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Physical processes involving Majorana neutrinos

L. F. Li* and Frank Wilczek
Institue for Theoretical Physics, University of California, Santa Barbara, California 93106
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The Feynman rules for Majorana neutrinos are reviewed and applied. The processes discussed are $\nu' \to \nu\gamma$, $\nu' \to \nu e^+ e^-$, and neutrino oscillations, especially with regard to the difference between Dirac and Majorana neutrinos. Some special features of CP violation in the neutrino sector are pointed out.

I. INTRODUCTION

Recently, there seems to be some indication that the neutrino mass might not be zero.[1,2] If that is the case, one important question is whether these neutrinos are described by Dirac fields or Majorana fields. The main difference in these two descriptions is that the massive Dirac field has four independent components and has a well-defined fermion number (or lepton number). Theoretically, it is more economical to have a two-component Majorana field[3] which can be identified with ν_L and its antiparticle $\bar{\nu}_R = (\nu_L)^C$. These are the components which are observed in the usual weak-interaction phenomenology. But for the massive Dirac neutrino we have to introduce additional new components, ν_R and its antiparticle $\bar{\nu}_L = (\nu_R)^C$, which have not been seen in the ordinary weak interactions. To distinguish these two types of neutrinos one might either look for lepton-number-violating processes which involve the Majorana neutrinos[4] or effects which depend on the number of components of the neutrinos. On the other hand, it is well known that as far as the weak interaction is concerned, in the limit of vanishing ν masses the Dirac field has only two components and is equivalent to the two-component Majorana field. This means that the lepton-number-violating processes which can distinguish Dirac neutrinos from Majorana neutrinos will be proportional to (m_ν/E), where E is the typical energy scale in the problem. Thus, if the neutrino masses are very small compared to the typical energy scale in the problem, practically there is no distinction between these two types of neutrinos. However, the present laboratory limits on the neutrino masses are $m_{\nu_e} \leq 60$ eV, $m_{\nu_\mu} \leq 500$ keV, $m_{\nu_\tau} \leq 250$ MeV. Hence neutrinos in the mass range of a few MeV which can decay into $e^+e^-\nu'$ are not ruled out and one might be able to see the difference between two types of neutrinos if the energy is not too high.

In this paper, we will study neutrino decays and neutrino oscillations to differentiate the Majorana type of neutrinos from the Dirac type of neutrinos. We will give the general formalism for the Majorana neutrino in Sec. II. Some peculiar aspects of CP violation in this context are noted. In Sec. III, we will study the decays $\nu \to e^+e^-\nu'$, $\nu \to \gamma\nu'$ and the neutrino oscillation of the type $\nu_e \to \nu_\mu$ and $\bar{\nu}_e \to \nu_\mu$ to illustrate the difference between the Dirac type of neutrino and the Majorana type of neutrino. Whether or not any of these processes is practically observable is problematic; we consider them mostly as exercises to understand the somewhat unfamiliar Majorana masses.

II. GENERAL FORMALISM OF MAJORANA NEUTRINOS

In this section, we will illustrate how can one go from the familiar Dirac field formalism to the Majorana fields. For convenience, we will use the Majorana representation where the Dirac matrices are all imaginary[5]:

$$\gamma_0 = \begin{bmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{bmatrix}, \quad \gamma^1 = \begin{bmatrix} i\sigma_3 & 0 \\ 0 & i\sigma_3 \end{bmatrix},$$

$$\gamma^2 = \begin{bmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{bmatrix}, \quad \gamma^3 = \begin{bmatrix} -i\sigma_1 & 0 \\ 0 & -i\sigma_1 \end{bmatrix},$$

$$\gamma_5 = \begin{bmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{bmatrix}.$$

In this representation, the charge-conjugation matrix c which has the property that $c(\gamma^\mu)^* c^{-1} = -\gamma^\mu$ is just a trivial phase factor which can be taken to be unity. Then the charge conjugation of the classical fields will just be the usual complex conjugate

$$\psi^c = \psi^* .$$

Thus the Majorana field x which is defined to be a self-conjugate field is just a real field

$$\chi = \chi^c = \chi^* .$$

To see the connection with the usual Dirac field, we start with the free Lagrangian for the left-handed Dirac field given by

$$L_0 = \tfrac{1}{2}\bar{\psi}_L i\overleftrightarrow{\partial}\psi_L ,$$

where

$$\psi_L = \tfrac{1}{2}(1-\gamma_5)\psi .$$

Suppose we add a Majorana mass term given by

$$L_M = \frac{m}{2}(\bar{\psi}_L^c \psi_L + \bar{\psi}_L \psi_L^c) ,$$

where

$$\psi_L^c \equiv (\psi_L)^c \equiv \tfrac{1}{2}(1+\gamma_5)\psi^* .$$

The combination defined by

$$\chi = \psi_L + \psi_L^c$$

is a Majorana field, i.e.,

$$\chi = \chi^c = \chi^* .$$

From the identities

$$\bar{\chi}\chi = \bar{\psi}_L^c \psi_L + \bar{\psi}_L \psi_L^c ,$$

$$\bar{\psi}_L i\overleftrightarrow{\partial}\psi_L = \bar{\psi}_L^c i\overleftrightarrow{\partial}\psi_L^c = \tfrac{1}{2}(\bar{\psi}_L i\overleftrightarrow{\partial}\psi_L + \bar{\psi}_L^c i\overleftrightarrow{\partial}\psi_L^c)$$
$$= \tfrac{1}{2}\bar{\chi} i\overleftrightarrow{\partial}\chi$$

we can write the Lagrangian as

$$L = \tfrac{1}{2}\bar{\psi}_L i\overleftrightarrow{\partial}\psi_L - \frac{m}{2}(\bar{\psi}_L^c \psi_L + \bar{\psi}_L \psi_L^c)$$
$$= \tfrac{1}{4}\bar{\chi} i\overleftrightarrow{\partial}\chi - \frac{m}{2}\bar{\chi}\chi .$$

This shows that the propagator for the χ field is just the usual one:

$$\langle 0 | T(\bar{\chi}(x)\chi(0)) | 0\rangle = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik\cdot x}}{k - m + i\epsilon} .$$

The left-handed Dirac field is just the left-handed projection of χ,

$$\psi_L = \tfrac{1}{2}(1-\gamma_5)\chi ,$$

so the weak currents in terms of Majorana fields take a simple form:

$$J_\mu = \bar{e}\gamma_\mu \tfrac{1}{2}(1-\gamma_5)\nu_e + \dots$$
$$= \bar{e}\gamma_\mu \tfrac{1}{2}(1-\gamma_5)\chi_e + \dots ,$$
$$J_\mu^\dagger = \bar{\nu}_e \gamma_\mu \tfrac{1}{2}(1-\gamma_5)e + \dots$$
$$= \bar{\chi}_e \gamma_\mu \tfrac{1}{2}(1-\gamma_5)e + \dots .$$

The current $J_\mu^\dagger$ can be written in terms of the charge-conjugate fields as

$$J_\mu^\dagger = \bar{e}^c \gamma_\mu \tfrac{1}{2}(1+\gamma_5)\nu_e^c + \dots$$
$$= \bar{e}^c \gamma_\mu \tfrac{1}{2}(1+\gamma_5)\chi_e^c + \dots .$$

From the fact that χ_e is a Majorana field $\chi_e^c = \chi_c$ this means that χ_e can produce either e^- or e^+, but with different chiralities. Since only the mass term can flip the chirality, in the zero-mass limit, where chirality is the same as the helicity, these two processes involving different chiralities will not interfere with each other and the Majorana field is equivalent to the Dirac field.

The Feynman rules for calculation with Majorana neutrinos of course can be read off from the above. It is then easy to see "by inspection" that even in nuclear decays like $H^3 \to He^3 + \bar{\nu} + e^+$, where the $\bar{\nu}$ is very soft, there will be no detectable difference between Dirac and Majorana neutrinos. Differences only arise the neutrino and antineutrino of opposite chiralities can interfere.

The extension to the cases of more than one ν is straightforward and will give rise to the presence of mixing angles in the lepton sector. The result is very similar to the quark sector except that unlike the quark fields, the Majorana ν's are real fields which cannot admit any phase transformation. This will give rise to additional CP-violation phases.[6] To see this, write the ν mass matrix in the form

$$L_M = \bar{\chi}_i M_{ij} \chi_j + \text{H.c.} .$$

A simple calculation gives

$$\bar{\chi} M_{ij} \chi_j = \bar{\chi} M_{ij} \chi_i ,$$
$$(\bar{\chi} M_{ij} \chi_j)^+ = \bar{\chi}_i M_{ji}^* \chi_j = \bar{\chi}_i M_{ij}^* \chi_j .$$

These imply that the mass matrix can be taken to be real and symmetric, and can be diagonalized by the orthogonal matrix

$$OMO_T = M_d ,$$

where M_d is diagonal. On the other hand, the charged lepton mass matrix is diagonalized by the biunitary transformation

$$V_L^\dagger M^\dagger V_R = M_d^\dagger$$

so the mixing matrix in the weak current is

$$U = O^T V_L ,$$

which is a unitary matrix. For the simple case of two generations, it has one angle and three phases. Redefining the charged lepton fields by the phase transformation can remove two phases, and one CP-violating phase will be left over in contrast to the absence of the CP-violating phase for two generations of quarks. In general, for n generations of leptons involving Majorana ν's there will be $\frac{1}{2}n(n-1)$ CP-violating phases in comparison to $\frac{1}{2}(n-1)(n-2)$ phases in the quark sector. Since in the zero-mass limit all the phases and angles can be transformed away, the effects due to the phases and angles are suppressed by a factor of (m_ν/E). Furthermore, the extra CP-violation phases due to the Majorana character of the neutrino fields will show up only in the lepton-number-violating processes which are of the order of (m_ν/E) compared to the lepton-conserving processes, since it is the Majorana term which violates the lepton number.

It is interesting to note that the Majorana field has the property

$$\bar{\chi}(x)\gamma_\mu \chi(x) = \bar{\chi}\sigma_{\mu\nu}\chi = 0 ,$$

which implies that the Majorana neutrinos cannot have a magnetic moment or charged radius, as expected from the charge-conjugation property of the Majorana fields.

III. NEUTRINO DECAYS AND NEUTRINO OSCILLATIONS

In this section, we will investigate the effects which can, in principle, distinguish the Majorana neutrino from the Dirac neutrino. The processes we will study are $\nu_1 \to \nu_2 \to +\gamma$, $\nu_1 \to \nu_2 + e^+ + e^-$, and neutrino oscillations.

A. $\nu_1 \to \nu_2 + \gamma$

If all the ν's are lighter than 1 MeV, this will be the main decay mode for the ν's. For simplicity, we will consider the limit that ν_2 is very light compared to ν_1 and parametrize the effective interaction as

$$L_{\nu\nu A} = \frac{f}{M}[\bar{\nu}_2 \sigma_{\mu\nu}(a + b\gamma_5)\nu_1 + \bar{\nu}_1 \sigma_{\mu\nu}(a^* + b^*\gamma_5)\nu_2]\partial^\nu A^\mu , \quad (3.1)$$

where M is the mass of ν_1 and f, a, b are constants. We will assume CP invariance so that a and b are real. The second term in (3.1) can be rewritten as

$$\bar{\nu}_2^c \sigma_{\mu\nu}(a - b\gamma_5)\nu_1^c .$$

Then the first term in (3.1) will give the decay $\nu_1 \to \nu_2 + \gamma$, while the second term gives the decay $\bar{\nu}_1 \to \bar{\nu}_2 + \gamma$ as shown in Fig. 1. These are two phsically distinct processes for the case of Dirac ν's. But for the case of Majorana ν's, these two processes are the same because ν_i's are self-conjugate and we have to add these two contributions. Thus, the effective interaction for the Dirac neutrino decay $\nu_1 \to \nu_2 + \gamma$ is

$$L_{\nu\nu A}^D = \frac{f}{M}[\bar{\nu}_2 \sigma_{\mu\nu}(a + b\gamma_5)\nu_1]\partial^\mu A^\nu , \quad (3.2)$$

while for the decay of the Majorana neutrino we have

$$L_{\nu\nu A}^M = \frac{f}{M}[\bar{\nu}_2 \sigma_{\mu\nu}\nu_1]\partial^\mu A^\nu . \quad (3.3)$$

It is straightforward to calculate the angular distribution for the decays of the Dirac neutrino to give

$$\frac{d\Gamma}{d(\cos\theta)} = \frac{f^2 M^2(|a|^2 + |b|^2)}{32\pi} \times \left[1 - \frac{2ab}{|a|^2 + |b|^2}\cos\theta\right] , \quad (3.4)$$

where θ is the angle between the momentum of the photon and the polarization of ν_1. It is clear that the angular distribution for the decays of a Majorana neutrino is isotropic, i.e.,

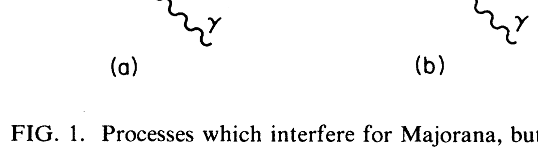

FIG. 1. Processes which interfere for Majorana, but not for Dirac, neutrinos.

$$\frac{d\Gamma}{d(\cos\theta)}=\frac{f^2M^2a^2}{8\pi} . \quad (3.5)$$

In general, the decay $\nu_1\to\nu_2+\gamma$ comes from the higher-order diagrams of the type shown in Fig. 2. If the neutrino coupling to the gauge bosons W is predominantly left-handed, as is usually the case, then we have

$$a\simeq -b . \quad (3.6)$$

The angular distribution for the case of the Dirac neutrino is then

$$\frac{d\Gamma}{d(\cos\theta)}\sim(1+\cos\theta) . \quad (3.7)$$

Thus, in principle, by measuring the angular distribution of the photon one can distinguish Dirac ν's from the Majorana ν's.[7]

B. $\nu_1\to\nu_2+e^++e^-$

If one of the ν's is heavier than 1 MeV, this will be the main decay mode.[8] The advantage of this process is that there are charged particles in the final states which can be measured rather easily. Also this decay can go through the usual lowest-order charged-current interaction and give the same rate as the usual weak decay except for the suppression of the phase space and the mixing angle. We parametrize the charged-current interaction between ν_i and e as

$$L_W=\frac{g}{2\sqrt{2}}[\bar{e}\gamma_\mu(1-\gamma_5)(\cos\theta\,\nu_2+\sin\theta\,\nu_1)]W^\mu + \text{H.c.} . \quad (3.8)$$

For the case of the Dirac ν's, the contributing diagram is shown in Fig. 3, where k_1, k_2 are the momenta of ν_1, ν_2, and q_-, q_+ are the momenta of e^-, e^+. In the rest frame of ν_1, the decay-angular distribution is then given by

$$\frac{d\Gamma}{d(\cos\theta)}=\frac{G_F^2\sin^2\theta\cos^2\theta m_1^2E_-dE_-}{48\pi^3}\times\left[3-\frac{4E_-}{m_1}\right](1-\alpha\beta\cos\theta) \quad (3.9)$$

with

$$\beta=\frac{(1-4E_-/m_1)}{(3-4E_-/m_1)} , \quad (3.10)$$

where α is the average polarization of ν_1, θ is the angle between the momentum a of the electron and the direction of the polarization of ν_1, E_- is the energy of the electron, and m_1 is the mass of ν_1. In these formulas, we have made the approximation $m_2=m_e=0$. Just as in the previous case, there are also two diagrams contributing to the decays of the Majorana neutrinos (Fig. 4). In the limit $m_e=m_2=0$, ν_2 from the first diagram is left-handed and ν_2 from the second diagram is right-handed. Thus there is no interference between these two diagrams. The angular distribution is then

$$\frac{d\Gamma}{d(\cos\theta)}\propto(1-\alpha\beta\cos\theta) \quad (3.11)$$

with

$$\beta=\frac{(7-16E_-/m_1)}{(9-16E_-/m_1)} . \quad (3.12)$$

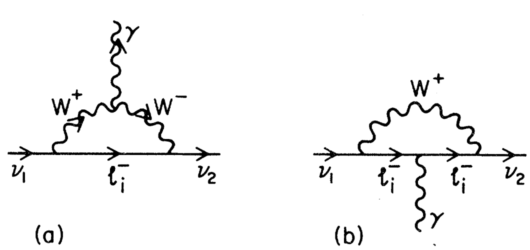

FIG. 2. Feynman diagram leading to radiative neutrino decay.

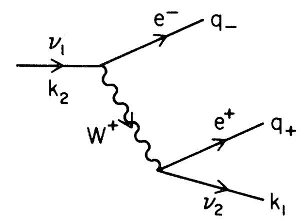

FIG. 3. $\nu_1\to e^+e^-\nu_2$, for a Dirac neutrino.

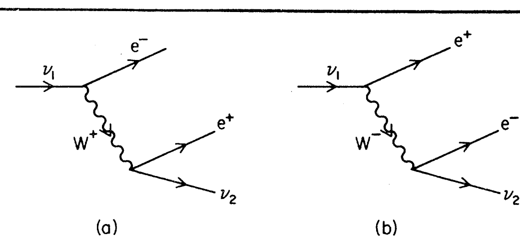

FIG. 4. Coherent amplitudes for $\nu_1\to e^+e^-\nu_2$, for a Majorana neutrino.

Thus, the angular distribution for the decays of the Majorana neutrino is readily distinguishable from that of the Dirac neutrino.[9]

C. Neutrino oscillations

As we have explained in Section II, for the case of the Majorana ν's, there are additional CP-violating phases, which are not present for the case of the Dirac ν's.[10] For illustrative purposes, we will take the simple case of two generations of ν's where there is one CP-violating phase for the Majorana ν's while thre is no CP violation for the Dirac ν's. We write the left-handed doublets as

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L = \begin{pmatrix} \cos\theta\ \nu_1 + \sin\theta\, e^{i\delta}\nu_2 \\ e \end{pmatrix} ,$$

$$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L = \begin{pmatrix} -e^{i\delta}\sin\theta\ \nu_1 + \cos\theta\ \nu_2 \\ \mu \end{pmatrix}_L . \tag{3.13}$$

To see the effect of the CP-violating phase δ, we will compare the oscillation $\nu_\mu \to \bar{\nu}_e$ with the CP-conjugate oscillation $\bar{\nu}_\mu \to \nu_e$.[11] More precisely, for the $\nu_\mu \to \bar{\nu}_e$ oscillation we consider the situation where ν_μ is produced from the π^+ decay at $t=0$ and at a later time t the neutrino will hit a nucleon target to produce e^+. We denote the matrix element for the decay $\pi^+ \to \mu^+ \nu_i$, $i=1, 2$ by

$$M_{i\mu} = \frac{G_F}{\sqrt{2}} f_\pi \bar{U}_i(k_i) \times [m_i(1-\gamma_5) - m(1+\gamma_5)]\nu_\mu(q) , \quad i=1,2 , \tag{3.14}$$

where f_π is the usual pion decay constant. Similarly, the amplitudes for the decays $\pi^- \to \mu^- \bar{\nu}_i$ are given by

$$M_{i\mu} = \frac{G_F}{\sqrt{2}} f\pi \bar{U}_\mu(q) \times [m_i(1+\gamma_5) - m(1-\gamma_5)]\nu_1(k_i) . \tag{3.15}$$

The amplitudes for the charged-current neutrino reactions are written as

$$T_{ie} = \frac{G_F}{\sqrt{2}} \langle N' | J_\nu | N \rangle \bar{U}_e(p)\gamma^\nu(1-\gamma_5)U_i(k_i) \tag{3.16}$$

for $\nu_i(k_i) + N \to e^-(p) + N'$, $i=1$, 2, and

$$T_{ie^-} = \frac{G_F}{\sqrt{2}} \langle N' | J_\nu^\dagger | N \rangle \bar{U}_i(k_i)\nu^\nu(1-\gamma_5)U_e(p) \tag{3.17}$$

for $\bar{\nu}_i(k_i) + N \to e^+(p) + N'$, $i=1, 2$. Then, for the $\nu_\mu \to \bar{\nu}_e$ oscillation the amplitude is

$$T_{\nu\mu,\bar{\nu}_e} = \tfrac{1}{2} \sin\theta \cos\theta\, e^{i\delta}[(M_{1\mu}T_{1\bar{e}} + M_{2\mu}T_{2\bar{e}})(-e^{-iE_1t} + e^{-2i\delta}e^{-iE_2t}) - (M_{1\mu}T_{1\bar{e}} - M_{2\mu}T_{2\bar{e}})(e^{-iE_1t} + e^{-2i\delta}e^{-iE_2t})] , \tag{3.18}$$

where $E_i = K_i{}^2 + m_i{}^2$. If we assume that

$$| M_{1\mu}T_{1\bar{e}} + M_{2\mu}T_{2\bar{e}} | >> | M_{1\mu}T_{1\bar{e}} - M_{2\mu}T_{2\bar{e}} , \tag{3.19}$$

we get

$$T_{\nu_\mu,\bar{\nu}_e} = \sin\theta \cos\theta \times e^{i\delta}M_{1\mu}T_{1\bar{e}}(-e^{-iE_1t} + e^{-2i\delta}e^{-iE_2t}) \tag{3.20}$$

and

$$| T_{\nu_\mu,\bar{\nu}_e} |^2 = \cos^2\theta \sin^2\theta \, | M_{1\mu}T_{1\bar{e}} |^2 \times 4 \sin^2(\tfrac{1}{2}\Delta E\ t - \delta) ,$$

where $\Delta E = E_1 - E_2$. Similarly, for the $\bar{\nu}_\mu \to \nu_e$ oscillations we get, in the same approximation,

$$| T_{\bar{\nu}_\mu,\nu_e} |^2 = \cos^2\theta \sin^2\theta \, | M_{1\bar{\mu}}T_{1\bar{e}} |^2 \times 4\sin^2(\tfrac{1}{2}\Delta E\ t + \delta) .$$

Thus, there is a phase difference between these two CP-conjugate oscillations. It is not hard to see from the structure of the matrix elements $M_{i\mu}$, $M_{i\bar{\mu}}T_{ie}$, $T_{i\bar{e}}$, given Eqs. (3.14)–(3.17) that these oscillations are suppressed by $(m_i/E)^2$ relative to the usual oscillations of the type $\nu_e \rightleftarrows \nu_\mu$. But it can be significant if the m_i's are not too small compared to the energies of ν beams. It is amusing that these oscillations are nonzero at $t=0$; a pure electron neutrino has a nonzero amplitude to be a muon antineutrino!

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*Permanent address: Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213

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[10]See also J. Schechter and J. Valle, Phys. Rev. D 23, 1666 (1981).

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