Exceptionally simple E(6) theory

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We present an E(6) unified theory where the Higgs scalars must be in a unique irreducible representation to solve the strong *CP* problem naturally. The superheavy symmetry breaking of E(6) to SU(5) and of the Peccei-Quinn U(1) is demonstrated. Phenomenological aspects are briefly discussed.

(1) For grand unification, the groups SU(*N*) have the disadvantage that in order to obtain a satisfactory solution of the strong *CP* problem with an invisible axion, the Higgs sector though very constrained must always contain at least two different nonzero *N*-alities.[1] This motivates us to consider the alternative groups SO(10) and E(6), and we shall here show that only the group E(6) works and further that it allows a unique and exceptionally simple answer. Note that $SO(4n+2)$ with $n \geqslant 3$ are uninteresting for grand unification unless the embedding of $SU(3)\times SU(2)\times U(1)$ is a nontrivial one (see, e.g., Ref. 3); we here restrict ourselves to the usual embedding.

Before discussing E(6) which is our main subject, let us therefore first dismiss SO(10). With fermions in the spinor 16 representation, one has mass terms

$$16\times 16 = 10_s + 120_a + 126_s \ , \tag{1}$$

and therefore one might reasonably consider scalars belonging to one or more of these three representations, and possibly to 16 and the adjoint 45. But the existence of, respectively, 10^2, 45^2, 120^2, and 126^4 eliminates four of the five fields since we demand a fully natural axial U(1) symmetry; this leaves only one 16 which is insufficient to break SO(10) down to $SU(3)\times SU(2)\times U(1)$. This eliminates SO(10). Since we agree, for the sake of argument, to avoid SU(*N*) this leaves only E(6).

(2) In E(6) the fermions are most naturally assigned to the defining representation 27. The Yukawa couplings are then of the form

$$27\times 27 = (\overline{27}+351')_s + 351_a \ . \tag{2}$$

Thus, we may consider these three representations, together with the adjoint 78, as possible Higgs scalars. The existence of singlet terms 27^3, 78^2, and $(351')^3$ disallows three out of these four; we believe the fourth possibility, the 351 representation, is of considerable interest and indeed this choice will underlie our model.

The model is thus exceptionally simply stated: the group is E(6), the scalars are in one, or at most two,[4] 351's, and the fermions[5] are in two or more 27's.

(3) We first show that a 351 of scalars can break E(6) to $SU(3)\times SU(2)\times U(1)$. E(6) has a maximal subalgebra

$$E(6) \supset SU(2)\times SU(6) \tag{3}$$

and let us embed, for book-keeping purposes, the standard $SU(3)\times SU(2)\times U(1)$ in an SU(5) subgroup of this SU(6). Then there are three SU(5) singlets in 351 contained in

$$(2,6)+(1,\overline{21}) \tag{4}$$

since $6=5+1$ and $\overline{21}=\overline{15}+\overline{5}+1$. Giving a vacuum expectation value (VEV) to either of the two singlets in (2,6) breaks E(6) to SU(5) (Ref. 6). Finally, 351 contains[7] two adjoints of SU(5):

$$351 = \overline{45}+2(45)+\overline{40}+2(24)+\overline{15}+4(\overline{10})+3(10)+4(5+\overline{5})+3(1) \ .$$

Only one 24 is necessary to break to $SU(3)\times SU(2)\times U(1)$ as required. Q.E.D.

(4) The Higgs potential for 351 has no cubic terms, but just one quadratic mass term $(351\times\overline{351})$ and six independent quartic terms all of the form $351^2\overline{351}^2$. There are no terms of the form $351^3\overline{351}$ or 351^4. Fortunately, we have at hand[8] the explicit Higgs potential. Writing 351 as the antisymmetric tensor $A^{\mu\nu}$ with $\mu,\nu=1,2,\ldots,27$ the potential is

$$\begin{aligned} V(A) = {} & M^2 A_{\mu\nu}\bar{A}^{\mu\nu} + h_1 (A_{\mu\nu}\bar{A}^{\mu\nu})^2 + h_2 A_{\mu\nu}\bar{A}^{\nu\sigma}A_{\sigma\tau}\bar{A}^{\tau\mu} + h_3 d^{\mu\nu\lambda}d_{\xi\eta\lambda}A_{\mu\sigma}A_{\nu\tau}\bar{A}^{\xi\sigma}\bar{A}^{\eta\tau} \\ & + h_4 d^{\mu\nu\alpha}d^{\sigma\tau\beta}d_{\xi\eta\alpha}d_{\lambda\rho\beta}A_{\mu\sigma}A_{\nu\tau}\bar{A}^{\xi\lambda}\bar{A}^{\eta\rho} + h_5 d^{\mu\nu\alpha}d^{\sigma\beta\gamma}d_{\xi\eta\beta}d_{\lambda\alpha\gamma}A_{\mu\sigma}A_{\nu\tau}\bar{A}^{\xi\lambda}\bar{A}^{\eta\tau} \\ & + h_6 d^{\mu\nu\alpha}d^{\sigma\tau\beta}d_{\alpha\beta\gamma}d^{\gamma\zeta\chi}d_{\xi\eta\zeta}d_{\lambda\rho\chi}A_{\mu\sigma}\bar{A}^{\xi\lambda}A_{\nu\tau}\bar{A}^{\eta\rho} \ . \end{aligned} \tag{5}$$

$V(A)$ and hence $\mathcal{L}$ has a global symmetry under

$$A_{\mu\nu} \to e^{i\theta} A_{\mu\nu}, \quad \psi_\mu \to e^{i\theta/2}\psi_\mu \tag{6}$$

and we will identify this with a Peccei-Quinn-type symmetry. What we will now show is that this U(1) can be *broken*, when we break E(6)→SU(5) at the superheavy scale; it will then follow that the resulting axion is an invisible one. Here, the U(1) is definitely color anomalous because each 27 of fermions gives a nonzero anomaly and there is no opportunity for cancellation.

The mass term in Eq. (5) cannot break the U(1) since after breaking to SU(5) it will obviously contain only 5-ality=zero quadratic terms. Similarly, the h_1 term which is nothing more than the square of the mass term cannot break the U(1). This leaves the five terms h_2 through h_6 which we shall study in turn.

Our first assertion is that the h_2 term will, in general, break the U(1) invariance. To prove this, let us introduce the following labeling of indices

$$27 \to 10 + \bar{5}_1 + 5 + \bar{5}_2 + 1 + 1 \ , \tag{7a}$$

$$\phi_\mu \to (\phi_L, \phi_M, \phi_N, \phi_P, \phi_{26}, \phi_{27}) \ , \tag{7b}$$

where $1 \leq L \leq 10$, $1 \leq M,N,P \leq 5$, and $\phi_L, \phi_M, \phi_N, \phi_P$ correspond to $10, \bar{5}_1, 5, \bar{5}_2$, respectively. One of our SU(5) singlets in $A_{\mu\nu}$ is[9] now $A_{26,27}$ so that giving a VEV v to this singlet gives rise to a piece

$$h_2 v \bar{A}^{26,\sigma} A_{\sigma\tau} \bar{A}^{\tau,27} \ . \tag{8}$$

The choices $\{\sigma, \tau\} = \{M,L\}, \{N,L\}$ give terms

$$2h_2 v(5_1 5 \overline{10}), 2h_2 v(\bar{5} \overline{10} \overline{10})$$

which are direct and indirect couplings sufficient to imply global charges $Q'(\bar{5}_1) = Q'(\bar{5}_2) = Q'(5) = Q'(10) = 0$. Since this sets $Q' = 0$ for the entire defining 27 of E(6), it follows that $Q'(351) = 0$, and hence that the U(1) *has* been broken.

This breaking of the U(1) in E(6) is to be contrasted with what would happen in SU(N) groups. For example, if the present theory were SU(27) for which $A^{\mu\nu}$ were the 351 antisymmetric second-rank representation the most general potential[6] would be exactly as Eq. (5) without the h_3 through h_6 terms, since the $d_{\mu\nu\sigma}$ tensor is no longer an invariant. But the global U(1) could never be broken in SU(27) due to a general theorem in SU(N) proved in Ref. 1; the reason is that the apparent U(1) breaking could be compensated by an SU(27) rotation. In the case of E(6), however, the compensating rotation is no longer available since the relevant SU(27) generator is not contained in the E(6) subgroup.

Now we consider the other four linearly independent quartic terms with couplings h_3 through h_6 in Eq. (5). These involve the auxiliary E(6) invariant tensor $d^{\mu\nu\lambda}$ and their analysis hence requires some further study of $d^{\mu\nu\lambda}$.

We wish to write $d^{\mu\nu\lambda}$ in an SU(5) basis; this is straightforward if we begin from the basis chosen by Cartan[10] (also considered later by Gantmacher[11]) which is based on the maximal subalgebra indicated in Eq. (3) above. The defining representation is written $(x_\alpha, y_\beta, z_{\gamma\delta})$ where $1 \leq \alpha, \beta, \gamma, \delta \leq 6$ and $z_{\gamma\delta} = -z_{\delta\gamma}$. Then the E(6) cubic invariant is[10]

$$x_\alpha y_\beta z_{\alpha\beta} - \sum_P (\alpha\beta\gamma\delta\epsilon\phi) z_{\alpha\beta} z_{\gamma\delta} z_{\epsilon\phi} \ . \tag{9}$$

Rewriting $x_\alpha = (\bar{5}_{1a}, 1_1)$, $y_\alpha = (\bar{5}_{2a}, 1_2)$, and $z_{\alpha\beta} = (10_{ab}, z_{6\beta} \equiv 5_\beta)$ in an obvious SU(6)→SU(5) notation with $1 \leq a,b \leq 5$ then the cubic E(6) invariant is

$$\bar{5}_{1a}\bar{5}_{2b}10^{ab} - \bar{5}_{1a}5^a 1_2 + \bar{5}_{2a}5^a 1_1 + 6\epsilon_{abcde}10^{ab}10^{cd}5^e \ . \tag{10}$$

Here the SU(5) notation is as in Eq. (7a), and we shall use below the letters L,M,N,P as defined by Eq. (7b). Then the nonvanishing $d^{\mu\nu\lambda}$ are (LMP), ($LL'N$), (MN,27), and (NP,26).

With this basis for the $d^{\mu\nu\lambda}$ tensor, we shall now consider whether each of the terms h_3–h_6 separately breaks the U(1), as the h_2 term did. Of course, naturalness dictates that all seven terms in $V(A)$ of Eq. (5) be present; our result here will be that the U(1) is broken at the superheavy scale by any of the last five of the seven terms in Eq. (5), even acting alone.

The h_3 term contains, putting ${}_0\langle A_{26,27}\rangle_0 = v$,

$$h_3 v[d^{26\nu\lambda} d_{\xi\eta\lambda} A_{\nu\tau} \bar{A}^{\xi,27} \bar{A}^{\eta\tau} - (26 \leftrightarrow 27)] \ . \tag{11}$$

The choices for $\{\nu, \lambda, \xi, \eta$, and $\tau\}$ in expression (11) can now give different SU(5) singlets within the E(6) singlet. The choices $\{N, P, N', 26$, and $N''\}$ and $\{N, P, L, M$, and $26\}$ in the first term give $(10\bar{5}\bar{5})$ and $(\overline{10}5_1 5)$, respectively (direct couplings). The choices $\{P, N, L, L'$, and $26\}$ and $\{M, N, L, L'$, and $26\}$ give, respectively, $(\bar{5}_2\overline{10}\,\overline{10})$ and $(\bar{5}_1\overline{10}\,\overline{10})$ in the first and second terms. These contributions already demand $Q' = 0$ for the entire (351) as required.

A similar analysis using the same VEV applied to the terms h_4, h_5, and h_6 shows that there are sufficient direct and indirect SU(5) couplings to break the U(1) in each case.

Note that here we have used only one of the three available SU(5) singlets in 351, that corresponding to $A_{26,27}$. The other two superheavy VEV's are independent from $A_{26,27}$ and hence do not alter our conclusion. Q.E.D.

(5) Before turning to the low-energy properties of our model, let us make one remark about overall asymptotic freedom. Asymptotic freedom allows only one 351 (not two) and up to nine 27's of fermions.[12] Thus, the combination of axion invisibility and asymptotic freedom of E(6) makes the Higgs sector unique.

(6) A number of authors have constructed E(6)

grand unified theories.[13] All such attempts that we are aware of used complicated Higgs sectors, always involving a 27 defining representation of scalars, in combination with higher representations such as 78, 351′, and 351. Thus, no previous work has studied the Higgs structure of our present model. Because our choice of Higgs structure is unique, and more strongly motivated than previous choices, it seems worth pursuing the phenomenological consequences of this symmetry-breaking pattern.

An immediate concern is therefore the masses of the light fermions but these are complicated in grand unified theories by several factors: (i) when there are superheavy fermions, such as $(5+\overline{5}) \subset 27$, these can mix freely with light fermions, (ii) the light masses are inextricably linked to the unsolved gauge hierarchy problem, and (iii) tree-level estimates may be completely changed by radiative corrections.

These uncertainties are present in our E(6) model, and make definite phenomenological predictions impossible at this time, especially without specifying the precise fermion content. A naive inspection of the up-quark mass matrix for, e.g., three 27 families, at the tree level, reveals an unacceptable antisymmetry in family space; hence the up quarks get mass only from an SU(5) 45 of Higgs bosons (not 5). This therefore leads us to introduce additional $(27+\overline{27})$ fermion components so that there are superheavy 10's to mix with the light ones, but this removes much of the predictivity. In addition to this problem, some authors[13] have appealed to radiative corrections to explain the fermion mass hierarchy.[14]

In summary, we have found an E(6) theory which must surely be the simplest containing a natural invisible axion: in particular, the Higgs scalars lie in a unique irreducible representation of E(6). If we had a solution of the gauge hierarchy problem, the present model would have strong predictive power.

Note added in proof. To achieve the required symmetry breaking at the superheavy scale, it is actually necessary to give vacuum expectation values to the SU(5) singlets in both (2,6) and $(1,\overline{21})$ of Eq. (4), as well as to the SU(5) adjoint in (2,84). This does not alter our conclusion.

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