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Weak gauge bosons above or straddling the standard-model masses in an SU(2)×U(1)×SU(2)′ gauge model

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We present a natural SU(2)×U(1)×SU(2)′ gauge model with low-energy predictions identical to those of the standard model, but which allows all gauge-boson masses to be above or straddling their standard values. We discuss the Higgs structure and its relationship to the naturalness of the model. Current measurements of the $e^+e^-\rightarrow\mu^+\mu^-$ cross section set upper limits on the lightest W and Z masses of 89 and 116 GeV, respectively; from data on $e^+e^-\rightarrow$ hadrons, the corresponding limits $M_{W_1}<87$ and $M_{Z_1}<108$ GeV are deduced. Some high-energy predictions for the model are explored. The model is extended to $\mathrm{U}(1)\times\mathrm{SU}(2)^N$ with all possible SU(2)×U(1) Higgs doublets and SU(2)×SU(2) Higgs quartets.

I. INTRODUCTION

The standard SU(2)×U(1) electroweak model[1] sucessfully describes all low-energy weak-interaction phenomena.[2] Alternative models based on the gauge group SU(2)×U(1)×G, where G is an arbitrary group, can be constructed[3–6] in which the predictions at low energies are identical to those of the standard model, but will differ at higher energies. In previously constructed models, the gauge bosons had masses less than the standard W and Z masses. A general analysis based on the SU(2) and U(1) couplings alone shows that, in principle, there is no upper bound on the lightest Z^0.[7] There is already a model[8] with all masses significantly greater than 100 GeV, but which requires an adjustment of parameters to achieve the proper charged-to-neutral-current strength ratio. In this paper we present a class of models which *naturally* reproduce the standard low-energy behavior and which allow the lightest weak bosons to be heavier than those of the standard model. We examine in detail the simplest of these models.[9] It is interesting to examine extended gauge groups in the event that the weak-boson mass spectrum turns out to have more structure than the one-Z, one-W prediction of the standard model. The group G could be connected with still-to-be-discovered heavy fermions.

The electroweak gauge group to be studied is SU(2)×U(1)×SU(2)′ with couplings g_0, $\frac{1}{2}g_1$, and g_2, respectively. The known quarks and leptons are assumed to transform only under the subgroup SU(2)×U(1), and in the same manner as the standard model. The additional SU(2)′ may couple to heavy fermions which are not detectable at presently available energies. To generate masses, we use the scalar doublets $\Phi=(\phi^+,\phi^0)$ and $\Psi=(\psi^+,\psi^0)$ and a scalar quartet $(\eta^+,\eta^-,\bar{\eta}^0,\eta^-)$ in the representations $(T,Y,T')=(\frac{1}{2},1,0)$, $(0,1,\frac{1}{2})$, and $(\frac{1}{2},0,\frac{1}{2})$, respectively. Spontaneous symmetry breakdown occurs with vacuum expectation values $\langle\phi^0\rangle,\langle\psi^0\rangle,\langle\eta^0\rangle$. This is similar to the Higgs structure of Ref. 4, except for the addition of the Ψ doublet. The Higgs mechanism is fully developed in Sec. II.

A consequence of this model is that the relative strength between the charged-current (CC) and neutral-current (NC) interactions is naturally the same as in the standard model, i.e., without adjusting any parameters. The naturalness comes about from the simple form of the weak-boson mass matrices. With our choice of Higgs structure the charged-boson mass-squared matrix $\mathscr{M}_W{}^2$ is found by removing from the neutral-boson mass-squared matrix $\mathscr{M}_Z{}^2$ the row and column corresponding to the B boson of the U(1) group. The matrix $\mathscr{M}_Z{}^2$ is given by $(\mathscr{M}_Z{}^2)_{ii}=\frac{1}{2}g_i{}^2\sum_{j\neq i}v_{ij}{}^2$ and $(\mathscr{M}_Z{}^2)_{ij} = -\frac{1}{2}g_ig_jv_{ij}{}^2$ for $i\neq j$, where v_{ij} is the vacuum expectation value of the Higgs multiplet which connects the ith and jth groups.

Since the known fermions are assumed to couple only to U(1) and to the first SU(2), their charged-current interaction is given by $2^{-1/2}g_0 j_\mu^{(+)}W_{1\mu}^+$, where W_1^+ is the charged boson associated with the first SU(2). The resulting effective low-energy interaction is $\frac{1}{2}g_0{}^2\langle W_1^+W_1^-\rangle$ where $\langle W_1^+W_1^-\rangle=(\mathscr{M}_W{}^{-2})_{11}$. The neutral-current interaction is given by $g_1(j_\mu^{\rm em}-j_\mu^{(3)})B_\mu+g_0 j_\mu^{(3)}W_{1\mu}^{(3)}$ and leads to the effective low-energy interaction $\frac{1}{2}a[(j_\mu^{(3)}-bj_\mu^{\rm em})^2+C(j_\mu^{\rm em})^2]$ where $a=g_1{}^2\langle BB\rangle-2g_0g_1\langle BW_1^{(3)}\rangle+g_0{}^2\langle W_1^{(3)}W_1^{(3)}\rangle$ and b is identified with $\sin^2\theta_W$. The factor $\frac{1}{2}a$ appears in the interaction because the product of neutral currents contributes in two possible orderings. The matrix $\mathscr{M}_Z{}^2$ is singular because one of the mass eignestates is the massless photon. By adding to $(\mathscr{M}_Z{}^2)_{ij}$ the term λ^2e^2/g_ig_j, the photon acquires an artificial mass, and the singularity is removed.[5] The propagators in the expression for a are $\langle BB\rangle=(\mathscr{M}_Z{}^{-2})_{11}$, $\langle BW_1^{(3)}\rangle=(\mathscr{M}_Z{}^{-2})_{01}$, and $\langle W_1^{(3)}W_1^{(3)}\rangle=(\mathscr{M}_Z{}^{-2})_{00}$. With the above mass-squared matrices, it can be demonstrated that $a=g_0{}^2\langle W_1^+W_1^-\rangle$ in the limit $\lambda=0$. This gives the standard-model relation between the effective CC and NC couplings.

The particular Higgs structure of our model is essential in obtaining the NC/CC ratio naturally. For an SU(2)$\times$U(1) Higgs multiplet (T,Y) the weak-boson masses arise from terms in the Lagrangian of the form $\langle\phi^\dagger\rangle(\frac{1}{2}g_1B_\mu^\dagger+g_i\vec{\rm T}_i\cdot\vec{\rm W}_\mu^\dagger)(\frac{1}{2}g_1B_\mu + g_i\vec{\rm T}_i\cdot\vec{\rm W}_\mu)\langle\phi\rangle$. A simple computation shows that the desired relation between $\mathscr{M}_W{}^2$ and $\mathscr{M}_Z{}^2$ occurs only when $T(T+1)-T_3{}^2=2T_3{}^2$, where T_3 is the SU(2) quantum number of the neutral Higgs field which acquires a nonzero vacuum expectation value. The relation holds for $T=\frac{1}{2}$ and for some higher values $(T=3,\frac{25}{2},\ldots)$. For SU(2)$\times$SU(2) Higgs multiplets (T,T'), more than one neutral Higgs field in the multiplet can break the symmetry and the situation is more complicated. The only simple examples which preserve the natural property are $T=T'$ with all nonzero vacuum expectation values in the multiplet equal. Only a $T=T'=\frac{1}{2}$ multiplet meets this last condition without the imposition of additional symmetries.

The physical gauge fields are related to the mass eigenstates in Sec. III. The six parameters of the model g_0, g_1, g_2, $\langle\phi^0\rangle\langle\psi^0\rangle$, and $\langle\eta^0\rangle$ can be expressed in terms of G_F, e, the angle θ_W of the standard model, and three other parameters which cannot be determined at low energies. Alternate choices of the parameters are three masses M_{Z_1}, M_{Z_2}, M_{Z_c} (or M_{W_1}, M_{W_2}, M_{W_c}), where M_{Z_1} and M_{Z_2} are physical guage-boson masses and M_{Z_c} is a critical mass in the model. Another choice of parameters is δ_0, δ_1, and C, where δ_0, δ_1 re related to the couplings g_0, g_1 and C is the coefficient of the $(j_\mu^{\rm em})^2$ term in the effective Lagrangian at low Q^2.

In Sec. IV we derive upper limits on the lightest W and Z masses for the model, based on $e^+e^-\to\mu^+\mu^-$ cross-section measurements. These limits are 89 and 116 GeV for the lightest W and Z, respectively. In Sec. V some phenomenology of the model is presented and compared with that of the standard model. From data on $e^+e^-\to$ hadrons, we deduce an improved upper limit on the parameter C with constrains the lightest W and Z to be below 87 and 108 GeV, respectively. Total widths for the weak bosons are calculated. We present predictions for the total cross section and asymmetry in e^+e^- annihilation and the dilepton mass spectrum for the Drell-Yan process in hadron collisions.

In Sec. VI, we extend the model to include an arbitrary number of SU(2) groups. We show that these extended models are natural and argue that the upper bounds on the lightest W and Z derived in Sec. IV are probably not raised substantially in the extended models.

II. THE HIGGS-BOSON STRUCTURE

The covariant derivative for the SU(2)$\times$U(1)$\times$SU(2)$'$ gauge group is given by

$$D_\nu=\partial_\nu-ig_0\vec{\rm T}\cdot\vec{\rm W}_\nu -ig_1\frac{Y}{2}B_\nu-ig_2\vec{\rm T}'\cdot\vec{\rm W}'_\nu\;, \tag{1}$$

where $\vec{\rm W}'_\nu$ are the new gauge bosons of SU(2)$'$ which are not coupled to the known fermions. The electric charge operator is $Q=T_3+\frac{1}{2}Y+T'_3$. The gauge-symmetry breaking is accomplished by the usual Higgs-boson doublet $(T=\frac{1}{2},Y=1,T'=0)$

$$\Phi=\begin{bmatrix}\phi^+\\ \phi^0\end{bmatrix}, \tag{2a}$$

a similar doublet for SU(2)′ ($T=0, Y=1, T'=\frac{1}{2}$),

$$\Psi=\begin{bmatrix}\psi^+\\ \psi^0\end{bmatrix}, \tag{2b}$$

and a Higgs-boson quartet ($T=\frac{1}{2}, Y=0, T'=\frac{1}{2}$) which is self-dual ($\eta=\tau_2\eta^*\tau_2$):

$$\eta=\begin{array}{cc} -\frac{1}{2} & +\frac{1}{2} \\ \end{array}\quad \begin{bmatrix}\eta^0 & -\eta^+\\ \eta^- & \bar{\eta}^0\end{bmatrix}\begin{array}{c}+\frac{1}{2}\\ -\frac{1}{2}\end{array} \tag{2c}$$

(here the rows correspond to different values of T_3, and the columns to different values of T'_3). In this representation

$$\vec{\mathrm{T}}\eta=\frac{\vec{\tau}}{2}\eta\,,$$

$$\vec{\mathrm{T}}'\eta=-\eta\frac{\vec{\tau}}{2}\,. \tag{3}$$

The most general SU(2)×U(1)×SU(2)′-invariant Lagrangian is

$$\mathscr{L}=(D_\nu\Phi)^\dagger(D_\nu\Phi)+(D_\nu\Psi)^\dagger(D_\nu\Psi)+TR(D_\nu\eta)^\dagger(D_\nu\eta)-V(\eta,\Phi,\Psi)\,, \tag{4}$$

where the Higgs potential is

$$V=\mu_1{}^2\Phi^\dagger\Phi+\lambda_1(\Phi^\dagger\Phi)^2+\mu_2{}^2\Psi^\dagger\Psi+\lambda_2(\Psi^\dagger\Psi)^2+m^2TR\,\eta^\dagger\eta+h(\mathrm{Tr}\eta^\dagger\eta)^2$$
$$+f_1\Phi^\dagger\eta\eta^\dagger\Phi+f_2\Psi^\dagger\eta^\dagger\eta\Psi+f_3\Phi^\dagger\Phi\Psi^\dagger\Psi+t(\Phi^\dagger\eta\Psi+\Psi^\dagger\eta^\dagger\Phi)\,. \tag{5}$$

Here Tr denotes the trace and the coefficients are real. Expanding this expression for the potential in terms of component fields, we obtain

$$\begin{aligned}V=&\mu_1{}^2(|\phi^0|^2+|\phi^+|^2)+\lambda_1(|\phi^0|^2+|\phi^+|^2)^2+\mu_2{}^2(|\psi^0|^2+|\psi^+|^2)\\&+\lambda_2(|\psi^0|^2+|\psi^+|^2)^2+2m^2(|\eta^0|^2+|\eta^-|^2)+4h(|\eta^0|^2+|\eta^-|^2)^2\\&+[f_1(|\phi^0|^2+|\phi^+|^2)+f_2(|\psi^0|^2+|\psi^+|^2)](|\eta^0|^2+|\eta^-|^2)\\&+f_3(|\phi^0|^2+|\phi^+|^2)(|\psi^0|^2+|\psi^+|^2)\\&+t(\phi^-\eta^0\psi^+-\phi^-\eta^+\psi^0+\bar{\phi}^0\eta^-\psi^++\bar{\phi}^0\bar{\eta}^0\psi^0+\phi^+\bar{\eta}^0\psi^--\phi^+\eta^-\bar{\psi}^0+\phi^0\eta^+\psi^-+\phi^0\eta^0\bar{\psi}^0)\,.\end{aligned} \tag{6}$$

With μ_1, μ_2, and m imaginary, the Higgs phenomenon occurs with vacuum expectation values

$$\langle\Phi\rangle=\begin{bmatrix}0\\ v\end{bmatrix},\quad \langle\Psi\rangle=\begin{bmatrix}0\\ w\end{bmatrix},\quad \langle\eta\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}u & 0\\ 0 & u^*\end{bmatrix}. \tag{7}$$

For vacuum stability we must impose the condition

$$h\geq 0\,. \tag{8}$$

Necessary conditions for a minimum of the potential with nonvanishing u, v, and w are

$$\mu_1{}^2+2\lambda_1v^2+\tfrac{1}{2}f_1u^2+f_3w^2+\frac{tuw}{\sqrt{2}v}=0\,,\quad \mu_2{}^2+2\lambda_2w^2+\tfrac{1}{2}f_2u^2+f_3v^2+\frac{tuv}{\sqrt{2}w}=0\,, \tag{9}$$
$$m^2+2hu^2+\tfrac{1}{2}f_1v^2+\tfrac{1}{2}f_2w^2+\frac{tvw}{\sqrt{2}u}=0\,.$$

Here we have ignored possible *CP*-violating effects and set $u=u^*$. Redefining the scalar fields as $\phi\to\phi+\langle\phi\rangle$, $\psi\to\psi+\langle\psi\rangle$, and $\eta\to\eta+\langle\eta\rangle$, we obtain the Higgs-boson mass-squared matrix from the quadratic terms of the potential. In the charged sector,

$$V=-\frac{t}{\sqrt{2}uvw}\,|uw\phi^+-uv\psi^++\sqrt{2}vw\eta^{-*}|^2\,. \tag{10}$$

Thus one linear combination of ϕ^+, ψ^+, and η^{-*} acquires mass. The two other independent states can be gauged away to become the longitudinal components of the charged vector gauge bosons.

The quadratic terms in the potential involving $\mathrm{Im}\phi^0$, $\mathrm{Im}\psi^0$, and $\mathrm{Im}\eta^0$ become

$$V=-\frac{t}{\sqrt{2}uvw}\,|\,uw\,\mathrm{Im}\phi^0-uv\,\mathrm{Im}\psi^0+\sqrt{2}vw\,\mathrm{Im}\eta^0\,|^2\,. \tag{11}$$

The two massless combinations are absorbed into longitudinal components of the two neutral gauge bosons. The mass-squared matrix of the remaining Higgs-boson states $\mathrm{Re}\phi^0$, $\mathrm{Re}\psi^0$, $\mathrm{Re}\eta^0$ is

$$V=\begin{bmatrix} -\frac{tuw}{\sqrt{2}v}+4\lambda_1 v^2 & 2f_3vw+\frac{ut}{\sqrt{2}} & \sqrt{2}uvf_1+wt \\ 2f_3vw+\frac{ut}{\sqrt{2}} & -\frac{tuv}{\sqrt{2}w}+4\lambda_2 w^2 & \sqrt{2}uwf_2+vt \\ \sqrt{2}uvf_1+wt & \sqrt{2}uwf_2+vt & -\sqrt{2}\frac{tvw}{u}+8hu^2 \end{bmatrix}. \tag{12}$$

To ensure that all observable Higgs bosons have positive mass squared, we require $t<0$ in addition to Eq. (8).

We remark briefly on the symmetry of the Higgs potential in Eq. (5). Without the t term, the symmetry is $O(4)\times O(4)\times O(4)$, which upon spontaneous symmetry breaking becomes $O(3)\times O(3)\times O(3)$. This yields nine Nambu-Goldstone bosons, of which six are absorbed into $W_1^{\pm},Z_1,W_2^{\pm},Z_2$ and three remain. With the t term, the symmetry of the potential is $SU(2)\times SU(2)\times SU(2)$, as can be verified by writing the t interaction in the form

$$\Phi^\dagger\eta\psi+\psi^\dagger\eta^\dagger\Phi=\mathrm{Tr}\begin{bmatrix}\phi^0 & -\phi^+\\ \phi^- & \bar{\phi}^0\end{bmatrix}\begin{bmatrix}\eta^0 & -\eta^+\\ \eta^- & \bar{\eta}^0\end{bmatrix}\begin{bmatrix}\bar{\psi}^0 & \psi^+\\ -\psi^- & \psi^0\end{bmatrix}. \tag{13}$$

After spontaneous breaking, the symmetry is reduced to SU(2) with six Nambu-Goldstone bosons that are absorbed. The residual SU(2) symmetry leaves a Higgs triplet, given by Eqs. (10) and (11). This symmetry is only approximate in that it will be broken by the Yukawa couplings to fermions.

III. MASS EIGENSTATES OF THE GAUGE FIELDS

In the neutral sector the mass-squared matrix in the $W^{(3)}$, $W'^{(3)}$, and B basis is

$$\mathcal{M}_Z{}^2=\frac{1}{2}\begin{bmatrix} g_0{}^2(v^2+u^2) & -g_0g_2u^2 & -g_0g_1v^2 \\ -g_0g_2u^2 & g_2{}^2(w^2+u^2) & -g_1g_2w^2 \\ -g_0g_1v^2 & -g_1g_2w^2 & g_1{}^2(v^2+w^2) \end{bmatrix}. \tag{14}$$

The matrix $\mathcal{R}^Z$ for which $\mathcal{R}^Z\mathcal{M}_Z{}^2\mathcal{R}^{Z\dagger}$ is diagonal is given by

$$\mathcal{R}^Z=\begin{bmatrix} \frac{n_0}{g_0} & \frac{n_0}{g_2} & \frac{n_0}{g_1} \\ \frac{n_1}{bg_0}\left[u^2-\frac{g_1{}^2K}{2M_{Z_1}{}^2}\right] & \frac{n_1}{bg_2}\left[-\frac{2M_{Z_1}{}^2}{g_0{}^2}+\left[u^2+v^2+\frac{g_1{}^2}{g_0{}^2}(v^2+w^2)\right]-\frac{g_1{}^2K}{2M_{Z_1}{}^2}\right] & \frac{n_1}{bg_1}\left[\frac{g_1{}^2w^2}{g_0{}^2}-\frac{g_1{}^2K}{2M_{Z_1}{}^2}\right] \\ \frac{n_2}{bg_0}\left[u^2-\frac{g_1{}^2K}{2M_{Z_2}{}^2}\right] & \frac{n_2}{bg_2}\left[-\frac{2M_{Z_2}{}^2}{g_0{}^2}+\left[u^2+v^2+\frac{g_1{}^2}{g_0{}^2}(v^2+w^2)\right]-\frac{g_1{}^2K}{2M_{Z_2}{}^2}\right] & \frac{n_2}{bg_1}\left[\frac{g_1{}^2w^2}{g_0{}^2}-\frac{g_1{}^2K}{2M_{Z_2}{}^2}\right] \end{bmatrix}, \tag{15}$$

where $K\equiv u^2v^2+u^2w^2+v^2w^2$ and $b=u^2-g_1{}^2w^2/g_0{}^2$. The normalizations n_0, n_1, and n_2 are chosen such that $\mathscr{R}^{Z\dagger}\mathscr{R}^{Z}=\mathscr{R}^{Z}\mathscr{R}^{Z\dagger}=1$:

$$n_0{}^2=\left[\frac{1}{g_0{}^2}+\frac{1}{g_1{}^2}+\frac{1}{g_2{}^2}\right]^{-1}, \quad \frac{n_1{}^2}{M_{Z_1}{}^2}+\frac{n_2{}^2}{M_{Z_2}{}^2}=\frac{2(u^2+w^2)}{K},$$
$$\frac{n_1{}^2}{M_{Z_1}{}^4}+\frac{n_2{}^2}{M_{Z_2}{}^4}=\frac{4[g_0{}^2u^4+g_1{}^2w^4+g_2{}^2(u^2+w^2)^2]}{K^2(g_0{}^2g_1{}^2+g_1{}^2g_2{}^2+g_2{}^2g_0{}^2)} . \tag{16}$$

In terms of the mass eigenstates A, Z_1, and Z_2, the covariant derivative of Eq. (1) for the neutral sector is

$$D_\nu^0=\partial_\nu-in_0QA_\nu-i\sum_{i=1,2}n_i\left[T^{(3)}+Q\frac{1}{b}\left[\frac{g_1{}^2w^2}{g_0{}^2}-\frac{g_1{}^2K}{2M_{Z_i}{}^2}\right]\right.$$
$$\left.+T'^{(3)}\frac{1}{bg_0{}^2}[-2M_{Z_i}{}^2+g_1{}^2v^2+g_0{}^2(u^2+v^2)]\right]Z_{i\nu} . \tag{17}$$

Hence n_0 is to be identified with the electric charge e. In the fermion sector the SU(2)′ group is inactive so that the interaction Hamiltonian is

$$\mathscr{H}=ej_\nu^{\mathrm{em}}A_\nu+\sum_{i=1,2}n_i\left[j_\nu^{(3)}-\frac{1}{b}\left[\frac{g_1{}^2K}{2M_{z_i}{}^2}-\frac{g_1{}^2w^2}{g_0{}^2}\right]j_\nu^{\mathrm{em}}\right]Z_{i\nu} , \tag{18}$$

where $\vec{j}$ is the usual left-handed weak fermion current and j^{em} is the electromagnetic current. The effective weak neutral-current interaction at low energy is then

$$\mathscr{H}_{\mathrm{eff}}^{\mathrm{NC}}=\frac{(u^2+w^2)}{K}\left\{\left[j_\nu^{(3)}-\left[\frac{u^2(1-e^2/g_1{}^2)+w^2(e^2/g_0{}^2)}{u^2+w^2}\right]j_\nu^{\mathrm{em}}\right]^2+C(j_\nu^{\mathrm{em}})^2\right\} , \tag{19}$$

where

$$C\equiv\frac{K}{2(u^2+w^2)}\left[\sum_{i=1,2}\frac{n_i{}^2}{b^2M_{Z_i}{}^2}\left[\frac{g_1{}^2K}{2M_{z_i}{}^2}-\frac{g_1{}^2w^2}{g_0{}^2}\right]^2\right]-\left[\frac{u^2(1-e^2/g_1{}^2)+w^2e^2/g_0{}^2}{u^2+w^2}\right]^2 . \tag{20}$$

To ensure that the first term of Eq. (19) reproduces the effective Hamiltonian of the standard model we require that

$$\frac{G_F}{\sqrt{2}}=\frac{(u^2+w^2)}{4K},$$
$$\sin^2\theta_W=\frac{u^2(1-e^2/g_1{}^2)+w^2e^2/g_0{}^2}{u^2+w^2} . \tag{21}$$

In terms of the gauge-boson masses M_W and M_Z of the standard model ($M_W=77.6$ GeV, $M_Z=88.5$ GeV for $\sin^2\theta_W=0.23$), Eq. (21) leads to the relation

$$\frac{Kg_1{}^2}{2b}\frac{(u^2c_W{}^2-w^2s_W{}^2)}{u^2+w^2}=M_Z{}^2s_W{}^2c_W{}^2 , \tag{22}$$

where $c_W\equiv\cos\theta_W$ and $s_W\equiv\sin\theta_W$.

It is convenient to define the quantities

$$\delta_0=\frac{s_W{}^2g_0{}^2}{e^2}-1 ,$$
$$\delta_1=\frac{c_W{}^2g_1{}^2}{e^2}-1 ,$$
$$D=\delta_0\delta_1+\delta_0s_W{}^2+\delta_1c_W{}^2 . \tag{23}$$

The first two measure the deviation of g_0 and g_1 from their standard-model values. Using the above relations, the right-hand side of Eq. (16) can be replaced by observable quantities:

$$n_0=e\ ,$$

$$\frac{n_1^2}{M_{Z_1}^2}+\frac{n_2^2}{M_{Z_2}^2}=\frac{8G_F}{\sqrt{2}}\ , \tag{24}$$

$$\frac{n_1^2}{M_{Z_1}^4}+\frac{n_2^2}{M_{Z_2}^4}=\frac{8G_F}{\sqrt{2}M_{Z_c}^2}\ ,$$

where the critical mass M_{Z_c} is defined as

$$M_{Z_c}^2=\frac{g_0^2g_1^2g_2^2}{2e^2}\frac{(u^2+w^2)K}{g_0^2u^4+g_1^2w^4+g_2^2(u^2+w^2)^2}$$

$$=M_Z^2(1-\delta_0\delta_1/D)^{-1}\ . \tag{25}$$

The values of n_1 and n_2 can thereby be expressed in terms of the critical mass and the Z_1, Z_2 masses:

$$n_i=\left[\frac{8G_F}{\sqrt{2}}\right]^{1/2}\frac{M_{Z_i}^2}{M_{Z_c}}\left|\frac{M_{Z_j}^2-M_{Z_c}^2}{M_{Z_2}^2-M_{Z_1}^2}\right|^{1/2}\ ,\quad i\neq j\ . \tag{26}$$

The expression for C in Eq. (20) can be written as

$$C=\frac{M_Z^4s_W^2c_W^2D}{M_{Z_1}^2M_{Z_2}^2}\ , \tag{27}$$

where D is defined in Eq. (23). The term in the effective NC Hamiltonian of Eq. (19) involving C is absent in the standard model. If Z_1 and Z_2 are labeled such that $M_{Z_1}<M_{Z_2}$, the reality of n_1 and n_2 requires

$$M_{Z_1}<M_{Z_c}<M_{Z_2}\ . \tag{28}$$

The charged-vector-boson mass-squared matrix is the same as the $W^{(3)},W'^{(3)}$ part of $\mathcal{M}_z^2$ in Eq. (14). By dropping terms involving n_0 and g_1 in Eq. (15) we can immediately write down the diagonalization matrix for $\mathcal{M}_W^2$:

$$R^W=\frac{1}{g_0}\begin{bmatrix}N_1 & N_2\\ N_2 & -N_1\end{bmatrix}\ , \tag{29}$$

where

$$\frac{N_1^2}{M_{W_1}^2}+\frac{N_2^2}{M_{W_2}^2}=\frac{2(u^2+w^2)}{K}=\frac{8G_F}{\sqrt{2}}\ ,$$

$$\frac{N_1^2}{M_{W_1}^4}+\frac{N_2^2}{M_{W_2}^4}=4\frac{g_0^2u^4+g_2^2(u^2+w^2)^2}{g_0^2g_2^2K^2} \tag{30}$$

$$\equiv\frac{8G_F}{\sqrt{2}}\frac{1}{M_{W_c}^2}\ .$$

The normalization constants expressed in terms of the W_1 and W_2 masses are

$$N_i=\left[\frac{8G_F}{\sqrt{2}}\right]^{1/2}\frac{M_{W_i}^2}{M_{W_c}}\left|\frac{M_{W_j}^2-M_{W_c}^2}{M_{W_2}^2-M_{W_1}^2}\right|^{1/2}\ ,\quad i\neq j \tag{31}$$

with the critical W-boson mass defined by

$$M_{W_c}^2=\frac{1}{2}\frac{g_0^2g_2^2(u^2+w^2)K}{g_0^2u^4+g_2^2(u^2+w^2)^2}$$

$$=M_W^2(1-\delta_0\delta_1c_W^2/D)^{-1}\ . \tag{32}$$

Labeling W_1 and W_2 such that $M_{W_1}<M_{W_2}$ the reality of N_1 and N_2 requires

$$M_{W_1}<M_{W_c}<M_{W_2}\ . \tag{33}$$

The charged-current Hamiltonian in the fermion sector is

$$\mathcal{H}=\frac{1}{\sqrt{2}}(N_1W_{1\nu}^{+}+N_2W_{2\nu}^{+})j_\nu^{(+)}+\text{ H.c.}\ , \tag{34}$$

where $j_\nu^{(+)}=j_\nu^{(1)}+ij_\nu^{(2)}$. The ratio of the charged-current to neutral-current weak coupling strengths is the same as in the standard model regardless of the value of the parameters in the theory.

From Eqs. (14) and (23) we find the mass relations

$$M_{W_1}M_{W_2}=M_{Z_1}M_{Z_2}\cos\theta_W(1+\delta_1)^{-1/2}\ , \tag{35}$$

$$M_{W_1}^2+M_{W_2}^2+M_W^2\tan^2\theta_W[1+\delta_1+\delta_1^2\cos^4\theta_W/(1+\delta_1)C]=M_{Z_1}^2+M_{Z_2}^2\ .$$

From Eq. (14) we find the mass inequalities

$$M_{W_1}<M_{Z_1}<M_{W_2}<M_{Z_2}\ . \tag{36}$$

However, $M_{Z_1}>M_Z$ does not necessarily require that $M_{W_1}>M_W$.

The covariant derivative of Eq. (17) for the neutral sector becomes

$$D_\nu^0=\partial_\nu-ieQA_\nu-i\sum_{i=1,2} n_i[T^{(3)}-\alpha_i Q+\beta_i T'^{(3)}] , \tag{37}$$

where

$$\alpha_i=(\delta_1-DM_Z{}^2/M_{Z_i}{}^2)/(\delta_0-\delta_1) ,$$

$$\beta_i=\frac{\delta_0 s_W{}^2}{D}(1+\delta_1)+\frac{(1+\delta_0)(1+\delta_1)M_Z{}^4}{M_{Z_1}{}^2M_{Z_2}{}^2(\delta_0-\delta_1)}\left[1+\delta_0 c_W{}^2+\delta_1 s_W{}^2+\delta_1 s_W{}^2-\frac{M_{Z_i}{}^2}{M_Z{}^2}\right] . \tag{38}$$

The ZWW vertex is

$$T(Z_{i\mu}\to W_{j\nu}(p)W_{k\lambda}^\dagger(q))=-\mathrm{i}f_{ijk}[g_{\nu\lambda}(q-p)_\mu+g_{\mu\nu}(2p+q)_\lambda-g_{\lambda\mu}(2q+p)_\nu] , \tag{39}$$

where

$$f_{ijk}=g_0\mathscr{R}_{j1}^W\mathscr{R}_{k1}^W\mathscr{R}_{i+1,1}^Z+g_2\mathscr{R}_{j2}^W\mathscr{R}_{k2}^W\mathscr{R}_{i+1,2}^Z \tag{40}$$

with $i,j,k=1$ or 2. The AWW vertex is obtained from Eq. (40) with $f_{ijk}=e$ and $j=k$.

One possible set of free parameters for the theory is δ_0, δ_1, and C. The original parameters of the theory are related to δ_0, δ_1, and C as follows [see also the first two equalities of Eq. (23)]:

$$g_2{}^2=\frac{e^2(1+\delta_0)(1+\delta_1)}{(\delta_0\delta_1+\delta_0 s_W{}^2+\delta_1 c_W{}^2)} , \quad u^2=\frac{2M_Z{}^2s_W{}^2c_W{}^2}{e^2(1+\delta_1)}\frac{\delta_0 s_W{}^2}{(1+\delta_0)^2}\frac{D}{C} ,$$
$$s^2=\frac{2M_Z{}^2s_W{}^2c_W{}^2}{e^2(1+\delta_0)}\frac{\delta_1 c_W{}^2}{(1+\delta_1)^2}\frac{D}{C} , \quad v^2=\frac{2M_Z{}^2s_W{}^2c_W{}^2}{e^2}\left[1-\frac{1}{C}\frac{\delta_0 s_W{}^2}{(1+\delta_0)}\frac{\delta_1 c_W{}^2}{(1+\delta_1)}\right] . \tag{41}$$

From the positivity of $g_2{}^2$, u^2, w^2, and v^2, δ_0 and δ_1 must both be non-negative in this $2Z$, $2W$ model, and C must satisfy the bound

$$C\geq\frac{\delta_0 s_W{}^2}{(1+\delta_0)}\frac{\delta_1 c_W{}^2}{(1+\delta_1)}\geq 0 . \tag{42}$$

In addition to the restriction of Eq. (28), Eqs. (25) and (41) imply

$$M_{Z_c}>M_Z . \tag{43}$$

Also, Eq. (42) implies a further restriction on M_{Z_1}, M_{Z_2}, and M_{Z_c}. We note that the parametrization of the theory using M_{Z_1}, M_{Z_2}, and M_{Z_c} is double valued. In the limit $M_{Z_c}\to M_Z$ the two realizations of the model correspond to the BKM model[4] and an SU(2)×SU(2)′×U(1) analog of the DGS model.[6]

We now examine the model for special cases in which one of vacuum expectation values, u,v,w goes to zero. At least two must be nonzero to give masses to all of the W and Z bosons.

The case $w=0$ corresponds to the SU(2)×U(1)×SU(2)′ model of BKM. From Eqs. (21) and (23) we see that this requires $\delta_1=0$. The case $u=0$ (i.e., $\delta_0=0$) gives a model with the same structure as the SU(2)×U(1)×U(1)′ model of DGS: the mass-squared matrix in the W sector is diagonal and only one W couples to the known fermions, giving an effective one-W, two-Z model. Both the $u=0$ and $w=0$ cases give $M_{Z_c}=M_Z$ for which the Z_1 and Z_2 mass straddle the Z mass.

The third limiting case is $v=0$. It gives the theory with the smallest value of C for given values of δ_0, δ_1 [see Eqs. (41) and (42)]. Because the known fermions receive their mass from Yukawa couplings with Φ, a theory with $v=0$ could not supply fermion masses in the standard way.

IV. UPPER LIMITS ON LIGHTEST W,Z MASSES

Since M_{W_1} and M_{Z_1} must be less than their corresponding critical masses, any restrictions on M_{W_c} and M_{Z_c} also place upper limits on the lightest weak-boson masses. Solving for M_{W_c} in terms of M_{Z_c},

$$M_{W_c}{}^2=\frac{M_W{}^2}{c_W{}^2M_Z{}^2/M_{Z_c}{}^2+s_W{}^2} , \tag{44}$$

we see that in this model M_{W_c} has an absolute upper bound of M_W/s_W.[10] There is no *a priori* theoretical restriction on the allowed range of M_{Z_c}.

Current measurements of the $e^+e^- \to \mu^+\mu^-$ cross section give the restriction $C \leq 0.027$.[11] This upper bound on C along with Eq. (42) constrains δ_0 and δ_1 and therefore the critical masses M_{Z_c} and M_{W_c} [see Eqs. (25) and (32)]. These limits are $M_{Z_c} \leq 121$ GeV, $M_{W_c} \leq 97$ GeV.[9]

More stringent upper limits on the lightest Z and W masses can be found by maximizing M_{Z_1} and M_{W_1} directly for given values of C. Using Eqs. (14), (23), and (41), we can write

$$\begin{aligned} &{M_{Z_i}}^4 - {M_{Z_i}}^2 {M_Z}^2[1+\delta_0 {c_W}^2(1+{s_W}^4/C)+\delta_1 {s_W}^2(1+{c_W}^4/C)] + M_Z^4 s_W^2 c_W^2 D/C = 0 \, , \\ &{M_{W_i}}^4 - {M_{W_i}}^2 {M_W}^2 {M_W}^2[1+\delta_0(1+{s_W}^4/C)+\delta_1 {s_W}^2 {c_W}^2/C(1+\delta_1)] + {M_W}^4 {s_W}^2 D/C(1+\delta_1) = 0 \, . \end{aligned} \tag{45}$$

From Eq. (45) it is straightforward but tedious to maximize M_{Z_1} and M_{W_1} with respect to δ_0 and δ_1 for fixed C. The results

$$\begin{aligned} (M_{Z_1})_{\max} &= \frac{M_Z s_W c_W}{({s_W}^2{c_W}^2 - C)^{1/2}} \left[1 + \left[\frac{(C+{s_W}^4)(C+{c_W}^4)}{C} \right]^{1/2} - \frac{{s_W}^2{c_W}^2 - C}{C^{1/2}} \right]^{1/2} , \\ (M_{W_1})_{\max} &= M_W \left[1 + \frac{C^{1/2}}{(C+{s_W}^4)^{1/2} + {s_W}^2} \right]^{1/2} . \end{aligned} \tag{46}$$

are shown in Figs. 1 and 2 as a function of C. For $C \leq 0.027$ the bounds are

$$M_{Z_1} \leq 116 \text{ GeV}, \quad M_{W_1} \leq 89 \text{ GeV} \, . \tag{47}$$

As C goes to zero, the mass limits approach the standard-model values. The values in Eq. (47) should be compared with the mass $M_Z = 88.5$ GeV of the standard model before radiative corrections.[12]

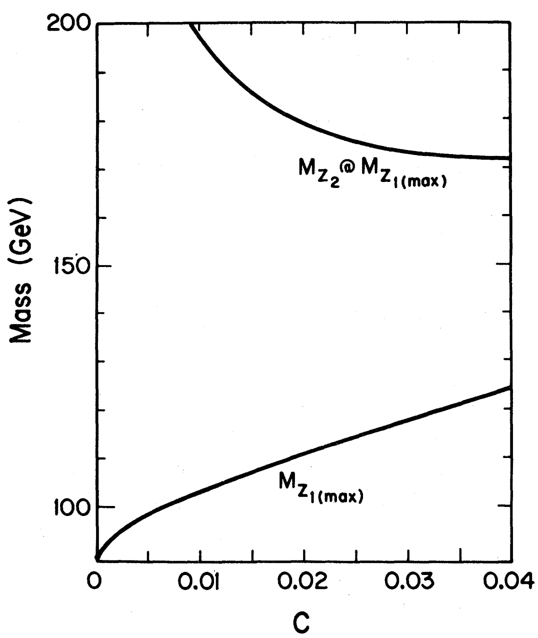

FIG. 1. Upper bound on the lightest Z mass versus the coefficient C of the $(j_\mu^{\rm EM})^2$ term in the effective Lagranian at low Q^2. Also shown is the value of M_{Z_2} required to achieve $(M_{Z_1})_{\max}$.

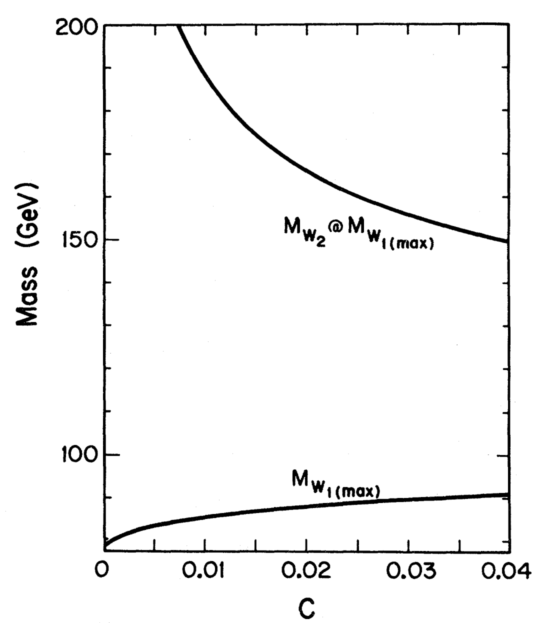

FIG. 2. Upper bound on the lightest W mass versus C. Also shown is the value of M_{W_2} required to achieve $(M_{W_1})_{\max}$.

V. PHENOMENOLOGY

In this section we examine the experimental implications of the model when all Z bosons have masses above that of the standard model. We use M_{Z_1}, M_{Z_2}, and M_{Z_c} as the three free parameters of the model. For the doublets (ν,e) and (u,d), the charged-current Hamiltonian is

$$\mathscr{H}_{\rm CC}=\frac{1}{2\sqrt{2}}[\bar{\nu}\gamma_\mu(1-\gamma_5)e+\bar{\mu}\gamma_\mu(1-\gamma_5)d_c]\sum_{i=1,2}N_i W_{i\mu}^\dagger\ , \tag{48}$$

where N_1 and N_2 are given in Eq. (31). The quantity d_C is the Cabibbo-rotated combination $d_C=d\cos\theta_C+s\sin\theta_C$. The neutral-current Hamiltonian is

$$\mathscr{H}_{\rm NC}=\sum_{i=1,2}(g_V^i\bar{\psi}\gamma_\mu\psi+g_A^i\bar{\psi}\gamma_\mu\gamma_5\psi)n_i Z_{i\mu}\ , \tag{49}$$

where n_1 and n_2 are given in Eq. (26) and

$$\begin{aligned} &g_V^i(\nu)=\tfrac{1}{4},\ g_A^i(\nu)=-\tfrac{1}{4},\ g_V^i(e)=-\tfrac{1}{4}+\alpha_i,\ g_A^i(e)=\tfrac{1}{4}\ ,\\ &g_V^i(u)=\tfrac{1}{4}-\tfrac{2}{3}\alpha_i,\ g_A^i(u)=-\tfrac{1}{4}\ ,\ g_V^i(d)=-\tfrac{1}{4}+\tfrac{1}{3}\alpha_i,\ g_A^i(d)=\tfrac{1}{4}\ . \end{aligned} \tag{50}$$

The quantity α_i is defined in Eq. (38). The couplings of successive generations of known quark or lepton doublets are identical to those of Eqs. (48)–(50).

A. Decay widths and branching fractions

The partial widths for fermion-antifermion decays of the weak bosons are

$$\begin{aligned} \Gamma(W_i\to f_1\bar{f}_2)&=\frac{cN_i^2}{48\pi M_{W_i}}\lambda^{1/2}(M_{W_i}{}^2,m_1{}^2,m_2{}^2)\left[1-\frac{1}{2}\frac{(m_1{}^2+m_2{}^2)}{M_{W_i}{}^2}-\frac{1}{2}\frac{(m_1{}^2-m_2{}^2)^2}{M_{W_i}{}^4}\right]\ ,\\ \Gamma(Z_i\to f\bar{f})&=\frac{cn_i^2}{12\pi M_{Z_i}}\lambda^{1/2}(M_{Z_i}{}^2,m^2,m^2)\left[(g_V^i)^2\left[1+\frac{2m^2}{M_{Z_i}{}^2}\right]+(g_A^i)^2\left[1-\frac{4m^2}{M_{Z_i}^2}\right]\right]\ , \end{aligned} \tag{51}$$

where c is the color factor and $\lambda(x,y,z)=x^2+y^2+z^2-2xy-2xz-2yz$. In addition, the Z_2 or W_2 may decay into lighter gauge particles via a ZWW vertex if kinematically allowed. These partial widths are

$$\begin{aligned} \Gamma(a\to bc)=\frac{f^2}{192\pi}\frac{\lambda^{1/2}(m_a{}^2,m_b{}^2,m_c{}^2)}{m_a{}^5m_b{}^2m_c{}^2}&(m_a{}^8+m_b{}^8+m_c{}^8+8m_a{}^6m_b{}^2+8m_a{}^6m_c{}^2+8m_b{}^6m_a{}^2+8m_b{}^6m_c{}^2\\ &+8m_c{}^6m_a{}^2+8m_c{}^6m_b{}^2-18m_a{}^4m_b{}^4-18m_a{}^4m_c{}^4-18m_b{}^4m_c{}^4\\ &-32m_a{}^4m_b{}^2m_c{}^2-32m_b{}^4m_a{}^2m_c{}^2-32m_c{}^4m_a{}^2m_b{}^2)\ , \end{aligned} \tag{52}$$

where a, b, and c are the gauge particles involved and f is the appropriate vertex factor taken from Eq. (40). In estimating these widths, we assume six flavors of leptons and quarks and use the masses $m_\tau=1.79$, $m_{\nu_\tau}=0$, $m_u=m_d=0.3$, $m_s=0.5$, $m_c=1.5$, $m_b=4.7$, and $m_t=30$ GeV. The results are fairly insensitive to the value of m_t. In Fig. 3 we show representative Z widths for the choices $M_{Z_2}=173$ GeV and $M_{Z_c}=M_{Z_1}+2$ GeV for a range of Z_1 masses around and above the standard Z mass. Figure 4 shows W widths for $M_{W_2}=168$ GeV and $M_{W_c}=M_{W_1}+5$ GeV. The widths are not appreciably different for other values of M_{Z_c}. The large width of the Z_2 at lower values of M_{Z_1} is caused principally by the decay $Z_2\to W_1W_1$. The Z_1 branching fractions are generally close to the standard-model values for Z_1 masses in the allowed range.

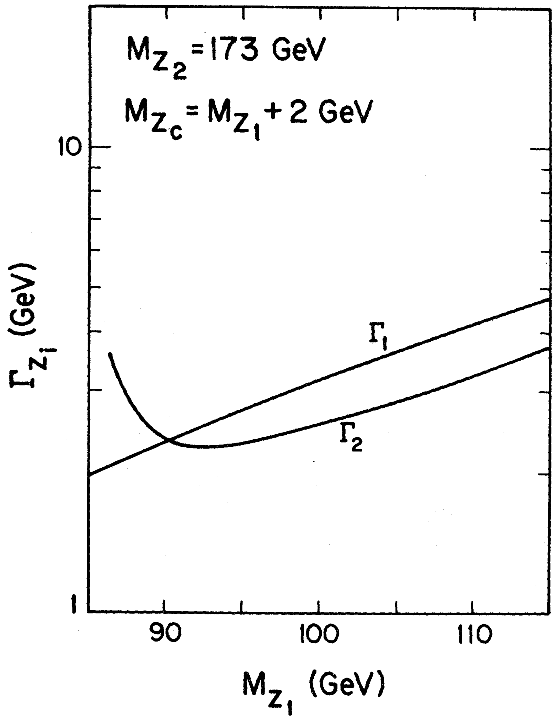

FIG. 3. Total decay widths of the Z_1 and Z_2 gauge bosons for representative masses.

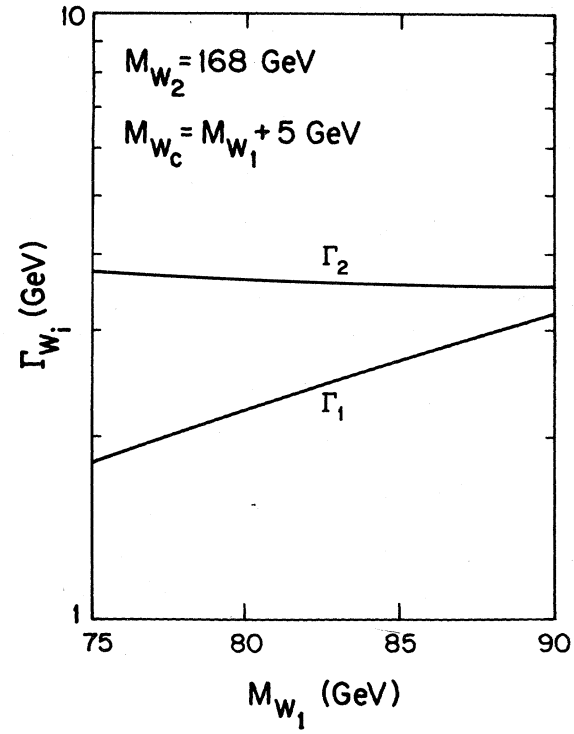

FIG. 4. Representative total decay widths of the W_1 and W_2 gauge bosons.

B. Predictions for $e^+e^- \to \mu^+\mu^-$

The reaction $e^+e^- \to \mu^+\mu^-$ provides a test of the standard model and places restrictions on the parameters of any expanded theory. At low energies $s << M_{Z_1}{}^2$ the cross section becomes

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}\left[1 - \frac{4G_F s}{\sqrt{2}\pi\alpha}[C + (\tfrac{1}{4} - \sin^2\theta_W)^2]\right]. \tag{53}$$

The latest results from PETRA at $\sqrt{s} \lesssim 36$ GeV give the limit $C \leq 0.027$ at the 95% confidence level.[11]

Predictions for the $e^+e^- \to \mu^+\mu^-$ total cross section and integrated forward-backward asymmetry are shown in Fig. 5 for an example with $M_{Z_1} > M_Z$. Current measurements[11] of the asymmetry $A(e^+e^- \to \mu^+\mu^-)$ at average $\sqrt{s} = 33$ GeV are compatible with the standard model but do not rule out alternatives. It is also possible to have a model different from the standard model yet with $M_{Z_1} = M_Z$. In that case, it can be shown that 73 GeV $< M_{W_1} <$ 80 GeV for $C < 0.027$. In Fig. 6, we show the integrated resonance contributions above background at the Z_1 for $M_{Z_1} = M_Z$, $M_{Z_2} = 200$ GeV and a range of values for M_{Z_c}. The standard-model results are at the left end of the curve where $M_{Z_c} = M_Z$. The relevant cross section formulas can be found in Ref. 4.

C. Predictions for $e^+e^- \to q\bar{q}$

Another process available for study in electron-positron annihilation is $e^+e^- \to q\bar{q} \to$ hadrons. For $s << M_{Z_i}{}^2$, the ratio

$$R = \sigma(e^+e^- \to \gamma, Z_i \to q\bar{q}) / \sigma(e^+e^- \to \gamma \to \mu^+\mu^-)$$

can be expressed as[13]

$$R=\left[3\sum_f Q_f{}^2+6s\frac{\sqrt{2}G_F}{16\pi\alpha}V+3s^2\left[\frac{\sqrt{2}G_F}{16\pi\alpha}\right]^2 W\right]\left[1+\frac{\alpha_s}{\pi}\right], \tag{54}$$

where Q_f is the quark charge and

$$V=16g_V(e)\sum_f Q_f g_V(f),\quad W=256[g_V(e)^2+g_A(e)^2]\sum_f[g_V(f)^2+g_A(f)^2], \tag{55}$$

and the sum is over the quark flavors which can be produced. The factor $1+\alpha_s/\pi$ represents the first-order QCD correction. Using the effective Hamiltonian of Eqs. (19) and (21), we find, for five flavors,

$$\begin{aligned}V&=\tfrac{1}{9}[(-21+128s_W{}^2-176s_W{}^4)-176C],\\ W&=\tfrac{1}{9}[(2-8s_W{}^2+16s_W{}^4)(90-168s_W{}^2+176s_W{}^4)+16(1-4s_W{}^2)(42-88s_W{}^2)(42-88s_W{}^2)C+2816C^2].\end{aligned} \tag{56}$$

Interpreting results on V and W from hadron production at PETRA,[14,15] we obtain the limit $C\leq 0.016$ at the 95% confidence level for $\sin^2\theta_W=0.23$. This translates via Eq. (46) into the limits $(M_{Z_1})_{\max}=108$ and $(M_{W_1})_{\max}=87$ GeV. Figure 7 shows the allowed region for V and W at 68% C.L.[14] The predictions of an effective Hamiltonian with a C term are given by the segmented curve.

D. Drell-Yan production

In hadron-hadron collisions, quark-antiquark annihilations can produce Z bosons, which can then be detected by muon pair production. The relevant cross section for $q\bar{q}$ annihilation is

$$\sigma(q\bar{q}\to\mu^+\mu^-)=\frac{m^2}{24\pi}(|H_-|^2+|H_+|^2+|H'_+|^2+|H'_-|), \tag{57}$$

where m is the muon-pair mass, and

$$\begin{aligned}H_\pm&=\frac{e_q e^2}{m^2}+\sum_{i=1,2}\frac{n_i{}^2[g_V^i(\mu)g_V^i(q)\pm g_A^i(\mu)g_A^i(q)]}{m^2-M_{Z_i}{}^2+iM_{Z_i}\Gamma_{Z_i}},\\ H'_\pm&=\sum_{i=1,2}\frac{n_i{}^2[g_V^i(\mu)g_A^i(q)\pm g_V^i(q)g_A^i(\mu)]}{m^2-M_{Z_i}{}^2+iM_{Z_i}\Gamma_{Z_i}}.\end{aligned} \tag{58}$$

Here e_q is the quark charge in units of e. For the inclusive production cross section $AB\to\mu^+\mu^-X$ the quark cross sections must be folded with the momentum distributions of the quarks in the initial hadrons:

$$\frac{d\sigma}{dy\,dm}=\frac{2x_+x_-}{3m}\sum_q f_q^A(x_+,m^2)f_{\bar{q}}^B(x_-,m^2)\sigma(q\bar{q}\to\mu^+\mu^-). \tag{59}$$

The summation is over all quark and antiquark flavors and $f_q^A(x,m^2)$ is the fractional momentum distribution of quark q in particle A. If y is the rapidity of the muon pair and $s=(p_A+p_B)^2$ is the c.m. energy squared, then $x_\pm=(m/\sqrt{s})\exp(\pm y)$. We use the QCD parametrization of Owens and Reya[16] for the parton distributions. Figure 8 shows the dimuon mass distribution at $y=0$ in $\bar{p}p\to\mu^+\mu^-X$ at $\sqrt{s}=540$ GeV for the case $M_{Z_1}=106$, $M_{Z_2}=173$, and $M_{Z_c}=108$ GeV.

VI. NATURAL EXTENSION TO $U(1)\times SU(2)^N$

In this section we consider a $U(1)\times SU(2)^N$ generalization of our two-W, two-Z model. We show that these extended models naturally reproduce the low-energy predictions of the standard model. With a symmetry assumption about couplings and vacuum expectation values, we also show that the maximum allowed values of the lightest W and Z for the gauge group $U(1)\times SU(2)^N$ are the same as

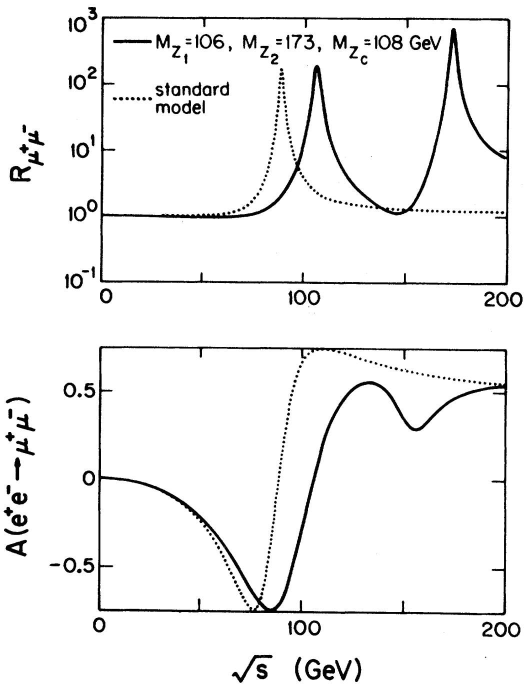

FIG. 5. Predictions of the $2W$, $2Z$ model in $e^+e^-\to\mu^+\mu^-$ for $R_{\mu^+\mu^-}=\sigma(e^+e^-\to\gamma^*,Z_1,Z_2\to\mu^+\mu^-)/\,\sigma(e^+e^-\to\gamma^*\to\mu^+\mu^-)$ versus $\sqrt{s}$, and the integrated forward-backward asymmetry parameter A versus $\sqrt{s}$. Parameters are based on masses $M_{Z_1}=106$ GeV, $M_{Z_c}=108$ GeV, and $M_{Z_2}=173$ GeV.

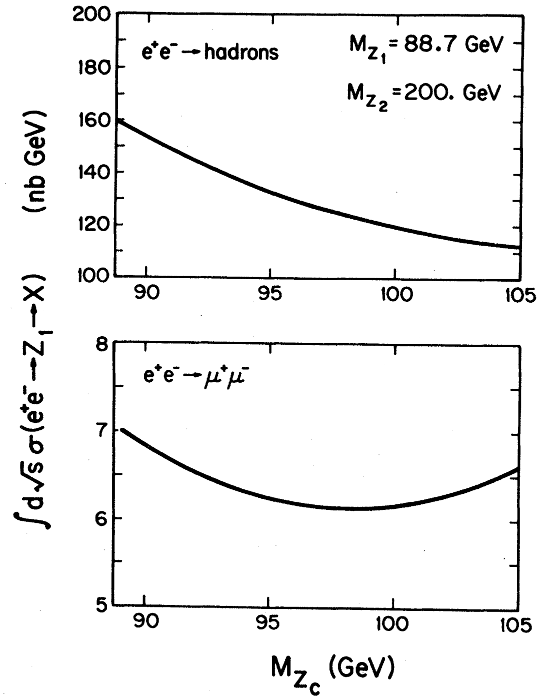

FIG. 6. Integrated Z_1-resonance contributions $\int d\sqrt{s}\;\sigma(e^+e^-\to Z_1\to X)$ versus M_{Z_c} for $M_{Z_1}=M_Z$, $M_{Z_2}=200$ GeV.

the corresponding limits for $U(1)\times SU(2)^{N-1}$, with $N\geq 3$. Then the limits $(M_{W_1})_{\max}$ and $(M_{Z_1})_{\max}$ derived in Sec. IV for the case $N=2$ are also upper bounds for an extended model of this type, regardless of the value of N.

Let the extended gauge group $U(1)\times SU(2)^N$ have the couplings $\frac{1}{2}g_0$, g_i and gauge bosons B, $W_i^{\pm,(3)}$, with $i=1,\ldots,N$. Note that this labeling of couplings to the groups differs from that in previous sections. The known fermions are assumed to couple only to U(1) and the first SU(2). Higgs doublets couple the U(1) and any one of the SU(2) groups; Higgs quartets couple any two SU(2)'s. We allow the most general Higgs doublet and quartet structure, with v_{ij} denoting the vacuum expectation value for the Higgs field coupling to the ith and jth groups, $i,j=0,\ldots,N$ and $i\neq j$. For the electric charge to be given by $Q=Y/2+\sum_i T_{3i}$, the photon must be of the form

$$A=\frac{e}{g_0}B+\sum_i\frac{e}{g_i}W_i^{(3)}\,, \tag{60}$$

where

$$e^{-2}=g_0^{-2}+\sum_i g_i^{-2}\,. \tag{61}$$

The photon is given mass λ by adding to each element in the neutral boson mass-squared matrix $(\mathcal{M}_Z{}^2)_{ij}$ a term λ^2e^2/g_ig_j.[5] The resulting symmetric matrix may then be written in component form as

$$\begin{aligned}(\mathcal{M}_Z{}^2)_{ii}&=\tfrac{1}{2}g_i{}^2\sum_{k\neq i}v_{ik}{}^2+\frac{\lambda^2e^2}{g_i{}^2}\,,\quad 0\leq i,k\leq N\,,\\ (\mathcal{M}_Z{}^2)_{ij}&=-\tfrac{1}{2}g_ig_jv_{ij}{}^2+\frac{\lambda^2e^2}{g_ig_j}\,,\quad 0\leq i,j\leq N\,.\end{aligned} \tag{62}$$

The charged-boson mass-squared matrix $\mathcal{M}_W{}^2$ is identical to $\mathcal{M}_Z{}^2$ without the row and column corresponding to the B boson, after setting $\lambda=0$.

The charge-current interaction $2^{-1/2}g_1j_\mu^{(+)}W_{1\mu}^+ +$ H.c. leads to the effective low-energy interaction $\frac{1}{2}g_1{}^2\langle W_1^+W_1^-\rangle$, where $\langle W_1^+W_1^-\rangle=(\mathcal{M}_W{}^{-2})_{11}$. The neutral-current interaction $g_0(j_\mu^{\rm EM}-j_\mu^{(3)})B_\mu+g_1j_{\mu}^{(3)}W_{1\mu}^{(3)}$ gives the effective low-energy interaction $\frac{1}{2}a[(j_\mu^{(3)}-bj_\mu^{\rm EM})^2+C(j_\mu^{\rm EM})^2]$, where

$$a = g_0{}^2\langle BB\rangle - 2g_0 g_1\langle BW_1^{(3)}\rangle + g_1{}^2\langle W_1^{(3)}W_1^{(3)}\rangle ,$$
$$ab = g_0{}^2\langle BB\rangle - g_0 g_1\langle BW_1^{(3)}\rangle ,$$

and

$$a(b^2+C) = g_0{}^2\langle BB\rangle - e^2/\lambda^2$$

(see Ref. 5). To establish that the effective strengths of the charged- and neutral-current interactions are naturally the same as in the standard model, we must show that

$$g_1{}^2\langle W_1^+ W_1^-\rangle = a = g_0{}^2\langle BB\rangle - 2g_0 g_1\langle BW_1^{(3)}\rangle + g_1{}^2\langle W_1^{(3)}W_1^{(3)}\rangle , \quad (63)$$

in the limit $\lambda=0$, where the individual terms in Eq. (63) can be written as $\langle W_1^+ W_1^-\rangle = (\mathcal{M}_W{}^{-2})_{11}$, $\langle BB\rangle = (\mathcal{M}_Z{}^{-2})_{00}$, $\langle BW_1^{(3)}\rangle = (\mathcal{M}_Z{}^{-2})_{01}$, and $\langle W_1^{(3)}W_1^{(3)}\rangle = (\mathcal{M}_Z{}^{-2})_{11}$. Note that $(\mathcal{M}^{-2})_{ij} = \mathrm{Cof}[(\mathcal{M}^2)_{ij}]/\mathrm{Det}(\mathcal{M}^2)$, where Cof denotes the cofactor of a given element in a matrix and Det the determinant. We first expand $\mathrm{Det}(\mathcal{M}_Z{}^2)$ in powers of λ^2:

$$\mathrm{Det}(\mathcal{M}_Z{}^2) = \lambda^2 e^2 \sum_{i,j} \frac{1}{g_i g_j} \mathrm{Cof}[(\mathcal{M}_Z{}^2)_{ij}(\lambda=0)] + O(\lambda^4) . \quad (64)$$

It is not difficult to show that

$$g_i g_j \mathrm{Cof}[(\mathcal{M}_Z{}^2)_{ij}(\lambda=0)] = g_0{}^2 Cof[(\mathcal{M}_Z{}^2)_{00}(\lambda=0)] = g_0{}^2 \mathrm{Det}(\mathcal{M}_W{}^2)$$

for all values of i and j. Then Eqs. (64) and (61) yield

$$\mathrm{Det}(\mathcal{M}_Z{}^2) = \lambda^2 e^2 g_0{}^2 \mathrm{Det}(\mathcal{M}_Z{}^2) \sum_{i,j} \frac{1}{g_i{}^2} \frac{1}{g_j{}^2} = \frac{\lambda^2 g_0{}^2}{e^2} \mathrm{Det}(\mathcal{M}_Z{}^2) . \quad (65)$$

The right side of Eq. (63) can now be expressed as

$$a = \frac{e^2}{\lambda^2 g_0{}^2 \mathrm{Det}(\mathcal{M}_W{}^2)} \{ g_0{}^2 Cof[(\mathcal{M}_Z{}^2)_{00}] - 2g_0 g_1 \mathrm{Cof}[(\mathcal{M}_Z{}^2)_{01}] + g_1{}^2 \mathrm{Cof}[(\mathcal{M}_Z{}^2)_{11}] \} . \quad (66)$$

In each cofactor term, the piece independent of λ is equal to $g_0{}^2\mathrm{Det}(\mathcal{M}_W{}^2)$ and thus cancels in the expression for a. Each cofactor in Eq. (66) is a determinant of an $N\times N$ submatrix of $\mathcal{M}_Z{}^2$, where each element

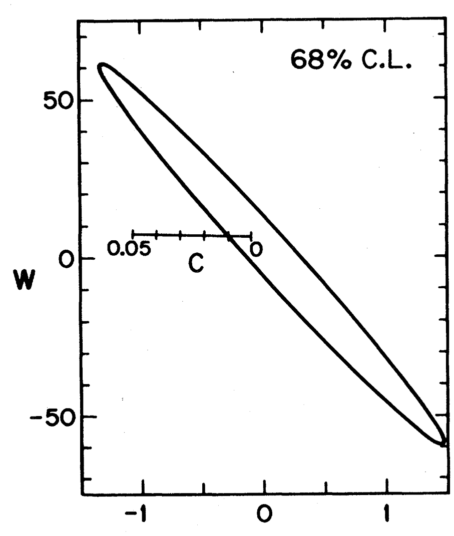

FIG. 7. Domain of V and W in Eq. (56) for $e^+e^-\rightarrow$hadrons; allowed region from data of Ref. 14 lies within the elliptical boundary. The segmented curve gives the predictions for an effective Hamiltonian with a C parameter in the range $0\le C\le 0.05$.

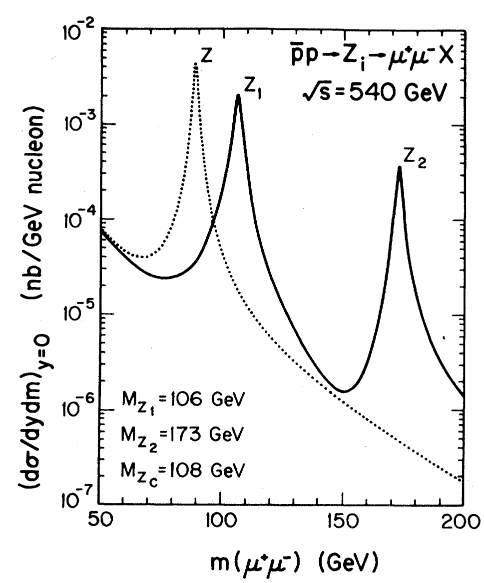

FIG. 8. Predictions for the cross section $d\sigma/dy\,dm$ at $y=0$ versus m of the Drell-Yan muon pair production process for $p\bar{p}$ colliding beams at $\sqrt{s}=540$ GeV.

in the submatrix has a λ^2 term. Therefore, each cofactor yields N^2 terms of order λ^2, analogously to the result for $\mathrm{Det}(\mathscr{M}_Z{}^2)$ in Eq. (64). It can be shown that Eq. (66) reduces to

$$a = g_1{}^2 \frac{\mathrm{Cof}[(\mathscr{M}_W{}^2)_{11}]}{\mathrm{Det}(\mathscr{M}_W{}^2)} . \tag{67}$$

The derivation of Eq. (67) is given in the Appendix. From $\langle W_1^+ W_1^- \rangle = (\mathscr{M}_W{}^{-2})_{11}$ we see that the naturalness condition of Eq. (63) is satisfied.

We now show that with the symmetry conditions of Eq. (70) below $(M_{W_1})_{\max}$ and $(M_{Z_1})_{\max}$ are the same for $\mathrm{U}(1)\times\mathrm{SU}(2)^N$ as for $\mathrm{U}(1)\times\mathrm{SU}(2)^{N-1}$ when $N \geq 3$. Consider the matrix $\mathscr{M}_Z{}^2$ of Eq. (62). By applying the rotation in $(W_{N-1}^{(3)}, W_N^{(3)})$ space given by the matrix

$$\begin{bmatrix} 1 & & & \\ & \ddots & & \\ & & 1 & & \\ & & & \cos\theta & -\sin\theta \\ & & & \cos\theta & \sin\theta \end{bmatrix} \tag{68}$$

to $\mathscr{M}_Z{}^2$, and setting $\tan\theta = -g_N/g_{N=1}$, the rotated (symmetric) neutral-boson mass-squared matrix elements $(\mathscr{M}_Z{}^2)'_{ij} = (\mathscr{R}\mathscr{M}_Z{}^2 R^\dagger)_{ij}$ for the Nth row are

$$\begin{aligned} &\tfrac{1}{2} g_j g'_{N-1} \left[\frac{g_{N-1}}{g_N} v_{j,N-1}{}^2 - \frac{g_N}{g_{N-1}} v_{j,N}{}^2 \right] \quad \text{for } i = N,\ 0 \leq j \leq N-2 , \\ &\tfrac{1}{2} g_{N-1}'{}^2 \left[\frac{g_N}{g_{N-1}} (v_{0,N-1}{}^2 + \cdots + v_{N-2,N-1}{}^2) - \frac{g_{N-1}}{g_N} (v_{0,N}{}^2 + \cdots + v_{N-2,N}{}^2) \right] \quad \text{for } i = N,\ j = N-1 , \end{aligned} \tag{69}$$

where $g'_{N-1}{}^{-2} = g_{N-1}{}^{-2} + g_N{}^2$. This transformation does not change G_F, $\sin^2\theta_W$, or C since they are calculated from $(\mathscr{M}_Z{}^{-2})_{00}$, $(\mathscr{M}_Z{}^{-2})_{01}$, $(\mathscr{M}_Z{}^2)_{11}$ and these elements are not affected by the rotation. We now make the assumption that $(M_{Z_1})_{\max}$ has a symmetric solution such that

$$g_{N-1} = g_N ,\quad v_{j,N-1} = v_{j,N} ,\ j = 0, 1, \ldots, N-2, \tag{70}$$

where $N-1, N$ is some pair chosen arbitrarily from the set $\{2, \ldots, N\}$. Using Eq. (70), we see that all elements in the Nth row and column of the $(\mathscr{M}_Z{}^2)'$ matrix become zero except for $(\mathscr{M}_Z^2)'_{NN}$. This corresponds to the case in which one Z decouples from the other Z bosons and from the known fermions. Hence the e, G_F, $\sin^2\theta_W$, and C calculated from the remaining submatrix are the same as for the full matrix. A similar statement holds for the charged bosons. For such symmetric sets of parameters, the upper bounds on the lightest weak-boson masses are the same for all values of $N \geq 2$.

In numerical evaluations of the $N=3$ case with $C < 0.04$, we found that $(\mathscr{M}_{Z_1})_{\max}$ can be at most 3% higher than the upper bound for $N=2$. Thus it seems likely that $(\mathscr{M}_{Z_1})_{\max}$ and $(\mathscr{M}_{W_1})_{\max}$ are not appreciably higher than the values in Eq. (46), if N is not too large.

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APPENDIX

Here we outline the proof of Eq. (67). We begin by writing the cofactors in Eq. (66) as

$$g_\alpha g_\beta \mathrm{Cof}[(\mathscr{M}_Z{}^2)_{\alpha\beta}] = g_0{}^2 g_1{}^2 \cdots g_N{}^2 \mathrm{Det}(C^{\alpha\beta})(-1)^{\alpha+\beta} \tag{A1}$$

with α, β either 0 or 1. The matrices $C^{\alpha\beta}$ are given by Eq. (62) as

$$(C^{\alpha\beta})_{ij}=\begin{cases}\frac{1}{2}\sum_{k\neq i}v_{ik}{}^2+\dfrac{\lambda^2e^2}{g_i{}^4}, & i=j\ ,\\ -\frac{1}{2}v_{ij}{}^2+\dfrac{\lambda^2e^2}{g_i{}^2g_j{}^2}, & i\neq j\ ,\end{cases} \tag{A2}$$

where i,j go from 0 to N with the row $i=\alpha$ and the column $j=\beta$ excluded. The expression for a in Eq. (66) becomes

$$a=\frac{e^2g_1{}^2\cdots g_N{}^2}{\lambda^2\mathrm{Det}(\mathscr{M}_W{}^2)}\tilde{a}\ , \tag{A3}$$

where

$$\tilde{a}\equiv\mathrm{Det}(C^{00})+2\,\mathrm{Det}(C^{01})+\mathrm{Det}(C^{11})\ . \tag{A4}$$

It is convenient to define coefficients D_{lm} such that $\tilde{a}$ can be written as

$$\tilde{a}=\lambda^2e^2\sum_{l,m}D_{lm}g_l{}^{-2}g_m{}^{-2}+O(\lambda^4)\ . \tag{A5}$$

Our next task is to determine the D_{lm}. By inspection of Eq. (A2), we observe that only C^{11} has a $g_0{}^{-4}$ term and hence

$$D_{00}=\mathrm{Cof}[(C^{11})_{00}] \tag{A6}$$

evaluated at $\lambda=0$. Identifying terms with $g_0{}^{-2}g_2{}^{-2}$ in Eq. (A2), we determine that for $\lambda=0$,

$$D_{02}=2\,\mathrm{Cof}[(C^{11})_{02}]+2\,\mathrm{Cof}[(C^{01})_{20}]\ . \tag{A7}$$

The factor of 2 in front of $\mathrm{Cof}[(C^{11})_{02}]$ includes the contribution from $\mathrm{Cof}[(C^{11})_{20}]$. In determinant form, all rows of $\mathrm{Cof}[(C^{11})_{02}]$ and $\mathrm{Cof}[(C^{01})_{20}]$ are identical except the first; therefore, we can combine the two determinants by adding the top rows. In the resulting determinant for D_{02}, we now add all other rows to the first row. The result is identical to the determinant form of $2D_{00}$. Similar manipulations can be used to show more generally that

$$\begin{aligned}&D_{ll}=D_{00}\ ,\\ &D_{lm}=2D_{00},\quad l\neq m\ ,\end{aligned} \tag{A8}$$

for any l. Hence $\tilde{a}$ of Eq. (A5) becomes

$$\begin{aligned}\tilde{a}&=\lambda^2D_{00}e^2\left[\sum_l g_l{}^{-2}\right]^2\\ &=\lambda^2D_{00}/e^2\ .\end{aligned} \tag{A9}$$

Combining Eqs. (A3), (A6), and (A9), we obtain

$$a=\frac{g_1{}^2\cdots g_N{}^2}{\mathrm{Det}(\mathscr{M}_W{}^2)}\mathrm{Cof}[(C^{11})_{00}]\ . \tag{A10}$$

By reference to Eqs. (A2) and (59), we conclude that

$$a=\frac{g_1{}^2\mathrm{Cof}[(\mathscr{M}_W{}^2)_{11}]}{\mathrm{Det}(\mathscr{M}_W{}^2)}\ . \tag{A11}$$

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