Strong-coupling expansion with fermions: The formalism and an application

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We review and discuss some properties of the strong-coupling expansion for a single scalar field with a quartic self-interaction. We generalize the method to the case of fermions and mesons, called, maybe unduly, nucleons and pions, interacting through a Yukawa coupling, and propose a renormalization program. We apply the ideas and the techniques to a leading-order calculation of the nucleon anomalous magnetic moments, with the result $\mu_p = -\mu_n = 2.33$.

I. INTRODUCTION

The main purpose of this paper is to generalize and extend to problems involving fermions the formalism for a strong-coupling expansion that we have developed for the scalar ϕ^4 theory in Refs. 1 and 2. This formalism and some modified versions of it were successfully applied to the one-dimensional anharmonic oscillator by Kaiser *et al.*,[3] Bender *et al.*,[4] and ourselves[2]; to the d-dimensional anharmonic oscillator by Ader *et al.*,[5] and to the d-dimensional ϕ^4 theory with an O(N) internal symmetry by Parga *et al.*[6]

In this paper we shall treat the case of a spinor-isospinor field ψ strongly coupled to an isovector-pseudoscalar field $\vec{\phi}$ through a Yukawa interaction. For simplicity, we call ψ and $\vec{\phi}$ the nucleon and the pion field, but we do not claim that the true nucleons and pions necessarily have something to do with our elementary fields, and the Lagrangian model that we study can be considered as nothing more than a simple support for the setting up of the formalism.

Certainly, according to the generally accepted ideas about the nature of the strong interactions, a major progress would be the development of methods for strong-coupling expansions in the framework of continuum non-Abelian gauge theories; in this respect, our work represents only a very preliminary step. However, our feeling is that ultimately the forces between hadrons will be described, at a phenomenological level, to some extent and to some approximation by simple Lagrangian models that, quite possibly, would not differ too much from the old-fashioned Yukawa-type or quartic interactions. From this point of view the method we propose here could be of interest in the understanding and in the calculation of low-energy strong effects. We have in mind, for instance, the production of low-lying resonances and the renormalization of static parameters related to weaker interactions. In particular, the first-order computation of the nucleon magnetic moments, which is the object of the last section of this paper, could be considered as the first step towards a realistic calculation.

Our paper is organized as follows. In Sec. II we review the known essential results for a single scalar field with a quartic self-interaction. The natural framework where to obtain strong-coupling expansions is provided by the path-integral formulation of quantum mechanics or field theory, and the very simple idea is to expand, not the interaction, but rather the free part of the Lagrangian after a field rescaling. Clearly the Green's functions will turn out expanded in negative powers of the coupling constant, but they exhibit two striking features: first, they are polynomials in the external momenta; second, the Green's functions critically depend on the volume Δ of the cells in space-time, whose discretization seems unavoidable in the computation of the functional integrals. These two features make difficult the connection with the usual renormalization program, but the work of Refs. 2–6 indicates that the situation is far from being hopeless.

In Sec. III we extend the formalism established for a scalar field to the more realistic situation where fermions are present and interact with mesons. We treat explicitly the case of nucleons and pions interacting through a Yukawa coupling, but consideration of other types of particles and of larger multiplets would require only a few modifications. An amusing result is that also in our context a pion self-coupling accompanies the Yukawa coupling; but, in contrast to what happens in usual perturbative calculations, the two interactions cannot be disentangled, even at the lowest order. In this case, the Green's functions turn out expanded in powers of the two parameters $\lambda^{1/2}g^{-2}$ and $\lambda^{-1/2}$ (λ and g are the π-π and the π-N coupling constants), and exhibit the same unusual features as in the case of a scalar field alone.

In Sec. IV we try to establish a connection with the traditional renormalization theory. The idea is again very simple: if our propagator is a polynomial, an appropriate Padé approximant can exhibit a pole, and from the pole we can define the physical mass and the wave-function renormalization constant. We shall not go beyond a second-order approximation for the nucleon propagator, but close examination of higher-order corrections will, it is hoped, suggest some more sophisticated procedure to restore in the propagator and in all other Green's functions the expected analytical structure.

Finally, in Sec. V, we put the strongly interacting nucleons and pions in a weak external electromagnetic field A_μ and face the old problem of computing the nucleon anomalous magnetic moments μ_p and μ_n. Our primary motivation is to test the ability of the formalism to treat, at least in an encouraging way, a real problem, and we limit our ambitions to a leading-order calculation. A nearly immediate result of our approach is that the isoscalar magnetic moment, $\mu_S=(\mu_p+\mu_n)/2$, is one order in $\lambda^{1/2}g^{-2}$ smaller than the isovector one, $\mu_V=(\mu_p-\mu_n)/2$; thus, to the leading order in that parameter, we have $\mu_p=-\mu_n$. To the leading order in $\lambda^{-1/2}$ also, we compute $\mu_p=-\mu_n=\frac{7}{3}$.

Section VI is devoted to some comments and discussions.

II. GENERAL FORMALISM FOR A SCALAR FIELD

Before considering the interaction between nucleons and pions, let us recall how the method works in the simpler case of a real scalar field ϕ, with a $\lambda\phi^4$ self-interaction. Our purpose here is to illustrate the major features of the strong-coupling expansion, and actually we condense and recast part of the material contained in Refs. 2–4.

The starting point is provided by $Z(J)$, the generating functional of the n-point unamputated Green's functions $G(x_1\cdots x_n)$, or by $W(J)$, the generating functional of the connected, unamputated Green's functions $G_c(x_1\cdots x_n)$:

$$Z(J)=\exp[W(J)]=N\int \mathfrak{D}\phi \exp\left(i\int dx(\mathfrak{L}_0-\lambda\phi^4+J\phi)\right), \tag{1}$$

$$G_c(x_1\cdots x_n)=\frac{1}{i^n}\left.\frac{\partial^n W(J)}{\partial J(x_1)\cdots\partial J(x_n)}\right|_{J=0}. \tag{2}$$

In Eq. (1), $\mathfrak{L}_0$ is the free Lagrangian

$$\mathfrak{L}_0=\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi-\tfrac{1}{2}\mu^2\phi^2$$

and J an arbitrary source; the coupling constant λ is supposed to be positive; N is a normalization coefficient, independent of J, such that $Z(J=0)=1$. In what follows we shall omit this coefficient, because it is inessential in computing the connected Green's functions, but we shall keep in mind that we can absorb undefined factors (independent of J) coming from the functional integration over ϕ.

We rewrite Eq. (1) in the equivalent form

$$Z(J)=\int \mathfrak{D}\phi \exp\left(\frac{i}{2}\int dy_1dy_2\phi(y_1)D(y_1-y_2)\phi(y_2)-i\lambda\int dx\,\phi^4+i\int dx\,J\phi\right), \tag{3}$$

where $D(x-y)$ is the inverse free propagator

$$D(x-y)=-(\Box_x+\mu^2)\delta(x-y)=\int\frac{d^4k}{(2\pi)^4}(k^2-\mu^2)e^{-ik(x-y)}. \tag{4}$$

The idea we want to exploit is very simple and attractive: instead of expanding as a perturbation the factor

$$\exp\left(-i\lambda\int dx\,\phi^4\right)=\sum_{m=0}^{\infty}\frac{1}{m!}(-i\lambda)^m\left(\int dx\,\phi^4(x)\right)^m,$$

we rescale the field ϕ so that $\lambda\phi^4\to\phi^4$, and consider rather as a perturbation the factor

$$\exp\left(\frac{i}{2}\frac{1}{\lambda^{1/2}}\int dy_1dy_2\phi(y_1)D(y_1-y_2)\phi(y_2)+\frac{i}{\lambda^{1/4}}\int dx\,J\phi\right). \tag{5}$$

Expansion of the source term leads immediately to

$$G(x_1\cdots x_n)=\frac{1}{\lambda^{n/4}}\int\mathfrak{D}\phi\,\phi(x_1)\cdots\phi(x_n)\exp\left(\frac{i}{2\lambda^{1/2}}\int dy_1dy_2\phi D\phi-i\int dx\,\phi^4\right) \tag{6}$$

and a further expansion of the $\phi D\phi$ term leads to

$$G(x_1\cdots x_n)=\frac{1}{\lambda^{n/4}}\sum_{m=0}^{\infty}\frac{1}{m!}\left(\frac{i}{2\lambda^{1/2}}\right)^m G^{(m)}(x_1\cdots x_n), \tag{7}$$

where the mth-order contribution $G^{(m)}(x_1\cdots x_n)$ is given by

$$G^{(m)}(x_1\cdots x_n)=\int \mathfrak{D}\phi\, \phi(x_1)\cdots\phi(x_n)\left(\int dy_1 dy_2\, \phi(y_1)D(y_1-y_2)\phi(y_2)\right)^m \exp\left(-i\int dx\, \phi^4\right). \tag{8}$$

The Green's functions (6) or (7) contain as a factor the vacuum-vacuum amplitude

$$\langle 0|0\rangle=\sum_{m=0}^{\infty}\frac{1}{m!}\left(\frac{i}{2\lambda^{1/2}}\right)^m V^{(m)}\equiv Z(J=0), \tag{9}$$

$$V^{(m)}=\int \mathfrak{D}\phi\left(\int dy_1 dy_2\, \phi(y_1)D(y_1-y_2)\phi(y_2)\right)^m \exp\left(-i\int dx\, \phi^4\right), \tag{10}$$

which differs from one if $Z(J)$ has not been correctly normalized. But once the $G^{(m)}$ are computed, it is easy by simple inspection to recognize and suppress disconnected contributions to $G^{(m)}$ coming either from vacuum diagrams $V^{(i)}$ or from products of Green's functions of lower dimensions. Thus Eqs. (7) and (8) are the basic relations to compute finally the connected Green's functions.

The next job is to give a meaning to the functional integrals (8) and compute them. As usual we adopt a discrete procedure by dividing the space-time in cells of volume Δ, transforming integrals into sums

$$\int dy \to \Delta \sum_i ,$$

and considering the x_i, y_i arguments of the ϕ's as labels of cells. This procedure causes no trouble in the usual perturbative expansion, essentially because there the functional integrals are of Gaussian type, and the parameter Δ disappears at the end of the calculations. In our formulation, this will not be the case, and Δ will play the role of an inverse cutoff.

Let us call occupied the cells labeled by one or more arguments of the ϕ's in Eq. (8), and unoccupied all other cells; let us call configuration a particular way of occupying the cells, namely a particular way of distributing the x_i, y_i arguments among the cells. Then Eq. (8) can be written as

$$G^{(m)}(x_1\cdots x_n)=\int \prod dy_j D(y_1-y_2)\cdots D(y_{2m-1}-y_{2m})F^{(m)}(x_1\cdots x_n;\, y_1\cdots y_{2m}) \tag{11}$$

(to simplify, we still use the integral notation), where

$$F^{(m)}(x_1\cdots x_n;\, y_1\cdots y_{2m})=\sum_{\text{config}}\prod_{\text{occup}}\int \phi^{p_j}e^{-i\Delta\phi^4}d\phi \prod_{\text{unocc}}\int e^{-i\Delta\phi^4}d\phi. \tag{12}$$

The factor $F^{(m)}$ arises from the functional integration over ϕ and is the sum over all configurations of the contributions of the unoccupied cells, and of the contributions of the cells occupied by p_j arguments of the ϕ's. We can divide $Z(J)$, and so all Green's functions, by the undefined factor (independent of J)

$$\prod_{\text{all cells}}\int e^{-i\Delta\phi^4}d\phi,$$

so that the new $F^{(m)}$ take the form

$$F^{(m)}(x_1\cdots x_n;\, y_1\cdots y_{2m})=\sum_{\text{config}}\prod_{\text{occup}}\left(\int \phi^{p_j}e^{-i\Delta\phi^4}d\phi\Big/\int e^{-i\Delta\phi^4}d\phi\right). \tag{13}$$

With this normalization, $V^{(0)}$ in Eq. (10), but still not $Z(J=0)$, will be equal to one.

We see that $F^{(m)}$ depends on x_i, y_i only through the way these arguments are distributed into the cells and not on their value; actually, because of the $\phi\to-\phi$ symmetry of the interaction, only cells occupied by an even number of arguments do contribute. Thus the $F^{(m)}$ contain as factors Kronecker δ's like δ_{y_i,y_j}, δ_{y_i,x_i}. In view of coming back later to the continuum, we replace Kronecker δ's by factors $\Delta\delta(y_i-y_j)$, $\Delta\delta(y_i-x_i)$. The consistency of this replacement can be checked by dimensional considerations and verified in situations like

$$\int dx\, dy\, f(x,y)\delta(x-y)\to\sum_i\sum_j \Delta^2 f(x_i,y_j)\frac{1}{\Delta}\delta_{x_i,y_j}=\sum_i \Delta f(x_i,x_i)\to\int dx\, f(x,x).$$

To evaluate the numerical coefficients in Eq. (13), we introduce a damping factor $\exp(-\epsilon\phi^2)$ (we could formulate the theory in Euclidean space as well), and we get

$$\int \phi^{2q} e^{-i\Delta\phi^4} d\phi \Big/ \int e^{-i\Delta\phi^4} d\phi = \frac{\Gamma((2q+1)/4)}{(i\Delta)^{q/2}\Gamma(\frac{1}{4})} \equiv \frac{1}{(i\Delta)^{q/2}} b_{2q}. \tag{14}$$

Equations (7), (8) and (11), (13) are all we need to formally compute, to any order, all Green's functions. As an example let us consider the two-point Green's function (whose connected part is the propagator) up to the second order. First we compute

$$F^{(0)}(x_1,x_2) = \frac{1}{(i\Delta)^{1/2}} \Delta\delta(x_1-x_2) b_2. \tag{15}$$

Here we have only one configuration: the two arguments x_1, x_2 are in the same cell. Next we compute

$$F^{(1)}(x_1x_2;\, y_1y_2) = \frac{1}{i\Delta}[\Delta^2\delta(x_1-x_2)\delta(y_1-y_2)b_2^2 + \Delta^2\delta(x_1-y_1)\delta(x_2-y_2)b_2^2 + \Delta^2\delta(x_1-y_2)\delta(x_2-y_1)b_2^2$$
$$- 3\Delta^3\delta(x_1-x_2)\delta(x_2-y_1)\delta(y_1-y_2)b_2^2 + \Delta^3\delta(x_1-x_2)\delta(x_2-y_1)\delta(y_1-y_2)b_4]. \tag{16}$$

In this case we have two kinds of configurations: the one (three of this kind) where the four points are distributed by pairs into two distinct cells [the first three terms of Eq. (16)], and the one where the four arguments are in the same cell [the last term in Eq. (16)]. The third term is a correction taking into account the possibility that the two cells of the first configurations "accidentally" coincide. We suppress the first term of Eq. (16) because after integration over y it will reproduce $F^{(0)}$ multiplied by the vacuum amplitude $V^{(1)}$, Eq. (10). From Eq. (11) we get

$$G_C^{(0)}(x_1,x_2) = F^{(0)}(x_1,x_2) = \frac{1}{(i\Delta)^{1/2}} \Delta\delta(x_1-x_2) b_2,$$
$$G_C^{(1)}(x_1,x_2) = \frac{1}{i\Delta}[2\Delta^2 D(x_1-x_2) b_2^2 + \Delta^3(3b_2^2 - b_4)D(0)\delta(x_1-x_2)] \tag{17}$$

and finally, from Eq. (7)

$$G_C(x_1-x_2) = \frac{1}{\lambda^{1/2}} \frac{1}{(i\Delta)^{1/2}} \Delta\delta(x_1-x_2) b_2 + \frac{1}{\lambda}\frac{i}{i\Delta}[\Delta^2 b_2^2 D(x_1-x_2) + \tfrac{1}{2}\Delta^3(3b_2^2-b_4)D(0)\delta(x_1-x_2)] + O\left(\frac{1}{\lambda^{3/2}}\right) \equiv i\Delta_F'(x_1-x_2). \tag{18}$$

The complexity of the expansion rapidly increases at higher orders, but most of the important features of the method are already apparent in the structure of the simple result (18), or can be deduced from inspection of the preceding formulas:

(i) The Green's functions show an explicit dependence on Δ; this parameter cannot be immediately set equal to zero because it multiplies singular factors whose singularities increase with the order of the expansion.

(ii) Apart from constant, even if undefined, coefficients, the Green's functions are made up not only with distributions like

$$\delta(x_1-x_2),\quad D(x_1-x_2),\quad \int dy\, D(x_1-y)D(y-x_2), \ldots,$$

but also with objects like

$$D^2(x_1-x_2),\quad \int dy\, D(x_1-y)D^2(y-x_2), \ldots,$$

which are not distributions.

(iii) As a consequence, when choosing some regularization procedure to handle such undefined quantities, the final independence of the results with respect to the particular regularization should be checked.

(iv) Whatever regularization we adopt, the Fourier transforms of the Green's functions turn out to be polynomials in the external momenta: they do not exhibit the analytical structure which is apparent in the first terms of the usual perturbation expansion.

(v) The expansion of the propagator starts with a term of order $\lambda^{-1/2}$; there is no "first-order approximation" surviving at the limit $\lambda \to \infty$.

In spite of this accumulation of difficulties, the work done in Refs. 2–6 shows that the situation is less dramatic than it would seem. If we treat as a strong interaction a mass term $\lambda\phi^2$ (our formalism can easily deal with this problem), we find, in a nontrivial way, that the propagator is given by

$$i\Delta_F'(k^2) = \frac{-i}{2\lambda}\sum_{n=0}^{\infty}\left(\frac{1}{2\lambda}\right)^n (k^2-\mu^2)^n = \frac{-i}{k^2-\mu^2-2\lambda},$$

as it must be. This suggests that Padé approximants could be an appropriate tool to restore, at least in an approximate way, the expected analytical properties of the Green's functions. More instructive is the study of the anharmonic oscillator in a $\lambda\phi^4$ potential (see Refs. 2–5). The regularization is achieved by replacing the δ distribution by a convenient function $\delta^{\rm reg}(x)$, such that

$$\Delta\,\delta^{\rm reg}(0)=1\,, \tag{19}$$

in order to respect the relation $\Delta\delta(x_i-x_j)\to\delta_{x_i,x_j}$. Of course, there are many choices for $\delta^{\rm reg}(x)$. Referring for more techniques and details to the quoted papers, we only remark here that the possibility of making Δ disappear, the final numerical independence on the regularization, and the good numerical agreement with the known values of the first energy levels indicate that the strong-coupling expansion does have a meaning and a predictive power. Also quite interesting is the work of Ref. 6, where the authors study the ϕ^4 quantum field theory with an $O(N)$ internal symmetry. In the large-N limit, the strong-coupling expansion is able to discover a critical point in $2<d<4$ dimensions and to compute correctly the critical exponent.

In this paper we work in a Minkowski four-dimensional space-time, and we want to preserve the Lorentz covariance; our choice for the regularized δ is

$$\delta^{\rm reg}(x)=\lim_{\epsilon\to0}\int\frac{d^4k}{(2\pi)^4}\,e^{-ikx}e^{-ik^2/\Lambda^2}\exp\left(-\epsilon\sum_{i=0}^{3}(k_i)^2\right)=\frac{i\Lambda^4}{16\pi^2}\exp\left(i\frac{\Lambda^2x^2}{4}\right), \tag{20}$$

where the "cutoff" Λ is related to Δ by condition (19):

$$\frac{i\Delta\Lambda^4}{16\pi^2}=1. \tag{21}$$

In this way, $D(x)$ is regularized through

$$D^{\rm reg}(x)=-(\Box+\mu^2)\delta^{\rm reg}(x)=-\mu^2\delta^{\rm reg}(x)+i\,\frac{d}{d(1/\Lambda^2)}\,\delta^{\rm reg}(x)$$

and, for instance,

$$D^{\rm reg}(0)=-\frac{1}{\Delta}(\mu^2+2i\Lambda^2).$$

III. STRONG INTERACTION BETWEEN NUCLEONS AND PIONS

In this section, we generalize the formalism we have developed for a single scalar field to the more complicated situation where nucleons and pions strongly interact through a Yukawa coupling

$$\mathcal{L}_{\pi N}=ig\bar\psi\gamma_5\vec\tau\psi\cdot\vec\phi\,. \tag{22}$$

In our context it is not evident, *a priori*, whether a pion self-interaction is really necessary or not, but anyway we can take it into account in the total Lagrangian. Thus the Z generating functional takes the form

$$Z(\vec{J},\eta,\bar\eta)=\int\mathfrak{D}\vec\phi\,\mathfrak{D}\psi\,\mathfrak{D}\bar\psi\,\exp\left[\frac{i}{g}\int dy_1dy_2\bar\psi(y_1)S^{-1}(y_1-y_2)\psi(y_2)+\frac{i}{g^{1/2}}\int dx(\bar\psi\eta+\bar\eta\psi)\right.$$
$$+\frac{i}{2}\int dy_1dy_2\,\vec\phi(y_1)D(y_1-y_2)\,\vec\phi(y_2)+i\int dx\,\vec{J}\cdot\vec\phi$$
$$\left.-\int dx\,\bar\psi\gamma_5\vec\tau\psi\cdot\vec\phi-i\lambda\int dx(\vec\phi\cdot\vec\phi)^2\right]. \tag{23}$$

We recognize in Eq. (23) a rescaling $g\bar\psi\psi\to\bar\psi\psi$. The matrix S^{-1} is the inverse free nucleon propagator, defined by

$$S^{-1}(x-y)=(i\gamma^\mu\partial_\mu-M)\delta(x-y)=\int\frac{d^4k}{(2\pi)^4}\,(\not k-M)e^{-ik(x-y)}, \tag{24}$$

while $D(x-y)$ is the same as in Eq. (4). Spin and isospin indices in the ψ, $\bar\psi$, η, $\bar\eta$ fields are understood, and the remarks of Sec. II concerning the normalization of Z at zero sources are still relevant. The fermionic nature of the nucleons is expressed in the fact that ψ, $\bar\psi$, η, $\bar\eta$ are anticommuting c numbers, and Grassmann algebra provides the framework in which to handle such quantities.[7] In what follows we shall need the rule

$$\int\left(\prod_i d\psi_i d\bar\psi_i\right)\exp\left(\sum_{ij}\bar\psi_iA_{ij}\psi_j+\sum_i(\bar\psi_i\eta_i+\bar\eta_i\psi_i)\right)=(\det A)\exp\left(\sum_{ij}\bar\eta_iA^{-1}{}_{ij}\eta_j\right). \tag{25}$$

The general idea is the same as in Sec. II, i.e., to keep the interaction factor $\exp(\bar\psi\psi\phi)$ as it stands and consider all other factors in Z, except possibly for the $\lambda\phi^4$, as perturbations. (We shall omit γ_5 and $\vec{\tau}$ matrices when they are irrelevant in the discussion.) But the presence here of two kinds of fields raises the problem of choosing the order of performing the integrations, either (i) first over the ϕ field, or (ii) first over the ψ, $\bar{\psi}$ fields. Let us examine option (i): still we must decide whether to treat the $\lambda\phi^4$ interaction (i_1) as a small perturbation or (i_2) as a strong interaction.

Case (i_1) is just the usual perturbative problem for a scalar field in the source

$$\vec{J} + i\bar{\psi}\gamma_5\vec{\tau}\psi ,$$

but the Gaussian integration over $\vec{\phi}$ produces, among others, a factor

$$\exp\left[-\frac{i}{2}\int dx\, dy (\bar{\psi}\gamma_5\vec{\tau}\psi)_x \Delta_F(x-y)(\bar{\psi}\gamma_5\vec{\tau}\psi)_y\right],$$

whose presence makes difficult the further integration over ψ, $\bar{\psi}$ (Δ_F is the free-boson propagator). So, we discard possibility (i_1).

Case (i_2) also is not interesting for us: actually, after the rescaling $\lambda\phi^4 \to \phi^4$, the Yukawa interaction should be expanded in powers of $\lambda^{-1/4}$, which is contrary to our philosophy. Actually, the method of Sec. II could be applied, but further integration over ψ, $\bar{\psi}$ would provide Green's functions expanded in positive powers of the coupling constant g.

We are thus left with option (ii). We expand the $\exp(\bar{\psi}S^{-1}\psi)$ factor in Eq. (23) and for the Z_ψ, the fermion-dependent part of Z, we get

$$Z_\psi = \sum_{m=0}^{\infty}\left(\frac{i}{g}\right)^m \frac{1}{m!} Z_\psi^{(m)}(\vec{\phi},\eta,\bar{\eta}), \tag{26}$$

where

$$Z_\psi^{(m)} = \int \mathfrak{D}\psi\mathfrak{D}\bar{\psi}\left[\int dy_1 dy_2 \bar{\psi}(y_1)S^{-1}(y_1-y_2)\psi(y_2)\right]^m \times F_\psi(\vec{\phi},\eta,\bar{\eta}) \tag{27}$$

and where F_ψ contains the Yukawa coupling and the fermion sources:

$$F_\psi = \exp\left(-\int dx\, dy\, \bar{\psi}(x)[\gamma_5\vec{\tau}\cdot\vec{\phi}(x)\delta(x-y)]\psi(y) + \frac{i}{g^{1/2}}\int dx(\bar{\psi}\eta + \bar{\eta}\psi)\right). \tag{28}$$

In order to evaluate the functional integrals in Eq. (27), and to compute Green's functions with fermions on the external lines, we essentially have to take appropriate derivatives with respect to $\bar{\eta}$ (to the left) and to η (to the right) of the basic integral

$$\begin{aligned}\int \mathfrak{D}\psi\mathfrak{D}\bar{\psi}F_\psi &= \det[\gamma_5\vec{\tau}\cdot\vec{\phi}\delta(x-y)]\exp\left(\int dx\, dy\, \frac{1}{g}\bar{\eta}(x)[\gamma_5\vec{\tau}\cdot\vec{\phi}\delta(x-y)]^{-1}\eta(y)\right)\\ &= \prod_K[\vec{\phi}(x_K)\cdot\vec{\phi}(x_K)]^4\exp\left(\frac{1}{g}\int dx\,\bar{\eta}(x)\frac{\gamma_5\vec{\tau}\cdot\vec{\phi}(x)}{\vec{\phi}^{\,2}(x)}\eta(x)\right)\end{aligned} \tag{29}$$

[see Eq. (25)]. As a very simple example, let us look at $Z_\psi^{(1)}$:

$$Z_\psi^{(1)} = -\int dy_1 dy_2 \sum_N \psi_\beta^N(y_2)S^{-1}{}_{\alpha\beta}(y_1-y_2)\bar{\psi}_\alpha^N(y_1)F_\psi ;$$

the minus sign comes from an anticommutation; $N=1,2$ refers to the proton and the neutron. Then we write

$$Z_\psi^{(1)} = -g\int dy_1 dy_2 \sum_N S^{-1}{}_{\alpha\beta}(y_1-y_2)\frac{\vec{\partial}}{\partial\bar{\eta}_\beta^N(y_2)}F_\psi\frac{\overleftarrow{\partial}}{\partial\eta_\alpha^N(y_1)} .$$

If we want $Z_\psi^{(1)}$ at $\eta=\bar{\eta}=0$, we find

$$\begin{aligned}Z_\psi^{(1)}(\vec{\phi};\eta=\bar{\eta}=0) &= -\sum_N\int dy_1 dy_2 \delta(y_1-y_2)S^{-1}{}_{\alpha\beta}(y_1-y_2)(\gamma_5)_{\beta\alpha}\left(\frac{\vec{\tau}\cdot\vec{\phi}(y_1)}{\vec{\phi}^{\,2}(y_1)}\right)^{NN}\prod_K[\vec{\phi}(x_K)\cdot\vec{\phi}(x_K)]^4\\ &= -\int dy\,\mathrm{Tr}[\gamma_5 S^{-1}(0)]\,\mathrm{Tr}\left(\frac{\vec{\tau}\cdot\vec{\phi}(y)}{\vec{\phi}^{\,2}(y)}\right)\prod_K[\vec{\phi}(x_K)\cdot\vec{\phi}(x_K)]^4 = 0 .\end{aligned}$$

It is amusing to recall that an expression similar to our result (29) can be found in the pioneering work of Hori.[8]

Two remarks are in order here. First we note that when computing the pth-order contribution, Eq. (27), to a Green's function with $2f$ fermions on the external lines, the derivatives with respect to η and $\bar{\eta}$ will essentially produce a factor

$\phi^{-(f+p)}$. As soon as $(f+p)>10$, the eight powers of ϕ in Eq. (29), and the two powers of ϕ in $\mathfrak{D}\vec{\phi}$ will be unable to avoid troubles at $\phi=0$. This is not really a serious defect because we could transfer the mass term $(-)M/g\bar{\psi}\psi$ into the interaction $i\bar{\psi}\psi\phi$, still integrate the so-modified Eq. (28), and get a new Eq. (29), complicated by mass terms, but without problems at $\phi=0$. We do not adopt this procedure here because we are interested only in lower-order calculations, and we do not like unnecessary complications in the ϕ integrals. The second remark is that the very presence in Eq. (25) of the factor $\prod_K \phi_K{}^8$ suggests we take into account a strong $\lambda\phi^4$ interaction. Actually, were this interaction absent or perturbatively treated, even in the simplest situations (leading order for Z_ψ, no fermions on the external lines), we should compute integrals of the type

$$\int \mathfrak{D}\vec{\phi} \prod_K [\vec{\phi}(x_K)\cdot\vec{\phi}(x_K)]^4 \phi_i(x_1)\phi_j(x_2)\cdots \exp\left[-\frac{i}{2}\int dy_1 dy_2 \vec{\phi}(y_1)D(y_1-y_2)\vec{\phi}(y_2)\right],$$

which do not have a clean x_i dependence. In contrast, when a strong $\lambda\phi^4$ interaction is present, we can apply the formalism of Sec. II, and we are led to integrals of the type

$$\int \mathfrak{D}\vec{\phi} \prod_K [\vec{\phi}(x_K)\cdot\vec{\phi}(x_K)]^4 \frac{1}{\vec{\phi}^{\,2}(x_1)}\,\frac{1}{\vec{\phi}^{\,2}(x_2)}\cdots\phi_i(x_3)\phi_j(x_4)\cdots\exp\left(-i\lambda\int dx\,\phi^4\right),$$

which can be easily computed.

The structure of the expansion of a Green's function for $2f$ fermions and m mesons on the external lines should now be clear. The f derivatives of expression (29) with respect to the couple η, $\bar{\eta}$ give rise to a factor g^{-f} and to a factor ϕ^{-f}; the pth order contribution to Z_ψ produces another g^{-p} and another ϕ^{-p}; after the rescaling $\lambda\phi^4\to\phi^4$ we get a factor $\lambda^{(f+p)/4}$; as in Sec. II, m external meson lines introduce a new $\lambda^{-m/4}$ and a new ϕ^m, while the qth order in the expansion of $\exp(\phi D\phi)$ contributes a $\lambda^{-q/2}$ and a ϕ^{2q}. Thus, finally, the ϕ dependence is essentially given by a factor $\phi^{(-f+m-p+2q)}$, by the $\exp(-\phi^4)$, and by $\prod_K\phi_K^8$. After the last integration, we get

$$G_{(2f,m)}^{(p,q)} \simeq g^{-f-p}\lambda^{(f+p-m-2q)/4}. \tag{30}$$

Of course, because of the symmetry $\vec{\phi}\to-\vec{\phi}$, $f+p-m$ must be even, p increases by steps of two units, and the relevant parameters in the expansion are $\lambda^{1/2}g^{-2}$ and $\lambda^{-1/2}$.

To show how the method works, let us consider the leading contribution to the nucleon propagator $iS'(x_1-x_2)$, which is the connected part of the two-point Green's function

$$G_2(x_1,x_2) = -\frac{\partial}{\partial\bar{\eta}(x_1)}\,Z(\vec{J},\eta,\bar{\eta})\,\frac{\partial}{\partial\eta(x_2)}\bigg|_{\vec{J}=\eta=\bar{\eta}=0}$$

$$=\langle 0|T[\psi(x_1)\bar{\psi}(x_2)]|0\rangle. \tag{31}$$

In this case we have $f=1$, $m=0$, and for the leading contribution, $p=1$, $q=0$. From Eqs. (26)–(29) we compute first

$$-\frac{i}{g}\,\frac{\partial}{\partial\bar{\eta}(x_1)}\,Z_\psi^{(1)}\,\frac{\partial}{\partial\eta(x_2)}\bigg|_{\eta=\bar{\eta}=0} = \frac{i}{g^2}\prod_K[\vec{\phi}(x_K)\cdot\vec{\phi}(x_K)]^4\gamma_5 S^{-1}(x_1-x_2)\gamma_5\,\frac{\vec{\tau}\cdot\vec{\phi}(x_1)}{\vec{\phi}^{\,2}(x_1)}\,\frac{\vec{\tau}\cdot\vec{\phi}(x_2)}{\vec{\phi}^{\,2}(x_2)}, \tag{32}$$

and then we look for the integral

$$I=\lambda^{1/2}\int\mathfrak{D}\vec{\phi}\prod_K[\vec{\phi}(x_K)\cdot\vec{\phi}(x_K)]^4\exp\left(-\int dx(\vec{\phi}\cdot\vec{\phi})^2\right)\frac{\vec{\tau}\cdot\vec{\phi}(x_1)}{\vec{\phi}^2(x_1)}\,\frac{\vec{\tau}\cdot\vec{\phi}(x_2)}{\vec{\phi}^2(x_2)}. \tag{33}$$

An easy way to evaluate this integral is to introduce "spherical" coordinates in the isospin space, i.e.,

$$\vec{\phi}=\phi\vec{n},\quad \vec{n}^2=1,\quad d\vec{\phi}=\phi^2 d\phi\, d\Omega,$$

so that

$$I=\lambda^{1/2}\int\mathfrak{D}\phi\,\mathfrak{D}\Omega\prod_K\phi_K{}^{10}\exp\left(-i\int dx\,\phi^4\right)\times\frac{\vec{\tau}\cdot\vec{n}(x_1)}{\phi(x_1)}\,\frac{\vec{\tau}\cdot\vec{n}(x_2)}{\phi(x_2)}. \tag{34}$$

The techniques developed in Sec. II can be applied to evaluate integral (34); we make discrete the space-time, factorize, and suppress the factor

$$\prod_{\text{all cells}}\int d\Omega_i\int d\phi_i e^{-i\Delta\phi_i{}^4}\phi_i{}^{10},$$

and we get (x_1 must coincide with x_2, and $[\vec{\tau}\cdot\vec{n}(x_1)]\times[\vec{\tau}\cdot\vec{n}(x_2)]\to 1$)

$$I=\lambda^{1/2}\Delta\,\delta(x_1-x_2)(i\Delta)^{1/2}C_8, \tag{35}$$

where the coefficients C, analogous to the b's of Eq. (14), are defined by

$$C_{2l}=\int d\phi\,\phi^{2l}e^{-\phi^4}\Big/\int d\phi\,\phi^{10}e^{-\phi^4}=\Gamma\left(\frac{2l+1}{4}\right)\Big/\Gamma(\tfrac{1}{4}). \tag{36}$$

Thus, the first approximation $iS'_{(1)}(x_1-x_2)$ to the nucleon propagator is

$$iS'_{(1)}(x_1-x_2)=\frac{i}{g^2}\,\Delta\delta(x_1-x_2)\lambda^{1/2}(i\Delta)^{1/2}(\gamma_5S^{-1}(0)\gamma_5)C_8. \tag{37}$$

The regularization can be achieved by the same procedure as in Sec. II; so, in Eq. (37)

$$S^{-1}(0)=-M\delta(0)=-M/\Delta \tag{38}$$

and, if we express Δ in terms of Λ according to Eq. (21),

$$iS'_{(1)}(x_1-x_2)=-\frac{i}{g^2}\,\frac{4\pi M}{\Lambda^2}\,\lambda^{1/2}C_8\delta(x_1-x_2). \tag{39}$$

IV. A HIGHER-ORDER CONTRIBUTION TO THE NUCLEON PROPAGATOR

In this section we compute a second-order contribution to the nucleon propagator, whose leading term is given by Eq. (39). We are interested in the contribution $p=1$, $q=1$, which arises from the expansion of the kinetic factor $\exp(\phi D\phi)$, and is of the same order in g^{-2} as the first one, but of the next order in $\lambda^{-1/2}$. Actually, for the computation of the nucleon magnetic moments, in the next section, we shall need just this approximation of the nucleon propagator. Furthermore, we illustrate the formalism and the regularization procedure in a still simple but nontrivial situation, and we show how it could be possible to establish a connection with the usual renormalization program.

With respect to the calculation of the preceding section, we see that instead of the integral (33) we must compute

$$\delta I=\frac{i}{2}\int \mathcal{D}\phi\,\mathcal{D}\Omega\prod_K\phi_K{}^{10}\exp\left(-\int dx\,\phi^4\right)\frac{\vec{\tau}\cdot\vec{n}(x_1)}{\phi(x_1)}\,\frac{\vec{\tau}\cdot\vec{n}(x_2)}{\phi(x_2)}\times\int dy_1dy_2\phi(y_1)\phi(y_2)\,\vec{n}(y_1)\vec{n}(y_2)D(y_1-y_2). \tag{40}$$

Integration over $\vec{\phi}$ produces five terms according to the following configurations:

$$\begin{aligned}
&(1)\ x_1=x_2,\ \ y_1=y_2,\ \ x_1\neq y_1\to\Delta^2\delta(x_1-x_2)\delta(y_1-y_2)C_8C_{12}[1-\Delta\delta(x_1-y_1)],\\
&(2)\ x_1=y_1,\ \ x_2=y_2,\ \ x_1=y_2,\ \ x_2=y_1,\ \ x_1\neq x_2\to\tfrac{2}{3}\Delta^2\delta(x_1-y_1)\delta(x_2-y_2)[1-\Delta\delta(x_1-x_2)],\\
&(3)\ x_1=x_2=y_1=y_2\to\Delta^3\delta(x_1-y_1)\delta(x_2-y_2)\delta(x_1-x_2).
\end{aligned} \tag{41}$$

Coefficients C have been defined in Eq. (36). For configurations (1) and (2), we recognize the corrections due to the possibility that the two different pairs of points "accidentally" coincide. The first of the two terms (1) reproduces the leading contribution to the propagator multiplied by a vacuum amplitude, so we suppress it and we get

$$i\delta S'_{(2)}(x_1-x_2)=-\frac{\Delta^2}{2g^2}\{\tfrac{2}{3}[\gamma_5S^{-1}(x_1-x_2)\gamma_5]D(x_2-x_1)+\Delta(\tfrac{1}{3}-C_8C_{12})[\gamma_5S^{-1}(0)\gamma_5]D(0)\delta(x_1-x_2)\}. \tag{42}$$

Coefficients $S^{-1}(0)$, $D(0)$ are already known, but the first, very singular term in Eq. (42) must be regularized. In momentum space, we must compute

$$\mathcal{T}=\int dx\,e^{ipx}\int\frac{dk}{(2\pi)^4}\,\gamma_5(\not k-M)\gamma_5e^{-ikx}e^{-ik^2/\Lambda^2}\times\int\frac{dl}{(2\pi)^4}\,(l^2-\mu^2)e^{ilx}e^{-il^2/\Lambda^2}. \tag{43}$$

In Eq. (43) we have understood the true convergence factors $\exp[-\epsilon\sum_\mu(k^\mu)^2]$. Performing the x integration, we obtain

$$\mathcal{T}=-\int\frac{dk}{(2\pi)^4}\,(\not k+\tfrac{1}{2}\not p+M)\times\left(k^2+\frac{p^2}{4}-k\cdot p-\mu^2\right)e^{-2ik^2/\Lambda^2}, \tag{44}$$

where a constant coefficient $\exp[-i(p^2/2\Lambda^2)]$ has already been replaced by one. Keeping in mind the presence of the convergence factors, the integration over k can be easily performed by application of simple rules like

$$\int dk\, k^2 e^{-i\alpha k^2} = i\frac{d}{d\alpha}\int dk\, e^{-i\alpha k^2}.$$

Collecting the leading term (39) and the second-order contribution (42), the Fourier transform of the nucleon propagator can be written as

$$iS'_{(2)}(p) = -i\frac{4\pi}{\Lambda^2}\frac{\lambda^{1/2}}{g^2}MC_8\left\{1+\frac{4\pi\lambda^{-1/2}}{\Lambda^2 MC_8}\left[\not p f_1(p^2)+f_2(p^2)\right]\right\} \tag{45}$$

with

$$f_1(p^2) = \left(\frac{p^2}{4}-\mu^2-\frac{i}{2}\Lambda^2\right)\Big/24,$$
$$f_2(p^2) = -\frac{M}{24}\left[12(\tfrac{1}{3}-C_8C_{12})(\mu^2+2i\Lambda^2) - 2\left(\frac{p^2}{4}-\mu^2-2i\Lambda^2\right)\right]. \tag{46}$$

The right-hand side of Eq. (45) hardly looks like a propagator, and clearly higher-order contributions will not improve the situation; but as we have remarked in Sec. II, a Padé approximant could help to restore a more correct analytical structure. To this order we can write

$$S'_{(2)}(p) \to \frac{\lambda M^2 C_8{}^2 g^{-2}}{-\Lambda^2\lambda^{1/2}MC_8/4\pi + f_2(p^2)+\not p f_1(p^2)} \tag{47a}$$

and identify the pole with the physical mass m of the nucleons

$$S'_{(2)}(p) \to \frac{R}{(\not p - m)\{1+[(\not p+m)/24f_1(m^2)](\not p/4+M/2)\}}, \tag{47b}$$

where

$$mf_1(m^2)+f_2(m^2) = \frac{\Lambda^2\lambda^{1/2}MC_8}{4\pi},\quad R=\frac{\lambda M^2 C_8{}^2}{g^2 f_1(m^2)}. \tag{48}$$

The wave-function renormalization constant $Z_2^{(2)}$, with

$$S'_{(2)}(p) = Z_2^{(2)} S'^R_{(2)}(p)$$

making the residue of the renormalized Padé approximant (48) equal to one, will be

$$Z_2^{(2)} = \frac{R}{1+[2m/24f_1(m^2)](m/4+M/2)} = \frac{24\lambda M^2 C_8{}^2 g^{-2}}{-\mu^2-(i/2)\Lambda^2+mM}. \tag{49}$$

In Eq. (49) we have already neglected m^2/Λ^2 with respect to mM/Λ^2.

More difficult is the introduction of a renormalization constant $Z_2^{(1)}$ for the first-order propagator $S'_{(1)}$, Eq. (39), which does not even depend on p. But we shall need it in the next section and we proceed as follows: We require that the (constant) value of the renormalized propagator $S'^R_{(1)}$ be the same as the value of $S'^R_{(2)}$ at $\not p = m$:

$$\frac{1}{Z_2^{(1)}}S'_{(1)} = \frac{1}{Z_2^{(2)}}S'_{(2)}(\not p = m). \tag{50}$$

This requirement is in some sense analogous to the more usual one that, to any order in perturbation theory, the renormalized propagators have a pole at the same location, with the same residue. Condition (50) determines $Z_2^{(1)}$, and we get

$$Z_2^{(1)} = \tfrac{1}{2} Z_2^{(2)}. \tag{51}$$

This procedure could be applied to the meson propagator, and with the additional pieces provided by the πNN and 4π vertices, it could be possible to set up a renormalization scheme along the traditional lines. Many difficulties arise in the realization of this program, and we shall comment on some of them in the last section. Let us only remark here that the dependence of the Green's functions on two expansion parameters, $\lambda^{1/2}g^{-2}$ and $\lambda^{-1/2}$, leaves us with the practical problem of discriminating a preferential parameter in powers of which to expand beyond the leading order. We shall see in the next section that, in the computation of the nucleon magnetic moments, this problem is easily solved.

We conclude this section with the technical remark that in many equations of this paper strange factors i appear. We do not worry about that because the origin of these factors lies in the particular choice (20) of the regularized $\delta(x)$, and all we ask of Λ^2 and of other bare parameters is they ultimately disappear in favor of true physical quantities.

V. NUCLEON MAGNETIC MOMENTS

The work we have done in the preceding sections enables us to compute strong corrections to properties of the nucleons and of the pions that are generated by weaker interactions. Here we are concerned with the electromagnetic form factors of the nucleons and in particular with the anomalous magnetic moments. Quite naturally, in this problem, we treat the electromagnetic field A^μ as a weak external perturbation to be retained only to the first order. Thus, the generating functional of Eq. (23) must be modified only by the insertion into the integrals of the factor

$$i S_{\text{em}} = -\frac{ie}{g} \int dx\, \bar{\psi}\gamma_\mu\left(\frac{1+\tau_3}{2}\right)\psi A^\mu + e\int dx\, \partial_\mu \phi_j\, Q_{jk}\, \phi_k A^\mu , \tag{52}$$

where

$$Q = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

is the charge matrix of the pion field $\vec{\phi}$. We recall that the parameter e in Eq. (52) is the physical charge of the proton and of the positive pion, since the A^μ field will not be renormalized. In the operator language, the quantity we are interested in would be

$$\langle 0 | T[\psi(x_1)\bar{\psi}(x_2)] | 0\rangle_A \equiv K(x_1, x_2), \tag{53}$$

i.e., the vacuum expectation value in presence of the external field A^μ of the time-ordered product $T[\psi(x_1)\bar{\psi}(x_2)]$. In the functional formulation, K is given by

$$K(x_1, x_2) = \int \mathfrak{D}\vec{\phi}\, \mathfrak{D}\psi\, \mathfrak{D}\bar{\psi}\, \frac{1}{g}\, \psi(x_1)\bar{\psi}(x_2)\, i S_{\text{em}} \exp(iS), \tag{54}$$

where S is the same action, containing the free part and the strong interactions, as in Eq. (23).

According to our method, to compute K we expand up to a convenient order the factor $\exp[(1/g)\bar{\psi}S^{-1}\psi]$ and integrate first over $\psi, \bar{\psi}$. Noting, again, that the integration over the fermion fields brings a factor ϕ^{-1} for each couple $\bar{\psi}\psi$, and taking into account the further integration over $\vec{\phi}$, we immediately see that the leading contributions to K of both the nucleon current and the pion current are of the same order g^{-2}. Thus, the integration over $\psi, \bar{\psi}$ gives

$$K_\psi = \frac{1}{g^2} K_\psi^{(1)} + O\left(\frac{1}{g^4}\right) \tag{55}$$

with

$$K_\psi^{(1)} = \int \mathfrak{D}\psi\, \mathfrak{D}\bar{\psi} \exp\left(-\int dx\, \bar{\psi}\gamma_5 \vec{\tau}\psi \cdot \vec{\phi}\right)(H_\pi + H_N) \tag{56}$$

and

$$H_\pi = i\, \psi(x_1)\bar{\psi}(x_2) \int dy_1 dy_2 \bar{\psi}(y_1) S^{-1}(y_1 - y_2)\psi(y_2), \tag{57}$$

$$H_N = -ie\psi(x_1)\bar{\psi}(x_2) \int dx\, \bar{\psi}\gamma_\mu\left(\frac{1+\tau_3}{2}\right)\psi A^\mu .$$

Simple inspection of Eq. (57) reveals an extremely pleasant feature of our method: H_N has the Lorentz structure $\gamma_\mu A^\mu$ and so, to the leading order g^{-2} in g, and to any order in λ, the nucleon current contributes only to the electric charge. As a consequence, to the same order, the anomalous magnetic moments receive a contribution only from the isovector pion current, and we can make the qualitative but important prediction $\mu_p \simeq -\mu_n$. These remarks make clear what the (optimistic) policy should be for a systematic calculation of the magnetic moments: first stay at order g^{-2} and go on with $\lambda^{-1/2}$ to get a good isovector magnetic moment, and then move to order g^{-4} to improve the isoscalar one.

Let us now perform the integrations (56); these calculations are very similar to those we have done to compute the first-order propagator, and we easily find

$$K_N = ie\delta(x_1 - x_2)\gamma_\mu A^\mu(x_1)\left[\frac{\vec{\tau}\cdot\vec{\phi}(x_1)}{\vec{\phi}^2(x_1)}\left(\frac{1+\tau_3}{2}\right)\frac{\vec{\tau}\cdot\vec{\phi}(x_2)}{\vec{\phi}^2(x_2)}\right] \times \prod_K [\vec{\phi}(x_K)\cdot\vec{\phi}(x_K)]^4, \tag{58}$$

$$K_\pi = i[\gamma_5 S^{-1}(x_1 - x_2)\gamma_5] \times\left[\frac{\vec{\tau}\cdot\vec{\phi}(x_1)}{\vec{\phi}^2(x_1)}\, \frac{\vec{\tau}\cdot\vec{\phi}(x_2)}{\vec{\phi}^2(x_2)}\right]\prod_K [(\vec{\phi}(x_K)\cdot\vec{\phi}(x_K)]^4 \tag{59}$$

as the contributions to $K_\psi^{(1)}$ of H_N and H_π.

Next comes the integration over $\vec{\phi}$. Remembering that the pion current multiplies the factor K_π, we note that with respect to the coupling λ, the leading contribution of the pion is of order one while that of the nucleon is of order $\lambda^{1/2}$. As the charge takes contributions from both currents, a consistent charge renormalization requires the expansion up to $\lambda^{-1/2}$ of the factor

$$\exp\left(i\, \frac{1}{2\lambda^{1/2}} \int dy_1 dy_2\, \vec{\phi}(y_1)\, D(y_1 - y_2)\vec{\phi}(y_2)\right)$$

when integrating K_N over $\vec{\phi}$. Thus, we compute N, the relevant contribution to K, from the nucleon current

$$N = \frac{ie}{g^2}\, \delta(x_1 - x_2)\gamma_\mu A^\mu(x_1)\, \frac{1}{3}\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \times\left[(i\Delta)^{1/2}\lambda^{1/2} C_8 + \frac{i}{2} D(0)\, \Delta^2(1 - C_8 C_{12}) + O\left(\frac{1}{\lambda^{1/2}}\right)\right]. \tag{60}$$

Integration over $\vec{\phi}$ of the pion contribution is a little more involved because of the derivative $\partial_\mu \vec{\phi}$; a good trick is to replace

$$A^\mu(x)\partial_\mu \vec{\phi}(x) \to \vec{\phi}(x + A) - \vec{\phi}(x),$$

integrate over $\vec{\phi}$, develop the result in powers of A^μ, and keep the linear term. We get

$$P=\frac{4}{9}\frac{ie}{g^2}\Delta^2\begin{pmatrix}1&0\\0&-1\end{pmatrix}\left\{\gamma_5[\delta(x_1-x_2)A^\mu(x_1)\partial_\mu S^{-1}(0)-A^\mu(x_1)S^{-1}(0)\partial_\mu\delta(x_1-x_2)]\gamma_5+O\left(\frac{1}{\lambda^{1/2}}\right)\right\} \tag{61}$$

as the relevant contribution to K from the pion current. Collecting terms P and N, applying our regularization procedure, which should now be familiar, and defining $K_\mu^{(1)}$ by

$$K(x_1,x_2)=K_\mu^{(1)}(x_1,x_2)A^\mu(x_1)+O\left(\frac{1}{g^4}\right) \tag{62}$$

we get, in momentum space,

$$K_\mu^{(1)}(p,p')=\frac{ie}{g^2}\frac{1}{3}\begin{pmatrix}1&0\\0&2\end{pmatrix}\frac{4\pi}{\Lambda^2}\gamma_\mu\left[C_8\lambda^{1/2}-\tfrac{1}{2}(\mu^2+2i\Lambda^2)\frac{4\pi}{\Lambda^2}(1-C_8C_{12})+O\left(\frac{1}{\lambda^{1/2}}\right)\right]$$
$$+\frac{e}{g^2}\frac{4}{9}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\frac{16\pi^2}{\Lambda^4}\left[\tfrac{1}{2}\Lambda^2\gamma_\mu-\frac{iM}{2}(p+p')_\mu+O\left(\frac{1}{\lambda^{1/2}}\right)\right]. \tag{63}$$

In Eq. (63), p and p' are, respectively, the momentum of the incoming and outgoing nucleon, and condition $\partial_\mu A^\mu=0$ has been taken into account.

To identify charges and magnetic moments, still we must amputate $K_\mu^{(1)}$ from the propagators lying on the nucleon external lines, put the result $\Gamma_\mu^{(1)}$ between free spinors $\bar{u}(p')$, $u(p)$ solutions of the Dirac equation with the physical mass m, and finally renormalize $\Gamma_\mu^{(1)}=Z_2\Gamma_\mu^{(1)}$. The relevant nucleon propagator has been computed in Sec. IV, Eq. (45), and we see that, in principle, $\Gamma_\mu^{(1)}$ will be made up of two terms, one of order $g^2\lambda^{-1/2}$ and one of order $g^2\lambda^{-1}$. Unfortunately, at the preliminary level of this work, the amputation of $K_\mu^{(1)}$ is a rather ambiguous operation; actually the term of order $\lambda^{1/2}$ in $K_\mu^{(1)}$ must be divided by second-order propagators where, to give a meaning to the notion of a physical mass, the pole at $\not p=m$ should be made apparent through the Padé approximant (47). Thus we face a problem like the one of attributing a value to $(1+x+\cdots)^{-2}$ at $x=1$, after having interpreted $(1+x+\cdots)$ as $(1-x)^{-1}$. However, fortunately enough, charge renormalization gets rid of this ambiguity; we can write

$$\Gamma_\mu^{(1)R}=A\gamma_\mu\begin{pmatrix}1&0\\0&2\end{pmatrix}+\frac{4}{9}\frac{eg^2}{\lambda M^2C_8^{\,2}}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\left((\tfrac{1}{2}\Lambda^2-iMm)\gamma_\mu-\frac{M}{2}\sigma_{\mu\nu}q^\nu\right)Z_2^{(1)}, \tag{64}$$

where coefficient A alone contains the ambiguous contribution of the nucleon current and all other terms come from the amputation, with first-order propagators, of the contribution from the pion current. We understand that $\Gamma_\mu^{(1)}$ is taken between free spinors, and we have as usual split the $(p+p')_\mu$ term into a γ_μ and a $\sigma_{\mu\nu}q^\nu$ term; here $q=p'-p$ and $\sigma_{\mu\nu}=(i/2)[\gamma_\mu,\gamma_\nu]$. We impose that the neutron charge vanishes, i.e.,

$$2A=\frac{4}{9}\frac{eg^2Z^{(1)}}{\lambda M^2C_8^{\,2}}(\tfrac{1}{2}\Lambda^2-iMm),$$

and we write

$$\Gamma_\mu^{(1)R}=\frac{3}{2}\frac{eg^2Z_2^{(1)}}{\lambda M^2C_8^{\,2}}(\tfrac{1}{2}\Lambda^2-iMm)\times\left[\gamma_\mu\begin{pmatrix}1&0\\0&0\end{pmatrix}+\frac{2}{3}\frac{iMm}{\tfrac{1}{2}\Lambda^2-iMm}\frac{i\sigma_{\mu\nu}q^\nu}{2m}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\right]. \tag{65}$$

Thus, for the anomalous magnetic moments we get

$$\mu_p=-\mu_n=\frac{2}{3}\frac{iMm}{\tfrac{1}{2}\Lambda^2-iMm} \tag{66}$$

in the usual units, provided that

$$\frac{3}{2}\frac{eg^2Z_2^{(1)}}{\lambda M^2C_8^{\,2}}(\tfrac{1}{2}\Lambda^2-iMm)=ie. \tag{67}$$

Condition (67), through Eqs. (49) and (51), still involves the ratio μ^2/Λ^2 and so does not provide a complete determination of iMm/Λ^2. We assume that the bare mass of the pion should not be of crucial importance in our lowest-order calculation (after all, it can be easily seen that the leading contribution to the pion propagator does not contain μ^2), so we set it equal to zero, and we find

$$iMm/\Lambda^2=\tfrac{7}{18}$$

and finally

$$\mu_p = -\mu_n = \tfrac{7}{3} , \qquad (68)$$

to be compared with the experimental values $\mu_p = 1.79$, $\mu_n = -1.91$.

VI. COMMENTS AND DISCUSSION

At first sight, the price to pay in order to obtain a strong-coupling expansion seems very high. The analytical structure of the Green's functions is lost and must be painfully restored to find again the familiar ground of renormalization theory. Essentially, this is because, to any finite order in our expansion, the Green's functions are too much local: they are combinations of δ functions and of their derivatives; but this feature is unavoidable and must be accepted to the extent that one accepts to expand the free Lagrangian instead of the interaction. From this point of view, the crucial dependence of the Green's functions on the Δ, or Λ, parameter is quite natural: increasing powers of the external momenta must be accompanied, for evident dimensional reasons, by appropriate powers of that parameter. Also, to establish the dependence on λ (let us think for simplicity to the scalar field) of the n-point unamputated Green's functions is a simple matter of inspection of the generating functional $Z(J)$, after rescaling of the field.

Thus, according to the preceding remarks in our strong-coupling approach, the propagator, say, for the scalar field must, *a priori*, have the structure

$$G_2(K^2) = \left(\frac{\Delta}{\lambda}\right)^{1/2} \sum_{m=0}^{\infty} \left(\frac{\Delta}{\lambda}\right)^{m/2} P_m(K^2, \mu^2, \Delta^{-1/2}) ,$$

where P_m is a homogeneous polynomial of degree m. Were we in a d-dimension space-time, we would modify the $\Delta^{-1/2}$ argument of P_m into $\Delta^{-2/d}$. Our task, in a sense, consists of computing the numerical coefficients of the polynomials, which do depend on the regularization and possibly on the dimension of the space-time.

The introduction of fermion fields interacting with mesons slightly complicated the situation, but a discussion along the same lines as before can be done in this case also. What is interesting in our opinion is that the effects of the two interactions, the Yukawa coupling and the pion self-interaction, are inextricably combined in the Green's functions. Of course when only pions are on the external lines, the expansion contains terms independent of the g coupling, but the numerical coefficients always depend, through the $\Pi\phi^8$ factor, on the presence of the π-N interaction.

In this paper we have introduced, and rather successfully applied, a renormalization procedure as close as possible to the traditional one. It could be that consideration of higher-order contributions forces us to adopt a procedure better suited to the formal structure of our expansion. We have in mind, in particular, the fact that when dealing with a high-order propagator, we have the choice between many different Padé approximants, and that for a given Green's function it is not evident in general which parameter among $\lambda^{1/2}g^{-2}$ and $\lambda^{-1/2}$ is the more relevant in a limited expansion. Furthermore, dealing with unamputated Green's functions is quite annoying, because the factors corresponding to the propagators on the external lines are not easily recognizable, imbedded as they are in the global polynomial structure, and because the amputation, as we have already remarked, is not free from ambiguities.

The calculation of the nucleon magnetic moments, which in our opinion is rather straightforward and clean, is a test for the predictive power of the method and an indication that it could be possible to overcome the difficulties still present in the formalism. The fact that the isoscalar anomalous magnetic moment is one order in g^{-2} smaller than the isovector one strongly suggests that our expansion is able indeed to explain the nature of strong corrections to some weak or electromagnetic quantities. An interesting comparison with our numerical result is provided by the lowest-order calculation in the usual perturbation theory, where one gets[9] $\mu_p = 0.54$, $\mu_n = -3.9$.

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