{-1}(p)\,|_{p^2=-M^2} \sim \not p - m(M)$, quark mass is m_Q $(M = 2m_Q)$. Then if a mass relation such as Eq. (15) holds at some high-energy scale (e.g., $\mu_0 \gtrsim 10^3$ GeV where S_4 may be unbroken) the predicted quark mass can be calculated by the lowest-order QCD correction:

$$m_i(\mu) = m_i(\mu_0) \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]^{4/[11-(2/3)f]} , \tag{3.9}$$

where $\mu = 2m_i$ and $f =$ the number of flavors. As found by a number of authors,[11] this makes $m_t \sim 20$ GeV. However, this procedure is not gauge invariant. Although this is usually regarded as a small effect, in principle it renders the result arbitrary. A gauge-invariant propagator was suggested by one of the authors (H.S.) recently and a calculation of the resulting corrected value of m_t is now in progress.[12]

Returning to diagonalization of M_i, the parameters ϕ and κ satisfy $1 >> \sin^2 2\phi >> \kappa^2$ and are given by

$$\sin^2 2\phi = \frac{16}{3} \left[\frac{m_c^2 m_d^2 - m_u^2 m_s^2}{m_t^2 m_d^2 - m_u^2 m_b^2} \right] , \tag{3.10}$$

$$\kappa^2 = \frac{2}{3} \left[\frac{m_c^2 m_d^2 - m_u^2 m_s^2}{m_t^2 m_s^2 - m_c^2 m_b^2} \right] .$$

The diagonalizing matrices are defined as

$$U_l M_l V_l^\dagger = \begin{bmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{bmatrix} \tag{3.11}$$

and similarly for U_+, V_+; U_-, V_-. Then the Kobayashi-Maskawa[13] matrix $U_{\rm KM}$ is

$$U_{\rm KM} = U_+ U_-^\dagger .$$

To order ϵ_i^2, $U_-^\dagger = U_+^T$ and hence

$$U_{\rm KM} = U_{\rm KM}^T$$

Explicitly U_+ is given by

$$U_+ = \begin{bmatrix} \dfrac{e^{i\eta_u}}{N_1} e^{-i\phi} & \dfrac{e^{i\eta_u}}{N_1} e^{-i\phi} & \dfrac{-ie^{i\eta_u}}{N_1} \dfrac{\sin 2\phi}{\kappa} \\ -\dfrac{e^{i\eta_c}}{N_2} e^{-i\phi}[e^{-2i\phi} + (\lambda_2 - 1)e^{2i\phi}] & \dfrac{e^{i\eta_c}}{N_2} e^{-i\phi}[e^{2i\phi} + (\lambda_2 - 1)e^{-2i\phi}] & \dfrac{e^{i\eta_c}}{N_2} 2\kappa(\lambda_2 - 2) \\ -\dfrac{e^{i\eta_t}}{N_3} e^{-i\phi}[e^{-2i\phi} + (\lambda_3 - 1)e^{2i\phi}] & \dfrac{e^{i\eta_t}}{N_3} e^{-i\phi}[e^{2i\phi} + (\lambda_3 - 1)e^{-2i\phi}] & \dfrac{e^{i\eta_t}}{N_3} 2\kappa(\lambda_3 - 2) \end{bmatrix} . \tag{3.12}$$

U_- has the same form with $\phi \to -\phi$, $\eta_u \to \eta_d$, etc. The η_i's are phases to bring $U_{\rm KM}$ to the KM form. N_i are given by

$$N_1^2 = 2 + \sin^2 2\phi / \kappa^2 ,$$

$$N_i^2 = \tfrac{8}{3} \{ [(2+\kappa^2)\lambda_i - (\sin^2 2\phi + 2\kappa^2)](1 + 2\kappa^2) - 3(\lambda_i - 1)(\sin^2 2\phi + 2\kappa^2) \} \quad (i = 2,3) ,$$

and λ_i are given by

$$\lambda_{2,3} = \frac{2}{3} \{ (2+\kappa^2) \mp [(2+\kappa^2)^2 - 3(2\kappa^2 + \sin^2 2\phi)]^{1/2} \} .$$

Now all elements of $U_{\rm KM}$ are completely fixed in terms of ϕ and κ and hence in terms of masses. We find

$$|U_{us}| = \sin\theta_1 \cos\theta_3 \sim \sin\theta_C = \left[\frac{m_\tau{}^2 m_d{}^2 - m_e{}^2 m_b{}^2}{m_\tau{}^2 m_s{}^2 - m_\mu{}^2 m_b{}^2}\right]^{1/2} \simeq m_d/m_s \left[1 - \frac{m_\mu{}^2}{m_\tau{}^2}\frac{m_b{}^2}{m_s{}^2}\right]^{-1/2} . \quad (3.13)$$

For $m_d/m_s \sim 1/18$, $m_b \sim 5$ GeV, and $m_s \sim 300$ MeV, this given $\theta_C \sim 0.22$. (The value of θ_C is sensitive to the precise value of m_s/m_b used because the denominator vanishes at $m_s/m_b = m_\mu/m_\tau$.) The square-root factor can be regarded as a corrective factor to the result m_d/m_s obtained[1] in S_3. We also find

$$\sin\theta_2 = \sin\theta_3 = \frac{1}{\sqrt{3}}\left[\frac{m_\mu{}^2 m_d{}^2 - m_e{}^2 m_s{}^2}{m_\tau{}^2 m_d{}^2 - m_e{}^2 m_b{}^2}\right]^{1/2}$$
$$\cong \frac{1}{\sqrt{3}}\frac{m_\mu}{m_\tau} \cong 0.034 ,$$
$$\sin\delta \sim 10^{-2} . \quad (3.14)$$

Hence we predict

$$\begin{aligned} |U_{cd}| &= |U_{us}| \simeq 0.22 ,\\ |U_{cb}| &= |U_{ts}| \cong 0.067 ,\\ |U_{ub}| &= |U_{td}| \cong 0.0075 ,\\ \alpha &= |U_{ub}|/|U_{cb}| \cong 0.11 . \end{aligned} \quad (3.15)$$

The full predicted matrix is then

$$U_{\mathrm{KM}} = \begin{bmatrix} 0.975 & -0.22 & -0.0075 \\ 0.22 & 0.973 & 0.067 \\ 0.0075 & 0.067 & -0.998 \end{bmatrix} . \quad (3.16)$$

With the value of $|U_{cb}| = 0.067$ and $|U_{ab}| = 0.0075$, the lifetime of B ($b\bar{u}, b\bar{d}$) mesons in the spectator model[14] is expected to be

$$\tau_B = \frac{\tau_\mu}{3.1}\left[\frac{m_\mu}{m_B}\right]^5 \frac{1}{(|U_{cb}|^2 + 2.5|U_{ub}|^2)}$$
$$\cong 6.60 \times 10^{-13} \text{ sec} . \quad (3.17)$$

Also with $|U_{td}| = 0.0075$ and $|U_{ts}| = 0.067$, the contribution of t quark to $\delta m_{K_L\text{-}K_S}$ is negligible and it is given essentially by the original Gaillard-Lee[15] estimate. Finally with $|s_2 s_3 \sin\delta| \sim 10^{-5}$ the amount of CP nonconservation in the KM matrix is too small[16] to account for the $K_L \to 2\pi$ rate. However, in this model there are flavor- and CP-nonconserving Higgs-boson couplings (to be discussed later) which could, in principle, account for $K_L \to 2\pi$.

If the Yukawa couplings are restricted to be real, we have spontaneous CP nonconservation. Also, in this case since $M_d M_u$ is real, θ_{QCD} is zero at tree level.[17]

IV. NEUTRINO MASSES AND MIXINGS

The pattern of neutrino masses and mixings depends on whether right-handed neutrinos exist, on their S_4 properties, and on the pattern of Majorana mass terms.

First consider the possibility that ν_R^i exist. Let there be as many ν_R^i as $(\nu_i, l_i)_L$. (An interesting phenomenological possibility is that the number of ν_R is not equal to that of ν_L; but in the spirit of the sequential model we will not consider it further here.) Then how do the ν_R^i transform under S_4? The simplest possibility (A) is that all of ν_R^i are invariant under S_4. This has some logic to it in that ν_R^i have no gauge interactions at the $SU(2)\times U(1)$ level. In this case the Yukawa coupling is

$$G[\,\overline{(\nu_i, l_1)}_L \tilde{\phi}_0 + \overline{(\nu_2, l_2)}_L \tilde{\phi}_1 + \overline{(\nu_3, l_3)}_L \tilde{\phi}_2]\nu_{3R} \quad (4.1)$$

and only one neutrino picks up a (Dirac) mass term. Then there is only one δm^2 relevant to ν oscillations and the mixing matrix is given by precisely

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix}_L = U_\nu \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}_L$$
$$= \begin{bmatrix} 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}_L \quad (4.2)$$

where $m_1 = m_2 = 0$. Another assignment is to let ν_i^R transform as l_i^R under S_4, i.e., as $\underline{2} \oplus \underline{1}$. In this case (B) the (Dirac) masses of neutrinos satisfy determinantal mass formulas as for charged fermions, viz.,

$$\begin{vmatrix} m_{\nu_1}{}^2 & m_{\nu_2}{}^2 & m_{\nu_3}{}^2 \\ m_e{}^2 & m_\mu{}^2 & m_\tau{}^2 \\ m_d{}^2 & m_s{}^2 & m_b{}^2 \end{vmatrix} = 0 . \quad (4.3)$$

This gives approximately

$$2.7 \times 10^3 m_{\nu_1}{}^2 + 300 m_{\nu_2}{}^2 \approx m_{\nu_3}{}^2 \quad (4.4)$$

In either case (A) or (B) we expect ν_R^i to get "large" $I=0$, S_4 singlet Majorana masses. (This is an example of a large mass scale entering.) Assuming for simplicity only one large mass say M, the light LH Majorana neutrinos masses become m_i^2/M [$m_i = m_\nu$(Dirac)] by a well known mechanism.[18] In the case (B) these masses now satisfy the linearized version of Eq. (27):

$$2.7\times 10^3 m_{\nu_1} + 300 m_{\nu_2} \approx m_{\nu_3} . \tag{4.5}$$

The mixing matrix in case A is U given by $U_\nu U_-^\dagger$ for either Dirac ν's or Majorana ν's (as long as there is only one heavy-mass scale). The mixing matrix in case B is $U \equiv U_{\rm KM}$ for Dirac ν's but not in general.

A second possibility, in a sense simpler, is that ν_R^i do not exist. The only way for ν_L^i to get masses is by coupling to an $I=1$ Higgs multiplet which may be regarded as being made up of two $I=1/2$ Higgs multiplets. Then the effective coupling looks like

$$(f/M)\overline{\psi^c_{L_k}}\vec{\tau}\psi_{L_l}\cdot\phi_i\vec{\tau}\phi_j . \tag{4.6}$$

Once again a new mass scale is called for. In general, there are four parameters since $\underline{3}\otimes\underline{3}=\underline{3}\oplus\underline{3}'\oplus\underline{2}\oplus\underline{1}$. But, for example, if the effective $I=1$ field transforms as S_4 singlet then all m_{ν_i} are equal. This is an interesting possibility because here is a case in which $m_{\nu_i}\neq 0$ but $\delta m^2{}_{ij}=0$ and there is no mixing and no oscillations.

V. *CP* NONCONSERVATION AND RARE PROCESSES

To study the effects due to Higgs-boson couplings, it is convenient to transform the Higgs doublets to a new basis ψ_1, ψ_2, ψ_3 defined by

$$\begin{bmatrix}\psi_1\\ \psi_2\\ \psi_3\end{bmatrix} = \begin{bmatrix} \dfrac{1}{(2+\kappa^2)^{1/2}}e^{-i\phi} & \dfrac{1}{(2+\kappa^2)^{1/2}}e^{i\phi} & \dfrac{1}{(2+\kappa^2)^{1/2}}\kappa \\ \dfrac{1}{[2(2+\kappa^2)]^{1/2}}\kappa e^{-i\phi} & \dfrac{1}{[2(2+\kappa^2)]^{1/2}}\kappa e^{i\phi} & \dfrac{1}{[2(2+\kappa^2)]^{1/2}}-2 \\ \frac{1}{2}e^{-i\phi} & -\frac{1}{2}e^{-i\phi} & 1 \end{bmatrix} \begin{bmatrix}\phi_0\\ \phi_1\\ \phi_2\end{bmatrix} . \tag{5.1}$$

The vacuum expectation values of ψ_i^0 are then

$$\langle\psi_1^0\rangle = \frac{\xi}{2}(2+\kappa^2)^{1/2} ,$$
$$\langle\psi_2^0\rangle = \langle\psi_3^0\rangle = 0 . \tag{5.2}$$

The coupling of ψ_i^0 to physical fermion fields is given by

$$\mathscr{L}_{\psi_i^0\text{-}f} = \frac{1}{\xi}\psi_1^0\sum_i m_{f_i}\bar{f}_i f_i + \frac{\sqrt{3}}{4\xi}\psi_2^0\left[m_\tau(\overline{e\mu\tau})_L A\begin{bmatrix}e\\ \mu\\ \tau\end{bmatrix}_R + m_b(\overline{dsb})_L A\begin{bmatrix}d\\ s\\ b\end{bmatrix}_R + m_t(\overline{uct})_L A\begin{bmatrix}u\\ c\\ t\end{bmatrix}_R\right]$$
$$+\frac{\sqrt{6}}{4\xi}\psi_3^0\left[m_\tau(\overline{e\mu\tau})_L B\begin{bmatrix}e\\ \mu\\ \tau\end{bmatrix}_R + m_b(\overline{dsb})_L B\begin{bmatrix}d\\ s\\ b\end{bmatrix}_R + m_t(\overline{uct})_L B\begin{bmatrix}u\\ c\\ t\end{bmatrix}_R\right] + \text{H.c.} \tag{5.3}$$

Here the matrices A and B are given by

$$A = \begin{bmatrix} 0 & -2e^{i(5/2)\phi} & \dfrac{-2}{\sqrt{3}}e^{i\phi/2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} ,$$
$$B = \begin{bmatrix} 0 & 2i\dfrac{\kappa}{\sin 2\phi}e^{-i\phi/2} & \dfrac{-i2}{\sqrt{3}}\dfrac{\kappa}{\sin 2\phi}e^{-i\phi/2} \\ 0 & \sqrt{2}ie^{i\phi} & i\dfrac{\sqrt{3}}{4\sqrt{2}}e^{i3\phi} \\ 0 & \dfrac{i\sin\phi}{\sqrt{2}}e^{2i\phi} & i\dfrac{5}{6}\dfrac{\sqrt{3}}{\sqrt{2}}\sin 2\phi\, e^{i(4/5)\phi} \end{bmatrix} . \tag{5.4}$$

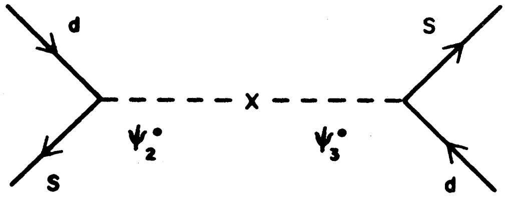

FIG. 1. CP violation through neutral-Higgs-boson exhange.

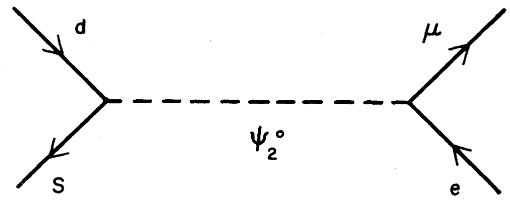

FIG. 2. Diagram for $K_L \to \mu e$ by neutral-Higgs-boson exchange.

In general the Higgs-boson mass eigenstates are mixtures of ψ_1^0, ψ_2^0, and ψ_3^0 and this will lead to CP violation in $\delta m_{k_L\text{-}K_S}$ as in the diagram in Fig. 1. If we parametrize

$$\langle 0 \,|\, T(\psi_i^0(x)\psi_j^0(0)) \,|\, 0 \rangle = \frac{-i\gamma_{ij}}{m_H{}^2}\delta(x) \tag{5.5}$$

(for $q^2 << m_H{}^2$), where γ_{ij} represents ψ_i^0-ψ_j^0 mixing and m_H is a characteristic Higgs-boson mass, then the contribution of Fig. 1 to $\mathrm{Im}\langle K^0 \,|\, H_{\mathrm{eff}} \,|\, \bar{K}^0 \rangle$ goes as

$$\mathrm{Im}\langle K^0 \,|\, H_{\mathrm{eff}} \,|\, \bar{K}^0 \rangle \simeq \frac{3\sqrt{2}}{4\xi^2} \frac{\kappa}{\sin 2\phi} \frac{m_b{}^2 f_K{}^2 \gamma_{23} m_K{}^2}{m_H{}^2} . \tag{5.6}$$

To reproduce the observed value of $K_L \to 2\pi$ rate (and $\epsilon \sim 10^{-3}$) we must have

$$\gamma_{23}/m_H{}^2 \sim (10^{-7} \text{ to } 10^{-8}) \text{ GeV}^{-2} . \tag{5.7}$$

If the parameter γ_{23} is of order 1 the characteristic mass scale for the Higgs-boson mass is a few TeV. From Eq. (5.3) the corresponding expressions for $\mathrm{Im}\langle D^0 \,|\, H_{\mathrm{eff}} \,|\, \bar{D}^0 \rangle$ and $\mathrm{Im}\langle B^0 \,|\, H_{\mathrm{eff}} \,|\, \bar{B}^0 \rangle$ are about $25 f_D{}^2/f_K{}^2$ and $\frac{1}{6} f_B{}^2/f_K{}^2$ times $\mathrm{Im}\langle K^0 \,|\, H_{\mathrm{eff}} \,|\, \bar{K}^0 \rangle$. Although with our predicted KM matrix we expect very little mixing[19] in D^0-$\bar{D}^0$ and B^0-$\bar{B}^0$ systems thus rendering the CP-violating charge asymmetries unobservable. We expect flavor-changing decays such as $K_L \to \mu e$, $\pi\mu e$, $\Sigma \to p\mu e$ due to diagrams such as in Fig. 2. We expect rates such as

$$\Gamma(K_L \to \mu^- e^+) \sim \frac{10^{-12}}{m_H{}^2} \text{ GeV} . \tag{5.8}$$

Hence $K_L \to \mu e$ should have a branching ratio $\sim 2.10^{-10}$ for $m_H \sim 10$ TeV to be compared to the present upper limit of 2.10^{-9}.

We have not discussed the effects of couplings of charged Higgs bosons. It can be shown that for charged Higgs bosons which have masses $\gtrsim m_W$ there are negligible contributions in the processes discussed above.

As in other models[20] where the dominant source of CP nonconservation is flavor-changing neutral Higgs bosons both ϵ' in K decay and the neutron electron dipole moment are expected to be small. They will be considered in detail elsewhere.

VI. CONCLUSION

We have pointed out that the sequential pattern seen in fermion families in $\mathrm{SU}(2) \times \mathrm{U}(1)$ is very suggestive of permutation symmetry. If this suggestion is valid, it may be a useful clue to the fermion mass pattern. In implementing the idea we found that mass scaling for all but the lightest generation is suggested:

$$m_\mu/m_\tau/m_L \ldots \simeq m_c/m_t/m_{t'} \ldots$$
$$\simeq m_s/m_b/m_{b'} \ldots .$$

The mixing matrix is symmetric and the quark-mass determinant is real. In the S_4 model for three families, we expect m_t to be between 20 and 30 GeV, predict the KM matrix to be

$$\begin{bmatrix} 0.975 & 0.22 & 0.0075 \\ -0.22 & 0.973 & 0.067 \\ -0.0075 & 0.067 & -0.998 \end{bmatrix}$$

This prediction for the KM matrix is certainly allowed by the data at present.[21] The source of CP violation in K decays is expected to be neutral Higgs bosons (which have to be several TeV in mass) and $K_L \to \mu e$ should occur at a level of 10^{-10} in branching ratio. All or some of these predictions can be tested in the near future.

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