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Baryon mass splittings in a quark model with broken symmetry

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A phenomenological model for spin-spin mass splittings in baryons is proposed in a quark model with broken symmetry. The quark-quark interaction which can distinguish between the even-wave quark pairs of spin 0 or 1 is shown to be responsible for a pattern of mass splittings in baryons in fair agreement with experimental data.

The main features of baryon spectroscopy have been explained in recent literature[1] as due to nonrelativistic medium-mass quarks moving in a confinement potential perturbed by short-range quantum-chromodynamics (QCD) effects.[2] In particular, the spin-spin δ-function color hyperfine interaction is shown to be mainly responsible for the mass splittings in baryons in the same multiplet. These calculations are essentially based on the $F\cdot FS\cdot S\,(m_i m_j)^{-1}$ coupling in quark-gluon exchange with color, and predict different patterns of spin-spin splittings for $(\Sigma\Lambda\Sigma^*)_s$ strange and $(\Sigma\Lambda\Sigma^*)_c$ charmed systems.[3]

However, it may be worthwhile to look for an alternative origin of mass splittings based on a symmetry-breaking potential in the confining region and evaluated perturbatively within the framework of effective Dalitz[4]-type classifications of (qqq) baryons. For this purpose, we consider the baryons as quark-diquark structures, with only even-wave diquarks in antisymmetric combinations in color, spin, and flavor space allowed by the Pauli antisymmetrization principle. These are $\mathrm{SU}(3)_{\mathrm{color}}=\bar{3}$ [spin 0, $\mathrm{SU}(3)_{\mathrm{flavor}}=\bar{3}$] or $\mathrm{SU}(3)_{\mathrm{color}}=\bar{3}$ [spin 1, $\mathrm{SU}(3)_{\mathrm{flavor}}=6$]. The pairwise potential V_{ij} is considered as different for the spin-1 and spin-0 diquarks,

$$V_{ij}=P_c^{(-)}(ij)[P_\sigma^{(+)}(ij)P_u^{(+)}(ij)+xP_\sigma^{(-)}(ij)P_u^{(-)}(ij)]V_0\,, \tag{1}$$

where V_0 is a constant and x is the symmetry-breaking parameter. The projection operator $P^{(\pm)}(ij)=\frac{1}{2}[1\pm(i\to j)]$ for states symmetric (or antisymmetric) under the exchange of color (c), spin (σ), and flavor (u) of i,j quarks operates on the (qqq) baryon wave functions of appropriate permutation symmetry. A similar potential had been considered earlier in an even-wave quark model (EWM).[5] We replace the spatial symmetric operator $P_x^{(+)}(ij)$ in EWM by the color-antisymmetric operator $P_c^{(-)}(ij)$ thus maintaining the Pauli antisymmetrization rule. In addition, the (qqq) wave functions for the $(\Sigma\Lambda\Sigma^*)_i$ system, $i=s$ or c, are considered in a symmetry broken at $\mathrm{SU}(4)_{\mathrm{flavor}}$ level, as discussed in Ref. 1, due to unequal quark masses.

To evaluate the matrix elements of $\sum_{i=1}^{3}\sum_{j=1}^{3}V_{ij}$ between appropriate (qqq) states, we consider at first the nonstrange (N,Δ) baryons. These are classified according to the $(56,21^+)$ and $(70,21+1^-)$ representations of $\mathrm{SU}(6)\otimes\mathrm{O}(3)$ in the quark-diquark model[6] based on only excitations of the $\vec{\lambda}=1/\sqrt{6}(-2\vec{r}_1+\vec{r}_2+\vec{r}_3)$ coordinate in the (qqq) system. The complete (qqq) wave functions are

$$\tilde{\Psi}_{56}^{d,q}(2l)=\psi_{2l}^s\left[\frac{1}{\sqrt{2}}(\chi^\rho\Phi^\rho+\chi^\lambda\Phi^\lambda);\,\chi^s\Phi^s\right], \tag{2}$$

$$\tilde{\Psi}_{70}^{d}(2l+1)=\tfrac{1}{2}[\Psi_{2l+1}^\rho(\chi^\rho\Phi^\lambda+\chi^\lambda\phi^\rho)+\Psi_{2l+1}^\lambda(\chi^\rho\phi^\rho-\chi^\lambda\phi^\lambda)]\,, \tag{3}$$

$$\tilde{\Psi}_{70}^{q}(2l+1)=\frac{1}{\sqrt{2}}\chi^s(\Psi_{2l+1}^\rho\Phi^\rho+\Psi_{2l+1}^\lambda\Phi^\lambda)\,, \tag{4}$$

$$\tilde{\Psi}_{70}^{d}(2l+1)=\frac{1}{\sqrt{2}}\Phi^s(\Psi_{2l+1}^\rho\chi^\rho+\Psi_{2l+1}^\lambda\chi^\lambda)\,. \tag{5}$$

Here, χ,Φ are the spin and unitary-spin wave functions, the superscripts ρ,λ denote mixed symmetry while s indicates total symmetry in three quarks.

For $(56,21^+)$ baryons,

$$\Delta_q-Nd=\tfrac{3}{2}(1-x)\beta \tag{6}$$

while for $(70,21+1^-)$ baryons,

$$\Delta_d-Nd=N_q-Nd=\tfrac{3}{4}(1-x)\beta\,. \tag{7}$$

For a constant potential V_0, the overlap factor β remains constant and $\Delta_q-Nd=300$ MeV can be compared with $\Delta_d-Nd=N_q-Nd=150$ MeV for $(70,1^-)$ states.

For $(70,3^-)$, large spin-orbit effects exist which make impossible any meaningful comparison.

For the $(\Sigma\Lambda\Sigma^*)_i$ systems ($i=s$ or c), we consider an (iud) basis and follow Ref. 1, so that isospin wave functions are

$$\Phi_\Lambda=\frac{i}{\sqrt{2}}(ud-du)\,,\quad \phi_\Sigma=\Phi_{\Sigma^*}=\frac{i}{\sqrt{2}}(ud+du)\,. \tag{8}$$

For $(\Xi,\Xi^*)_i$, which are similar to $(\Sigma,\Sigma^*)_i$, the basis is (dii) or (uii). In the usual notation for SU(3) functions, let Φ^a, Φ^ρ, Φ^λ, and Φ^s denote the $\underline{1}$, $\underline{8}(\rho)$, $\underline{8}(\lambda)$, and $\underline{10}$ states, so that

$$\Phi_\Lambda = \frac{1}{\sqrt{3}}(\Phi^a - \sqrt{2}\,\Phi^\rho)\,, \quad \Phi_\Sigma = \Phi_{\Sigma^*} = \frac{1}{\sqrt{3}}(\Phi^s + \sqrt{2}\,\Phi_\lambda)\,. \tag{9}$$

The complete wave functions are given by

$$\tilde{\Psi}_\Lambda = \Psi^\lambda \chi^\rho \Phi_\Lambda\,, \quad \tilde{\Psi}_\Sigma = \Psi^\lambda \chi^\lambda \Phi_\Sigma\,, \quad \tilde{\Psi}_{\Sigma^*} = \Psi^\lambda \chi^s \Phi_{\Sigma^*}\,,$$
$$\tilde{\Psi}_\Xi = \Psi^\lambda \chi^\lambda \Phi_\Xi\,, \quad \tilde{\Psi}_{\Xi^*} = \Psi^\lambda \chi^s \Phi_{\Xi^*}\,. \tag{10}$$

The Ψ^λ spatial wave functions are symmetric in (u,d) quarks and include excitations of $\vec{\lambda}$ coordinates. For these states $(\Lambda_d, \Sigma_d, \Sigma_q^*)_i$, we obtain

$$\Sigma_i - \Lambda_i = \tfrac{1}{2}(1-x)\beta = \tfrac{1}{3}(\Delta_q - N_d)\,, \tag{11}$$

$$\Sigma_i^* - \Sigma_i = \Xi_i^* - \Xi_i = \tfrac{3}{4}(1-x)\beta\,, \tag{12}$$

$$\Sigma_i^* - \Lambda_i = \tfrac{5}{4}(1-x)\beta\,. \tag{13}$$

In addition, we recover the QCD result

$$\frac{(2\Sigma_i^* + \Sigma_i)}{3} - \Lambda_i = \tfrac{2}{3}(\Delta_q - N_d)\,. \tag{14}$$

These results are predicted for all positive as well as negative parity Ψ^λ excitations. While the ratio $(\Sigma^* - \Sigma)/(\Delta_q - N_d) = \frac{1}{2}$ is lower than the QCD ratio 2:3 for ground states, we recover QCD results in Eqs. (11) and (14) without any parameter fitting. For $L^P = 1^-$, the $\Lambda(1520)$, $\Sigma(1620)$, $\Sigma^*(1765)$ show the expected pattern. The Λ_q^* quartet spin state is associated with Ψ^ρ, ρ-type excitations[1] and cannot be compared without further dynamical assumptions. For $L^P = 2^+$, $(\Sigma - \Lambda) = 100$ MeV while $(\Sigma^* - \Sigma)/(\Delta_q - N_d) = 115/262 = 0.44$ favoring the 1:2 ratio. These states, however, show large mixing[1] in 2_8, 4_8 states of the same J which, along with spin-orbit effects, can be more important than spin-spin effects.

For the charmed baryons, the pattern of mass splittings is predicted to be the same as for the strange system, in contrast to QCD predictions which depend on quark masses. This however, awaits experimental confirmation. The mesons cannot be considered in the present formulation.

The main conclusion of this work is that the confining potential plays a major role in baryon spectroscopy, and any interaction which can distinguish between spin-1 and spin-0 diquarks can contribute significantly to the spin-spin splittings of baryon masses. The spin-dependent qq forces in confining region[7] have been recently investigated in a relativistic framework,[8] and also independently in a Bethe-Salpeter formulation.[9] In the former work,[8] the authors have assumed a $(m_i m_j)^{-1} S \cdot S F \cdot F$ force in addition to the confining potential of harmonic-oscillator type, in an exact relativistic formalism. In the present work, we assume that the interaction is small enough to be considered perturbatively and there is no dependence on quark charge or mass. The mass splittings are also insensitive to the radial dependence of the interaction as obtained from the approximately constant value of overlap factor β. Thus a specific δ-function (contact-type) dependence of spin-dependent hyperfine interaction is open to question, and behavior of the charmed-baryon wave function at the origin cannot be obtained on the basis of such models.[10]

[1] L. A. Copley, N. Isgur, and G. Karl, Phys. Rev. D 20, 768 (1979); 23, 817(E) (1981); N. Isgur and G. Karl, Phys. Rev. D 18, 4187 (1978); Phys. Rev. D 19, 2653 (1979); 23, 817(E) (1981).

[2] A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).

[3] See F. Close, *An Introduction to Quarks and Partons* (Academic, New York, 1979), p. 393.

[4] R. H. Dalitz, in *Proceedings of the XIII International Conference on High Energy Physics, Berkeley, 1966* (Univ. of California Press, Berkeley and Los Angeles, 1967).

[5] A. N. Mitra, Phys. Rev. D 11, 3270 (1975).

[6] R. H. Capps, Phys. Rev. Lett. 33, 1637 (1974).

[7] D. A. Liberman, Phys. Rev. D 15, 1572 (1977).

[8] R. K. Bhaduri, L. E. Cohler, and Y. Nogami, Phys. Rev. Lett. 44, 1369 (1980).

[9] A. N. Mitra, Z. Phys. C 38, 25 (1981).

[10] V. Barger, J. P. Leveille, and P. M. Stevenson, Phys. Rev. Lett. 44, 226 (1980).