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ρ-meson coupling in the chiral bag model

W. Ferchländer*
Department of Physics, State University of New York, Stony Brook, New York 11794
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The ρNN coupling is calculated within a model which combines the chiral bag model with features of the usual Lagrangian field theory. We couple the ρ meson to the bag via two-pion decay as well as via a surface interaction and get reasonable values for the effective coupling constant for bag radii of about 0.5 fm.

I. INTRODUCTION

The chiral bag model[1-4] has proven to be a useful and rather successful method of describing pionic effects in hadronic physics without the necessity of caring about the dynamical structure of the individual pion. But there are still some drawbacks to this model, and one of them comes from the fact that there are only pions and quarks (and in the linear version, unphysical σ mesons) involved, and that it is, therefore, only possible to couple other mesons to the bag if one goes beyond the model. In the case of the ρ meson there is a strong hint from experiment, as well as from dispersion theory, how to proceed: it is well known that this meson appears only in $\pi\pi$ scattering as a resonance where it governs the whole P wave up to an energy of about 1000 MeV.[5] This means that other strong ρ decays are not allowed on shell, and that it should be a realistic model to couple the ρ meson to the nucleon bag mainly via the pion part of the vector current, which is an operator between quark states in the chiral bag model.[2] The ρ cannot couple to the quark vector current inside the bag because the resulting ρ-exchange interaction would violate the condition of asymptotic freedom; it seems that asymptotic freedom is even lost if we consider the ρ field as a Yang-Mills field getting its mass via the Higgs mechanism.[6] But there is no reason why there should not be a kind of additional surface interaction between the ρ meson and the quarks, and the quark vector current will serve us as a guide when we construct the corresponding interaction Lagrangian.

The pion field of which the outer current is constructed is calculated in Sec. II. This pion field acts on quark states, and so does the resulting effective $\rho\pi\pi$ Lagrangian density, which is the starting point for our calculation of the ρNN vertex function in Sec. III. There are still some parameters left in our theory. Using the asymptotic structure of the pion field and the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation,[7] we can easily fix these parameters and compare the result with the usual ρNN vertex function. This will be done in Sec. IV. Section V gives some concluding remarks.

Before starting with our theory we want to make a short remark about masses. Chiral invariance implies zero masses of quarks and pions, and it seems to be rather difficult to break this symmetry in a consistent way within the framework of the chiral bag model. Therefore, our results are—strictly speaking—only correct for massless quarks and pions. But it is well known[8] that chiral-symmetry breaking for the quarks alone results in very small u- and d-quark masses (about 4–8 MeV). For this reason we regarded it as a good approximation to a consistent symmetry-breaking procedure to set the quark masses identically to zero and to introduce pion masses by adding a mass term to the total Lagrangian density.

II. THE PION FIELD

In spite of many attempts it seems to be impossible to solve the equations of motion,[2,9] resulting from the chiral bag model, without serious approximations. But if we are only interested in the lowest order of the pion field, the corresponding equations become much simpler:

$$(\Box+m^2)\phi_i(x)=0 \quad \text{for} \quad r>R\,,$$
$$n^\mu\partial_\mu\phi_i(x)=\frac{1}{2f_\pi}\sum_c \bar{q}_c(x)i\gamma^5\tau^i q_c(x) \quad \text{for} \quad r=R\,. \tag{1}$$

In these equations i is an isospin index, m is the mass of the pion, R is the bag radius, n^μ is the outward normal to the bag surface, and $1/f_\pi$ is the coupling strength, which should be related to the pion decay constant.[2,10] $q_c(x)$ is the quark field operator for a spherical bag which is identical with the MIT solution[2,9] for zero quark masses:

$$q_c(x)=\sum_{fs}N\begin{pmatrix} ij_0(\omega r) \\ -j_1(\omega r)\vec{\sigma}\cdot\hat{r}\end{pmatrix}u_{fs}e^{-i\omega t}b(c,f,s)\,,$$

with

$$N^{-2}=8\pi R^3\left(1-\frac{1}{\omega R}\right)j_0{}^2(\omega R)\,,$$
$$\hat{r}=\vec{r}/r\,, \tag{2}$$
$$\omega R=2.04\,.$$

u_{fs} is the double spinor for isospin (flavor) and spin, and $b(c,f,s)$ is the quark destruction operator with color, flavor, and spin indices.

Solving Eq. (1) means to solve the Klein-Gordon equation with Dirichlet boundary conditions at infinity and Neumann boundary conditions at the bag surface. We want to preserve the two-phase picture of hadronic matter, which means that chiral symmetry is realized in the Wigner-Weyl mode inside and in the Nambu-Goldstone mode outside the bag[2,9] and, therefore, we cannot allow the pion to enter into the bag. Because it is still an open question whether the two-phase picture is justified or not, there are versions of the chiral bag model[1,4] with pion contributions inside the bag; asymptotic freedom is guaranteed in these models as well as in our model by restricting the interaction between quarks and pions to the surface. Our result is

$$\phi_i(\vec{r})=\frac{g(r,R)}{8\pi f_\pi(1-1/\omega R)}\sum_c \tau_c^i\vec{\sigma}_c\cdot\hat{r}\,,$$

with

$$g(r,R)=\frac{e^{mR}}{1+(1+mR)^2}\left(\frac{m}{r}+\frac{1}{r^2}\right)e^{-mr}\quad\text{for}\quad r\geqslant R\,,$$
$$\tau_c^+\sigma_c^+=4b^+(c,u,\uparrow)b(c,d,\downarrow)\,, \tag{3}$$

and the corresponding definitions for the other $\tau_c^m\sigma_c^n$ operators. We see that this field behaves asymptotically like the usual Yukawa field, and later on we will use this behavior to fix the value of f_π.

III. THE EFFECTIVE ρNN INTERACTION

The vector current resulting from Eq. (3) is an operator in quark space:

$$\vec{V}_i(\vec{r})\equiv\tfrac{1}{2}\epsilon_{ijk}\{[\vec{\nabla}\phi_j(\vec{r})]\phi_k(\vec{r})-\phi_j(\vec{r})[\vec{\nabla}\phi_k(\vec{r})]\}$$
$$=\frac{1}{[8\pi f_\pi(1-1/\omega R)]^2}$$
$$\times\frac{1}{2r}g^2(r,R)\sum_{c'c}\epsilon_{ijk}\tau_{c'}^j\tau_c^k(\vec{\sigma}_{c'}\vec{\sigma}_c\cdot\hat{r}-\vec{\sigma}_{c'}\cdot\hat{r}\vec{\sigma}_c)\,. \tag{4}$$

With a little spin algebra and the Wigner-Eckart theorem, we can transform this current into an operator in nucleon space by simply using

$$\sum_{c'c}\epsilon_{ijk}\tau_{c'}^j\tau_c^k\vec{\sigma}_{c'}\cdot\vec{A}\vec{\sigma}_c\cdot\vec{B}\to 2i\tau^i\vec{A}\cdot\vec{B}-\tfrac{22}{3}\tau^i\vec{\sigma}\cdot\vec{A}\times\vec{B}\,, \tag{5}$$

where τ^i and $\vec{\sigma}$ now act on nucleon spinors. But we must be careful with the interpretation of the transformed current. Since we are using static pion fields, matrix elements of the current between nucleons are only correct in the zero-momentum limit. And if we now couple a ρ meson to these nucleons, we can only trust our result for small momentum transfer.

Motivated by the form of the quark vector current, we now construct the source term for the surface interaction:

$$\vec{W}_i(\vec{r})\equiv\frac{1}{2f}\sum_c\bar{q}_c(\vec{r},t)\tau^i\vec{\gamma}q(\vec{r},t)\delta(R-r)$$
$$=\frac{1}{16\pi fR^3(1-1/\omega R)}i\sum_c\tau_c^i(\vec{\sigma}_c\vec{\sigma}_c\cdot\hat{r}-\vec{\sigma}_c\cdot\hat{r}\vec{\sigma}_c)\,, \tag{6}$$

where f is an unknown coupling constant with the dimension of an energy. Replacing[3]

$$\sum_c\tau_c^i\vec{\sigma}_c\cdot\hat{r}\to\tfrac{5}{3}\tau^i\vec{\sigma}\cdot\hat{r}\,, \tag{7}$$

we can transform Eq. (6) into an operator between nucleon states.

The operator for an incoming ρ meson with spin s, polarization vector $\vec{\epsilon}_s(\vec{k})$, isospin vector $\hat{\rho}_i$ and energy $\Omega_k=(m_\rho{}^2+\vec{k}^2)^{1/2}$ looks like

$$\vec{\rho}_s^{\,i}(\vec{k},\vec{r})=\frac{1}{(2\pi)^{3/2}(2\Omega_k)^{1/2}}\vec{\epsilon}_s(\vec{k})e^{i\vec{k}\cdot\vec{r}}\hat{\rho}_i\,. \tag{8}$$

We couple this field to $\vec{V}_i$ and $\vec{W}_i$ and construct an effective interaction Hamiltonian by integrating over the space:

$$H'=\int d^3r[f_\rho\vec{V}_i(\vec{r})\theta(r-R)+\vec{W}_i(\vec{r})\delta(r-R)]\cdot\vec{\rho}_s^{\,i}(\vec{k},\vec{r})\,. \tag{9}$$

Doing this we have left the limits of the chiral bag model, and we have to pay for that step because we are now dealing with two phenomenological parameters: the coupling strengths f_ρ and $1/f$.

After expanding the ρ-meson operator (8) in spherical polar coordinates, we can easily perform the integration in Eq. (9) because we have only to care about small ρ momenta k. The result is

$$H'=\frac{\tau^i\hat{\rho}_i}{4\pi M\sqrt{m_\rho}}\left\{\frac{f_\rho}{\sqrt{4\pi}}\,\frac{11M}{36\pi f_\pi{}^2(1-1/\omega R)^2}\right.$$
$$\times\frac{1}{[1+(1+mR)^2]^2}\left(\frac{1}{R}+\frac{m}{2}\right)$$
$$\left.+\frac{5M}{gf\sqrt{4\pi}\,(1-1/\omega R)}\right\}i\vec{\sigma}\cdot\vec{k}\times\vec{\epsilon}_s(\vec{k})\,, \tag{10}$$

where M is the mass of the nucleon. This expression contains the ρNN vertex functions, but—as mentioned above—we can only take them seriously in the long-wavelength limit.

IV. FIXING THE PARAMETERS AND RESULTS

There are still four parameters (f_π, f_ρ, f, and R) to be fixed before we can compare H' with the usual ρNN interaction in order to get the value of the coupling constant. The simplest choice for f_π would be to identify it with the pion decay constant: $f_\pi = 93$ MeV. But it is well known[2,9,10] that this would lead to a pion-nucleon coupling constant of $g^2/4\pi \simeq 20$, which is much too large. We therefore prefer to calculate an effective value of f_π from the condition that our pion field as an operator between nucleon states should asymptotically approach the usual Yukawa field. We simply have to use Eq. (7) in Eq. (3) and compare the result with

$$\phi_Y^i(\vec{r}) = \frac{g}{8\pi M}\left(\frac{m}{r} + \frac{1}{r^2}\right)e^{-mr}\tau^i\vec{\sigma}\cdot\hat{r}\,, \tag{11}$$

where $g^2/4\pi \simeq 14.5$. We get

$$f_\pi = \frac{5M}{3g(1-1/\omega R)}\;\frac{e^{mR}}{1+(1+mR)^2}\,, \tag{12}$$

which shows a small dependence on R for massive pions.

Similar to f_π, we could fix the strong ρ-decay constant by taking the experimental value $f_\rho{}^2/4\pi \simeq 2.9$. But this value is measured at the mass of the ρ, and we need it at threshold. There are several methods of calculating f_ρ for zero ρ momentum.[11] We prefer the KSRF relation,[7,11] which is based on current algebra and the assumption of a universal coupling of the ρ meson to the nucleon and pion isospin current:

$$\frac{f_\rho{}^2}{4\pi} = \frac{1}{2}\,\frac{g^2}{4\pi}\left(\frac{m_\rho}{g_A M}\right)^2 \simeq 3.1 \tag{13}$$

with the axial-vector charge g_A. We cannot use (13) as a real constraint in our model because we do not couple the ρ meson to the total vector current; therefore, we consider the KSRF relation only as a hint how the experimental f_ρ has to be changed at threshold.

The third parameter of our theory, f, cannot be related to other constants in a simple way, because the source term $\vec{W}_i(\vec{r})$ is no vector current. But if we assume that the surface interactions of the chiral bag model[2,9] do not differ very much in their coupling strengths, we can simply identify f with f_π, thus getting a small R dependence for $\vec{W}_i(\vec{r})$ too.

We can now rewrite Eq. (10) in terms of the fixed coupling constants as a function of the bag radius R and compare the result with the usual ρNN interaction[12,13] for small momentum transfer:

$$H' = \frac{\tau^i\hat{\rho}_i}{4\pi M\sqrt{m_\rho}}\left[\frac{f_\rho}{\sqrt{4\pi}}\,\frac{g^2}{4\pi}\,\frac{11}{25M}\left(\frac{1}{R}+\frac{m}{2}\right)e^{-2mR} + \frac{g}{\sqrt{4\pi}}\,\frac{1+(1+mR)^2}{3}\,e^{-mR}\right]i\vec{\sigma}\cdot\vec{k}\times\vec{\epsilon}_s(\vec{k})\,, \tag{14}$$

$$H_{\rho NN} = \frac{\tau^i\hat{\rho}_i}{4\pi M\sqrt{m_\rho}}\;\frac{g_V}{\sqrt{4\pi}}\left(1+\frac{g_t}{g_V}\right)i\vec{\sigma}\cdot\vec{k}\times\vec{\epsilon}_s(\vec{k})\,. \tag{14'}$$

Comparing the coefficients of these two equations, we get the effective coupling constant for the ρNN vertex as a function of the bag radius R:

$$\begin{aligned} F(R) &\equiv \frac{g_V}{\sqrt{4\pi}}\left(1+\frac{g_T}{g_V}\right) \\ &= \frac{f_\rho}{\sqrt{4\pi}}\,\frac{g^2}{4\pi}\,\frac{11}{25M}\left(\frac{1}{R}+\frac{m}{2}\right)e^{-2mR} \\ &\quad + \frac{g}{\sqrt{4\pi}}\,\frac{1+(1+mR)^2}{3}\,e^{-mR}\,. \end{aligned} \tag{15}$$

Thus we can predict the total coupling constant $F(R)$, but not the vector coupling constant g_V and the ratio between tensor and vector coupling g_T/g_V separately. This is a consequence of the static approximation of our bag model and of the fact that our equations are only correct in the long-wavelength limit.

Figure 1 shows the result of our calculation of

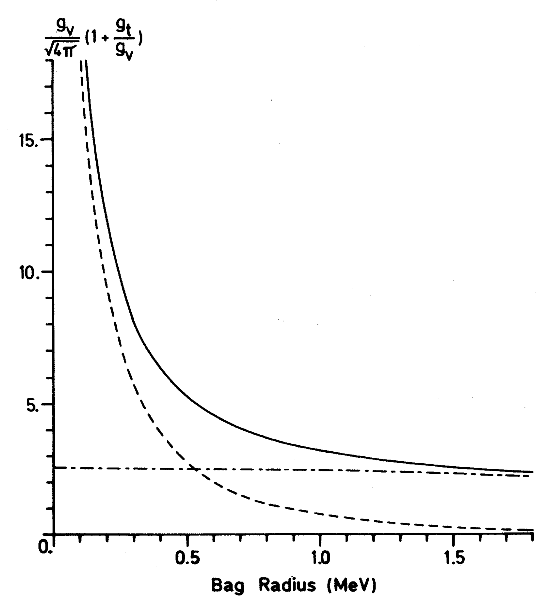
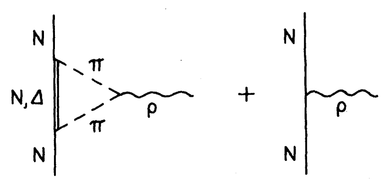


FIG. 1. Effective ρNN coupling constant as a function of the bag radius R. The solid curve is the sum of the two-pion contribution (dashed curve) and the surface interaction (dot-dashed curve).

$F(R)$. We see that the total coupling is dominated by the two-pion contribution for small bag radii, whereas for larger bag radii $F(R)$ is nearly totally determined by the surface interaction. It is well known from dispersion theoretical[14] and one-boson-exchange calculations[12] of the nucleon-nucleon interaction that the ρNN coupling constant should be $F^2(R) \simeq 27$ or separately $g_V^2/4\pi = 0.55$ and $g_T/g_V = 6$. These values are reached at about $R = 0.5$ fm, thus preferring kind of "little bag" picture. In the chiral limit ($m = 0$) we would get $R = 0.9$ fm; but because we believe that there is only one pion in nature and not a Goldstone pion and a $q\bar{q}$ pion, this is clearly an unphysical case.

Before taking our value of the bag radius too seriously, we should keep in mind that it may be too straightforward to use $F^2(R) = 27$ for the fit. We have to take into account that our model is only an approximation to the more realistic diagrams:



We are using the correct spin-isospin structure in our model, but in contrast to the left diagram we are treating all intermediate states in a static limit, and do not allow the particles to propagate. This procedure prefers the right side of this diagram where we have coupled the ρ meson to the vector current, and suppresses the left side where most of the tensor coupling should come from. Therefore, it might be more appropriate to use a smaller $F^2(R)$ for the fit, thus shifting the bag radius to larger values. Another source of uncertainty is the identification $f \equiv f_\pi$, which might lead to a too strong surface interaction. But using a larger f would bring the bag radius back to smaller values, and therefore we expect the real R not too far above 0.5 fm.

V. CONCLUDING REMARKS

In this paper we have shown an effective model of coupling the ρ meson to the chiral bag via the pion part of the vector current and an additional surface interaction. We have used the KSRF relation to fix the ρNN vector coupling constant, and got as a result the experimental coupling constant at reasonable values for the bag radius.

Since we have introduced the pion mass by hand, our calculations for massive pions can only be regarded as an approximation to a consistent symmetry-breaking procedure. But the result underlines the importance of such a procedure, because only in that way can we give a physical meaning to quantities like the bag radius.

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*Present address: Institut für Theoretische Kernphysik der Universität Bonn, Nussallee 14-16, D-5300 Bonn, W. Germany.

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