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Mass effects in weak decays of heavy particles

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Mass effects in weak-decay rates of heavy particles are computed. A natural explanation of the experimental D^+ semileptonic branching ratio arises when mass and quantum-chromodynamic corrections are both taken into account. The experimental results on semileptonic branching ratio of b-flavored B mesons allows the prediction of the lifetime ratio $\tau(B^-)/\tau(\bar{B}^0) \leq 3.5$ in a model-independent way.

INTRODUCTION

In the lowest order of the weak interaction and the zeroth order of quantum chromodynamics (QCD), the inclusive weak-decay rate of a heavy particle (say a charmed or b-flavored one) is governed by two classes of diagrams. The first one (Fig. 1), usually designated as the spectator decay mechanism (SP), corresponds to the disintegration $Q \to q_1+q_2+\bar{q}_3$ of the heavy quark Q, and the light constituent quark q is simply inert. In the second class of diagrams (Fig. 2), called the nonspectator mechanism (NSP), both heavy and light quarks participate in the decay via W-boson exchange in either the t or s channel. The latter case is sometimes designated as the annihilation mechanism.

When the QCD hard-gluon interaction is switched on, then not only do there arise corrections[1] to these decay rates but also there emerges a new class of diagrams[2] called the "penguin" diagrams (Fig. 3).

The first diagram contributes to the nonleptonic as well as to the semileptonic modes. The nonspectator mechanism (W-boson exchange in the t channel) and the penguin diagram are responsible for pure hadronic decays. Only the annihilation mechanism (s-channel W exchange) can contribute to all modes: pure leptonic, pure hadronic, and semileptonic. The latter case is operative mainly[3,4] in the soft-gluon regime (Fig. 5).

For the spectator mechanism ($Q \to q_1+q_2+\bar{q}_3$), up to and including the order α_s, computations of the leading-logarithm (LL) and of the next-to-LL QCD effects have been achieved recently by the work of Altarelli, Curci, Martinelli, and Petrarca.[5] For the penguin diagram, the next-to-leading effects have also been discussed by Galic.[6] These calculations are done with final massless quarks. To our surprise, even at zeroth order in QCD, the corrections due to all final masses are not yet computed and one of our purposes is to fulfill this gap. When the mass effect is taken into account, we find that the hadronic width is strongly reduced in the spectator diagram. Subsequently, we will apply the mass and QCD corrections to charmed- and b-flavored-meson decays for which recent data are available.[7] Discussions on semileptonic branching ratios of D^+ and B^- mesons follow. For these charged mesons, contributions from the spectator mechanism dominate by far all other mechanisms (including the penguin diagram). The nonspectator contribution is Cabibbo suppressed in D^+; for B^- it is suppressed by the presumed small weak coupling V_{bu}. Anyway, our analysis of B-meson decays in Sec. II is independent of the weak couplings. QCD corrections as calculated by Altarelli *et al.*[5] show that the hadronic width is enhanced with respect to LL and *a fortiori* with respect to the free-quark limit. On the other hand, at the order α_s, the semileptonic width is known to be reduced.[8] Therefore, the semileptonic branching ratio of D^+ is found[5] to be about 10% which is considerably smaller than 20% as in the free-quark case. If experiments continue to confirm the value[9] close to $(21^{+4}_{-2})\%$ for the D^+ semileptonic branching ratio, then our mass corrections offer a natural and quantitative explanation without evoking other *Ansätze* such as the interference model.[10]

Finally, using the preliminary data on the semileptonic branching ratio of B mesons, the lifetimes of B^- and $\bar{B}^0$ are found by a model-independent analysis to be rather similar, $\tau(B^-)/\tau(\bar{B}^0) \leqslant 3.50$. Even the possibility for $\tau(B^-) \leqslant \tau(\bar{B}^0)$ is not excluded.

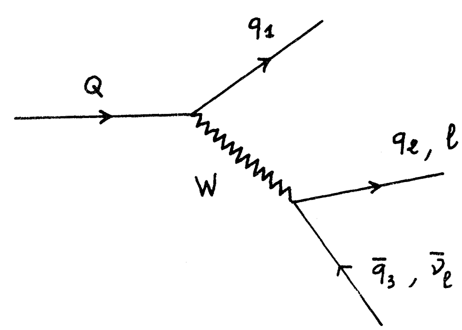

FIG. 1. Spectator diagram.

I. MASS CORRECTIONS TO THE DECAY WIDTH

We start with the effective Hamiltonian

$$H_W=\frac{G}{\sqrt{2}}V_{Qq_1}V_{q_2q_3}\bar{\psi}(q_1)\gamma_\mu(1-\gamma_5)\psi(Q)$$
$$\times\bar{\psi}(q_2)\gamma_\mu(1-\gamma_5)\psi(q_3)\,,$$

where the Cabibbo-type angles are put into V_{Qq_1} and $V_{q_2q_3}$.

A. Spectator diagram

We compute the hadronic decay width through the mechanism $Q\to q_1+q_2+\bar{q}_3$ where all final-quark masses m_i $(i=1,2,3)$ are kept. The result (including the factor 3 of color) is

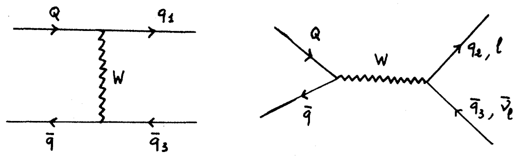

FIG. 2. Nonspectator diagrams for meson decay.

$$\Gamma_{\rm SP}^{\rm had}=3\frac{G^2}{192\pi^3}M^5|V_{Qq_1}|^2|V_{q_2q_3}|^2I(x_1,x_2,x_3)\,,\quad(1)$$

where $x_i=m_i/M$ and M is the initial-heavy-quark mass:

$$I(x_1,x_2,x_3)=12\int_{(x_1+x_2)^2}^{(1-x_3)^2}\frac{ds}{s}(s-x_1{}^2-x_2{}^2)(1+x_3{}^2-s)$$
$$\times\{[s-(x_1-x_2)^2][s-(x_1+x_2)^2][(1+x_3)^2-s][(1-x_3)^2-s]\}^{1/2}\,.\quad(2)$$

The function $I(x_1,x_2,x_3)$ is symmetrical in the interchange $x_1\leftrightarrow x_2$; it can be expressed in terms of complicated elliptic functions and reduces successively to elementary functions for appropriate limits of the arguments x_1,x_2,x_3:

$$I(0,0,0)=1\,,$$
$$I(x,0,0)=I(0,x,0)=I(0,0,x)=1-8x^2+8x^6-x^8-24x^4\ln x\,,\quad(3)$$
$$I(x,x,0)=(1-4x^2)^{1/2}[1-14x^2-2x^4-12x^6]+24x^4(1-x^4)\ln\frac{1+(1-4x^2)^{1/2}}{1-(1-4x^2)^{1/2}}\,,\quad(4)$$

$$I(x,y,0)=[1-2(x^2+y^2)+(x^2-y^2)^2]^{1/2}[1-7(x^2+y^2)+6x^2y^2-2(x^2+y^2)^2$$
$$-6x^2y^2(x^2+y^2)+(x^2-y^2)(x^4-y^4+5y^2-5x^2)]$$
$$+12(x^4+y^4-2x^4y^4)\ln\frac{1-x^2-y^2+[1-2(x^2+y^2)+(x^2-y^2)^2]^{1/2}}{2xy}$$
$$+12(x^4-y^4)\ln\frac{x^2+y^2-(x^2-y^2)^2-(x^2-y^2)[1-2(x^2+y^2)+(x^2-y^2)^2]^{1/2}}{2xy}\,,\quad(5)$$

$$I(x,0,y)=[1-2(x^2+y^2)+(x^2-y^2)^2]^{1/2}[1-7(x^2+y^2+x^4+y^4+x^2y^4+x^4y^2)+12x^2y^2+x^6+y^6]$$
$$+12(x^4-2y^4+x^4y^4)\ln\frac{1+y^2-x^2-[1-2(x^2+y^2)+(x^2-y^2)^2]^{1/2}}{2y}$$
$$+12x^4(1-y^4)\ln\frac{(1-y^2)^2-x^2(1+y^2)+(1-y^2)[1-2(x^2+y^2)+(x^2-y^2)^2]^{1/2}}{2yx^2}\,.\quad(6)$$

The relation (3) is well known.[11] The relation (4) has been derived in previous work.[12] The others are new. The mass corrections given by Eq. (2) are obviously relevant for the decays such as $b\to cs\bar{c}$, $t\to bc\bar{s}$, $b\to c\tau\bar{\nu}$, but even for the light-quark case $c\to su\bar{d}$ the corrections are not negligible as shown in Fig. 4 and Table I. Since $I(x_1,x_2,x_3)<1$ the mass corrections always act to lower the width. The formula given by Eq. (2) is calculated for the case $(V-A)\times(V-A)$. For $(V+A)\times(V+A)$ the result is exactly the same, and finally for $(V\mp A)\times(V\pm A)$ one has only to change $I(x_1,x_2,x_3)$ by $I(x_1,x_3,x_2)$.

A remark here is in order. It is not clear which mass, "current" or "constituent," should be used for the light up and down quarks. However, it is clear that all three final quarks must be considered on the same footing. For charmed-meson decays $c\to su\bar{d}$, since the s-quark mass is taken as the constituent mass ($\simeq$500 MeV) by Cabibbo and Maiani,[8] then the up- and down-quark masses must also be taken as constituent masses. The hadronization of u,d quarks into physical final states suggests also that an "effective" quark field would be used.[13] In such a case, a massive quark might be more plausible. In the following we will take m_u and m_d between 100 and 300 MeV. Some typical values of $I(x_1,x_2,x_3)$ are given in Table I.

B. Nonspectator diagram

To zeroth order of QCD, the total hadronic width calculated in t-channel W-boson exchange is given by

$$\Gamma(Q\bar{q}\to q_1\bar{q}_3)=\frac{1}{3}\frac{G^2}{8\pi}|V_{Qq_1}|^2|V_{qq_3}|^2 f_{Q\bar{q}}^{\,2}M(m_1^{\,2}+m_3^{\,2})\left[1-\frac{(m_1^{\,2}-m_3^{\,2})^2}{M^2(m_1^{\,2}+m_3^{\,2})}\right]\left\{\left[1-\left(\frac{m_1+m_3}{M}\right)^2\right]\left[1-\left(\frac{m_1-m_3}{M}\right)^2\right]\right\}^{1/2}. \tag{7}$$

For the s-channel W exchange, we get

$$\Gamma(Q\bar{q}\to q_2\bar{q}_3)=3\frac{G^2}{8\pi}|V_{Qq}|^2|V_{q_2q_3}|^2 f_{Q\bar{q}}^{\,2}M(m_2^{\,2}+m_3^{\,2})\left[1-\frac{(m_2^{\,2}-m_3^{\,2})^2}{M^2(m_2^{\,2}+m_3^{\,2})}\right]\left\{\left[1-\left(\frac{m_2+m_3}{M}\right)^2\right]\left[1-\left(\frac{m_2-m_3}{M}\right)^2\right]\right\}^{1/2}. \tag{8}$$

From Eq. (7) we realize that the mass effect due to m_3 increases the width with respect to its value when m_3 is neglected. The corrections for the nonspectator diagram go always in the opposite direction of the spectator diagram. For example, in D^0 decay via the t-channel W exchange $\Gamma(c\bar{u}\to s\bar{d})$, the effect of d-quark mass tends to increase the width, contrary to the case of the spectator diagram for which the d mass acts as a damping factor. Of course, the mass effect alone cannot explain the large difference in the D^0 and D^+ lifetimes (for D^+ only the spectator diagram contributes at the Cabibbo-favored level).

II. B-MESON DECAYS

We confine our discussions to the lightest B mesons: $B^-\equiv(b\bar{u})$, $\bar{B}^0\equiv(b\bar{d})$ for which recent data are available.[7] First of all, we must check whether or not the penguin contributions might be neglected here. This because, contrary to the charmed case for which the penguin contribution is always suppressed by Cabibbo-type angles with respect to the spectator one, here for the b-flavored case (as for the strangeness case), this is not true. The effective QCD-corrected Hamiltonian for the $\Delta S=1$ case has been extensively discussed in the literature[14]; for our purpose of B decays, a slight modification is sufficient. Among the six operators arising when hard QCD is switched on, four are relevant to the penguin contribution and among them the last two operators, noted as O_5 and O_6 in the Gilman-Wise notation,[14] contribute dominantly because of their structure $(V-A)\times(V+A)$ that enhances the nonspectator matrix element (the helicity suppression factor $m_{q'}^{\,2}/m_b^{\,2}$ disappears). Denoting by c_5 and c_6 the coefficients of the corresponding operators, we find

$$\Gamma_{\text{penguin}}(b\bar{q}\to s\bar{q})=\tfrac{1}{3}(c_5+3c_6)^2\frac{G^2}{8\pi}|V_{bt}|^2|V_{ts}|^2 f_B^{\,2}m_b^{\,3}. \tag{9}$$

The coefficients c_5, c_6 are calculated following

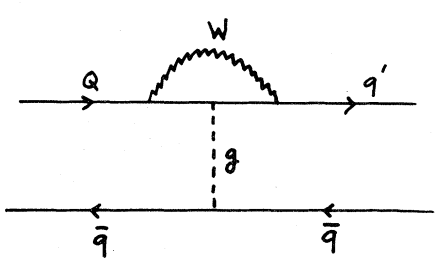

FIG. 3. Penguin diagram for meson decay. Dashed line, gluon.

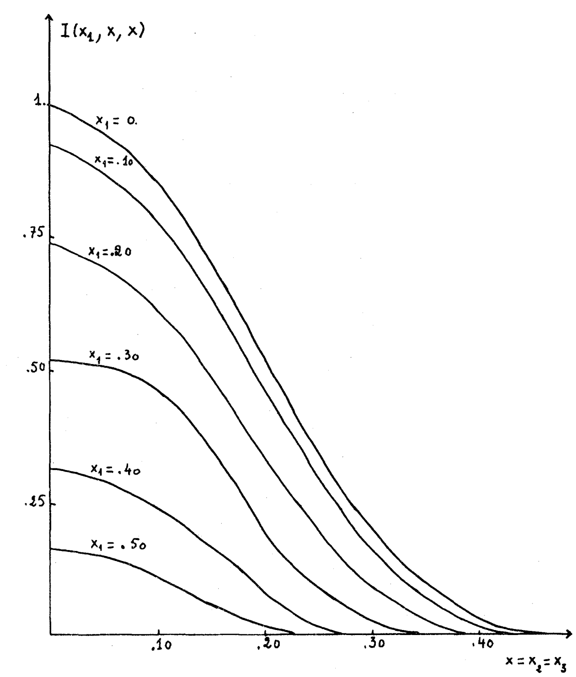

FIG. 4. Plot of $I(x_1,x,x)$ as a function of x for different values of x_1.

the method given in Ref. 14 and we get $c_5 = 0.02$, $c_6 = -0.04$. The penguin contributions for B decays are indeed negligible (a few per cent) with respect to the spectator diagram (even for $f_B \simeq 500$ MeV). Let us write down explicitly, within this class of diagram, all decay widths. In units of $G^2 m_b^5/192\pi^3$, we have

$$\Gamma_{\rm SP}(B\to e\bar{\nu}X) = |V_{bc}|^2 I\left(\frac{m_c}{m_b},0,0\right)\left[1-\frac{2\alpha_s}{3\pi}f\left(\frac{m_c}{m_b},0,0\right)\right] + |V_{bu}|^2\left[1-\frac{2\alpha_s}{3\pi}f(0,0,0)\right], \qquad (10)$$

$$\Gamma_{\rm SP}(B\to \tau\bar{\nu}X) = |V_{bc}|^2 I\left(\frac{m_c}{m_b},\frac{m_\tau}{m_b},0\right)\times\left[1-\frac{2\alpha_s}{3\pi}f\left(\frac{m_c}{m_b},\frac{m_\tau}{m_b},0\right)\right] + |V_{bu}|^2 I\left(0,\frac{m_\tau}{m_b},0\right)\times\left[1-\frac{2\alpha_s}{3\pi}f\left(0,\frac{m_\tau}{m_b},0\right)\right]. \qquad (11)$$

The function $f(m_c/m_b,0,0)$ is obtained by integrating the radiative corrections to the electron spectrum in μ decay. It has been given explicitly in Ref. 15 and numerically by Cabibbo and Maiani.[8] For the function $f(m_c/m_b, m_\tau/m_b, 0)$ it can also be obtained by modification of the matrix element in the radiative μ decay where the neutrino mass is taken different from zero (see Lenard, Ref. 15). Note that $I(x,0,0)$ decreases more quickly than $f(x,0,0)$ when x increases (see Ref. 8); it is likely that the same thing happens when one compares $I(x,y,0)$ to $f(x,y,0)$.

TABLE I. Values of $I(x_1,x_2,x_3)$ for different choices of x_1, x_2, x_3. We take $M_b = 4.5$ GeV, $M_c = 1.5$ GeV.

	$M_s = 0.5$ GeV $M_u = 0.3$ GeV	$M_s = 0.3$ GeV $M_u = 0.1$ GeV
$c\to su\bar{d}$	0.15	0.68
$c\to s\nu_e e^+$	0.45	0.74
$c\to s\nu_\mu\mu^+$	0.41	0.71
$b\to cd\bar{u}$	0.40	0.44
$b\to cs\bar{c}$	0.08	0.10
$b\to ud\bar{u}$	0.90	0.99
$b\to us\bar{c}$	0.36	0.42
$b\to c\tau\bar{\nu}_\tau$	0.06	0.06
$b\to u\tau\bar{\nu}_\tau$	0.30	0.32

Then, in units of $G^2 m_b^5/192\pi^3$, we obtain numerically

$$\Gamma_{\rm SP}(B\to e\bar{\nu}X) \simeq \Gamma(B\to \mu\bar{\nu}X) = 0.38|V_{bc}|^2 + 0.77|V_{bu}|^2, \qquad (12)$$

$$\Gamma_{\rm SP}(B\to\tau\bar{\nu}X) = 0.06|V_{bc}|^2 + 0.28|V_{bu}|^2. \qquad (13)$$

The hadronic width of B is given by

$$\Gamma_{\rm SP}(B\to {\rm had}) = (2f_+^2+f_-^2)J\left\{|V_{bc}|^2\left[I\left(\frac{m_c}{m_b},0,0\right)+I\left(\frac{m_c}{m_b},\frac{m_s}{m_b},\frac{m_c}{m_b}\right)\right] + |V_{bu}|^2\left[1+I\left(0,\frac{m_s}{m_b},\frac{m_c}{m_b}\right)\right]\right\}, \qquad (14)$$

where J is the next-to-leading QCD correction given by Altarelli *et al.* Using their formulas, we obtain $J = 1.06$, $f_- = 1.38$, $f_+ = 0.85$ (for $\Lambda_{\rm QCD} = 0.25$ GeV), and numerically

$$\Gamma_{\rm SP}(B\to {\rm had}) = 1.9|V_{bc}|^2 + 5.1|V_{bu}|^2. \qquad (15)$$

If only spectator diagrams contribute to the decay of B mesons, then their semileptonic branching ratio is given by

$$B_{\rm SL} \equiv \frac{\Gamma_{\rm SP}(B\to e\bar{\nu}X)}{\Gamma_{\rm SP}^{\rm tot}} = \frac{0.38|V_{bc}|^2+0.77|V_{bu}|^2}{2.72|V_{bc}|^2+6.92|V_{bu}|^2}. \qquad (16)$$

Whatever the couplings V_{bc} and V_{bu} are, $B_{\rm SL}$ is bounded by[16]

$$\frac{0.77}{6.92} = 0.11 \le B_{\rm SL} \le 0.14 = \frac{0.38}{2.72}. \qquad (17)$$

It is amusing to remark that in the massless-free-quark limit (which is far from being realized here) $B_{\rm SL} = \frac{1}{9}$, very close to the bounds [Eq. (17)]. Mass and QCD corrections seem to conspire here to mutually cancel their effects.

We now come to the nonspectator diagrams that contribute differently to $\bar{B}^0$ and B^- cases.

For $\bar{B}^0$, since only pure hadronic decay modes occur (assuming the absence of flavor-changing neutral current $b\bar{d}Z$), the total width for $\bar{B}^0$ must be equal to or greater than that of the spectator mechanism as given by the denominator of Eq. (16). Since the semileptonic partial width [numerator of Eq. (16)] remains unchanged by nonspectator contributions, the $\bar{B}^0$ semileptonic branching ratio ($\bar{B}^0_{\rm SL}$) is bounded from above by 0.14 independently on how big the nonspectator (penguin included) contributions are.

For B^-, both pure hadronic and semileptonic modes can occur (as for the F^+ or D^+ of charmed mesons) in the soft-gluon regime[3,4] (see Fig. 5).

Denoting[3,4] by $\mathcal{S}$ and $\mathcal{O}$, respectively, the color-singlet and -octet parts in the system ($b\bar{u}$) of the B^- meson, the semileptonic and hadronic widths can be parametrized as (always in units $G^2 m_b^5/192\pi^3$)

$$\Gamma_{\rm NSP}(b\bar{u}\to l\bar{\nu}+\text{gluons})=|V_{bu}|^2\mathcal{S}\left(1-\frac{m_l^2}{m_b^2}\right)^2, \tag{18}$$

$$\Gamma_{\rm NSP}(b\bar{u}\to q_1\bar{q}_2+\text{gluons})=|V_{bu}|^2\left[3\left(\frac{2f_++f_-}{3}\right)^2\mathcal{S}+(f_+-f_-)^2\mathcal{O}\right]$$
$$\times\left[1-\frac{(m_1^2-m_2^2)^2}{m_b^2(m_1^2+m_2^2)}\right]\left[\left(1-\frac{m_1^2+m_2^2}{m_b^2}\right)^2-\frac{4m_1^2m_2^2}{m_b^4}\right]^{1/2}. \tag{19}$$

Here $q_1\bar{q}_2$ are $d\bar{u}$ and $s\bar{c}$.

Putting all contributions together [Eqs. (10), (11), (14), (18), and (19)], we get numerically

$$B_{\rm SL}^-\equiv\frac{\Gamma(B^-\to e\bar{\nu}X)}{\Gamma_{\rm SP}^{\rm tot}+\Gamma_{\rm NSP}^{\rm tot}(B^-)}=\frac{0.38|V_{bc}|^2+(0.77+\mathcal{S})|V_{bu}|^2}{2.72|V_{bc}|^2+(6.92+8.27\mathcal{S}+0.48\mathcal{O})|V_{bu}|^2}. \tag{20}$$

Whatever the parameters $\mathcal{S},\mathcal{O}$, $|V_{bc}|^2$, $|V_{bu}|^2$ are, we obtain the upper bound[17]

$$B_{\rm SL}^-\leq 0.14. \tag{21}$$

This bound depends only on $x_i=m_i/m_b$, f_+, f_-, J and not at all on $\mathcal{S},\mathcal{O},|V_{bu}|^2,|V_{bc}|^2$. This result is quite interesting because of its independence of the parameters of the nonspectator soft-gluon mechanism. Furthermore this upper bound is not affected by the penguin contributions whatever they are.

Equipped now with this rather stringent bound $B_{\rm SL}^-\leq 0.14$, we analyze further data to obtain an inequality between total lifetimes of $\bar{B}^0$ and B^-. Our analysis is the following. Experimental data[7] on the semileptonic branching ratio of $\bar{B}^0$ and B^- into electron + muon are given as[18]

$$\tfrac{1}{2}(\bar{B}_{\rm SL}^0+B_{\rm SL}^-)_{e+\mu}=(24\pm6)\%.$$

From this experimental number and Eq. (21) we deduce (assuming equal rates of electron and muon)

$$\bar{B}_{\rm SL}^0\geq(10\pm6)\%.$$

Since

$$\bar{B}_{\rm SL}^0\equiv\frac{\Gamma_{\rm SP}(\bar{B}^0\to e\bar{\nu}X)}{\Gamma_{\rm tot}(\bar{B}^0)}=\frac{\Gamma_{\rm SP}(\bar{B}^0\to e\bar{\nu}X)}{\Gamma_{\rm SP}^{\rm tot}+\Gamma_{\rm NSP}^{\rm tot}(\bar{B}^0)},$$

from the spectator contribution Eqs. (16) and (17) we have

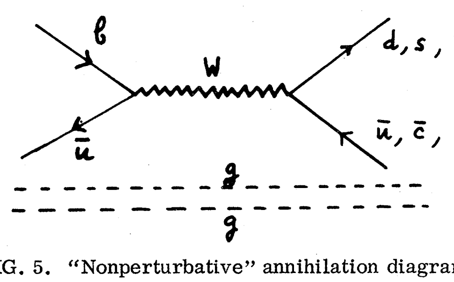

FIG. 5. "Nonperturbative" annihilation diagram for charged mesons (D^+,F^+,B^-) decays. The gluons here play the spectator role.

$$0.10\pm0.06\leq\bar{B}_{\rm SL}^0\leq\frac{0.14}{1+\beta}, \tag{22}$$

where

$$\beta\equiv\frac{\Gamma_{\rm NSP}^{\rm tot}(\bar{B}^0)}{\Gamma_{\rm SP}^{\rm tot}}.$$

The inequality (22) is satisfied (experimental errors included) only if $\beta\leq 2.5$, or

$$\Gamma_{\rm tot}(\bar{B}^0)\leq 3.5\Gamma_{\rm SP}^{\rm tot}.$$

Moreover, by definition

$$\Gamma_{\rm tot}(B^-)\equiv\Gamma_{\rm SP}^{\rm tot}+\Gamma_{\rm NSP}^{\rm tot}(B^-)\geq\Gamma_{\rm SP}^{\rm tot}.$$

We then get

$$\frac{\tau(B^-)}{\tau(\bar{B}^0)}\equiv\frac{\Gamma_{\rm tot}(\bar{B}^0)}{\Gamma_{\rm tot}(B^-)}\leq 3.5,$$

whatever the nonspectator contributions in $\bar{B}^0$ and B^- are. The bound $\tau(B^-)/\tau(\bar{B}^0)\leq 3.5$ reflects uncertainties on the present experimental results; it can be lowered when more accurate data on the semileptonic branching ratio are given.

While charged and neutral charmed mesons D^+, D^0 have a big difference in their lifetimes $[\tau(D^+)/\tau(D^0)\sim 5-10]$ it is remarkable that a prediction $\tau(B^-)/\tau(\bar{B}^0)\leq 3.5$ can be obtained only from the semileptonic branching ratio $\frac{1}{2}(\bar{B}_{\rm SL}^0+B_{\rm SL}^-)_{e+\mu}=(24\pm6)\%$.

Moreover, since the lower bound of $\tau(B^-)/\tau(\bar{B}^0)$ cannot be obtained without further assumptions on the nonspectator decay widths, even the possibility for $\tau(B^-)\leq\tau(\bar{B}^0)$ is not excluded. This could happen in the case $|V_{bu}|\geq|V_{bc}|$. However data on the K/π ratio[7] of B decays strongly support the converse; therefore we guess that $\tau(B^-)$ is slightly higher than $\tau(\bar{B}^0)$. Our result is at variance with that of Ref. (19), according to which the nonperturbative effects dominate the spectator diagram, even for large M.

III. THE D^+ CASE

QCD corrections[5] beyond the leading-logarithmic ones show that the hadronic decay width in the spectator mechanism always increases with respect to the one computed in zeroth order of QCD. On the other hand, the partial semileptonic width is known also[8] to decrease when QCD and mass corrections are taken into account. Consequently, the electronic branching ratio $D^+_{\rm SL} \equiv \Gamma(D^+ \to e^+\nu X)/\Gamma_{\rm tot}(D^+)$ is found to be much lower than that given by the naive free-quark limit (10% instead of 20%).

If experiments continue to confirm[9] $D^+_{\rm SL} = (21^{+4}_{-2})\%$, then the effect has been conjectured[5] to be attributable to the nonpartonic sector, in particular, to the interference between the two $\bar{d}$ antiquarks in the final state of D^+ decay, as suggested in Ref. 10.

We point out here that an alternative is possible by our mass corrections. In units of $G^2 m_c^{\,5}/192\pi^3$, we write

$$\Gamma(D^+ \to l^+\nu X) = I\left(\frac{m_s}{m_c}, 0, \frac{m_l}{m_c}\right)\left[1 - \frac{2\alpha_s}{3\pi} f\left(\frac{m_s}{m_c}, 0, \frac{m_l}{m_c}\right)\right],$$

$$\Gamma(D^+ \to \text{hadrons}) = (2\tilde{f}_+^{\,2} + \tilde{f}_-^{\,2}) J_c I\left(\frac{m_s}{m_c}, \frac{m_u}{m_c}, \frac{m_d}{m_c}\right),$$

with $J_c = 1.20$; $\tilde{f}_+ = 0.72$, $\tilde{f}_- = 1.93$ ($\Lambda_{\rm QCD} = 0.25$ GeV). Since $I(x, y, z)$ decreases quite quickly (see Table I and Fig. 4), the electronic branching ratio $D^+_{\rm SL} \simeq 20\%$ can be easily accounted for with $m_u/m_c = m_d/m_c \simeq 0.1-0.2$.

CONCLUDING REMARKS

In this paper, we compute first, at zeroth order of QCD, the corrections due to final-quark and -lepton masses. This must be considered as a compulsory step to be achieved in any reliable calculation of decay widths. It is certainly relevant for future heavy-quark decays such as $b \to c\tau\bar{\nu}$, $b \to cs\bar{c}$, $t \to bc\bar{s}$, etc.

For the charmed case $c \to su\bar{d}$, $c \to s\nu\bar{\mu}$, QCD and muon and up- and down-quark mass corrections provide a natural explanation of the D^+ semielectronic branching ratio.

Finally applying QCD and mass corrections to B-meson decays and using as input the experimental results on semileptonic decays of the sum $\bar{B}^0 + B^-$ into $e + \mu$, we predict $\tau(B^-)/\tau(\bar{B}^0) \leq 3.5$ in a model-independent way. Even the surprising possibility $\tau(B^-) \leq \tau(\bar{B}^0)$ is not excluded although unlikely.

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[16]Expressions such as $\rho \equiv \sum_i \lambda_i D_i / \sum_j \mu_j D_j$, where all λ, μ, D are positive or zero, are bounded by min $(\lambda_i/\mu_i) \leq \rho \leq \max(\lambda_i/\mu_i)$.

[17]We assume that $O(\alpha_s)$ corrections to $\Gamma_{\rm NSP}$ [Eqs. (18), (19), and (20)] do not modify our analysis leading to

the bound [Eq. (21)]; one expects that these corrections decrease the semileptonic branching ratio in the same way as in the spectator case, and consequently support our assumption.

[18]We understand that cuts in momentum of e and μ are such that these data correspond to direct b decays and not to cascade ones such as

$$b \to c\bar{\nu}\tau, \quad \tau \to e\nu\bar{\nu} \qquad \text{or} \qquad b \to c q_2 \bar{q}_3, \quad c \to e\bar{\nu}s .$$

Also, we assume equal production cross sections for $B^0\bar{B}^0$ and B^+B^- above the Υ''' resonance.

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