Chromostatics of two-quark systems

Kimball A. Milton,* William F. Palmer, and Stephen S. Pinsky
The Ohio State University, Department of Physics, Columbus, Ohio 43210
(Received 11 September 1981)

We study an estimate for the mean-field potential between two heavy quarks (qq) using Adler's chromostatics. To do so we work out the pseudocolor charge algebra for the qq system in SU(n) of color, which has not been correctly presented previously. Using the leading-logarithm, renormalization-group-improved Euclidean action for the gluon fields, we find that the mean-field potential depends crucially on the algebraic properties of the sources, and that while the quark-antiquark ($q\bar{q}$) system possesses an at-least-linear potential, as Adler showed, the qq system has infinite energy, and hence is decoupled from the physical spectrum. The physical states exhibit color screening.

I. INTRODUCTION

Adler[1] has recently proposed a semi-classical model for bag formation by heavy quark systems. Using the leading-logarithm, renormalization-group improvement to the local Euclidean Lagrange function,[2–4]

$$\mathscr{L}_{\text{eff}}(F^2)=\tfrac{1}{8}b_0F^2\ln(F^2/e\kappa^2)\ , \tag{1}$$

where b_0 is the first-order β-function coefficient, he minimizes the Euclidean action in the presence of static color sources—"quarks." That minimization, when the sources are *classical objects,* yields (i) an at-least-linear mean-field potential for antiparallel color charges, and (ii) infinite action for parallel color charges. This suggests flux-tube confinement of the quark-antiquark system, and the absence of quark-quark states from the physical spectrum.

However, to approximate the realistic quark system, it is necessary to employ color charges that satisfy the SU(3) color charge algebra.[5,6] Adler schematically indicated[1] how the argument goes through for the $q\bar{q}$ case, where the algebra was first worked out by Giles and McLerran.[7] However, it is not obvious what happens for the quark-quark system, and one might fear that this mechanism leads to automatic confinement of all quark states, through the generation of a permittivity rather similar to that postulated in the MIT bag model.[8] The difficulty of seeing what occurs here is compounded by the apparently large number of transcription sign errors in the published quark-quark charge algebra.[9] Beyond this, there remained the important question of where $q\bar{q}$ confinement occurs, in the singlet or octet channel.

In this paper, we will start by recomputing the SU(n), $n\geq 3$, qq and $q\bar{q}$ color-charge algebra. In the process we note a hitherto unnoticed symmetry between the two algebras. Then we will adapt, with some detail, Adler's analysis to this matrix charge space ("pseudocolor"). This will enable us to derive the results: flux-tube confinement for the $q\bar{q}$ system, but infinite energy for qq. Moreover, we show that for the confined configuration the total (quark + gluon) color charge is zero, so that confinement indeed occurs in a color-singlet channel of the system, which in an appropriate gauge is the quark color-singlet channel.

II. PSEUDOCOLOR ALGEBRA

The essence of Adler's approach[5,6] is that in the static limit the quark color charges must still satisfy the non-Abelian color algebra of SU(n) QCD, and that causality requires that spatially separated quark charges commute. This recognition is implemented by requiring that the two charges lie in an $n^2\times n^2$ direct-product space, the direct product of the $n\times n$ U(n) charges,

$$Q^a=\frac{\lambda^a}{2},\ \ \bar{Q}^a=-\frac{\lambda^{a*}}{2}\ ,$$
$$a,b,c=0,1,2,\ldots,n^2-1,\ \ \lambda^0=\left[\frac{2}{n}\right]^{1/2}1\ . \tag{2}$$

Bimatrix-valued sources imply, through the field equations, bimatrix-valued gluon fields. Thus the static limit of the non-Abelian configuration of two quarks with their associated gluon fields leads naturally to a bimatrix-valued "chromostatics."

The U(n) algebra of each charge is given by

$$\mathrm{tr}Q^aQ^b=\mathrm{tr}\bar{Q}^a\bar{Q}^b=\tfrac{1}{2}\delta^{ab}\ , \qquad (3)$$

$$Q^aQ^b=q^{abc}Q^c\ , \qquad (4a)$$

$$\bar{Q}^a\bar{Q}^b=\bar{q}^{abc}\bar{Q}^c\ , \qquad (4b)$$

$$q^{abc}=\tfrac{1}{2}(d^{abc}+if^{abc})\ , \qquad (5a)$$

$$\bar{q}^{abc}=\tfrac{1}{2}(-d^{abc}+if^{abc})\ , \qquad (5b)$$

in terms of the symmetric and antisymmetric structure constants of SU(n), and

$$q^{0bc}=\tfrac{1}{2}\left[\frac{2}{n}\right]^{1/2}\delta^{bc}\ . \qquad (5c)$$

The algebra of the direct-product space is defined in terms of the SU(n) outer product

$$[u,v]^A\equiv\frac{i}{2}f^{ABC}(u^Bv^C+v^Cu^B)\ , \qquad (6)$$

where $A,B,C,\ldots=1,2,3,\ldots,n^2-1$. We choose as the basis for the qq and $q\bar{q}$ pseudocolor algebra, respectively, the following:

$$\alpha_i^A=G_i^{Abc}Q_1^b\otimes Q_2^c\ , \qquad (7a)$$

$$\beta_i^A=G_i^{Abc}Q_i^b\otimes\bar{Q}_2^c\ , \qquad (7b)$$

where

$$G_1^{Abc}=q^{Abc},\quad G_2^{Abc}=\bar{q}^{Abc}\ ,$$
$$G_3^{Abc}=\sqrt{2n}\,\delta^{Ab}\delta^{c0},\quad G_4^{Abc}=\sqrt{2n}\,\delta^{Ac}\delta^{b0}\ . \qquad (7c)$$

The qq algebra is given by

$$[\alpha_i,\alpha_j]^A=A_{ij}^k\alpha_k^A\ , \qquad (8)$$

where the A_{ij}^k are computed most easily using the q^{abc} identities given in Table I of Ref. 5. The A's are tabulated in Table I. The $q\bar{q}$ algebra turns out to differ from this only by an overall sign:

$$[\beta_i,\beta_j]^A=-A_{ij}^k\beta_k^A\ . \qquad (9)$$

Actually, it should be noted that there are only three independent products in Table I, say those corresponding to A_{12}^k, A_{13}^k, and A_{34}^k, since the others are determined by charge interchange symmetry and complex conjugation.

The pseudocolor algebra of (8) and (9) has the structure of SU(2)$\times$U(1). A normalized[10] diagonal basis is, for qq

$$e_1^A=\frac{2}{n}(\alpha_3^A-\alpha_4^A)=\frac{2}{n}(Q_1^A\otimes 1-1\otimes Q_2^A)\ ,$$
$$e_2^A=\frac{4}{n}(\alpha_1^A-\alpha_2^A)=\frac{4}{n}d^{ABC}Q_1^B\otimes Q_2^C+\frac{4}{n^2}(Q_1^A\otimes 1+1\otimes Q_2^A)\ ,$$
$$e_3^A=-\frac{4}{n}i(\alpha_1^A+\alpha_2^A)=\frac{4}{n}f^{ABC}Q_1^B\otimes Q_2^C\ ,$$
$$e_4^A=2(n^2-4)^{-1/2}\left[\alpha_3^A+\alpha_4^A-\frac{4}{n}(\alpha_1^A-\alpha_2^A)\right]=2\frac{(n^2-4)^{1/2}}{n^2}[Q_1^A\otimes 1+1\otimes Q^A]-8\frac{(n^2-4)^{-1/2}}{n}d^{ABC}Q_1^B\otimes Q_2^c\ , \qquad (10)$$

in terms of which the SU(2)$\times$U(1) algebra is manifest:

$$[e_i,e_j]^A=i\epsilon_{ijk}e_k^A,\quad \{i,j,k\}=\{1,2,3\}\ ,$$
$$[e_i,e_4]^A=0\ , \qquad (11)$$

and the basis satisfies the trace normalization condition

$$\mathrm{Tr}e_i^Ae_j^B=(4/n)\delta_{ij}\delta^{AB}\ . \qquad (12)$$

The normalized diagonal basis for $q\bar{q}$, $\{\tilde{e}_i\}$, is obtained by replacing α_i^A by β_i^A, and introducing an extra (-1) factor in the definition of $\tilde{e}_1^A$. Thus

$$\tilde{e}_1^A=\frac{2}{n}(Q_1^A\otimes 1+1\times\bar{Q}_2^A)\ ,$$
$$\tilde{e}_2^A=\frac{4}{n}d^{ABC}Q_1^B\otimes\bar{Q}_2^C-\frac{4}{n^2}(Q_1^A\otimes 1-1\otimes\bar{Q}_2^A)\ ,$$
$$\tilde{e}_3^A=\frac{4}{n}f^{ABC}Q_1^B\otimes\bar{Q}_2^C\ ,$$
$$\tilde{e}_4^A=\frac{-2(n^2-4)^{1/2}}{n^2}(Q_1^A\otimes 1-1\otimes\bar{Q}_2^A)-\frac{8(n^2-4)^{-1/2}}{n}d^{ABC}Q_1^B\otimes\bar{Q}_2^C\ . \qquad (13)$$

TABLE I. $8A_{ij}^k$ for qq pseudocolor algebra.

$ij \backslash k$	1	2	3	4
12	0	0	$-\frac{n}{2}$	$\frac{n}{2}$
13	$-2n$	0	1	-1
14	$2n$	0	1	-1
23	0	$2n$	1	-1
24	0	$-2n$	1	-1
34	8	8	0	0

The principal result we shall need in the following is the expansion of the charges in the pseudocolor basis. For qq, we have

$$Q_1^A \otimes 1 = \frac{n}{4} e_1^A + \tfrac{1}{2} e_2^A + \tfrac{1}{4}(n^2-4)^{1/2} e_4^A \,, \qquad 1 \otimes Q_2^A = -\frac{n}{4} e_1^A + \tfrac{1}{2} e_2^A + \tfrac{1}{4}(n^2-4)^{1/2} e_4^A \,, \tag{14}$$

while for $q\bar{q}$,

$$Q_1^A \otimes 1 = \frac{n}{4} \tilde{e}_1^A - \tfrac{1}{2} \tilde{e}_2^A - \tfrac{1}{4}(n^2-4)^{1/2} \tilde{e}_4^A \,, \qquad 1 \otimes \bar{Q}_2^A = \frac{n}{4} \tilde{e}_1^A + \tfrac{1}{2} \tilde{e}_2^A + \tfrac{1}{4}(n^2-4)^{1/2} \tilde{e}_4^A \,. \tag{15}$$

The reader will note at once the crucial differences between qq and $q\bar{q}$. $Q_1^A \otimes 1$ and $1 \otimes Q_2^A$ can be rotated by a local SU(2) pseudocolor gauge transformation so that they are parallel while $Q_1^A \otimes 1$ and $1 \otimes \bar{Q}_2^A$ cannot; they can only be made antiparallel by such a transformation.[11]

III. MEAN-FIELD POTENTIAL

As stated in the Introduction, when the gauge currents j_μ^A describe two static heavy quarks or a quark and antiquark, the resulting chromostatics is given in terms of sources and gauge fields that are bimatrices, belonging to the SU(2) $\otimes$ U(1) algebra constructed in the previous section.[12] The appropriate Euclidean action functional is[1]

$$W[j_\mu^A] = \int d^3x \, [\mathscr{L}_{\text{eff}}(F^2) - \mathscr{L}_{\text{eff}}(\kappa^2)] - \frac{1}{n^2} \text{Tr} \int d^3x c_\mu^A(x) j_\mu^A(x) \,, \tag{16}$$

where $\mathscr{L}_{\text{eff}}$ is given by (1) and

$$F^2 = \frac{1}{n^2} \text{Tr}[E^{A,j}E^{A,j} + B^{A,j}B^{A,j}], \quad j = 1,2,3 \,, \tag{17}$$

and where $E^{A,j}$ and $B^{A,j}$ are the color electric and magnetic fields, which for statics are

$$E^{A,j} = -\partial_j c^{A,0} + i[c^j, c^0]^A \,, \qquad B^{A,j} = \epsilon^{jkl} \left[\partial_k c^{A,l} - \frac{i}{2}[c^k, c^l]^A \right] . \tag{18}$$

All colored quantities are to be expanded in the pseudocolor basis ($\{e_i\}$ for qq, $\{\tilde{e}_i\}$ for $q\bar{q}$) as follows:

$$c^{A,\mu} = c_r^\mu e_r^A, \quad c_r^\mu = (\vec{c}^{\,\mu}, c_4^\mu) \,, \tag{19}$$

and then

$$\vec{E}^j = -\mathscr{D}_j \vec{c}^{\,0}, \quad \mathscr{D}_j = \partial_j + \vec{c}^{\,j} \,, \qquad E_4^j = -\partial_j c_4^0 \,, \qquad \vec{B}^j = \epsilon^{jkl}(\partial_k \vec{c}^{\,l} + \tfrac{1}{2} \vec{c}^{\,k} \times \vec{c}^{\,l}) \,, \qquad B_4^j = \epsilon^{jkl} \partial_k c_4^l \,, \tag{20}$$

and

$$F^2 = 4 \frac{n^2-1}{n^3} (\vec{E}^j \cdot \vec{E}^j + \vec{B}^j \cdot \vec{B}^j + E_4^j E_4^j + B_4^j B_4^j) \,. \tag{21}$$

Thus, effectively, the SU(n) color problem is reduced to an SU(2)$\times$U(1) chromostatics one.

The static sources are taken to be, in the pseudocolor basis, $j_r^\mu = (\vec{j}^{\,0}, j_4^0)$, where

$$\vec{j}^{\,0}(x) = \vec{Q}_1 \delta^3(x - x_1) + \vec{Q}_2 \delta^3(x - x_2) \,, \qquad j_4^0(x) = Q_1^4 \delta^3(x - x_1) + Q_2^4 \delta^3(x - x_2) \,, \tag{22}$$

where, according to (14),

$$\vec{Q}_1 = \left[\frac{n}{4}, \frac{1}{2}, 0 \right], \quad \vec{Q}_2 = \left[-\frac{n}{4}, \frac{1}{2}, 0 \right], \qquad Q_1^4 = \tfrac{1}{4}(n^2-4)^{1/2}, \quad Q_2^4 = \tfrac{1}{4}(n^2-4)^{1/2} \,, \tag{23}$$

for the qq configuration, and, according to (15),

$$\vec{Q}_1 = \left[\frac{n}{4}, -\frac{1}{2}, 0 \right], \quad \vec{Q}_2 = \left[\frac{n}{4}, \frac{1}{2}, 0 \right], \qquad Q_1^4 = -\tfrac{1}{4}(n^2-4)^{1/2}, \quad Q_2^4 = \tfrac{1}{4}(n^2-4)^{1/2} \,, \tag{24}$$

for the $q\bar{q}$ configuration. The field equations which follow from variation of the action (16) are

$$\mathscr{D}_j \epsilon \vec{E}^j = \vec{j}^{\,0}, \quad \partial_j \epsilon E_4^j = j_4^0 \,, \tag{25}$$

where

$$\epsilon(F^2) = \partial \mathscr{L}_{\text{eff}}(F^2)/\partial(\tfrac{1}{2}F^2) = \tfrac{1}{4} b_0 \ln(F^2/\kappa^2) \,, \tag{26}$$

and

$$\epsilon^{ijk} \mathscr{D}_j \epsilon \vec{B}^k = \vec{c}^{\,0} \times \epsilon \vec{E}^i, \quad \epsilon^{ijk} \partial_j \epsilon B_4^k = 0 \,. \tag{27}$$

Applying $\mathscr{D}_i$ to the $\vec{B}^k$ equation implies

$$\vec{c}^{\,0} \times \vec{j}^{\,0} = 0 \,; \tag{28}$$

that is, at the charges, the SU(2) parts of the scalar potential and the pseudocolor charge are parallel or antiparallel. In fact Eq. (25) implies that these are parallel. In view of the observation made at the end of the previous section, it appears natural to perform a local SU(2) rotation so that, for qq,

$$\vec{Q}_1=(0,0,|Q|),\ \vec{Q}_2=(0,0,|Q|)\ , \tag{29}$$

and, for $q\bar{q}$,

$$\vec{Q}_1=(0,0,-|Q|),\ \vec{Q}_2=(0,0,|Q|)\ , \tag{30}$$

where $|Q|=\frac{1}{4}(n^2+4)^{1/2}$. However such a gauge rotation does not preserve the commutation relations of the charges at x_1 and x_2. In fact no pseudocolor gauge rotation will preserve these commutation relations.[13] The commutation relations should be viewed as a constraint that picks out the physical solution from the set of pseudocolor gauge equivalent solutions. Therefore the physical solution for the problem at hand will be obtained by making a pseudocolor rotation on our solution that reverses the rotation that we have just performed on the charges.[14] Without further loss of generality we can take the gauge where

$$\vec{c}^{\,0}(x)=(0,0,c(x))\ . \tag{31}$$

Then

$$\vec{j}^{\,0}\cdot\vec{c}^{\,0}=j(x)c(x)\ , \tag{32}$$

where

$$j(x)=|Q|[\delta^3(x-x_2)\pm\delta^3(x-x_1)]\ , \tag{33}$$

where $\pm$ refers to qq, $q\bar{q}$, respectively. Here, the antisymmetric form of j for $q\bar{q}$ corresponds to a color singlet state.

Once this gauge choice is made, F^2 takes the form

$$F^2=N[(\partial_j c)^2+(\vec{c}^{\,j}\times\hat{z})^2c^2+\vec{B}^j\cdot\vec{B}^j+E_4^jE_4^j+B_4^jB_4^j]\ , \tag{34}$$

where $N=4(n^2-1)/n^3$. Following Adler[1] we minimize W, which, of course, is equivalent to solving the field equations. Since B_4^j obeys a homogeneous field equation, we may take $B_4^j=0$. Then, as in Adler's case, $\vec{c}^{\,j}$ may be taken to be parallel to $\hat{z}$. Now we define some two-vector quantities:

$$\underline{c}=(c,c_4^0),\ \underline{j}=(j,j_4^0),\ \underline{E}^j=-\partial_j\underline{c}\ . \tag{35}$$

In terms of these we seek to minimize

$$W[\underline{c},B^2]=\int d^3x\{\mathscr{L}_{\rm eff}[N(\underline{E}^2+B^2)]-\mathscr{L}_{\rm eff}(\kappa^2)-N\underline{c}\cdot\underline{j}\}\ , \tag{36}$$

where

$$\underline{E}^2=\underline{E}^j\cdot\underline{E}^j,\ B^2=\vec{B}^j\cdot\vec{B}^j\ . \tag{37}$$

Adler next minimizes W with respect to $\vec{B}^j$ variations:

$$\tilde{\mathscr{L}}_{\rm eff}(N\underline{E}^2)=\min\{\mathscr{L}_{\rm eff}[N(\underline{E}^2+B^2)]-\mathscr{L}_{\rm eff}(\kappa^2)\}\ , \tag{38a}$$

where

$$\tilde{\mathscr{L}}_{\rm eff}(N\underline{E}^2)=\begin{cases}0 \text{ if } N\underline{E}^2\le\kappa^2\ , & (38b)\\ \mathscr{L}_{\rm eff}(N\underline{E}^2)-\mathscr{L}_{\rm eff}(\kappa^2) \text{ if } N\underline{E}^2\ge\kappa^2\ ,\end{cases}$$

where in the first case, B^2 fills in to minimize $\mathscr{L}_{\rm eff}$, that is,

$$N(\underline{E}^2+B^2)=\kappa^2\ , \tag{38c}$$

while in the second

$$B^2=0\ . \tag{38d}$$

Finally we must minimize

$$\tilde{W}[\underline{c}]=\int d^3x[\tilde{\mathscr{L}}_{\rm eff}(N(\partial_j\underline{c})^2)-N\underline{c}\cdot\underline{j}]\ , \tag{39}$$

which implies the flux conservation equation

$$\partial_k\underline{D}^k=\underline{j}\ , \tag{40}$$

where

$$\underline{D}^j=\tilde{\epsilon}\underline{E}^j\ , \tag{41}$$

$$\tilde{\epsilon}=\begin{cases}\epsilon(N\underline{E}^2), & N\underline{E}^2\ge\kappa^2\ ,\\ 0, & N\underline{E}^2\le\kappa^2\ .\end{cases} \tag{42}$$

Note the similarity of the permittivity $\tilde{\epsilon}$ to that of the bag model.[8] The seemingly simple equation (40) has not yet been solved.[15] Instead Adler[1] follows a flux conservation argument of 't Hooft.[16] Adapting that argument we find the following inequality satisfied by the mean-field potential

$$V=-\tilde{W}\ge\kappa N\int d^3x D\ , \tag{43}$$

where $D=(\underline{D}^j\cdot\underline{D}^j)^{1/2}$, provided the source term in (39) is integrated by parts, and the surface term discarded. Now

$$\int d^3x\, D=\int dl\, dA D=\int dl\, d\,|\underline{\Phi}|\ge|\underline{\Phi}|\,l_{\min}\ , \tag{44}$$

where $l_{\min}$ is the length of the shortest flux line (which may be ∞), and $\underline{\Phi}$ is the flux emanating from a single charge.

Now we refer explicitly to the qq, $q\bar{q}$ systems, where the sources are given in terms of the charge algebras by (14) and (15). After carrying out the charge rotation given in (29) and (30) we find for the total flux at infinity

$$\underline{\Phi}_{\rm tot}=\int d^3x\underline{j}(x)$$
$$=(\tfrac{1}{4}(n^2+4)^{1/2},\tfrac{1}{4}(n^2-4)^{1/2})$$
$$\times\int d^3x[-\delta^3(x-x_1)+\delta^3(x-x_2)]=0 \tag{45a}$$

for $q\bar{q}$, but

$$\underline{\Phi}_{\rm tot}=\int d^3x\underline{j}(x)=(\tfrac{1}{2}(n^2+4)^{1/2},\tfrac{1}{2}(n^2-4)^{1/2}) \tag{45b}$$

for qq. Thus, for the $q\bar{q}$ case, the flux lines begin on one charge and terminate on the other; the minimum flux line has a length equal to the separation between the charges, $R=|\vec{x}_1-\vec{x}_2|$, so (43) implies for the mean-field potential

$$V\geq\kappa N^{1/2}|\underline{Q}|R \tag{46}$$

where the magnitude of the charge on either quark is

$$|\underline{Q}|=\tfrac{1}{4}[(n^2+4)+(n^2-4)]^{1/2}=\frac{\sqrt{2}n}{4}\ . \tag{47}$$

Thus, the potential between quark and antiquark is at least linear. On the other hand, for the quark-quark system, Gauss's law (45b) implies that the flux lines are semi-infinite; hence the action is infinite, and the qq state does not lie in the physical spectrum.

However, although this conclusion for qq is correct, the argument is a bit more subtle. In fact, $-\int d^3x\,\underline{c}\cdot\underline{j}$ is infrared finite, and indeed simple asymptotic estimates

$$[D\sim 1/\rho^2,(E-\kappa N^{-1/2})\sim 1/\rho^2]$$

show that the surface term omitted above cancels the IR divergence of the $-\int d^3x\,\underline{E}_j\cdot\underline{D}_j$ term obtained on partial integration. $\int d^3x\tilde{\mathscr{L}}_{\rm eff}$ is IR finite, since $\tilde{\mathscr{L}}(\kappa^2)=\tilde{\mathscr{L}}'(\kappa^2)=0$. The point is that for unbounded flux configurations the above form of the action cannot be identified with the energy, but instead the canonical energy density is[2,17]

$$\theta_{00}=N\underline{E}_j\cdot\underline{D}_j-\tilde{\mathscr{L}}_{\rm eff}\ . \tag{48}$$

($\theta_{\mu\nu}$ gives the correct trace anomaly.) Integrating this over a large spherical volume of radius ρ, we find, from the first term

$$\epsilon\sim\kappa N^{1/2}|\underline{Q}|\rho\ , \tag{49}$$

linearly diverging as $\rho\to\infty$. Thus the qq state is a physically inaccessible one of infinite energy.

IV. COLOR SCREENING

We should point out that the flux arguments are not special to the particular gauge we have been using with $\vec{c}^{\,j}$ and $\vec{c}^{\,0}$ proportional to $\hat{z}$. In general the total color charge is carried not only by the quarks but by the gluons as well. The pseudocolor charge of the latter is found by varying $\int d^4x\mathscr{L}_{\rm eff}$ with respect to

$$\delta\vec{c}_\mu=\delta\vec{\lambda}(x)\times\vec{c}_\mu\ , \tag{50}$$

$$\int d^4x\,\delta\mathscr{L}_{\rm eff}=-\int\frac{\partial\mathscr{L}_{\rm eff}}{\partial(\frac{1}{2}F^2)}(\partial^0\delta\vec{\lambda})\cdot\vec{E}^j\times\vec{c}^{\,j}\ , \tag{51}$$

whence the gluon pseudocolor charge is

$$\vec{Q}^g=\int d^3x\epsilon\vec{E}^j\times\vec{c}^{\,j}\ . \tag{52}$$

The quark pseudocolor charge is obtained by integrating (25):

$$\vec{Q}^q=\int d^3x\,\mathscr{D}_j\epsilon\vec{E}^j$$
$$=\int d^3x[\partial_j(\epsilon\vec{E}^j)+\vec{c}_j\times\epsilon\vec{E}_j]\ . \tag{53}$$

Then the total charge,

$$\vec{Q}_{\rm tot}=\int d\sigma^j(\epsilon\vec{E}^j)\ , \tag{54}$$

vanishes for any configuration with vanishing flux at infinity.[18] Thus the $q\bar{q}$ case with antiparallel pseudocolor charges corresponds to confinement in the singlet channel of the composite quark and gluon system. It was only a matter of convenience that we chose $\vec{c}^{\,j}$ and $\vec{c}^{\,0}$ proportional to $\hat{z}$ everywhere. In a general gauge the cancellation of the flux at infinity would reflect the non-Abelian charge-carrying attributes of the gauge fields, as well as that of the static charges. In the particular gauge used for this calculation, the gluon contribution to this flux is zero.

V. CONCLUSION

In summary, the effective one-loop-renormalization-group-improved action coupled with Adler's pseudocolor charge algebra seems to embody a great deal of the essential physics of the QCD of heavy quarks (presumably), linear confinement of a quark and antiquark in the color-singlet channel, together with decoupling from the physical spectrum of nonsinglet states.[19] The next step is to investigate three quark states, which is an order of magnitude[20] more difficult an undertaking, both algebraically and physically.

ACKNOWLEDGMENTS

We are grateful to Stephen Adler for helpful conversations. This work was supported in part by the U. S. Department of Energy.

---

*Present address: Department of Physics, Oklahoma State University, Stillwater, Oklahoma 74078.

[1]S. L. Adler, Phys. Rev. D 23, 2905 (1981), and 5th Johns Hopkins Workshop on Current Problems in Particle Theory, 1981 (unpublished).

[2]H. Pagels and E. Tomboulis, Nucl. Phys. B143, 485 (1978).

[3]G. K. Savvidy, Phys. Lett. 71B, 133 (1977).

[4]There is a formal similarity to the work of R. Fukuda, Phys. Rev. D 21, 485 (1980); R. Fukuda and Y. Kazama, Phys. Rev. Lett. 45, 1142 (1980). However, their effective action is a function of the condensate $\langle F^2 \rangle$. It is related also to the "Copenhagen vacuum": see N. K. Nielsen and P. Olesen, Nucl. Phys. B144, 376 (1980) and J. Ambjørn and P. Olesen, Nucl. Phys. B170, 60 (1980), for example.

[5]S. L. Adler, Phys. Rev. D 17, 3212 (1978).

[6]S. L. Adler, Phys. Lett. 86B, 203 (1979); Phys. Rev. D 20, 3273 (1979); for a review see S. L. Adler, in *Proceedings of the European Conference on Unification of the Fundamental Interactions, Erice, 1980,* edited by S. Ferrara, J. Ellis, and P. van Nieuwenhuizen (Plenum, New York, 1980).

[7]R. Giles and L. McLerran, Phys. Lett. 79B, 447 (1978).

[8]A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D 9, 3471 (1974). For a review see P. Hasenfratz and J. Kuti, Phys. Rep. 40C, 75 (1978).

[9]S.-C. Lee, Phys. Rev. D 20, 1951 (1979).

[10]Note that our normalization differs from that of Ref. 1.

[11]Of course, this distinction disappears in SU(2) since quarks and antiquarks are equivalent there. Indeed, the algebra for $n = 2$ is SU(2), so all charges can be made parallel or antiparallel at will.

[12]Adler shows (Ref. 1) that only the constrained field components are bi-matrices. This allows the use of canonical quantization.

[13]K. A. Milton and S. Pinsky (unpublished).

[14]S. L. Adler (private communication).

[15]S. L. Adler, IAS report (unpublished), and S. L. Adler and T. Piran (unpublished) can now show that the same equations result from either a Euclidean or Minkowskian formulation, since in the interior of the bag $B^2 = 0$. They have made considerable progress in solving the flux equation and have proved that the bag is indeed of finite cross section.

[16]G. 't Hooft, CERN Report No. TH 1902, 1974 (unpublished).

[17]Coincidentally this is the same form one gets for the action when one integrates by parts and discards the surface term.

[18]The SU(2) analog of this was proved by I. Bender, D. Gromes, and H. J. Rothe, Z. Phys. C 5, 151 (1980).

[19]The SU(n) decomposition of $\underline{n \times \bar{n}}$ into $\underline{1}$ and $\underline{n^2 - 1}$, and of $\underline{n \times n}$ into $\underline{\frac{1}{2}(n+1)}$ and $\underline{\frac{1}{2}n(n-1)}$ is explicitly carried out by the aid of suitable projection operators in K. A. Milton, W. Wilcox, W. F. Palmer, and S. S. Pinsky, Oklahoma State University report (unpublished). The results are qualitatively unchanged.

[20]S.-C. Lee, Phys. Rev. D 21, 466 (1980); P. Cvitanovič, R. J. Gonsalves, and D. E. Neville, *ibid.* 18, 3881 (1978).