Perhaps proton decay violates Lorentz invariance

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Perhaps physics is not Lorentz invariant at the very short distance scales over which the proton decays.

Experimental discovery of proton decay would provide a stunning vindication of modern ideas on the unity of the fundamental forces.[1] Proton decay is uniquely significant in that the process probes physics[2] at a distance of 10^{-29} cm or shorter, distances far beyond what has been hitherto accessible to experimental physics. Thanks to that complex of notions in quantum field theory known as renormalization and soft symmetry breaking we are able to lift ourselves out of our dreary existence amidst the low-energy debris of a broken symmetry to catch a glimpse of the possible physics at 10^{-29} cm. Conversely, proton decay can teach us about low-energy symmetries.[3–5]

In this note, we would like to raise and to discuss the possibility that proton decay may violate Lorentz invariance.[6] This is certainly a very speculative suggestion but is perhaps not totally far-fetched and outrageous. To begin with, the distance scale in question is so many orders of magnitude smaller than what has been probed experimentally that one may justifiably feel that "anything" can happen. In recent years, gauge theories formulated on space[7] or space-time[8] lattices have been vigorously studied. The introduction of a lattice is normally regarded as merely a mathematical trick which enables us to study the strong-coupling limit and to perform computations. It is, however, certainly a logical possibility that the lattice is physical and really there. Finally, the notion of space and time being related is so counterintuitive even after almost a century that it may not surprise us entirely to witness Lorentz invariance broken to rotation invariance at some distance scale.

Incidentally, up to now direct experimental tests[9] of Lorentz invariance have consisted mostly in measuring time dilation and in checking relativistic kinematics in high-energy particle experiments. Also, the agreement between calculation and measurement of extreme-precision quantum-electrodynamic quantities such as $(g-2)$ indicates that Lorentz invariance holds up to something like 10^{-17} cm.

We would now like to discuss the manifestation of Lorentz noninvariance in proton decay. Our discussion will be kinematical and independent of any specific theory of grand unification, but if we need to be specific we will refer to the SU(5) theory.[1] We envisage the decay to occur over a very small distance (which we will denote by a_{decay}) after which the outgoing particles will propagate normally. We suppose that the effective decay vertex violates Lorentz invariance. For instance, proton decay may be mediated, as in grand unified theories, by some supermassive boson (with mass $\sim 1/a_{\text{decay}}$) and the Lagrangian may be Lorentz invariant except for those terms describing the propagation of the supermassive boson. We certainly expect Einstein's energy-momentum relation $E=(\vec{p}^{\,2}+m^2)^{1/2}$ to hold for the outgoing decaying particles. We also expect energy and momentum conservation to hold. For a regular lattice of spacing a, energy-momentum conservation would hold modulo a unit of energy-momentum equal to $2\pi/a$.

We first suppose that even rotational invariance breaks down. This would be the case if there really is a lattice. The experimental manifestations should be fairly striking. In a decay like $p\to\pi^0\mu^+$ the pion and the muon would emerge only along the x,y,z axes or a Cartesian coordinate system. (We assume a regular and orthogonal lattice.) The decay is envisaged to occur over a very small distance a_{decay} after which the outgoing particle will propagate along the lattice in some fashion, over a distance much larger than a_{decay}, all the while acted on by weak final-state $SU(3)\times SU(2)\times U(1)$ forces. We expect that these final-state effects, including those which turn the outgoing quark into a pion, will wash out the signature for Lorentz noninvariance only slightly. In the absence of a theory, it is difficult to estimate the amount of "washout." A rather imperfect analogy might be β decay in which the fundamental parity-violating process occurs over $\sim 10^{-16} - 10^{-17}$ cm and in which parity violation is ultimately observed at a much larger distance scale. The fact that even the largest detectors conceivable may see only one proton-decay event every few days may also present a problem. During the time between two subsequent events, the detector will have hurtled through space due to the rotation of the earth, the revolution of the earth around the sun, the rotation of the galaxy, and the motion of the galaxy. The lattice would provide the modernday version of the ether against which absolute motion is to be defined. Presumably, these effects are well understood by astronomers and can be subtracted out. In other words, the experimenters should measure the decay axis in each $p \to \pi^0 \mu^+$ event relative to the "fixed stars," not relative to the mine shaft. The tendency for the decay axis to cluster around three orthogonal directions presumably depends on the dimensionless parameter $\xi \equiv a_{\text{lattice}} / a_{\text{decay}}$ where a_{lattice} denotes the lattice spacing. If ξ is much smaller than 10^{-1}, the effect may be very difficult to observe. An optimistic experimenter, on the other hand, sees no logical impediment to the possibility that ξ may be large. The successful calculation[10] of $\sin^2\theta$ perhaps suggests that ξ cannot be much larger than unity.

The observation of somewhat bizarre decays such as

$$p \to \pi^+ \pi^0 , \quad (1)$$

$$p \to \pi^+ \pi^+ \pi^- , \quad (2)$$

$$p \to \mu^+ \nu \quad (3)$$

would certainly provide a spectacular manifestation of the breakdown of rotational invariance. The experimenters would have to make sure in (1) and (2) that a massless half-integral spin particle does not escape undetected, with a kinematic analysis reminiscent of what experimenters went through when β decay was first studied.

We remark in passing that the experiment of Cocconi and Salpeter[11] to test Mach's principle also provides a test for rotational invariance. Briefly, the experiment involves measuring the photoabsorption spectrum of a Li^7 nucleus which has $J = \frac{3}{2}$ in the ground state. The four $J = \frac{3}{2}$ states are split by a magnetic field and if rotational invariance holds the levels should be equally spaced and the spectrum should show only one line. That one line would split into three lines if rotational invariance fails. The negative result of this experiment establishes that rotational invariance holds, but on a distance scale much larger than that relevant for proton decay.

The reader will have noticed that all the processes described above test the complete breakdown of Lorentz invariance. We cannot exclude the logical possibility that Lorentz invariance may be broken at a decay, but rotational invariance continues to hold. As remarked above, the notion of time in special relativity is so unlike our psychological notion of time that it may be fun to discover that after all, fundamental physics may be only rotational invariant. We could easily realize this possibility by again imagining proton decay to be mediated by some supermassive boson and by supposing those terms describing the propagation of the boson to be rotational, but not Lorentz, invariant.

Unfortunately, the obvious suggestion of comparing the lifetime of a proton in flight and that of a proton at rest is patently absurd[12] from an experimenter's point of view. We have to look for suitable tests involving a proton decaying at rest. Consider for instance the decay $p \to \pi_1 \pi_2 \mu^+$ where π_1, π_2 denote two spinless mesons with momentum k_1, k_2, respectively. The amplitude contains four Lorentz invariants:

$$\bar{u}'[a + b\gamma_5 + (c\gamma_\mu + d\gamma_\mu\gamma_5)(k_1 - k_2)^\mu]u \ .$$

Here u and u' are the Dirac spinors for p and μ^+, respectively; a,b,c,d are functions of $p \cdot k_1$, $p \cdot k_2$, and $k_1 \cdot k_2$. At issue is whether this Lorentz-restricted form leads to predictions on the decay distributions which do not follow from rotation invariance alone. Well, it is easy to see that rotation invariance allows the amplitude to consist also of four invariants:

$$\chi'^\dagger[\alpha + \vec{\sigma} \cdot (\beta \vec{k}_1 + \gamma \vec{k}_2) + \delta \vec{\sigma} (\vec{k}_1 \times \vec{k}_2)]\chi \ .$$

Here χ and χ' are the Pauli spinors for p and μ^+, respectively; $\alpha,\beta,\gamma,\delta$ are functions of E_1, E_2, and θ_{12}, the energies of the two mesons and the angle between them. It is easy to see that, for a proton at rest, $\alpha,\beta,\gamma,\delta$ are just related to a,b,c,d. We can put this slightly differently. One might have thought that if Lorentz invariance is broken to rotational invariance additional amplitudes such as $\eta^\mu \bar{u}'\gamma_\mu u$ with $\eta^\mu = (1,\vec{0})$ are allowed. But for a proton at rest $\eta^\mu \bar{u}'\gamma_\mu u$ is the same as $\bar{u}'u$. We have also examined processes such as

$p \rightarrow \pi_1 \pi_2 \pi_3 \mu^+$ and come to the same negative conclusion. Essentially, this follows because the counting of independent amplitudes could also be made using the helicity formalism.[13] With Lorentz invariance broken, there is a special frame, which we take to be the rest frame of the proton, in which rotational invariance holds.

In a sense, it is rather mysterious that Lorentz invariance and rotational invariance hold so well. An equivalent question is why the spacing of the space-time lattice, if it really exists, is so small compared to the length scales so far observed. The point of this paper is precisely that the spacing is perhaps not small compared to the distance scale of proton decay. It is instructive to contrast the situation with our understanding of baryon- and lepton-number violation. Because of color and Lorentz invariance, the minimal dimension of operators violating B and L is six[3,4] and hence the effects of such operators may be argued to be suppressed. In contrast, in the days when fundamental theories are written in terms of meson and baryon fields, one can easily write down a dimension-four operator describing proton decay.[14] Similarly, in our present-day theories it is easy to write down a Lorentz-noninvariant term with dimension less than four. Is there any reason why the coefficient of such a term should be small? Perhaps the suggestion of Nielsen and collaborators[15] that such a coefficient would be driven to zero in the long-distance limit offers a clue.

Note added. The possibility that proton decay violates Lorentz invariance has been considered independently by J. Ellis, M. Gaillard, D. Nanopoplous, and S. Rudaz, Nucl. Phys. B176, 61 (1980). We thank J. Ellis for calling our attention to this paper and to Ref. 15.

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[12] In principle one can compare the lifetime of a nucleon in a heavy nucleus with that of a nucleon in a light nucleus, noting that the Fermi motions are different and assuming that nuclear effects can be subtracted out. The deviation from the usual time-dilation formula should be of order momentum divided by the nucleon mass.

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