Decays of charmed mesons

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The inclusive and two-body hadronic decays of the charmed mesons D^0, D^+, and F^+ are studied. The important nonspectator process is analyzed in terms of contributions from the color-octet and -singlet currents generated by soft-gluon emission from the initial and final states. Analysis of existing data shows that the color-octet term dominates but data on Cabibbo-suppressed decays and the lifetime of F^+ require that the color-singlet term must have a nonzero effect. All two-body and inclusive decay data can be numerically accounted for in a model containing two parameters which characterize the strengths of the two color currents.

I. INTRODUCTION

The decay of the charmed D^0, D^+, and F^+ mesons is a topic that has recently witnessed a great deal of experimental[1-3] and theoretical[4-12] activities. The observations reported are at variance with the predictions of the decay model that has been widely accepted as standard until very recently. The basic assumption of this early model[4] is that the c quark of the initial mesons converts into light quarks via the weak interaction while its companion antiquark plays the role of a spectator, and that gluons participate in the hadronic decay only indirectly in the sense that they affect the important short-distance renormalization of the four-fermion weak vertex.[13] The model unambiguously predicts equal D^0 and D^+ lifetimes and the decay rate for $D^0 \to K^-\pi^+$ to be an order of magnitude larger than that for $D^0 \to \bar{K}^0\pi^0$. However, recent data from the DELCO and MARK II groups yielded the following semileptonic branching ratios[1,2]:

$$B_e^0 \equiv \Gamma(D^0 \to e\nu X)/\Gamma(D^0 \to \text{all}) \begin{cases} < 4.0\% \\ = (5.2 \pm 3.3)\% \end{cases} \quad (1)$$

and

$$B_e^+ \equiv \Gamma(D^+ \to e\nu X)/\Gamma(D^+ \to \text{all}) \begin{cases} = (22.0^{+4.4}_{-2.2})\% \\ = (15.8 \pm 5.3)\% \,. \end{cases} \quad (2)$$

Since it follows from isospin symmetry and the smallness of Cabibbo-suppressed decay rates that D^0 and D^+ have equal semileptonic rates, these measurements imply that the D^+ lifetime is significantly greater than the D^0 lifetime: $\tau(D^+)/\tau(D^0) > 4.3$. This observation is in agreement with the direct lifetime measurements made in the Fermilab emulsion experiments,[3]

$$\tau(D^+) = (10.3^{+10.5}_{-4.1}) \times 10^{-13} \text{ sec} \quad (3)$$

and

$$\tau(D^0) = (1.01^{+0.43}_{-0.27}) \times 10^{-13} \text{ sec}\,. \quad (4)$$

Furthermore, among the large number of branching ratios measured by the MARK II group,[2] it has been found that the rates for $D^0 \to K^-\pi^+$ and $D^0 \to \bar{K}^0\pi^0$ are very similar.

In attempts to save the quark model several ideas have been put forward: to break the SU(3) symmetry by masses or leptonic decay constants[5]; to introduce new mixing angles of heavy quarks[6]; to vary the renormalization coefficients of the effective Hamiltonian[7]; to include soft-gluon radiation.[8] On a somewhat different track, the effects of conventional final-state interactions in the form of resonances have also been considered.[9] These methods so far have not achieved the success that was sought for. A more radical departure from the model seems to be necessary. In particular, it has been suggested that the annihilation or nonspectator mechanism, in which the constituent $c\bar{q}$ pair of the disintegrating meson annihilates to create a light $q\bar{q}$ pair, and which was previously thought to be suppressed[4] by helicity arguments, could enhance some decays.[10-12] In fact, an unsuppressed annihilation contribution would follow from the assumption that the charmed meson contains not only the usual zero spin $(c\bar{q})$ component but also an admixture of gluons and spin-1 $(c\bar{q})$ component.[10] Alternatively the initial $c\bar{q}$ pair could acquire spin 1 by emitting a gluon, and the annihilation mechanism could proceed unimpeded.[11,12] In this paper we adopt the view that gluons are emitted during the decay process. However, since the long-range gluonic component cannot yet be calculated reliably,[14] because perturbation theory cannot be applied with confidence here, we adopt a phenomenological approach.

In Sec. II, we analyze partial widths of hadronic two-body D and F decays assuming color-octet dominance in the annihilation mechanism. The limitations of this model then lead us to introduce in Sec. III a model in which both color-singlet and octet currents contribute. We find that in order to fit the data for Cabibbo-suppressed decays, the ratio g of the singlet to octet strengths must not be zero. We also argue that kinematically there is no reason to expect g to be of order $m_K^2/m_D^2 \ll 1$. In Sec. IV we gain more insight about our model by examining it in the SU(3) limit. Among other things we find that there is no evidence to support the supposition of sextet dominance and that the imaginary, absorptive parts of the SU(3) matrix elements are probably quite small. In Sec. V, we apply the model to the consideration of inclusive branching ratios and find some simple but rather interesting relations between the leptonic and Cabibbo-suppressed hadronic branching ratios. In Sec. VI we apply the model to compute total lifetimes. We show that the aforementioned ratio g can be determined from either of the two lifetime ratios τ^0/τ^+ and τ^0/τ^F, with the latter being much more sensitive to g. Present data on lifetimes do not allow this parameter to be precisely determined, but definitely favor a small value for it. We find that the values of color strengths determined from two-body decay data also give a quantitative account of all inclusive data. Section VII contains our concluding remarks.

II. COLOR-OCTET DOMINANCE

In the free-quark model the effective weak Hamiltonian for charm-changing processes is given by

$$H_w^f(\Delta C = -1) = \frac{G}{\sqrt{2}} (\bar{s}'c)(\bar{u} d'), \qquad (5)$$

where $d' = d\cos\theta_C + s\sin\theta_C$, $s' = -d\sin\theta_C + s\cos\theta_C$, θ_C being the usual Cabibbo angle (we neglect small differences between different mixing angles which arise in the six or more quark models). We have also used the abbreviated notation

$$(\bar{s}c) = \sum_i \bar{s}^i \gamma_\mu (1+\gamma_5) c^i, \qquad (6)$$

where i is the color index.

The Hamiltonian in Eq. (5) can be decomposed into a term transforming as the $\underline{20}$ representation of flavor SU(4) and a term transforming as the $\underline{84}$ representation, both of which have equal weights: $c_{\underline{84}} \equiv c_+ = 1$ and $c_{\underline{20}} \equiv c_- = 1$. Because quarks are strongly interacting particles, it is expected that (5) will be renormalized by the strong interaction. It has been shown[13] that in an asymptotically free theory such as quantum chromodynamics (QCD), the renormalization due to short-distance effects still leaves the effective Hamiltonian a linear combination of $\underline{20}$ and $\underline{84}$ four-fermion local operators, but the $\underline{20}$ component is enhanced and the $\underline{84}$ component is suppressed: $c_+ < 1 < c_-$. The renormalized Hamiltonian then becomes

$$H_w = \frac{G}{\sqrt{2}} \{c_- \tfrac{1}{2}[(\bar{u} d')(\bar{s}' c) - (\bar{u} c)(\bar{s}' d')] + c_+ \tfrac{1}{2}[(\bar{u} d')(\bar{s}' c) + (\bar{u} c)(\bar{s}' d')]\}. \qquad (7)$$

In detailed computations, H_w is contracted in all possible ways, and it is convenient then to use a Fierz identity to reorder the fermion fields:

$$(\bar{q}_1 q_2)(\bar{q}_3 q_4) = \tfrac{1}{3}(\bar{q}_1 q_4)(\bar{q}_3 q_2) + \tfrac{1}{2} \sum_a (\bar{q}_1 \lambda^a q_4)(\bar{q}_3 \lambda^a q_2), \qquad (8)$$

where λ^a ($a = 1, \ldots, 8$) are the generators of the color SU(3) group of QCD. Alternatively one can include the effect of the Fierz transformation and, depending on how the $q\bar{q}$ pairs are grouped, either write the Hamiltonian as

$$H_w = \frac{G}{\sqrt{2}} [\chi_+(\bar{s}'c)(\bar{u} d') + \tfrac{1}{2}\xi_-(\bar{s}'\lambda^a c)(\bar{u}\lambda^a d')], \qquad (9a)$$

or as

$$H_w = \frac{G}{\sqrt{2}} [\chi(\bar{u}c)(\bar{s}' d') + \tfrac{1}{2}\xi_+(\bar{u}\lambda^a c)(\bar{s}'\lambda^a d')], \qquad (9b)$$

where $\chi_\pm = \frac{1}{3}(2c_+ \pm c_-)$ and $\xi_\pm = \frac{1}{2}(c_+ \pm c_-)$.

We now study the two-body decays in a model (hereafter referred to as model 1) where the spectator mechanism is supplemented by a nonspectator contribution that derives only from the color-octet currents manifest in the second term in (9a) and (9b). Since these terms cannot contribute to the allowed D^+ decays, it is hoped that not only the total D^0 rate would be appreciably enhanced over the total D^+ rate, but also that the partial width for the mode $D^0 \to \bar{K}^0\pi^0$ would be increased to the level of that for $D^0 \to K^-\pi^+$.

In the spectator mechanism (Fig. 1) the decay amplitude is calculated under the usual assumption of factorization of the product operators and

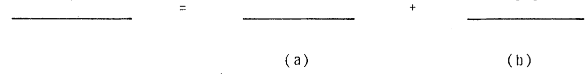

FIG. 1. The spectator decay processes. The final mesons are color matched in (a) and color mismatched in (b). The wavy line represents a QCD-renormalized W boson.

saturation of the intermediate states by the vacuum. The matrix elements of bilinear quark operators are then related to the leptonic-decay constant F_M having the dimension of mass, or to the dimensionless semileptonic-decay scalar form factor f_s^c:

$$\langle 0|\bar{q}'\gamma_\mu\gamma_5 q|M(k)\rangle = ik_\mu F_M, \tag{10}$$

$$k_\mu\langle P|\bar{q}'\gamma_\mu q|D\rangle = (m_D{}^2 - m_P{}^2)f_s^c(k^2), \tag{11}$$

where m_D and m_P are the charmed- and noncharmed-pseudoscalar-meson masses, respectively. Contributions from the spectator mechanism can then be read off from the first terms of (9a) and (9b) with the results

$$A_{sp} = F_M f_s^c \chi_\pm, \tag{12}$$

where the coefficient χ_+ is associated with a color-matching quark decay, χ_- with a color-mismatch decay.

The second terms in (9a) and (9b) contribute to the nonspectator process represented by Fig. 2(a), where the initial $(c\bar{q})$ pair is in a spin-1 color-octet state. The diagram in Fig. 2(a) is not a Feynman diagram in the sense that the gluon represents the sum of soft-gluon effects that make the initial state into a color octet. The factorization approximation may again be used to evaluate the matrix element, with the intermediate state now saturated by the gluon described above. From (9a) and (9b) it is seen that the amplitude for D^0 decays contains the factor ξ_- and those for D^+ and F^+ decays contain the factor ξ_+. We thus write the amplitude as

$$A_{ns} = F_\pi f_s^c h\xi_\pm, \tag{13}$$

where h is a parameter expressing the relative strength of the color-octet term.

Expressions for the spectator and nonspectator contributions for various two-body decay modes are listed in Table I. Here we have introduced further notations: $x = \chi_-/\chi_+$, $y_1 = \xi_+/\chi_+$, and $y_2 = \xi_-/\chi_+$. The amplitudes so given are such that their relations to the widths read

$$\Gamma = \frac{(Gm_D{}^2F_\pi)^2}{32\pi m_D}(f_s^c\chi_+\cos^2\theta_C)^2|A|^2$$

$$= 2.0(f_s^c\chi_+)^2|A|^2\times 10^{11}\ \text{sec}^{-1}. \tag{14}$$

For simplicity, we have neglected light-meson masses, and have taken equal D and F meson masses $m_D \approx m_F = 1.86$ GeV, equal K and η decay constants $f = F_K/F_\pi = F_\eta/F_\pi = 1.23$, the pion decay constant $F_\pi = 0.96\,m_\pi$, and finally the Cabibbo angle such that $\tan\theta_C = 0.23$. From QCD it is estimated that $c_- \approx 2.1$ and $c_+ \simeq 0.7$. However, analyses of kaon and other strange-particle decays[15] indicate that a further enhancement is needed. We will therefore adopt the value $c_+ = (0.7)/2 = 0.35$ in the following. Throughout this discussion, we will keep the above parameters fixed, because we believe that, within reasonable limits, variations in their values do not play a decisive role in the general conclusions of the present analysis. The remaining two parameters f_s^c and h are determined by fitting the two partial widths[2,3] $\Gamma(D^+\to\bar{K}^0\pi^+) = 0.22\times10^{11}\ \text{sec}^{-1}$ and $\Gamma(D^0\to K^-\pi^+) = 3.0\times10^{11}\ \text{sec}^{-1}$ with the constraint that $\Gamma(D^0\to\bar{K}^0\pi^0)$ is consistent with data, and we obtain

$$f_s^c = 0.93, \quad h = -1.84. \tag{15}$$

These values are then used to calculate the rates of other decay modes. Although the D decay constant f_s^c is not precisely known, existing analyses of D-meson leptonic decays[16] indicate that it is of the order of unity.

In the second column of Table II we give the available experimental partial widths computed from lifetime[3] and branching-ratio[1,2] data. In the next column are the partial widths calculated in the present model. The detailed expressions found in Table I show that in order for both $\Gamma(D^0\to K^-\pi^+)$ and $\Gamma(D^0\to\bar{K}^0\pi^0)$ to have reasonable values in this model, the parameter h must have a relatively large negative value. However, for any $h \lesssim -1$, the ratio $\Gamma(D^0\to K^-K^+)/\Gamma(D^0\to\pi^-\pi^+)$ is smaller than 1, contrary to observations. On the other hand, the F^+ decay widths appear to be in accordance with the observed lifetime relations $\tau_{D^0} < \tau_{F^+} < \tau_{D^+}$.

To summarize, the octet-dominance model succeeds in the task it was designed for, namely to enhance the ratios $\Gamma(D^0)/\Gamma(D^+)$ and $\Gamma(D^0\to\bar{K}^0\pi^0)/\Gamma(D^0\to K^-\pi^+)$ over earlier predictions, but encounters difficulties with the richer phenomenon of the Cabibbo-suppressed decays.

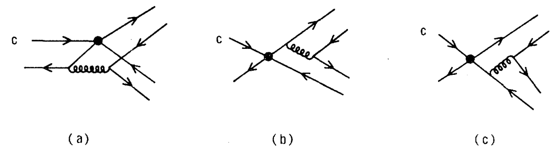

FIG. 2. Nonspectator two-body decays to lowest order in gluon exchange. The emission of a gluon (coiled line) from the initial state (a) generates a color-octet term in the Hamiltonian. When the gluon is exchanged in the final state, (b) and (c), only color-singlet terms are generated.

III. COLOR-SINGLET CONTRIBUTIONS

In this section we relax the assumption of color-octet currents dominating the nonspectator process,

TABLE I. Two-body decay amplitudes of charmed mesons. Define $\xi_\pm = \frac{1}{2}(c_+ \pm c_-)$, $\chi_\pm = \frac{1}{3}(2c_+ \pm c_-)$; then $x = \chi_-/\chi_+$, $y_1 = \xi_+/\chi_+$, $y_2 = \xi_-/\chi_+$, $f = F_K/F_\pi = 1.23$, $t = \tan\theta_C = 0.23$, and $\mu = m_u/m_s \approx 0.62$.

Decay modes	Spectator	Nonspectator Model 1	Nonspectator Model 2	SU(3)
$D^0 \to K^-\pi^+$	1	$y_1 h$	$(1+\mu)xg_s + y_1 g_o$	$2T+E-S$
$\bar{K}^0\pi^0$	$\frac{1}{\sqrt{2}}xf$	$-\frac{1}{\sqrt{2}}y_1 h$	$-\frac{1}{\sqrt{2}}[(1+\mu)xg_s + y_1 g_o]$	$\frac{1}{\sqrt{2}}(3T-E+S)$
$\bar{K}^0\eta$	$\frac{1}{\sqrt{6}}xf$	$-\frac{1}{\sqrt{6}}y_1 h$	$-\frac{1}{\sqrt{6}}[(1+\mu)xg_s + y_1 g_o]$	$\frac{1}{\sqrt{6}}(3T-E+S)$
$D^+ \to \bar{K}^0\pi^+$	$1+xf$	0	0	$5T$
$F^+ \to \bar{K}^0 K^+$	xf	$y_2 h$	$2g_s + \mu y_2 g_o$	$2T+E+S$
$\eta\pi^+$	$-(\frac{2}{3})^{1/2}$	$(\frac{2}{3})^{1/2} y_2 h$	$(\frac{2}{3})^{1/2}(2g_s + \mu y_2 g_o)$	$-(\frac{2}{3})^{1/2}(3T-E-S)$
$D^0 \to K^-K^+$	tf	$ty_1 h$	$t(2\mu x g_s + y_1 g_0)$	$t(2T+E-S)$
$\bar{K}^0 K^0$	0	0	$-2t(1-\mu)xg_s$	0
$\pi^-\pi^+$	t	$ty_1 h$	$t(2xg_s + y_1 g_0)$	$t(2T+E-S)$
$\pi^0\pi^0$	$\frac{1}{\sqrt{2}}tx$	$-\frac{1}{\sqrt{2}}ty_1 h$	$-\frac{1}{\sqrt{2}}t(2xg_s + y_1 g_0)$	$\frac{1}{\sqrt{2}}t(3T-E+S)$
$\eta\pi^0$	$\frac{1}{\sqrt{12}}tx(1-3f)$	$\frac{1}{\sqrt{3}}ty_1 h$	$\frac{1}{\sqrt{3}}t(2xg_s + y_1 g_0)$	$-\frac{1}{\sqrt{3}}t(3T-E+S)$
$\eta\eta$	$-\frac{1}{\sqrt{2}}txf$	$\frac{1}{\sqrt{2}}ty_1 h$	$\frac{1}{3\sqrt{2}}t[(4\mu-1)2xg_s + 3y_1 g_0]$	$-\frac{1}{\sqrt{2}}t(3T-E+S)$
$D^+ \to \bar{K}^0 K^+$	tf	$-ty_2 h$	$-t(2g_s + y_2 g_0)$	$t(3T-E-S)$
$\pi^0\pi^+$	$\frac{1}{\sqrt{2}}t(1+x)$	0	0	$\frac{1}{\sqrt{2}}t5T$
$\eta\pi^+$	$\frac{1}{\sqrt{6}}t(1+3xf)$	$(\frac{2}{3})^{1/2}ty_2 h$	$(\frac{2}{3})^{1/2}t(2g_s + y_2 g_0)$	$\frac{1}{\sqrt{6}}t(9T+2E+2S)$
$F^+ \to K^0\pi^+$	$-t$	$ty_2 h$	$t[(1+\mu)g_s + \mu y_2 g_0]$	$-t(3T-E-S)$
$K^+\pi^0$	$\frac{1}{\sqrt{2}}tx$	$\frac{1}{\sqrt{2}}ty_2 h$	$\frac{1}{\sqrt{2}}t[(1+\mu)g_s + \mu y_2 g_0]$	$\frac{1}{\sqrt{2}}t(2T+E+S)$
$K^+\eta$	$\frac{1}{\sqrt{6}}tf(2+3x)$	$\frac{1}{\sqrt{6}}ty_2 h$	$\frac{1}{\sqrt{6}}t[(1+\mu)g_s + \mu y_2 g_0]$	$\frac{1}{\sqrt{6}}t(12T+E+S)$

and examine the contribution of color-singlet currents. This contribution arises from the first terms in (9a) and (9b), and is represented to lowest order in gluon exchange by Fig. 2(b). Combining the contributions from color-singlet and color-octet currents we can write the nonspectator amplitude as

$$A_{ns} = \chi_- S + \xi_+ O \qquad (16)$$

for D^0 decays, and

$$A_{ns} = \chi_+ S + \xi_- O \qquad (17)$$

for D^+ and F^+ decays. Since the singlet and octet contributions, S and O, depend on the propagators of the intermediate quarks, it is convenient to factor out explicitly the numbers of quarks, n_s and n_o, in the decay meson that can contribute to the nonspectator process. Because the charm quark is much heavier than the other quarks, we suppress processes where the gluon is emitted from the c quark. In the same notion, we assign a mass factor $\mu = m_u/m_s \approx 0.62$ to terms involving the propagator of an s quark; in this way the flavor SU(3) symmetry is naturally broken. Then (16) and (17) may be rewritten as

$$A_{\rm ns} = \chi_\pm n_s g_s + \xi_\mp n_o g_o , \tag{18}$$

where the parameters g_s and g_o measure the color-singlet and -octet transition strengths relative to the quark-decay strength, and are to be determined by the data. Detailed expressions for various modes calculated from (18) (hereafter referred to as model 2) are given in the fourth column of Table I. Adding the nonspectator to the spectator amplitudes we calculate the widths with the parameters

$$f_s^c = 0.93, \quad g_s = -0.90, \quad g_o = -2.40 , \tag{19}$$

which are determined from the experimental widths[1-3] for $D^0 \to K^-\pi^+$, $D^0 \to \overline{K}^0\pi^0$, and $D^+ \to \overline{K}^0\pi^+$. The resulting widths are given in column 4 of Table II.

Again in this model the measured ratio of the rates of the two decays $D^0 \to K^-\pi^+$ and $D^0 \to \overline{K}^0\pi^0$ requires that both g_s and g_o be negative and that $|g_s| < |g_o|$. The introduction of the color singlet does not change that ratio but it affects the calculated widths for the Cabibbo-suppressed decays in a favorable manner. The relative widths for $D^0 \to \pi^-\pi^+$ and $D^+ \to \overline{K}^0K^+$ are now in agreement with the data, and the ratio $\Gamma(D^0 \to K^-K^+)/\Gamma(D^0 \to \pi^-\pi^+)$ has increased over that given in model 1 although it still appears to be somewhat smaller than the measured ratio. Overall, the model gives a good account of all two-body-decay data.

As for the F^+ decays, for which no data are presently available, the model predicts relatively large rates for both $F^+ \to \overline{K}^0K^+$ and $\eta\pi^+$. This point obviously needs experimental confirmation, but it has a certain bearing on the relatively large branching for the Cabibbo-suppressed mode $D^+ \to \overline{K}^0K^+$, which is already known.[2] The close relationship between the amplitudes for this decay and the allowed F^+ decays can be inferred from the explicit expressions for their nonspectator amplitudes given in Table I; in the SU(3) limit these amplitudes are identical to within a constant factor (when $T=0$, see Table IV). Numerically the correlation is illustrated in Table III, where the widths of some decays are given as functions of the color-octet strength g_o and the color-singlet-to-octet ratio $g = g_s/g_o$. It can be seen that for the three values of g that we have examined, the widths remain essentially stable with the exception of $F^+ \to \overline{K}^0K^+$, $\eta\pi^+$, and $D^+ \to \overline{K}^0K^+$. This correlation allows us to have some faith in predicting the widths of the allowed F^+ decays to be of the order of 1 to 2 $\times 10^{11}$ sec^{-1}. In Sec. VI we shall show that the magnitude of g is essentially determined by the lifetime ratio $\tau(D^0)/\tau(F^+)$.

Although we do not intend to discuss the dynamical origin of g_o and g_s in detail, we would like to argue that kinematically g_s need not be zero. The matrix element for the color-singlet term has a factor that is analogous to (11),

TABLE II. Two-body decay widths of charmed mesons (in units of 10^{11} sec^{-1}). Experimental rates are extracted from measured lifetimes (Ref. 3) and branching ratios (Ref. 2). Model 1: $f_s^c = 0.93$, $h = -1.84$. Model 2: $f_s^c = 0.93$, $g_s = -0.90$, $g_o = -2.40$.

Decay modes	Expt. rates	Model 1	Model 2	SU(3)
$D^0 \to K^-\pi^+$	$3.0^{+2.2}_{-2.6}$	3.0 [a]	3.0 [a]	3.0 [a]
$\overline{K}^0\pi^0$	$2.2^{+2.6}_{-2.1}$	2.4	2.4 [a]	2.2 [a]
$\overline{K}^0\eta$		0.80	0.80	0.74
$D^+ \to \overline{K}^0\pi^+$	$0.22^{+0.25}_{-0.15}$	0.22 [a]	0.22 [a]	0.22 [a]
$F^+ \to \overline{K}^0K^+$		1.80	1.50	1.5 [a]
$\eta\pi^+$		0.52	1.90	0.63
$D^0 \to K^-K^+$	$0.40^{+0.46}_{-0.36}$	0.11	0.15	0.16
$\overline{K}^0K^0$		0.0	0.01	0.0
$\pi^-\pi^+$	$0.10^{+0.15}_{-0.09}$	0.15	0.12	0.16
$\pi^0\pi^0$		0.14	0.12	0.12
$\eta\pi^0$		0.08	0.06	0.08
$\eta\eta$		0.13	0.17	0.12
$D^+ \to \overline{K}^0K^+$	$0.05^{+0.08}_{-0.03}$	0.02	0.05	0.05 [a]
$\pi^0\pi^+$		0.01	0.01	0.01
$\eta\pi^+$		0.09	<0.01	0.08
$F^+ \to K^0\pi^+$		0.04	0.09	0.05
$K^+\pi^0$		0.06	0.01	0.04
$K^+\eta$		0.07	<0.01	0.04

[a] Values used in fitting.

TABLE III. Dependence of the decay rates on color strengths.

g_o	−2.40	−2.28	−2.0
g $(=g_s/g_o)$	0.375	0.30	0.125
$D^0 \to K^-\pi^+$	3.0	3.0	3.0
$\overline{K}^0\pi^0$	2.4	2.4	2.4
K^-K^+	0.15	0.14	0.12
$\pi^-\pi^+$	0.12	0.13	0.15
$D^+ \to \overline{K}^0\pi^+$	0.22	0.22	0.22
$\overline{K}^0K^+$	0.05	0.02	<0.01
$F^+ \to \overline{K}^0K^+$	1.50	0.70	<0.01
$\eta\pi^+$	1.90	1.14	0.10

$$(k-k')_\mu \langle P(k)|\bar{q}'\gamma_\mu q|P'(k')\rangle$$
$$=(m_P{}^2-m_{P'}{}^2)f_+(m_D^2)+m_D^2 f_-(m_D^2)\,, \quad (20)$$

where $f_\pm$ are the form factors of the noncharmed pseudoscalar meson. Because of the small value of $m_P{}^2/m_D{}^2$, the first term on the right-hand side of (20) is relatively very small compared to (11), which is partially responsible for the so-called helicity suppression of the nonspectator process. However, the second term is not suppressed in this way. It is known[17] that $f_+(t)\approx 1$ and $f_-(t)\approx 0$ at $t\approx m_\pi{}^2\approx 0$, but the behavior of these form factors is not known when t is large ($\approx m_D{}^2$) and timelike; in particular there is no reason to believe $f_-(m_D{}^2)$ should be of order $m_P{}^2/m_D{}^2$. Since our color-singlet strength g_s is directly proportional to $f_-(m_D{}^2)$, it need not *a priori* be suppressed by a factor of m_P^2/m_D^2.

IV. THE SU(3) LIMIT

In weak processes, the flavor SU(3) symmetry is not strictly obeyed. Nevertheless, the simplicity in the SU(3) limit can give us some insight on the models used in the last two sections.

The effective weak Hamiltonian in (7) is a linear combination of terms which transform as the $\underline{6}^*$ and $\underline{15}$ representations of the SU(3) group. If we ignore the existence of all but a single flavor-mixing angle, as has been done throughout this study, and if we assume strict SU(3) symmetry, then only three SU(3) matrix elements are required to describe the decays of D and F mesons.[18] They are $T=\langle \underline{27}|\underline{15}|\underline{3}^*\rangle$, $E=\langle \underline{8}|\underline{15}|\underline{3}^*\rangle$, and $S=\langle \underline{8}|\underline{6}^*|\underline{3}^*\rangle$. The decay amplitudes in terms of these matrix elements are given in the last column of Table I. We note that E and S always appear in the linear combination $E-S$ in D^0 decays, and $E+S$ in D^+ and F^+ decays. Three of the five unknown numbers, $|T|^2$, $|E-S|^2$, and $\mathrm{Re}[T^*(E-S)]$, can be determined from the three known widths of $D^0\to K^-\pi^+$, $\bar{K}^0\pi^0$, and $D^+\to\bar{K}^0\pi^+$. For the remaining two, $|E+S|^2$ and $\mathrm{Re}[T^*(E+S)]$, we have at our disposal only the width of $D^+\to\bar{K}^0K^+$. We simply adopt the additional assumption that $\Gamma\,(F^+\to\bar{K}^0K^+)=1.5\times10^{11}\ \mathrm{sec}^{-1}$. The five unknown real numbers are listed in the last column of Table IV, and the widths calculated with these values are given in the last column of Table II.

Since in our models we neglect the mass difference of D and F mesons and ignore the masses of light mesons, the SU(3) symmetry is broken by the decay constants and the quark masses. Thus the SU(3) limit of model 1 is attained simply by setting $f=1$, which yields the result

$$T=\tfrac{1}{5}(1+x)\,, \quad (21a)$$

$$E=\tfrac{1}{2}(1+x)(\tfrac{1}{5}+\tfrac{3}{4}h)\,, \quad (21b)$$

$$S=-\tfrac{1}{2}(1-x)(1+\tfrac{3}{2}h)\,. \quad (21c)$$

Similarly the SU(3) limit of model 2 corresponds to $f=1$, $\mu=1$, yielding the amplitudes

$$T=\tfrac{1}{5}(1+x)\,, \quad (22a)$$

$$E=\tfrac{1}{2}(1+x)(\tfrac{1}{5}+2g_s+\tfrac{3}{4}g_0)\,, \quad (22b)$$

$$S=-\tfrac{1}{2}(1-x)(1-2g_s+\tfrac{3}{2}g_0)\,. \quad (22c)$$

An indication on the range of their values can be obtained by adopting the parameters $h=-1.84$, Eq. (15), and $g_s=-0.90$, $g_o=-2.40$, Eq. (19), which we now use without attempting any new fitting. Then we have for model 1, $T=0.1$, $E=-0.30$, $S=1.32$, and for model 2, $T=0.1$, $E=-0.85$, $S=0.60$. From these values or their equivalents given in Table IV, one can draw a number of conclusions. Contrary to common belief, there is no sextet dominance, i.e., E is not much smaller than S, although T is. The color-singlet currents affect $E+S$ more strongly than $E-S$; thus the two models, which differ on their predictions on $E+S$, differ also on their predictions on the suppressed D^+ decays as well as on most of the F^+ decays. The SU(3) amplitudes of the quark models are real, while those determined experimentally are complex. However, as can be inferred from Table IV, their imaginary, absorptive parts are small. This and the small residual enhancement still needed in our model for the ratio $\Gamma(D^0\to K^-K^+)/\Gamma(D^0\to\pi^-\pi^+)$ concur to suggest that the final-state interactions among hadrons[9] are necessary, but not preponderant, effects.

V. INCLUSIVE BRANCHING RATIOS

Our model (hereafter we concentrate on model 2) is easily applied to inclusive decays. We write the total width as

$$\Gamma=\Gamma_c(\gamma_L+\gamma_{sp}+\gamma_{ns})\,, \quad (23)$$

where

$$\Gamma_c=G^2m_c{}^5/192\pi^3\approx 2.7\times10^{11}\ \mathrm{sec}^{-1} \quad (24)$$

with $m_c\approx1.5$ GeV. $\gamma_L=2$ is the semileptonic reduced width, $\gamma_{sp}=2c_+{}^2+c_-{}^2$ is the hadronic reduced

TABLE IV. Amplitudes in the SU(3) limit.

	Model 1	Model 2	Model 3
$\lvert T\rvert^2$	0.01	0.01	<0.01
$\lvert E-S\rvert^2$	2.608	2.103	2.358
$\lvert E+S\rvert^2$	1.051	0.063	0.824
$\mathrm{Re}[T^*(E-S)]$	−0.162	−0.145	−0.091
$\mathrm{Re}[T^*(E+S)]$	0.102	−0.025	0.040

width for spectator processes, and γ_{ns} is the reduced width for nonspectator processes. Following our considerations in Sec. III, the nonspectator decays derive from the incoherent sum of the three gluon emission processes shown in Fig. 3. Neglecting the doubly Cabibbo-suppressed contributions we have ($c = \cos\theta_C$, $s = \sin\theta_C$)

$$\gamma_{ns} \equiv C\bar{\gamma}_{ns} \approx C\,(c^4\gamma_A + c^2 s^2 \gamma_S)\,. \tag{25}$$

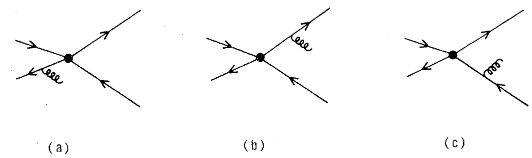

FIG. 3. Gluon-emitting nonspectator processes considered in inclusive decays. Gluon emission from the c quark is suppressed by the latter's heavy mass.

With the superscripts 0, +, and F denoting D^0, D^+, and F^+ decays, the Cabibbo-allowed and -suppressed contributions γ_A and γ_S read

$$\gamma_A^0 = (1+\mu)^2 x^2 g^2 + y_1^2\,, \quad \gamma_S^0 = 4(1+\mu^2)x^2 g^2 + 2y_1^2\,, \tag{26a}$$

$$\gamma_A^+ = 0\,, \quad \gamma_S^+ = 4g^2 + y_2^2\,, \tag{26b}$$

$$\gamma_A^F = 4g^2 + \mu^2 y_2^2\,, \quad \gamma_S^F = (1+\mu)^2 g^2 + \mu^2 y_2^2\,, \tag{26c}$$

where μ, x, $y_{1,2}$, and $g = g_s/g_o$ are as defined in Table I. The constant factor C introduced in Eq. (25) is $3\chi_+^2 g_o^2 K$, where K depends on a form factor and a phase-space integral, and is numerically close to 1. In computing the absolute lifetimes we shall use $K = 1$. However, the branching ratios and the lifetime ratios to be considered below are independent of this factor.

The Cabibbo-suppressed hadronic branching ratio can be derived from (23) and (25) to be

$$B_S = c^2 s^2 \{2B_e\gamma_{sp} + [1 - B_e(2+\gamma_{sp})](\gamma_S/\bar{\gamma}_{ns})\}\,, \tag{27}$$

where $B_e = \Gamma_c/\Gamma$ is the semileptonic ($e\nu X$) branching ratio. Setting $\mu = 1$ and dropping smaller terms, we can obtain from (26) and (27) the following approximate relations:

$$B_S^0 \approx 2c^2 s^2(1 - 2B_e^0)\,, \tag{28a}$$

$$B_S^+ \approx c^2[1 - 2B_e^+(1 + \tfrac{1}{2}c^2\gamma_{sp})]\,, \tag{28b}$$

$$B_S^F \approx s^2[1 - 2B_e^F(1 - \tfrac{1}{2}c^2\gamma_{sp})]\,, \tag{28c}$$

which are to be compared with the relation $B_S = 2c^2s^2(1-2B_e)$ which would hold in the absence of nonspectator decays. Thus the presence of a nonspectator mechanism has changed radically the structure of B_S^+ and B_S^F. The approximation $\mu = 1$ used to derive (28) is excellent for D mesons, but poor for the F meson as can be seen from (26). Remarkably, the relations (28) are completely independent of the color parameters g_o and g_s, which implies that the exact branching ratio given by (27) should also depend very weakly on the details of the nonspectator mechanism. It is clear in the case of D^+ meson, because $\gamma_A = 0$ implies $\bar{\gamma}_{ns}/\gamma_S = c^2s^2$, the ratio B_S depends only on the renormalization coefficients through the presence of γ_{sp}.

Using $g = g_s/g_o = 0.375$ and the renormalization coefficients $c_+ = 0.35$ and $c_- = 2.1$, we obtain from (26) and (27)

$$B_S^0 = -0.2\,B_e^0 + 0.1\,, \tag{29a}$$

$$B_S^+ = -6.22\,B_e^+ + 1.0\,, \tag{29b}$$

$$B_S^F = 0.19\,B_e^F + 0.04\,. \tag{29c}$$

With $B_e^0 = 0.05$ and $B_e^+ = 0.15$ the above equations yield $B_S^0 = 0.09$ and $B_S^+ = 0.07$. Experimentally only the lower limits for B_S^0 and B_S^+ are known from measuring the K^+ contents of inclusive D^0 and D^+ decays[2]: $B_S^0 > (7.9 \pm 2.6)\%$ and $B_S^+ > (5.6 \pm 2.9)\%$. B_e^F has not been measured, but from the lifetime of F^+ (Ref. 3) it is safe to assume $0.025 < B_e^F < 0.10$, which implies $0.045 < B_S^F < 0.06$. Of course, Eqs. (29) can also be used in the reverse order to provide limits on the range of values of B_e. This is particularly useful in the case of the D^+ meson because of the apparent difference in the DELCO and MARK II results, Eq. (2), and because Eq. (29b) yields meaningful limits. In fact, for $c_+ = 0.35$ and $c_- = 2.1$, $B_S^+ > 0$ requires $B_e^+ < 16\%$, while an arbitrary upper limit $B_S^+ < 20\%$ entails the condition $B_e^+ > 13\%$. Similarly narrow, but nonoverlapping, ranges of B_e^+ obtain for the standard renormalization ($c_+ = 0.7$, $c_- = 2.1$) and in the absence of renormalization ($c_+ = c_- = 1$): $12\% < B_e^+ < 14.5\%$ and $17\% < B_e^+ < 21\%$, respectively. Our model with strong renormalization favors the branching ratio measured by MARK II but cannot pick out a particular set of renormalization coefficients unless B_S^+ is more accurately known.

An equivalent way of determining the renormalization coefficients is through the relation

$$\gamma_{sp} = 2c_+^2 + c_-^2 = B_A^+/(c^4 B_e^+)\,, \tag{30}$$

which results directly from (23). Here B_A is the hadronic Cabibbo-allowed branching ratio. Again B_A^+ is not accurately known, but assuming $B_A^+ = 0.61 \pm 0.19$ from the K^- and $\bar{K}^0$ contents of D^+ decays[2] and B_e^+ given by Eq. (2), we obtain

$$2c_+^2 + c_-^2 = \begin{cases} 3.1 \pm 1.4 & \text{(DELCO)} \\ 4.3 \pm 2.6 & \text{(MARK II)}\,. \end{cases} \tag{31}$$

Again the MARK II result is consistent with our value $\gamma_{sp} = 4.7$.

TABLE V. Results for total and inclusive decay rates. Experimental values in parentheses.

	Lifetime (10^{-13} sec) ($m_0=1.5$ GeV)		Inclusive branching ratio (%) Leptonic (B_e)	Hadronic Allowed (B_A)	Hadronic Suppressed (B_S)
D^0	1.2	($1.01^{+0.43}_{-0.27}$ [a]; <2.1 [b])	3.0 (<4 [b]; 5.2 ± 3.3 [c])	84 (76 ± 14 [c])	9.5 ($>7.9\pm2.6$ [c])
D^+	5.4	($10.3^{+10.5}_{-4.1}$ [a]; $10.4^{+4.9}_{-2.9}$ [b])	13 ($22^{+4.4}_{-2.2}$ [b]; 15.8 ± 5.3 [c])	54 (61 ± 19 [c])	20 ($>5.6\pm2.9$ [c])
F^+	2.1	($2.2^{+2.8}_{-1.0}$ [a])	5.0	84	5.2

[a] Reference 3.
[b] Reference 1.
[c] Reference 2.

VI. LIFETIMES AND LIFETIME RATIOS

The lifetimes for the charmed mesons, calculated from (23) to (26), with $m_c=1.5$ GeV and the color strength g_o and g_s given in (19), are listed in the second column of Table V. It is gratifying that these lifetimes agree with the measured lifetimes as well as they do, since in addition to (23) being entirely different from (14), the parameters g_o and g_s were determined by the consideration of exclusive and therefore coherent amplitudes. Having the lifetimes we can calculate the semileptonic branching ratio $B_e=\Gamma_c/\Gamma$, and with the help of (23) and (29) the hadronic branching ratios B_A and B_S. The results are also presented in Table V.

Interestingly, a relation between lifetime ratios which depends only on the ratio of color strengths $g=g_s/g_o$ can be derived from (23):

$$\frac{1-\tau^0/\tau^+}{1-\tau^0/\tau^F}=\frac{1-\gamma^+_{\rm ns}/\gamma^0_{\rm ns}}{1-\gamma^F_{\rm ns}/\gamma^0_{\rm ns}}, \tag{32}$$

where the $\gamma_{\rm ns}$'s are given in (26). This relation is exhibited in Fig. 4, where contour lines for fixed g^2 are plotted in the τ^0/τ^+ versus τ^0/τ^F plane. We emphasize that these lines depend only on g^2, but not on the other parameters g_o, m_c, etc. It is clearly seen in Fig. 4 that τ^0/τ^+ is not sensitive to g^2, but that τ^0/τ^F has a very strong dependence on it. This observation corroborates our conclusion at the end of Sec. III. Existing experimental limits[1-3] on lifetime ratios

$$0.036<\tau^0/\tau^+<0.23\,,$$

$$0.15<\tau^0/\tau^F<1.21$$

delineated in Fig. 4 are not sufficiently restrictive to give us a useful bound on g^2, but if $\tau^0/\tau^F\approx0.5$ as the emulsion data[3] seem to suggest, then $g^2\approx0.2$; in Sec. III we have obtained from two-body-decay data the value $g^2=0.14$. Notice that color-octet dominance ($g^2=0$) is ruled out provided $\tau^F<2.9\,\tau^0$.

In Fig. 4 we have also plotted contour lines for fixed $|g_o|$ and lines giving the experimental lifetime bounds[3]

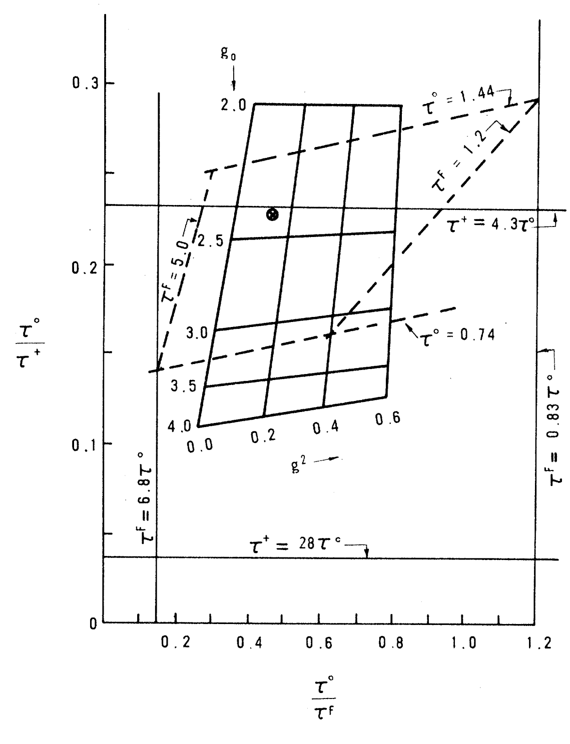

FIG. 4. Contour lines determined by Eqs. (23)–(26) for fixed values of g^2 and of $|g_0|$ are plotted in the plane with τ^0/τ^+ and τ^0/τ^F as Cartesian coordinates; the experimental limits on the two ratios are also shown. The dashed lines are contours of the experimental limits of τ^F and τ^0; they may also be treated as contours with respect to the generalized coordinates g^2 and $|g_o|$. The circled cross corresponds to the values for g and g_o given in (19).

$$\tau^0 = (1.01^{+0.43}_{-0.27}) \times 10^{-13} \text{ sec},$$

$$\tau^F = (2.2^{+2.8}_{-1.0}) \times 10^{-13} \text{ sec}.$$

We plot these mainly to display the sensitivity of τ^0/τ^+, and also τ^0, to the color-octet strength $|g_o|$; they are very insensitive to g^2. On the other hand, τ^F has a strong dependence on both g^2 and $|g_o|$. It can also be seen that existing lifetime bounds provide more restrictive constraints on the parameters than bounds from lifetime ratios. The circled cross in Fig. 4 shows the lifetime ratios obtained from the values for g and g_o given in (19).

VII. CONCLUSION

We have studied the two-body hadronic and inclusive decays of the charmed D^0, D^+, and F^+ mesons in some detail. In particular we have addressed ourselves to examining the roles of color-octet and -singlet currents, generated from soft-gluon emission, in the nonspectator decay processes. We have analyzed existing data in terms of a model containing two unknown parameters, g_o and g_s, which characterize the contributions of color-octet and -singlet currents to the decay amplitude relative to the contribution from the spectator process. One can immediately infer from the lifetime ratio $\tau^+/\tau^0 \gg 1$ that $|g_o| > 1$. We have found the following from our analysis of existing two-body and inclusive decay data.

(i) The color-octet and -singlet nonspectator amplitudes interfere destructively with the spectator amplitude in two-body decays.

(ii) The color-octet current is dominant but $g = g_s/g_o \approx 0.3$ to 0.4 if the Cabibbo-suppressed two-body decays are to be explained.

(iii) The ratio $\tau^F/\tau^0 \approx 2$ requires that $g^2 \approx 0.2$, in particular $g^2 = 0$ would be excluded if $\tau^F < 2.9\,\tau^0$.

(iv) In terms of flavor-SU(3) amplitudes, there is no evidence for sextet dominance in the two-body data; however, the matrix element $\langle \underline{27} | \underline{15} | \underline{3}^* \rangle$ is indeed very small.

(v) More accurate measurements of the inclusive leptonic and hadronic decays of the D^+ meson will help to fix the renormalization coefficients; as of now $c_+ = c_- = 1$ is consistent with $0 < B_S^+ < 0.20$ and $0.17 < B_e^+ < 0.21$, and $c_+ = 0.35$, $c_- = 2.1$ with $0 < B_S^+ < 0.20$ and $0.13 < B_e^+ < 0.16$.

(vi) Exclusive and inclusive decay widths and branching ratios calculated with $g_s = -0.90$ and $g_o = -2.40$ are consistent with all existing data.

Our calculated results for two-body decays are shown in the fourth column of Table II, and for inclusive decays, in Table V. In particular we predict $\Gamma(F^+ \to \overline{K}^0 K^+, \eta\pi^+) = (1 \text{ to } 2) \times 10^{11}$ sec^{-1}, $B(F^+ \to e\nu X) \approx 5\%$, and the branching ratios for Cabibbo-suppressed decays of D^0, D^+, and F^+ to be $\approx 10\%$, 20%, and 5%, respectively. We hope that these will be tested by experiments in the near future.

Although it is certain that our model is but a simplification of the real process at work, we believe that phenomenologically the model has isolated the most important aspects of the decay of charmed mesons, and that our analysis has shown quite conclusively the presence of soft gluons in these decays.

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