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Charge-retention sum rules

Leo Stodolsky
*Laboratoire de Physique Théorique et Hautes Energies, Paris, France**
and Max-Planck Institut für Physik, Munich, Germany†
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If "charge retention," the idea that the average charge of a jet is the quark charge, holds, then there are useful sum rules in deep-inelastic lepton scattering for charge-weighted cross sections. For example, in electron scattering, $d\langle q\sigma\rangle/dQ^2=\frac{5}{9}\times 4\pi\alpha^2/Q^4$.

"Charge retention," the idea that the nonintegral charge of a quark may be seen as the average value of a multiparticle jet[1] in deep-inelastic processes is one of the most intriguing ideas in the quark-parton model. Experimental efforts to verify the idea so far have, in various ways, attempted to establish the average charge of a jet, but have yielded uncertain results due to the problem of extrapolating to the high-energy limit. Recently, the trick of taking charge-flow ratios appears to give stable results with ν and $\bar{\nu}$ data for the *ratio* of quark charges in agreement with the quark model.[2] In this paper we would like to recall[3] that if charge retention holds there are interesting sum rules in deep-inelastic lepton scattering.

We introduce the "charge-weighted cross section"

$$\frac{d\langle q\sigma\rangle}{dQ^2dx}\equiv\sum_n q_F^n\frac{d\sigma^n}{dQ^2dx}\ , \qquad (1)$$

where q_F^n is the total forward charge of the hadronic final state in deep-inelastic lepton scattering. "Forward" is defined as, e.g., the virtual-photon direction in the hadronic center of mass. Experimentally, (1) is to be constructed at a given value of the deep-inelastic scattering variables[4] Q^2 and x by taking the total cross section for events with forward charge (+1) and weighting it with (+1), taking those with forward charge (−2) and weighting them with (−2), and so forth.

Now the forward charge q_F^n represents the charge of the struck quark in the parton model. If charge retention holds, we can replace the average over the charge in Eq. (1) by the charge of the struck quark itself. Using the standard parton-model formula this allows us to write for electron or muon scattering

$$\frac{d\langle q\sigma\rangle^{\rm EM}}{dQ^2dx}=\frac{4\pi\alpha^2}{Q^4}\left[\sum_q q_q{q_q}^2 f_q(x)\right]\frac{1+(1-y)^2}{2}\ , \qquad (2)$$

where the index q refers to the quark type and q_q means the quark charge. The variable y is Q^2/xs, and $f_q(x)$ is the parton distribution function. We write $q_q{q_q}^2$ instead of ${q_q}^3$ since q_q is the "retained" charge while ${q_q}^2$ is the dynamic charge giving the scattering strength, and the two might differ.

Now let us consider the integral of Eq. (2) over x. At small x the integral should converge since the number of quarks and antiquarks become equal and $q_q{q_q}^2$ is odd in the quark charge. In the limit of small y, that is $Q^2/s<x_0$, where x_0 is the value of x by which the integral has converged, we have simply

$$\int_0^1\frac{d\langle q\sigma\rangle}{dQ^2dx}dx=\frac{4\pi\alpha^2}{Q^4}\int_0^1\left[\sum_q q_q{q_q}^2 f_q^{\rm valence}(x)\right]dx\ , \qquad (3)$$

since in the "sea" there are equal amounts of quarks and antiquarks. But since

$$\int_0^1 f_{\rm up}^{\rm valence}(x)dx=2$$

and

$$\int_0^1 f_{\rm down}^{\rm valence}(x)dx=1$$

in the proton, we have simply

$$[(2\times\tfrac{2}{3}\times\tfrac{4}{9})-(\tfrac{1}{3}\times\tfrac{1}{9})]=\tfrac{5}{9}$$

for the integral, so

$$\frac{d\langle q\sigma\rangle^{\rm EM}}{dQ^2}\equiv\int_0^1\frac{d\langle q\sigma\rangle}{dQ^2dx}dx=\frac{4\pi\alpha^2}{Q^4}\left[\frac{5}{9}\right]. \tag{4}$$

On the deuteron we would have

$$3[(\tfrac{2}{3}\times\tfrac{4}{9})-(\tfrac{1}{3}\times\tfrac{1}{9})]=\tfrac{7}{9}\,,$$

giving

$$\frac{d\langle q\sigma\rangle^{\rm EM}}{dQ^2}=\frac{4\pi\alpha^2}{Q^4}\left[\frac{7}{9}\right] \tag{5}$$

and so on for heavier nuclei.

To apply the same idea of using the charge average to eliminate the sea quarks in neutrino scattering, we must start from a quark-antiquark-symmetric cross section, so we define for charge-current reactions

$$\frac{d\langle q\sigma\rangle^{\rm CC}}{dQ^2dx}\equiv\frac{d\langle q\sigma\rangle^{\nu\to\mu^-}}{dQ^2dx}+\frac{d\langle q\sigma\rangle^{\bar\nu\to\mu^+}}{dQ^2dx}\,. \tag{6}$$

With the same proviso as previously that $Q^2/s<x_0$, the sea quarks cancel in the integral over x, giving

$$\frac{d\langle q\sigma\rangle^{\rm CC}}{dQ^2}=\frac{G^2}{\pi}(0) \tag{7}$$

on the proton, and

$$\frac{d\langle q\sigma\rangle^{\rm CC}}{dQ^2}=\frac{G^2}{\pi}(1) \tag{8}$$

on the deuteron.

On the other hand, for neutral-current reactions at small y,

$$\frac{d\langle q\sigma\rangle^{\rm NC}}{dQ^2dx}\equiv\frac{d\langle q\sigma\rangle^{\nu\to\nu}}{dQ^2dx}=\frac{d\langle q\sigma\rangle^{\bar\nu\to\bar\nu}}{dQ^2dx}\,, \tag{9}$$

with the results

$$\frac{d\langle q\sigma\rangle^{\rm NC}}{dQ^2}=\frac{G^2}{\pi}\{\tfrac{4}{3}[(\epsilon_L^{\rm up})^2+(\epsilon_R^{\rm up})^2]-\tfrac{1}{3}[(\epsilon_L^{\rm down})^2+(\epsilon_R^{\rm down})^2]\} \tag{10}$$

on the proton, and

$$\frac{d\langle q\sigma\rangle^{\rm NC}}{dQ^2}=\frac{G^2}{\pi}\{2[(\epsilon_L^{\rm up})^2+(\epsilon_R^{\rm up})^2]-[(\epsilon_L^{\rm down})^2+(\epsilon_R^{\rm down})^2]\} \tag{11}$$

on the deuteron.

For experimental self-consistency it would seem that two conditions must be met. First the integral over x should converge smoothly for small x, and second, there is the related point that the results should not be sensitive to the exact definition of forward in constructing q_F^n. One might try, for example, using the Breit frame instead of the hadronic c.m. in making up q_F^n. If there is indeed a "neutral central region" between the forward jet and the backward jet as is usually supposed, then these conditions should be met, at least at very high energy. At finite energies, the major problem is likely to be that some of the forward charge overlaps into the backward direction and vice versa. The fact that charge flow appears to scale in the λ variable,[2,5] implies that such overlap effects should disappear as $1/W$, or after the x integration, $1/\sqrt{Q^2}$. Combining this with the effects of finite y, which go as Q^2/s, this suggests a parametrization of the numerical coefficients in the parentheses as

$$C_0+\frac{C_1}{\sqrt{Q^2}}+C_2\frac{Q^2}{s} \tag{12}$$

for the approach to high energy.

Assuming that well defined C_0 emerge from the analysis, it is possible that they are near, but not exactly equal to, the quark-model values. This could be due to "charge leakage," where the hadronization process is not entirely neutral. This charge leakage is thought[6] to simply result in a reduction in the retained charge of all quarks by a small common amount l (for antiquark increased by l). In this case the numerical factors change as follows: Eq. (4): $(\frac{5}{9}-l)$; Eq. (5): $(\frac{7}{9}-\frac{5}{3}l)$; Eq. (7): $(0-3l)$; Eq. (8): $(1-6l)$. In Eqs. (10) and (11) the factor in curly brackets receives an additional term

$$[(\epsilon_L^{\rm up})^2+(\epsilon_R^{\rm up})^2+(\epsilon_L^{\rm down})^2+(\epsilon_R^{\rm down})^2]$$

times $(-3l)$ for the proton and $(-6l)$ for the deuteron. The value of l may be around 0.05 (Ref. 6) or 0.07 (Ref. 7). The consistency of the assumption of a common l could be checked by comparing the various sum rules. It would of course be most interesting if definite numbers were to arise and they would be far from the quark-model values.

Verification of these sum rules, however, with perhaps a small charge-leakage factor, would provide one of the nicest successes of the quark-parton model in its full naivete.

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*Laboratoire associé au CNRS. Postal address: Université Pierre et Marie Curie (Paris VI), Tour 16, 1er étage, 4 place Jussieu, 75230 Paris Cedex 05, France.

†Permanent address. Postal address: Föhringer Ring 6, 8000 Munich 40, Germany.

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[3]The existence of such sum rules is implicit in the discussion around Eq. (56.1) of Ref. 1.

[4]We use the formulas and notation of Particle Data Group, Rev. Mod. Phys. 52, S42 (1980) for the parton model.

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