Dispersive $K_L \to \mu^+\mu^-$ contributions, neutral-kaon mixing, and mass of the top quark

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We estimate the $K_L \to 2\mu$ weak amplitude from data, by relating the dispersive $K_L \to 2\gamma \to 2\mu$ amplitude to that of $\eta \to 2\mu$. Allowed bands of mixing angles θ_2, θ_3 are determined from this weak amplitude. The bands overlap those from analyses of K^0-$\bar{K}^0$ mixing, with rather loose constraints on the t-quark mass which depend sensitively on the bag factor B. For $B=0.4$, m_t is likely to be less than 75 GeV.

Recently it was pointed out by Buras[1] that the $K_L \to \mu^+\mu^-$ rate and the K_L-K_S mass difference together serve as a probe of the mass of the yet undiscovered top quark. There it was assumed that the dispersive 2γ contribution to $K_L \to \mu^+\mu^-$ was relatively unimportant. Hence the direct weak-interaction contribution, which depends critically on m_t,[2,3] could be given as a difference between the experimental $K_L \to \mu^+\mu^-$ rate[4] and the absorptive 2γ contribution,[5,6] with the latter determined by the experimental $K_L \to \gamma\gamma$ rate and the purely electromagnetic process $\gamma\gamma \to \mu^+\mu^-$. However, neglect of the dispersive 2γ contribution to $K_L \to \mu^+\mu^-$ may not be justified, as we show in the following by relating it to the corresponding $\eta \to \mu^+\mu^-$ contribution. Having thus isolated the $K_L \to \mu^+\mu^-$ weak amplitude, we obtain allowed bands for the weak mixing angles[7] θ_2, θ_3. These bands depend only on the t-quark mass m_t. The bands overlap those obtained from K^0-$\bar{K}^0$ mixing analyses,[8] for rather wide ranges of m_t. These m_t ranges depend sensitively on the bag factor[9] which enters in the evaluation of the K^0-$\bar{K}^0$ transition matrix element. Our results are summarized in Figs. 1 and 2.

Consider the major contributions to the $K_L \to \mu^+\mu^-$ rate. The absorptive part is dominated by the 2γ intermediate state, and it is given by[5,6]

$$\Gamma_{\text{abs}}/\Gamma(K_L \to \gamma\gamma) = 1.2\times 10^{-5} \,. \tag{1}$$

The dispersive part consists mainly of two terms. One is purely weak, i.e., $K_L \to Z \to \mu^+\mu^-$ and $K_L \to W^+W^- \to \mu^+\mu^-$; the other is both weak and electromagnetic, i.e., $K_L \to \gamma\gamma \to \mu^+\mu^-$, the photons being off-shell. Calling their respective amplitudes A_{weak} and A_{em}, we note that while A_{weak} is reliably calculated in the quark model[2,3] and depends crucially on m_t, A_{em} is not. Therefore, in order to determine m_t, an assumption has to be made regarding A_{em}. Previous authors[1,10] have based their numerical analyses on the assumption that A_{em} is no bigger than about one-half of its absorptive counterpart.[11] However, is that estimate really valid? If not, the bound on m_t obtained in Ref. 1 will have to be changed accordingly.

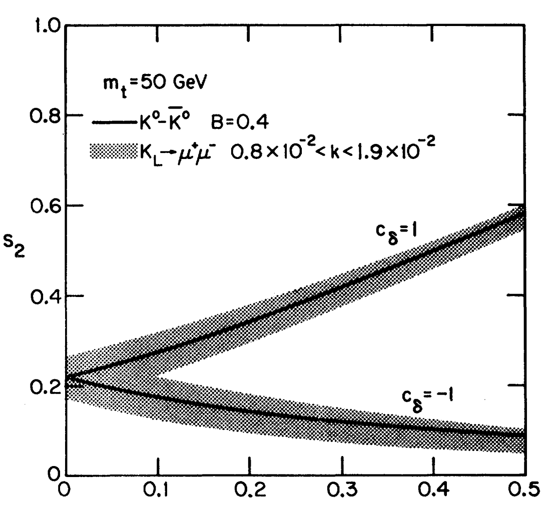

FIG. 1. Allowed values of s_2 vs s_3 from $K_L \to \mu^+\mu^-$ [shaded bands represent Eqs. (9a) and (10a)] and from K^0-$\bar{K}^0$ mixing [solid curves], for $m_t = 50$ GeV and bag factor $B=0.4$.

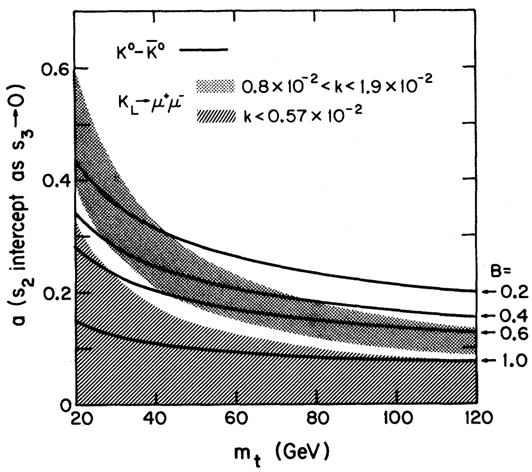

FIG. 2. Dependence of the parameter a of Eq. (12) on m_t for $K_L \to \mu^+\mu^-$ [dotted shading represents Eqs. (9a) and (10a) and diagonal shading represents Eq. (10b)] and K^0-$\bar{K}^0$ mixing [solid curves]. Results for various bag factors B are shown.

To determine A_{em}, we note that the process $\eta \to \mu^+\mu^-$ has exactly the same kinds of contributions. The absorptive part is again dominated by the 2γ intermediate state, with

$$\Gamma_{abs}/\Gamma(\eta \to \gamma\gamma) = 1.1 \times 10^{-5} . \quad (2)$$

The dispersive part is again composed of two terms, but the purely weak-interaction contribution, i.e., $\eta \to Z \to \mu^+\mu^-$, is at most only 10^{-4} times the experimental rate, and can therefore be safely ignored.[12] The remaining dispersive contribution must be dominantly due to the virtual 2γ intermediate state, and is simply given by the difference between the $\eta \to \mu^+\mu^-$ rate and 1.1×10^{-5} times the $\eta \to \gamma\gamma$ rate. Experimentally,[13] there are two separate measurements of $\eta \to \mu^+\mu^-$, but they do not agree: the branching fraction as given by Hyams *et al.* is $(2.2 \pm 0.8) \times 10^{-5}$, whereas that given more recently by Dzhelyadin *et al.* is $(6.5 \pm 2.1) \times 10^{-6}$. Hence there are two possible cases,

$$|A_{disp}/A_{abs}| = 2.08^{+0.43}_{-0.55} \quad (3)$$

and

$$|A_{disp}/A_{abs}| = 0.74^{+0.28}_{-0.51} . \quad (4)$$

We now argue that the ratio of dispersive to absorptive 2γ contributions is the same for $K_L \to \mu^+\mu^-$ as for $\eta \to \mu^+\mu^-$. The $K_L \to 2\gamma$ process is dominated by low-energy contributions, with π^0, η, η' one-particle intermediate states accounting for the observed $K_L \to 2\gamma$ rate.[14] In calculating $K_L \to \mu^+\mu^-$, we use pole dominance together with the reasonable assumption that the ratio of dispersive to absorptive contribution is the same for each pole.[15] The weak-interaction dependence and the pole-model factors then cancel in the dispersive 2γ to absorptive ratio for $K_L \to \mu^+\mu^-$ and the result is the same as that for $\eta \to \mu^+\mu^-$. Therefore, the dispersive 2γ contribution A_{em} for $K_L \to \mu^+\mu^-$ should also be given by Eqs. (3) and (4).

Using the experimental branching fractions[4] $(9.1 \pm 1.9) \times 10^{-9}$ and $(4.9 \pm 0.5) \times 10^{-4}$ for $K_L \to \mu^+\mu^-$ and $K_L \to \gamma\gamma$, respectively, and subtracting off the absorptive 2γ contribution via Eq. (1), we find

$$\Gamma_{disp}/\Gamma(K_L \to \gamma\gamma) = (0.66 \pm 0.43) \times 10^{-5} . \quad (5)$$

This corresponds to

$$\left| \frac{A_{weak} + A_{em}}{A_{abs}} \right| = 0.74^{+0.21}_{-0.30} . \quad (6)$$

Comparing with Eqs. (3) and (4), we see that A_{weak} is either very small, or is comparable in magnitude but opposite in sign to A_{em}. The relative sign between A_{weak} and A_{em} is calculable[2,3,14] in the standard electroweak gauge model,[16] and is given by the sign of $-c_1 s_2 c_3 + c_2 s_3 \cos\delta$, where the Kobayashi-Maskawa[7] (KM) parametrization of mixing angles for six quarks has been used. This factor is indeed negative for all acceptable values of the mixing angles.[8] However, we cannot rule out a change in sign in the $K_L \to \gamma\gamma$ amplitude,[14] which is dominated by low-energy contributions, and hence subject to large strong-interaction corrections.[17]

Let $\Gamma_{disp} = |A_{weak} + A_{em}|^2$, and noting that there is still an ambiguity in the overall sign, we find two possible solutions for $|A_{weak}|$, i.e.,

$$\frac{|A_{weak}|}{[\Gamma(K_L \to \gamma\gamma)]^{1/2}} = \begin{cases} (4.6^{+2.1}_{-1.7}) \times 10^{-3} , \\ (9.8^{+2.1}_{-1.7}) \times 10^{-3} , \end{cases} \quad (7)$$

for Eq. (3), and

$$\frac{|A_{weak}|}{[\Gamma(K_L \to \gamma\gamma)]^{1/2}} = \begin{cases} (5.1^{+2.0}_{-1.2}) \times 10^{-3} , \\ (0.0^{+2.0}_{-0.0}) \times 10^{-3} , \end{cases} \quad (8)$$

for Eq. (4). In comparison, the upper bound on the same quantity assumed previously[1,10] was

3.3×10^{-3}. Adopting the notation of Ref. 1 and recognizing that the u and c contributions to $A_{\rm weak}$ are negligible,[1,10] we find

$$k \equiv \frac{G(x_t)\eta\,|\,{\rm Re}A_t\,|}{|\,s_1c_3\,|}$$

$$= \begin{cases} (1.24^{+0.57}_{-0.48})\times10^{-2}\,, & (9a)\\ (2.62^{+0.57}_{-0.48})\times10^{-2} & (9b)\end{cases}$$

for Eq. (3), and

$$k = \begin{cases} (1.38^{+0.54}_{-0.33})\times10^{-2}\,, & (10a)\\ (0.0^{+0.57}_{-0.0})\times10^{-2} & (10b)\end{cases}$$

for Eq. (4). Here ${\rm Re}A_t = -s_1s_2(c_1s_2c_3 - c_2s_3\cos\delta)$, and[2,3]

$$G(x_t) = \frac{x_t}{1-x_t} - \frac{x_t^2}{4(1-x_t)} + \frac{3x_t^2\ln x_t}{4(1-x_t)^2}\;. \qquad (11)$$

The parameter η represents quantum-chromodynamic (QCD) corrections and is set equal to 0.9, as in Ref. 1, and $x_t \equiv m_t^2/M_W^2$.

The constaint on the KM angles implied by Eq. (9) is

$$t_3 \equiv \frac{s_3}{c_3} = \frac{c_1(s_2^2 - a^2)}{s_2c_2c_\delta}\,, \qquad (12)$$

where $c_\delta \equiv \cos\delta$ and

$$a \equiv \left|\frac{k}{c_1\eta G(x_t)}\right|^{1/2}. \qquad (13)$$

Since all three angles $\theta_{1,2,3}$ are by convention in the first quadrant, Eq. (9) implies that the solution of s_3 vs s_2 has the two branches $s_2^2 > a^2$ for $c_\delta > 0$, and $s_2^2 < a^2$ for $c_\delta < 0$. For the $K_L \to \mu^+\mu^-$ analysis of $A_{\rm weak}$, we assume as in Ref. 10 that $|\,c_\delta\,| \simeq 1$, which is the result of the Δm_K and CP-nonconservation analysis given in Ref. 8 for all but very small values of s_3, i.e., $s_3 << 0.1$. The allowed bands of s_2 vs s_3 from Eq. (9a) and Eq. (10a) are nearly the same: the combination of these bands is illustrated in Fig. 1 for $m_t = 50$ GeV. The m_t dependence of the bands is completely specified by the single parameter a (the s_2 intercept for $s_3 \to 0$), which is displayed in Fig. 2 for Eqs. (9a) and (10a) and for the upper bound of Eq. (10b).

For the K^0-$\bar{K}^0$ analysis of Δm_K and ϵ, we follow Ref. 1 and use the exact formulas[3] as well as QCD corrections[18] in modifying the results of Ref. 8. We then observe that the allowed (s_2,s_3) regions so obtained can be parameterized, as in the $K_L \to \mu^+\mu^-$ analysis, by Eq. (12) to a very good approximation. Hence we can also plot an effective a value as a function of m_t for different values of the bag factor[9] used in the K^0-$\bar{K}^0$ analysis, as illustrated in Fig. 2. The overlap between the curve for a particular value of the bag factor B and the allowed region from the $K_L \to \mu^+\mu^-$ analysis then gives us the allowed range of values for m_t. Using the preferred value[1,9] $B = 0.4$, we find from Fig. 2 that $m_t = 50 \pm 25$ GeV for Eqs. (9a) or (10a) and $m_t < 20$ GeV for Eq. (10b). [Since there has to be an almost complete cancellation between two terms for the solution in Eq. (9b), which gives $m_t > 80$ GeV, it is probably not the solution to take.] In Fig. 1, the solid curve corresponds to $B = 0.4$ and $m_t = 50$ GeV for the K^0-$\bar{K}^0$ analysis without approximating it by Eq. (12). Notice that it lies entirely within the allowed region from the $K_L \to \mu^+\mu^-$ analysis for all values of s_2 and s_3. The constraint $s_3 < 0.5$ is taken from Ref. 19. The $B = 1.0$ curves in Fig. 2 correspond to the vacuum-insertion approximation.

Since the parameter a changes very slowly as a function of m_t for $m_t \gtrsim 40$ GeV in both the K^0-$\bar{K}^0$ and the $K_L \to \mu^+\mu^-$ analyses, the overlap of the two a parameters is extremely sensitive to the experimental uncertainties on k and to the exact value of B used. If we allow B to vary between 0.2 and 0.6, then as seen in Fig. 2, there is essentially no useful limit on m_t. This uncertainty overshadows anything else that may be variable in the model, such as whether $m_c = 1.2$ or 1.5 GeV (the latter value being the one used here), or whether $M_W = 77.8$ or 80.5 GeV (the former value being the one used here for $\sin^2\theta_W = 0.23$), or small additional contributions to $K_L \to \mu^+\mu^-$ and $\eta \to \mu^+\mu^-$ such as the $\pi\pi\gamma$ intermediate states.

The corridors for the mixing angles obtained from the $K_L \to 2\mu$ analysis are relatively model-independent, with the t-quark mass the only unknown. It is reassuring that the more model-dependent K^0-$\bar{K}^0$ mixing analyses[20] yield compatible constraints over a wide range of m_t for plausible choice of the bag factor, giving added confidence in the validity of the derived corridors from the $K_L \to \mu^+\mu^-$ analysis. Additional methods of obtaining constraints on the KM parameters have been discussed elsewhere.[21]

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