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Radiative muon decays and lower bounds on the mass scale in a composite model of leptons

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(Received 15 December 1981)

The experimental upper bounds on rates for the muon radiative decays $\mu \to e+\gamma+\gamma$ and $\mu \to e+\gamma$ are used to give lower bounds on the mass scale of the constituent particles in a composite model of leptons.

The proliferation of leptons and quarks makes us suspect that there might be a substructure in them.[1–8] There are several questions, however, to be resolved in composite models of leptons and quarks. Some of them are listed below: (1) whether the rapidly increasing mass spectra of the observed leptons (and quarks in a sense) can be understood, (2) why orbital-angular-momentum excited states and higher-spin states are not observed, and (3) how the electron and the muon can have Dirac magnetic moments with the accuracy shown by precision measurements of the $g-2$ factor.[9]

The last question[2] was answered by several groups[3]: If the constituent particles of which the leptons are composite systems have large mass ($\gtrsim 10^7$ GeV), the present $g-2$ experiment is compatible with composite models. On the other hand, questions (1) and (2) are puzzling and further study is needed. Recently the author,[4] and independently Akama and Terazawa,[5] have computed the mixing matrix for quarks and leptons in a composite model.[6–8] In Ref. 4, the author has assumed that the potential is a sum of a local potential and a portion which depends on energy and angular momentum,

$$V(r,E,l)=V(r)+V_l(E) \quad , \tag{1}$$

thereby circumventing[10] the difficulty of questions (1) and (2). In this Communication, we discuss another question[6] for composite models: (4) how flavor-changing radiative processes, such as $\mu \to e+\gamma$ and $\mu \to e+\gamma+\gamma$, can be small.[9]

In a composite model[6–8] where leptons and quarks are assumed to be S ($l=0$) excited states of spin-$\frac{1}{2}$ and spin-0 particles, the observed leptons e, μ, and τ are $1S$, $2S$, and $3S$ states. In the case of the hydrogen atom, the metastable $2S$ state decays into the ground state predominantly by two-photon emission (with a decay time $\frac{1}{7}$ sec), while one-photon emission by the magnetic-dipole transition is a rather slow process[11] (~ 2 days). Below we examine analogous processes for composite leptons in which the photons are emitted purely electrodynamically (i.e., without considering the effect of weak interactions and neutrino mixing).

I. $\mu \to e+\gamma+\gamma$

The rate for the transition ${}_{2S}\mathrm{H} \to {}_{1S}\mathrm{H}+\gamma+\gamma$ is computed in Ref. 11,

$$\Gamma({}_{2S}\mathrm{H} \to {}_{1S}\mathrm{H}+\gamma+\gamma)=\frac{3^4}{8^5}\left(\frac{e^2}{\hbar c}\right)^6\left(\frac{e^2}{2ah}\right)I \quad , \tag{2}$$

where $a=\hbar^2/me^2$ is the Bohr radius and

$$4.4<I<7.1 \quad . \tag{3}$$

If we assume that the potential of the composite system is given by Eq. (1) with

$$V(r)=-\frac{\alpha_C \hbar c}{r} \quad , \tag{4}$$

where $\alpha_C \hbar c$ is the coupling strength of the Coulomb-type interaction, the decay rate for $\mu \to e+\gamma+\gamma$ can be read off from Eq. (2) ($\hbar=c=1$),

$$\Gamma(\mu \to e\gamma\gamma)=\frac{\alpha^2}{12\pi}\omega^5\left(\frac{1}{M\alpha_C}\right)^4 I \quad , \tag{5}$$

where M is the reduced mass of the constituent particles and the excitation energy ω is given by

$$\omega = m_\mu - m_e \ . \qquad (6)$$

The power of the length scale $1/M\alpha_C$ in Eq. (5) is dictated by the fact that each electric-dipole emission contributes a factor $e/M\alpha_C$ in the amplitude. The numerical factor in Eq. (5) is determined by the condition that Eq. (5) reduces to Eq. (2) when the substitution $\omega = E_{2S}^{(H)} - E_{1S}^{(H)} = \frac{3}{8} m\alpha^2$ and $M\alpha_C = m\alpha$ is made.

From Eqs. (5) and (6), it follows that

$$M\alpha_c = m_\mu \left[\frac{\alpha^2 I}{12\pi} \frac{m_\mu}{\Gamma(\mu \to e\gamma\gamma)} \right]^{1/4} \ . \qquad (7)$$

Using current experimental data,[9]

$$\Gamma(\mu \to e\gamma\gamma) < \frac{4\times 10^{-6}}{2.20\times 10^{-6}}\ \mathrm{sec}^{-1} = 1.82\ \mathrm{sec}^{-1} \ , \qquad (8)$$

and inequality (3), we obtain

$$M\alpha_C > 2.9\times 10^3\ \mathrm{GeV} \ . \qquad (9)$$

II. $\mu \to e + \gamma$

The magnetic-dipole transition rate for ${}_{2S}\mathrm{H} \to {}_{1S}\mathrm{H} + \gamma$ is given by[11]

$$\Gamma({}_{2S}\mathrm{H} \to {}_{1S}\mathrm{H} + \gamma) = \frac{2}{729} \left(\frac{e^2}{\hbar c} \right)^9 \omega \ . \qquad (10)$$

The power of c in Eq. (10) indicates that the magnetic-dipole transition is a relativistic effect and is proportional to $\beta^4 = (v/c)^4$. [Note that the factor $1/c^3$ is associated with one-photon emission since $k^2dk = (1/c^3)\omega^2 d\omega$, and $(e/mc)^2$ is attributed to the magnetic-moment operator squared.] The rate should be proportional to the length scale squared. Hence, the transition to the composite-model calculation reads ($\hbar = c = 1$),

$$\Gamma(\mu \to e + \gamma) = \frac{128}{9^4} \alpha\beta^4\omega^3 \left(\frac{1}{M\alpha_C} \right)^2 \ . \qquad (11)$$

The coefficient of Eq. (11) is determined by the condition that Eqs. (10) and (11) coincide when the substitution $\beta = \alpha$, $\omega = \frac{3}{8} m\alpha^2$, and $M\alpha_C = m\alpha$ is made in the latter.[12] Noting that

$$\beta = \alpha_C \qquad (12)$$

for the composite system, and using Eqs. (6) and (11), we obtain

$$\frac{M}{\alpha_C} = \frac{16}{81} m_\mu \left[\frac{1}{2} \frac{\alpha m_\mu}{\Gamma(\mu \to e + \gamma)} \right]^{1/2} \ . \qquad (13)$$

The experimental data,

$$\Gamma(\mu \to e + \gamma) < \frac{1.9\times 10^{-10}}{2.2\times 10^{-6}}\ \mathrm{sec}^{-1} = 0.86\times 10^{-4}\ \mathrm{sec}^{-1} \ , \qquad (14)$$

gives a bound

$$M/\alpha_C > 5.5\times 10^{10}\ \mathrm{GeV} \ . \qquad (15)$$

Eliminating α_C from Eqs. (7) and (13) or from Eqs. (10) and (15), we obtain

$$M = \frac{4m_\mu}{9} \left[\frac{I\alpha^4 m_\mu{}^3}{48\pi\Gamma(\mu \to e\gamma\gamma)[\Gamma(\mu \to e\gamma)]^2} \right]^{1/8} \qquad (16)$$

or

$$M > 1.3\times 10^7\ \mathrm{GeV} \ . \qquad (17)$$

The bound given in Eq. (17) is independent of the magnitude of the coupling strength α_C.

If one makes the assumption that

$$\alpha_C \approx \alpha \ , \qquad (18)$$

Eqs. (7) and (13) give bounds on M,

$$M > 4.0\times 10^5\ \mathrm{GeV} \ , \qquad (9')$$

$$M > 0.40\times 10^9\ \mathrm{GeV} \ , \qquad (15')$$

respectively, while the assumption

$$\alpha_C \approx 1 \qquad (19)$$

leads to

$$M > 2.9\times 10^3\ \mathrm{GeV} \qquad (9'')$$

and

$$M > 0.55\times 10^{11}\ \mathrm{GeV} \ , \qquad (15'')$$

respectively.

The bound given in Eq. (17) corrresponds to the value of the coupling strength

$$\alpha_C \approx \frac{1}{32}\alpha \ , \qquad (20)$$

which is very unlikely. Assuming that

$$\alpha_C > \alpha \ , \qquad (21)$$

we conclude that

$$M > 10^9\ \mathrm{GeV} \ . \qquad (22)$$

The above bounds on the mass scale for composite particles should be compared with the one obtained from the precision measurement of the $g-2$ factor of the muon,[3]

$$M > 10^7\ \mathrm{GeV} \ .$$

Earlier works[13] give an estimate of 1.7×10^5 GeV for the lower bounds of the composite-system mass scale from the $\mu \rightarrow e\gamma$ experiment.

We have presented lower bounds on the mass scale of a composite model from a constraint given by the experimental upper bounds of $\mu \rightarrow e\gamma$ and $\mu \rightarrow e\gamma\gamma$. Although our discussion is based on nonrelativistic calculations, the conclusion is expected to be valid in relativistic calculation, since the gross features of the estimate are dictated by dimensional factors. Of course, the constraints discussed in this text are peculiar to the radial-excitation model of the lepton generations.

Finally, it should be pointed out that the problem of finding a consistent picture of a composite model which gives the small mass scale of the observed particle spectra from a much larger mass scale of the composite system stays with us as an unsolved question.

The author is greatly indebted to Bob Lewis, Joe Sucher, Bill Williams, Tom Sterling, and R. Akhoury for useful discussions. The work is supported in part by the U.S. Department of Energy.

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[10]Obviously this is not a solution to the problem but a device to postpone a confrontation with these questions and still be able to make a prediction for the mixing matrix. A more sensible approach would be to start with a relativistic formulation for bound states, such as the Bethe-Salpeter equation and find the mass spectrum as well as the mixing matrix.

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[12]The value of β is equal to α/n for the nS state. Since we deal with the transition between $2S$ and $1S$ states, it is more appropriate to use some kind of average value, e.g., $\beta^4 = \alpha^2(\alpha/2)^2$. Then Eq. (10) should be multiplied by 4. In this case, however, Eq. (12) should be replaced by $\beta^4 = \alpha_C/4$. The resulting formula Eq. (13) is identical.

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