PHYSICAL REVIEW D VOLUME 25, NUMBER 7 1 APRIL 1982

Phenomenology of an $f(1270)$-glueball mixture

John F. Donoghue
Department of Physics and Astronomy, University of Massachusetts, Amherst, Massachusetts 01003
(Received 9 October 1981)

Possible experimental signatures of a 2^{++} $Q\bar{Q}$ and glueball mixture near the $f(1270)$ resonance are studied using the P-matrix and K-matrix formalisms.

I. INTRODUCTION

One of the low-lying bound states of gluons which is expected in many models has $J^{PC}=2^{++}$. In particular the bag model[1–3] predicts the 2^{++} glueball near $M=1.29$ GeV. While there is considerable uncertainty in any mass estimate, there do not appear to be candidates for this state within a reasonable range around this estimate. This led the authors of Ref. 1 (called DJL below) to suggest that it may be essentially where it is predicted but that it is hidden underneath the broad $f(1270)$ meson. DJL suggested a partial decoupling mechanism which may make one of the two physical states difficult to detect in pion-induced reactions. However, this mechanism was presented using a nonunitary procedure of diagonalizing a mass matrix. Rosner[4] has also studied this system using a simple mixing scheme. It is the purpose of this paper to attempt to uncover the phenomenological implications of such a two-particle mixture by use of more realistic P-matrix[5] and K-matrix[6] techniques.

The primary goal is to help experimenters test this hypothesis. However, the results may be of interest to theorists concerned with the mixing of two states in general, because they reveal some unexpected effects when the two poles are highly overlapping. In addition, some may find the usage of the relatively new P-matrix formalism instructive.

The plan of the paper is as follows. In the next section we set up the P- and K-matrix formalisms, while in Sec. III we explain how we will use them. The decoupling mechanism of DJL is studied in Sec. IV. A further exploration, utilizing the lack of total decoupling, is made in Sec. V in order to uncover possible experimental signals for a two-state mixture. Comparison with the mixing scheme is discussed briefly in Sec. VI, while Sec. VII contains a summary of results and suggestions for useful experiments.

II. P MATRIX and K MATRIX

While the physics that motivates the P-matrix and K-matrix formalisms are quite different, for the purposes of this paper they are similar. In this section we will show how, if one is far from the location of a four-quark "primitive," they may be similarly parametrized and hence lead to the similar physics.

The physics which we wish to model in this paper is clearly most appropriate for the P matrix. Jaffe and Low[5] have argued that the P matrix describes a system which is artificially confined and not allowed to couple to open channels. This is just the approximation in which the quark model is always used. $Q\bar{Q}$, GG, and $QQ\bar{Q}\bar{Q}$ states are prepared without considering that such states in general will decay into lighter systems. In particular the $QQ\bar{Q}\bar{Q}$ states can simply fall apart into two $Q\bar{Q}$ systems; they were never confined in the first place. However, even $Q\bar{Q}$ and GG states couple to open channels, although less trivially. When the P matrix is used to generate the S matrix, spurious poles, such as is often the case with $QQ\bar{Q}\bar{Q}$ states, do not appear as poles in the S matrix, while true resonances do.

The definition of the P matrix in the lth partial wave is[5]

$$\bar{P}_l=[e_l^{-\prime}(x)-e_l^{+\prime}(x)S_l][e_l^-(x)-e_l^+(x)S_l]^{-1} \tag{1}$$

or

$$S_l=[e_l^{+\prime}(x)-\bar{P}_l e_l^+(x)]^{-1}[e_l^{-\prime}(x)-\bar{P}e_l^-(x)] \tag{2}$$

with $\bar{P}_l$ a dimensionless reduced P matrix

$$\bar{P}_l = P_l / x \tag{3}$$

and

$$x = kb \tag{4}$$

with b being the "matching radius" of Jaffe and Low (we will use $b = 7\ \text{GeV}^{-1}$). $e_l^{\pm}(x)$ are related to Hankel functions; specifically

$$\begin{aligned} e_0^{\pm}(x) &= e^{\pm ix} , \\ e_1^{\pm}(x) &= \pm ie^{\pm ix}(1 \pm 1/x) , \\ e_2^{\pm}(x) &= -e^{\pm ix}(3/x^2 \mp 3i/x + 1) . \end{aligned} \tag{5}$$

A smoothly varying phase shift corresponds to a tower of poles in the P matrix. In particular for $S = 1$, one obtains

$$\bar{P}_l^0 = \frac{e_l^{-\prime} - e_l^{+\prime}}{e_l^- - e_l^+} , \tag{6}$$

which has poles whenever $\text{Im} e_l^- = 0$. When studying the effect of other poles, it is important to put in these compensation poles in addition in order to obtain a resonance on a smooth background.

In contrast the K matrix is related to the S matrix by

$$S_l = (1 + i\bar{K}_l)(1 - i\bar{K}_l)^{-1} \tag{7}$$

with

$$\bar{K} = k^{1/2} K k^{1/2} . \tag{8}$$

Both K and P are real.

When one is near a compensation energy, the P-matrix description has different properties than the K matrix. The lowest such energy occurs at $x = \pi$ for $l = 0$, $x = 4.49$ for $l = 1$, and $x = 5.67$ for $l = 2$, corresponding to center-of-mass energies ($E = 2x/b$), $E = 0.90$ GeV, 1.28 GeV, and 1.65 GeV, respectively (for $b = 7\ \text{GeV}^{-1}$). Our work for $l = 2$ will be below this energy. To keep $S = 1$ in the absence of either the $Q\bar{Q}$ or GG primitives, one can write (dropping l indices)

$$\bar{P} = \bar{P}^0 + Q , \tag{9}$$

so that

$$\begin{aligned} S &= \{ (e^{+\prime} - \bar{P}^0 e^+)[1 - (e^{+\prime} - \bar{P}^0 e^+)^{-1} Q e^+] \}^{-1} (e^{-\prime} - \bar{P}^0 e^-)[1 - (e^{-\prime} - \bar{P}^0 e^-)^{-1} Q e^-] \\ &= [1 + (e^{+\prime} - \bar{P}^0 e^+)^{-1} Q e^+]^{-1} (e^{+\prime} - \bar{P}^0 e^+)^{-1} (e^{-\prime} - \bar{P}^0 e^-)[1 - (e^{-\prime} - \bar{P}^0 e^-)^{-1} Q e^-] \\ &= \left[1 - \frac{\text{Im} e^-}{\text{Im}(e^- e^{+\prime})} Q e^+ \right]^{-1} \left[1 - \frac{\text{Im} e^-}{\text{Im}(e^- e^{+\prime})} Q e^- \right] . \end{aligned} \tag{10}$$

If

$$e^+ \equiv i\rho e^{-i\beta} \tag{11}$$

with ρ real, then the kinematic factors can be absorbed into Q by defining

$$\bar{Q} = \frac{\text{Im} e^-}{\text{Im}(e^- e^+)} Q \rho , \tag{12}$$

such that

$$S = (1 - i\bar{Q} e^{-i\beta})^{-1} (1 + i\bar{Q} e^{i\beta}) . \tag{13}$$

This is similar to the K-matrix expression. However, instead of the resonance energy corresponding to $\delta = \pi/2$, it will occur near $\delta = \pi/2 + \beta$, appearing similar to a background phase shift plus resonance. In our region of interest $\beta \approx 45°$. This will lead to an apparent decrease in the mass of the resonance.

III. THE "EXPERIMENT"

The physics of a two-particle mixture is not well appreciated. To explore it we can perform a model "experiment" by constructing the S matrix on a computer and studying the resulting properties. The goal is to study the range of physical properties which one should realistically expect to observe. By use of the K-matrix or P-matrix formalisms, one can obtain a unitary S matrix, a feature which is not obtained in the simpler mass-matrix mixing schemes. In fact we will find that a two-particle S matrix is considerably more subtle than naive mixing schemes would lead us to believe.

We will include two "bare" poles, i.e., two states before any mixing occurs. These should be thought of as the $Q\bar{Q}$ and GG states, but they will be labeled simply 1 ($\approx Q\bar{Q}$) and 2 ($\approx GG$). There will be four open decay channels, which will be labeled $\pi\pi$, $K\bar{K}$, $Q\bar{Q}$, and GG. The first two are obvious. The latter two are meant to represent other open channels for the two states, and the labeling only indicates that state 1 has open channels primarily into the mode $Q\bar{Q}$, while state 2 uses GG. In principle these channels could be two-body modes like $\eta\eta$ or multiparticle final states. However, in the formalism they will be treated as two-body channels. The "experiment" consists of studying the 4×4 S matrix describing the 16 reactions of all possible combinations of

$$(\pi\pi,K\bar{K},Q\bar{Q},GG)\to(\pi\pi,K\bar{K},Q\bar{Q},GG)$$

mediated by the two s-channel poles, 1 and 2. By CPT, this amounts to ten independent reactions.

The imput consists of the masses of 1 and 2 and their coupling to each of the four decay modes, a total of ten numbers. The coupling can be given in terms of a coupling V_α^i where $i=1,2$ and $\alpha=\pi\pi$, $K\bar{K},Q\bar{Q}$, and GG. These can be normalized to include any phase-space factors, such that the total decay rate would be

$$\Gamma(i\to\alpha)=|V_\alpha^{(i)}|^2 \qquad (14)$$

if the other pole were not nearby. The inclusion of phase space is important when comparing the $\pi\pi$ and $K\bar{K}$ modes. In presenting the results, we will use vectors

$$\Gamma^{(i)}\equiv(|V_{\pi\pi}^{(i)}|^2,|V_{K\bar{K}}^{(i)}|^2,|V_{Q\bar{Q}}^{(i)}|^2,|V_{GG}^{(i)}|^2) \qquad (15)$$

to give the couplings. The relative phases of the $V_\alpha^{(i)}$ is observable. However, we have studied the effects of the phases and not found any significant effect worth separate mention. All $V_a^{(i)}$ below will be chosen positive.

With the normalization, the K matrix is simply

$$K_{ij}=\frac{V_i^{(1)}V_j^{(1)}}{E-M_1}+\frac{V_i^{(2)}V_j^{(2)}}{E-M_2}, \qquad (16)$$

where $i,j=\pi\pi$, $K\bar{K}$, $Q\bar{Q}$, and GG. In the P-matrix formalism we will choose the P matrix to be that "compensation" matrix P_0, which produces zero phase shift in the absence of any extra pole, plus a background $\bar{Q}$ as expressed in Eq. 9. In particular we will choose

$$\bar{Q}_{ij}=K_{ij}\ . \qquad (17)$$

Note that there is no reason to attempt to diagonalize a mass matrix to obtain the mass eigenstates. This procedure is not meaningful in the above formlism, where the only observables are the scattering-matrix elements. These are correctly given without any diagonalization.

In the next section we will vary the masses of the two poles in order to study the decoupling mechanism suggested by DJL. We will find that the mechanism will tend to hide the second state when the dominant production and decay channel is used. However, decoupling is not total. We then, in Sec. V, use a variety of couplings in order to explore the possible signals of a two-channel system. The couplings of the expected states are not known. We therefore need to consider several trial couplings, spanning a physically interesting range, in order to learn about how the two-state system *may* manifest itself.

IV. THE DECOUPLING MECHANISM

In DJL it was noted that, in a mass-mixing formalism, it is natural for two nearby states to mix in such a way that they essentially decouple from the dominant channel. This mechanism is crucial for the suggestion that there may be a second state (glueball) hiding behind the $f(1270)$, as the dominant $\pi\pi$ mode shows no hint at all of two states. If two exists, only one may be strongly couple to $\pi\pi$. The appropriate mixing happens automatically. The mass matrix is

$$H=M+i\Gamma\ , \qquad (18)$$

with M and Γ Hermitian. One can choose a basis where M is diagonal, such that the remaining mixing is governed by the width matrix

$$\Gamma_{ij}=2\pi\sum_I\langle i\,|\,V\,|\,I\rangle\langle I\,|\,V\,|\,j\rangle\delta(E_i-E_j)\ . \qquad (19)$$

If the decays are dominated by a single two-body mode (such as $\pi\pi$), then Γ_{ij} has a zero eigenvalue, since $|\Gamma_{12}|^2=\Gamma_{11}\Gamma_{22}$. This in turn implies that as $m_1\to m_2$ that one eigenvalue of H has zero width, i.e., no coupling to the dominant channel. Examples of this mechanism are given in Refs. 1 and 2. The dominance of $\pi\pi$ is assured in the 2^{++} system by D-wave phase space, which suppresses $K\bar{K}$ relative to $\pi\pi$ by a factor of 10 even if the $\pi\pi$ and $K\bar{K}$ couplings were equal.

This mechanism can be studied here without resorting to mass-matrix diagonalization. One simply feeds masses and couplings of "bare" poles into a K or P matrix and studies $\pi\pi$ scattering to see if the cross section looks like a single or double pole. We have done this for many sets of parameters, with typical results to be described below. The decoupling mechanism works very well, and is sometimes operable in other channels as well.

In Figs. 1 and 2 the couplings are kept constant at

$$\Gamma^{(1)}=(160,10,40,0)\ \text{MeV}\ ,$$
$$\Gamma^{(2)}=(63,5,0,40)\ \text{MeV}\ , \tag{20}$$

while the mass difference is varied from zero to 600 MeV with the average mass being held fixed at $\bar{M}=1.28$ GeV. The resulting cross sections for all the $\pi\pi$ channels and selected other modes shown for the K and P matrices in Figs. 1 and 2.

One can easily see that the desired effect occurs. In $\pi\pi\rightarrow\pi\pi$ one sees only one resonance when the states are very close. As the states separate slight deviations from a standard Breit-Wigner shape develop and become progressively larger until at large separation two separate states are seen. A determination of the phase shifts also supports this picture. In Fig. 3, the phase shifts for $\pi\pi\rightarrow\pi\pi$ are give for a couple of mass differences. In both cases, the phase shift goes through 90° only once, as is appropriate for a single resonance. If the masses were separated further, the phase shift would eventually go through 90° and 270° as it passes the poles.[2]

It is somewhat unexpected, however, to notice that when the two poles are close together even nondominant channels appear to indicate only a single resonance. This is especially striking when the two states are on top of each other. Instead of interference effects, the signature that two states are present is that the peak has different widths in different channels.

It is also instructive to use these results to compare the P-matrix and K-matrix formalisms. One can see that, when the input is treated in the manner of this paper, the P and K matrices lead to very similar physical results. The resonance peaks are shifted somewhat with respect to the input masses by the P matrix. This is an interesting phenomena which may be important when comparing experimental results to quark-model calculations.

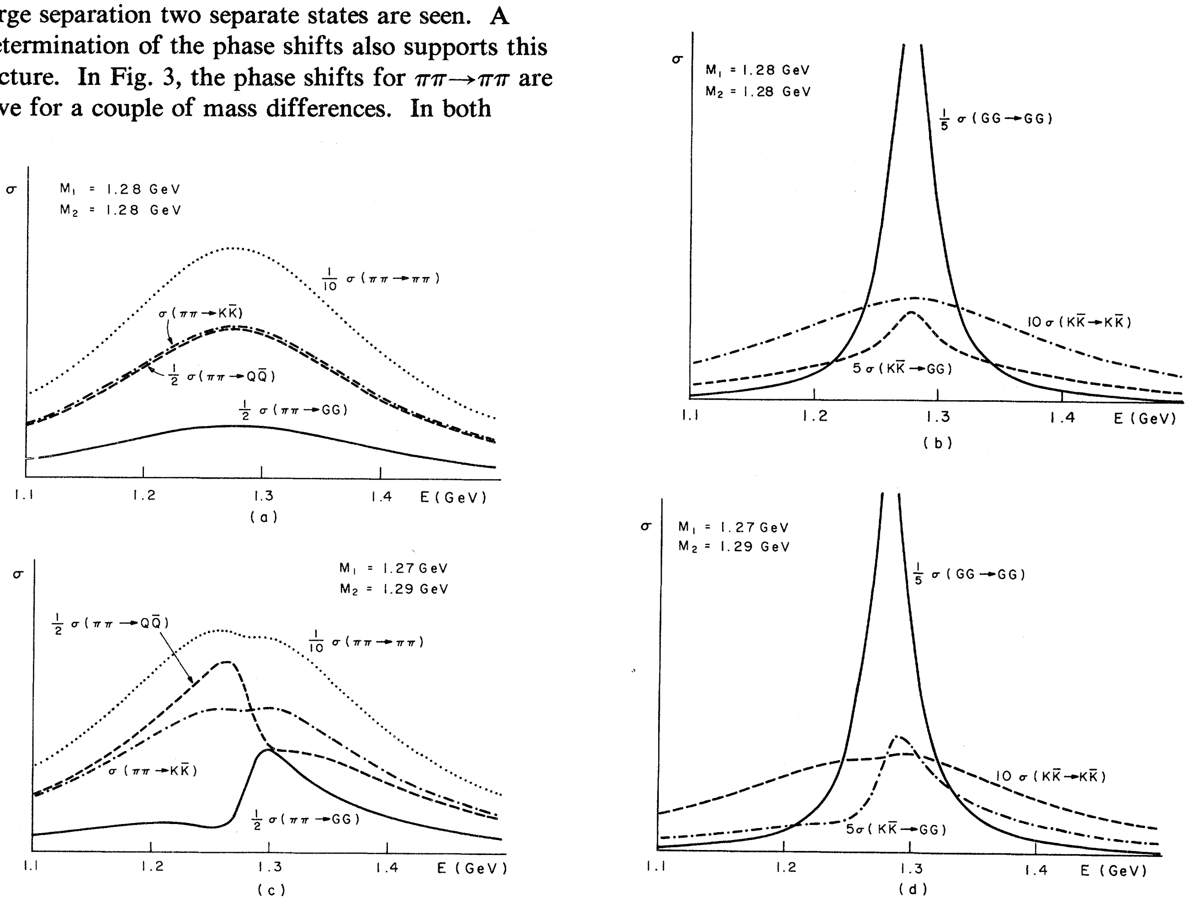

FIG. 1. K-matrix results for scattering cross sections using the couplings $\Gamma^{(1)}=(160,10,40,0)$ MeV and $\Gamma^{(2)}=(63,5,0,40)$ MeV, for a variety of masses.

In addition, the cross sections are forced down at higher energies by the P matrix, as when $\beta \rightarrow \pi/2$ (at the compensation energy) $S=1$ automatically.

V. VISIBLE EFFECTS IN A TWO-STATE SYSTEM

The main motivation for this paper is to study the possible means for confirming or refuting the hypothesis of an f-glueball mixture. As we have little indication of the strength of the various couplings in the problem, we must study a range of possible couplings. In this section we run our "experiment" with a variety of imput parameters and attempt to make sense of the results. In the last section we varied the masses of the states. Here we will keep the masses fixed (at 1.27 and 1.29 GeV) and study the role of coupling strengths. The variety of effects uncovered in this way is no smaller than if we had let both the masses and couplings change.

To simplify the presentation we will give the results in terms of the cross section for various processes. After studying several of the parameter sets it became clear that it was more difficult to obtain a feel for the trend of the results if one studied the phase shifts directly. It was more enlightening to convert this information into cross sections in which the structure near resonance was visible. We are not aware of any information essential for our purposes in this paper which is lost by such a presentation

In addition, all of the results presented here will be given using the P-matrix formalism. In many cases both K- and P-matrix techniques were studied, for the same input. The differences between the two methods are small enough that, for our purposes, to present both would be redundant.

The most common effect is a distortion of the Breit-Wigner shape. Figures 2(d) and 2(e) of the last section illustrates this property in several ways. Then in $\pi\pi \rightarrow \pi\pi$, for example, there is a slight doubling of the peak which could be visible in a sensitive enough experiment. More dramatic are the lopsided peaks in $\pi\pi \rightarrow Q\bar{Q}$ and $\pi\pi \rightarrow GG$.

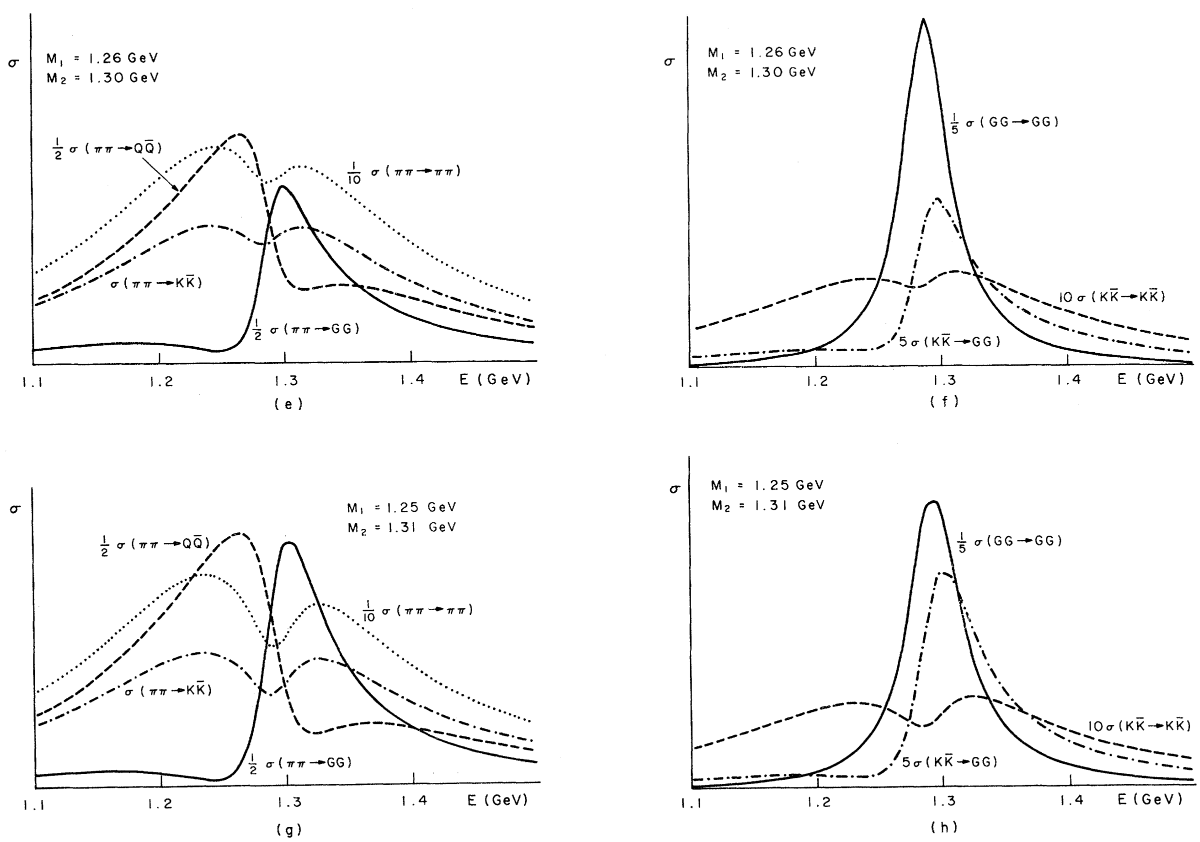

FIG. 1. (*Continued.*)

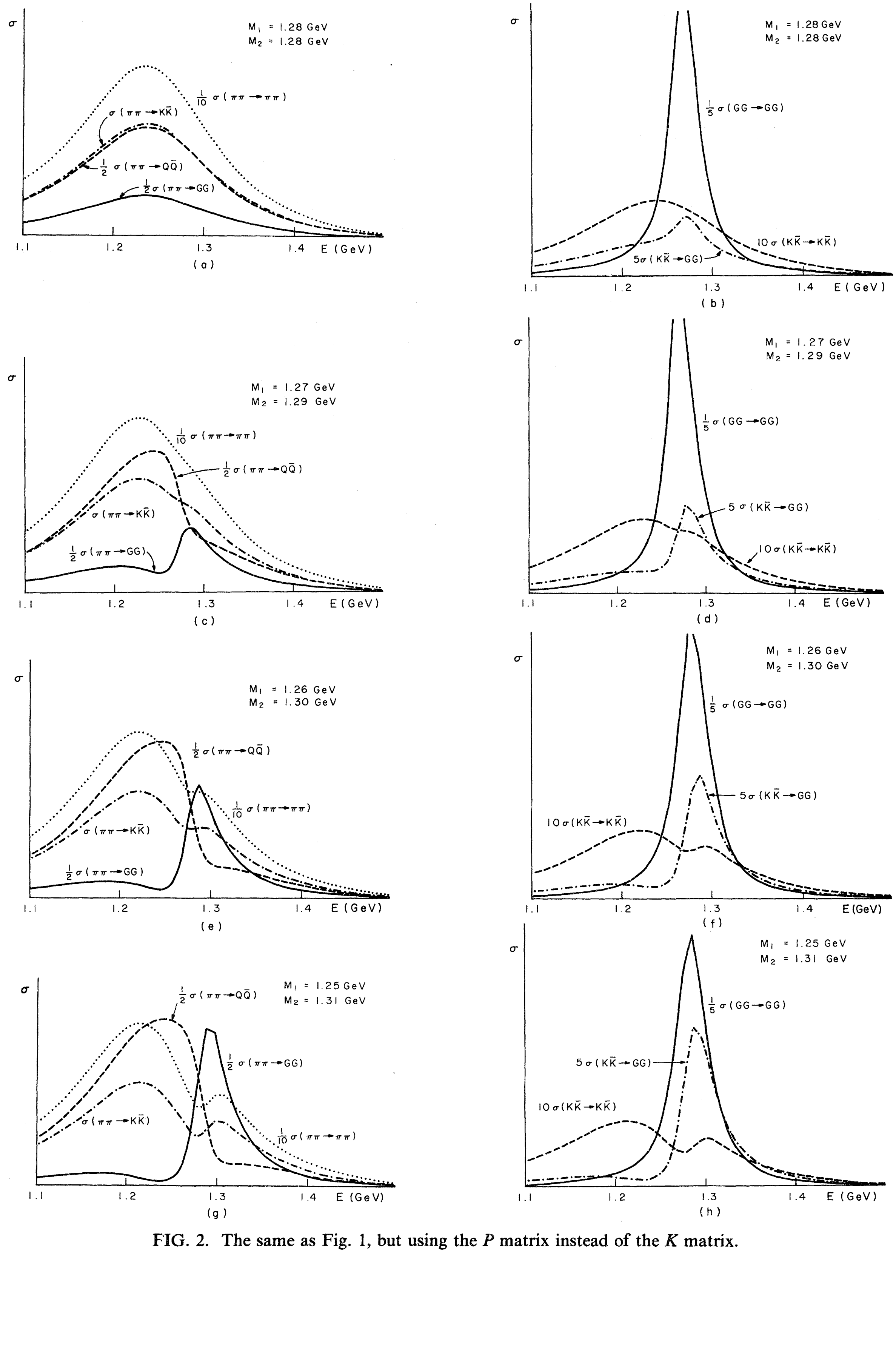

FIG. 2. The same as Fig. 1, but using the P matrix instead of the K matrix.

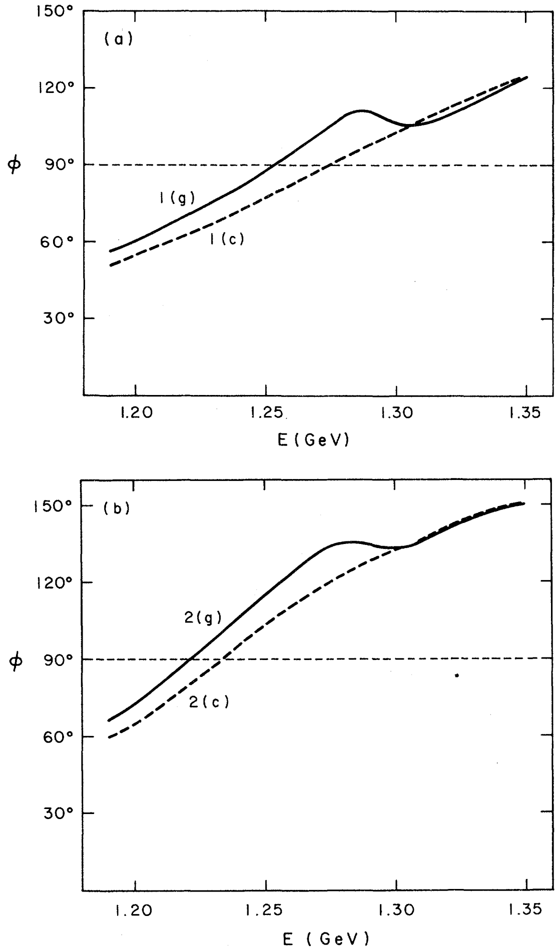

FIG. 3. The $\pi\pi\rightarrow\pi\pi$ phase shift corresponding to the cross sections given in (a) Figs. 1(c) and 1(g), and (b) Figs. 2(c) and 2(g).

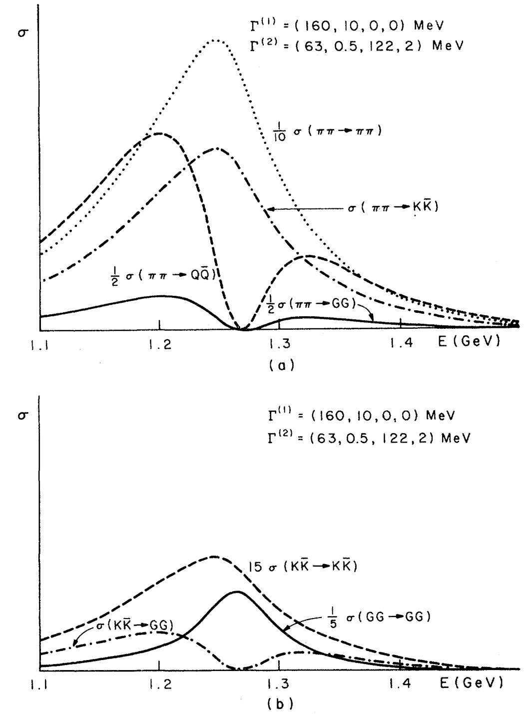

FIG. 4. P-matrix results using $M_1=1.27$ GeV, $M_2=1.29$ GeV, and the couplings $\Gamma^{(1)}=(160,10,0,0)$ MeV and $\Gamma^{(2)}=(63,0.5,122,2)$ MeV.

They would show up as narrower resonances displaced from the standard mass value. In addition some of the channels which do not couple to $\pi\pi$ show a single narrow peak. However, $K\bar{K}\rightarrow K\bar{K}$ retains the general shape of the $\pi\pi\rightarrow\pi\pi$ mode.

Figure 4 shows a radically different picture. Here the various reactions show one of only two types of behavior. In $\pi\pi\rightarrow\pi\pi$, $\pi\pi\rightarrow K\bar{K}$, $K\bar{K}\rightarrow K\bar{K}$, and $GG\rightarrow GG$, a similar undistorted Breit-Wigner shape is seen. However, in $\pi\pi\rightarrow GG$, $\pi\pi\rightarrow Q\bar{Q}$, and $GG\rightarrow K\bar{K}$, very dramatic double-peak structures are seen, with the minimum being near where the central mass of the resonance should be. Such a signal, if seen, would be hard to mistake.

The most uncertain aspect of this system is clearly the gluonic state's couplings. We can estimate the bare $Q\bar{Q}$ state's couplings by using SU(3) or SU(6) applied to the 2^{++} nonet as a whole. To acquire a more systematic, if not necessarily more reliable, look at our problem, we can fix the one set of couplings at this estimate and vary the second set.

Rosner[4] has given the $\pi\pi$ and $K\bar{K}$ couplings for a nonstrange-quark state in the 2^{++} nonet, using SU(3) and SU(6). We will use his estimates for these and add 10 MeV of width in the $Q\bar{Q}$ channel, and none in the GG channel, so that our width vector is

$$\Gamma^{(1)}=(115,3,10,0)\text{ MeV}\ . \tag{21}$$

The other width vector is varied from

$$\Gamma^{(2)}=(10,1,0,40) \tag{22}$$

to

$$\Gamma^{(2)}=(90,9,0,90) \tag{23}$$

with the $Q\bar{Q}$ channel fixed at zero coupling. In addition, the couplings to $K\bar{K}$ and $\pi\pi$ are set equal aside from D-wave phase-space suppression of the $K\bar{K}$, as might be appropriate for a gluonic resonance. The results are given in Figs. 5 and 6. In

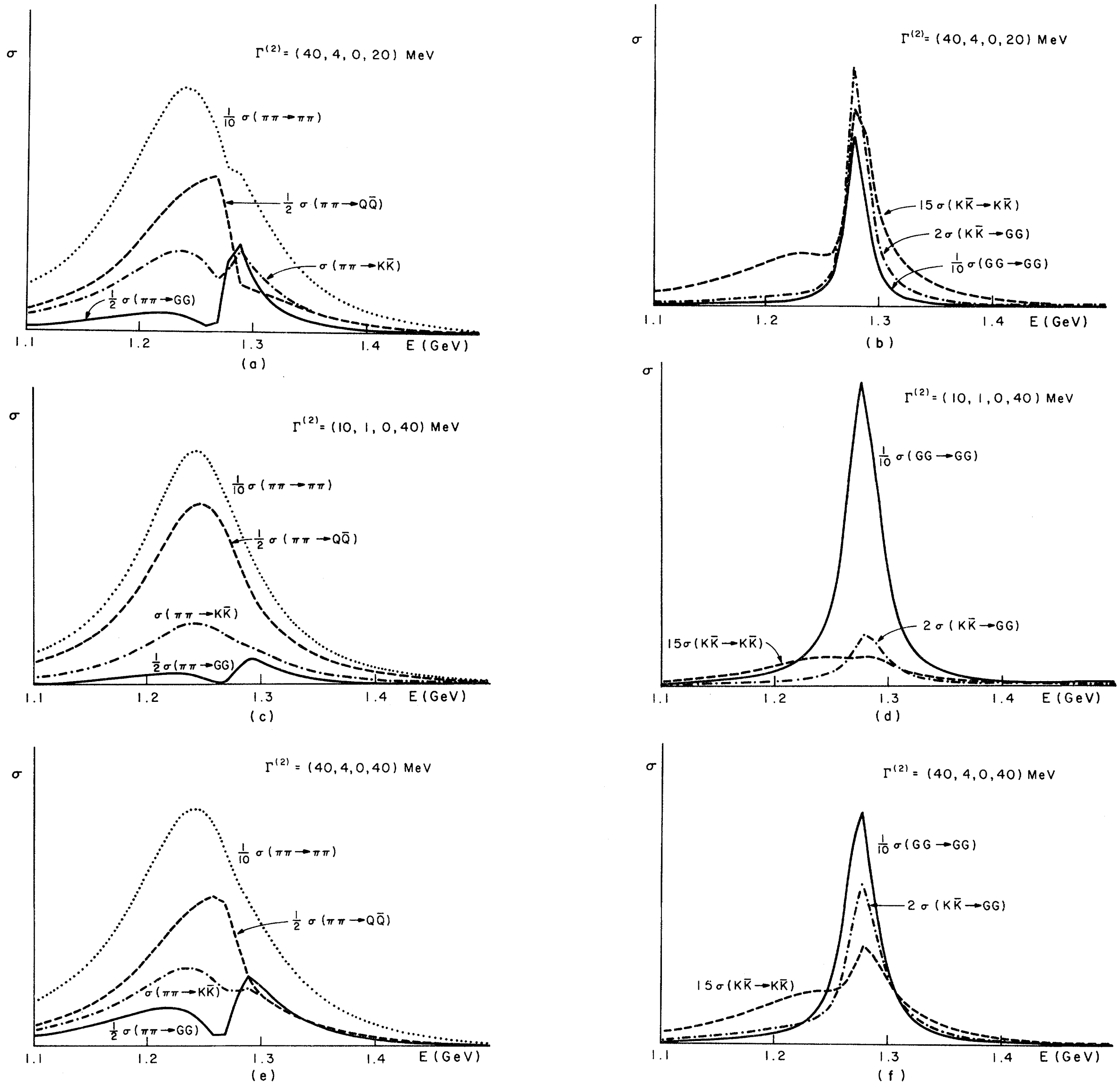

FIG. 5. Cross sections resulting from a P-matrix analysis using $M_1=1.27$ GeV, $M_2=1.29$ GeV, $\Gamma^{(1)}=(115,3,10,0)$ MeV, and a variable $\Gamma^{(2)}$.

all cases there are observable effects in some of the less dominant channels. However, no single effect needs be present all of the time.

There is interesting and usual physics in the branching ratios. One of the hallmarks of a single resonance is that it decays into a set of final states with fixed branching ratios. However, a two-pole system may have branching ratios which are process dependent. For example, the resonance seen in $\pi\pi\to X$ may populate the various final states X in different ratios than the resonance seen in $K\bar{K}\to X$, because the two resonances are different combinations of the bare poles.

In general we can describe the "effective" branching ratios in some resonance region by specifying the initial state, integrating the reaction cross section over the resonance and normalizing

$$B(I,F)=\frac{\sigma(I\to F)}{\sum_X \sigma(I\to X)}\ , \tag{24}$$

where I,F,X are the channels $\pi\pi, K\bar{K}$, etc. If there is only a single resonance, then $B(I,F)$ is independent of I. We find substantial dependence on I of

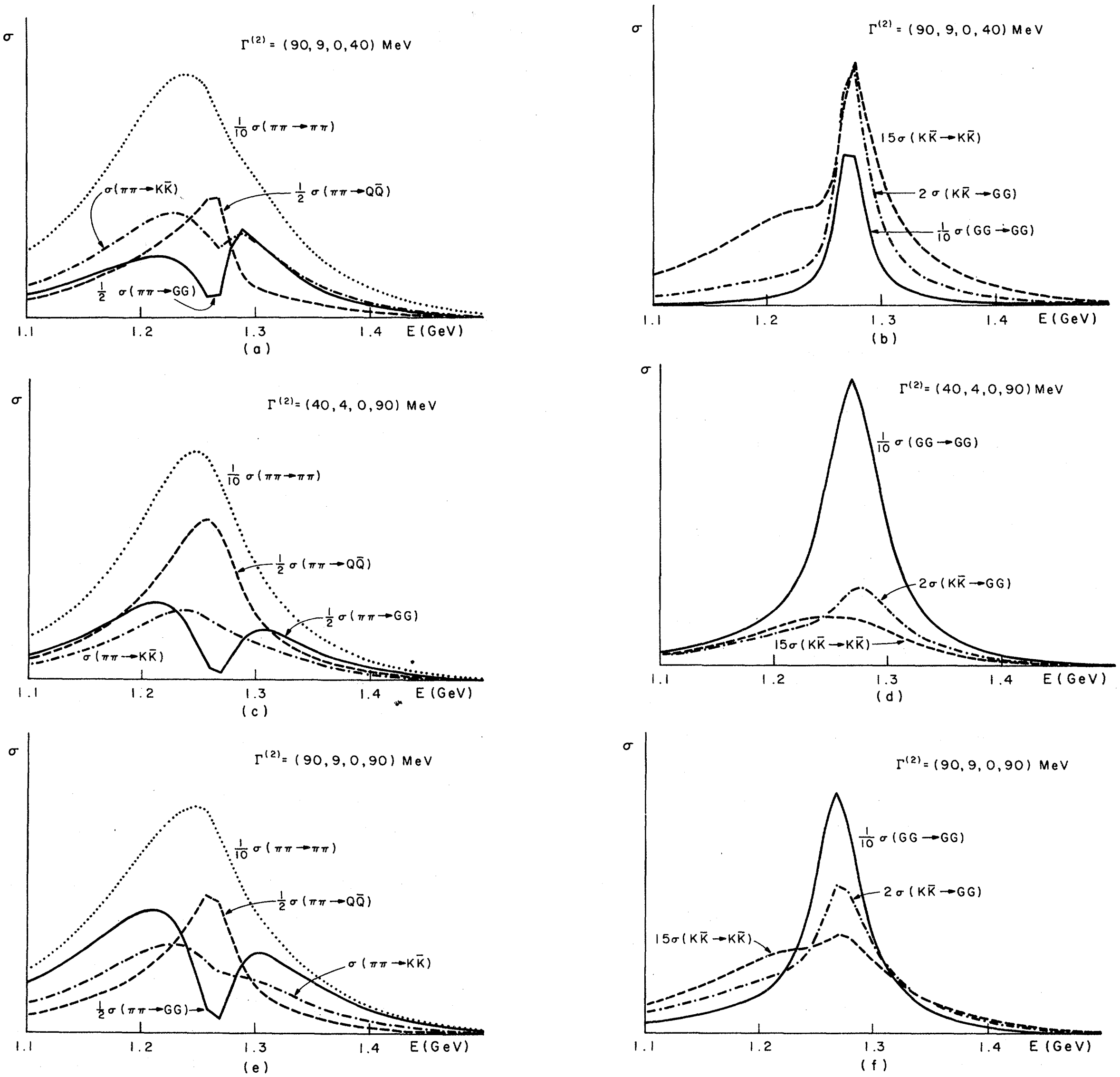

FIG. 6. Cross sections resulting from a P-matrix analysis using $M_1=1.27$ GeV, $M_2=1.29$ GeV, $\Gamma^{(1)}=(115,3,10,0)$ MeV and a variable $\Gamma^{(2)}$.

$B(I,F)$ in our "experiments." For example, a not untypical set of values is given in Table I, corresponding to the cross section shown in Fig. 5(e). Experimentally, the most accessible of the branching fractions is the ratio of $K\bar{K}$ to $\pi\pi$ in the final state. Let us define

$$\rho_I^{K/\pi}=\frac{B(I\to K\bar{K})}{B(I\to\pi\pi)}\ . \qquad (25)$$

Typically this shows variations of about 50%, but sometimes there is a larger fluctuation such as in the last row of Table I. This may be the most useful signal, especially if an experiment has poor mass resolution.

VI. COMPARISON WITH RESONANCE-MIXING SCHEMES

Rosner has studied the same system using a standard method of describing two states which are linear combinations of two bare states, with the mixing described by a single mixing angle. DJL's original suggestion was motivated in this framework. It is the conventional intuition for a two-state mixture. While some of the effects found in such a scheme agree with those presented above, others do not. In this section a comparison of the two methods is undertaken.

One of the more counterintuitive results of the

TABLE I. Branching ratios $B(I \to F)$ for the parameters of Fig. 5(e). The last column is the K-to-π ratio.

$I \backslash F$	$\pi\pi$	$K\bar{K}$	$Q\bar{Q}$	GG	$\rho^{K/\pi}$
$\pi\pi$	0.84	0.03	0.09	0.04	0.04
$K\bar{K}$	0.64	0.04	0.04	0.28	0.06
$Q\bar{Q}$	0.57	0.01	0.19	0.23	0.02
GG	0.10	0.03	0.09	0.78	0.30

present paper is that resonances can merge together and overlap to form what appears to be a single resonance in several (but not all) channels if the spacing of the poles is small compared to their widths. This appears to be governed by unitarity. Rescattering, or reresonating, effects determine the shape of many channels and it is not possible to think of one channel in isolation from the others. Such an effect is missing in the mixing schemes. The simplicity of mixing schemes is best suited for resonances which are somewhat far apart compared to their widths or where at least one state is very narrow.

A related point is that it does not appear permissible in our framework to think of two orthogonal physical states. For some of the couplings there do in fact appear to be two more or less superposed states, but for others (such as Fig. 4) there appears no reason to describe what is seen as two states. In such a case, we know of no operational definition of "physical." The bare states and their couplings are in principle the inputs from theory, and the S-matrix elements are the experimental observables. What is of interest is not to find a second physical state but to find the effect of a second bare state (or "primitive").

Finally, we have found that we can shift the resonance mass around considerably without moving the bare states. This may account for part of the A_2-f mass difference in a fashion that is not modeled by the mixing schemes.

The conventional mixing formalism has the advantage that it is easy to handle and conceptually simple. However, in detail, it may not provide an accurate description of the physics of two highly overlapping resonances.

V. CONCLUSIONS

We have provided a phenomenological study of some of the aspects of a two-pole system when the poles are located close to each other on a scale set by their widths. While some of the results are general, our primary motivation was to learn about signals to detect a $Q\bar{Q}$ and glueball mixture near the $f(1270)$ resonance. There has been no single feature found which by its absence could rule out this effect. However, there are many signals which could indicate a two-pole mixture. These include (1) distorted or split resonace structure, (2) a process-dependent width of a resonace, (3) a shift in the apparent mass of a resonance, and (4) process-dependent branching ratios. In general several of these effects are present when one can study all channels, and in no case studied was there a lack of a signal of the two-state nature.

What is needed most experimentally is new ways to study this system. In the past most experiments have used a pion beam and worked at low transverse momentum, a situtation probably dominated by pion exchange. The use of other ways to produce the f would be expected to be more sensitive to the above features. Perhaps the best from the point of new physics would be a careful study of f production in J/ψ radiative decays. This is in effect an initial two-gluon channel. We would expect the K/π ratio to be different from the pion-produced f and perhaps some unusual structure would exist in the shape of the resonance. This would probably be especially visible in the $K\bar{K}$ or $\eta\eta$ final states; however, the $\pi\pi$ final state may also be useful. Kaon beams could be very useful for studying the system, especially if unusual final states can be studied. The reaction $K^- p \to f\Lambda \to \eta\eta\Lambda$ (via K^+ exchange) would be excellent. Finally, even pion-initiated experiments can be useful if one gets away from the pion pole in the exchange channel by going to high transverse momentum. If experiments such as these do not see any unusual signals, the hypothesis of an f-glueball mixture would be strongly disfavored.

ACKNOWLEDGMENT

I would like to thank R. L. Jaffe for useful conversations. This work was supported in part by the National Science Foundation.

[1]J. F. Donoghue, K. Johnson, and B. A. Li, Phys. Lett. 99B, 416 (1981).

[2]A review of bag-model glueballs and preliminary results of the present paper were given by J. F. Donoghue, in *Gauge Theories, Massive Neutrinos, and Proton Decay,* proceedings of Orbis Scientiae 1981, edited by A. Perlmutter and L. Scott (Plenum, New York, 1981), p. 85.

[3]K. Johnson and C. Thorn, Phys. Rev. D 13, 1934 (1976); C. Thorn (unpublished).

[4]J. L. Rosner, Phys. Rev. D 24, 1347 (1981).

[5]R. L. Jaffe and F. E. Low, Phys. Rev. D 19, 2105 (1979).

[6]See, for example, H. Pilkuhn, *The Interactions of Hadrons* (North-Holland, Amsterdam, 1967).