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Spectroscopy of the three-heavy-quark system

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We describe a way to include asymptotic freedom in the potential energy for a baryon composed of three heavy quarks, which is based on the Born-Oppenheimer approximation to the MIT bag model. The running coupling constant appears in both the two-body Coulomb terms and the three-body confining term of the potential energy. The Schrödinger equation with this potential is solved using a hyperspherical expansion, and spectra are obtained for the baryons b^3, c^3, and $\Omega^-(s^3)$. Comparisons of these systems with the corresponding $q\bar{q}$ systems are made using the same running coupling constant and bag parameters for the baryons and the mesons.

I. INTRODUCTION

Since the discovery of the charmonium ($c\bar{c}$) system a good deal of progress has been made towards understanding the main features of the static interaction between a heavy quark and a heavy antiquark. There now exist derivations[1,2] of the spin-independent part of the potential energy based on the Born-Oppenheimer approximation[3] to the MIT bag model,[4] where it is assumed that the glue field adjusts rapidly compared to the motion of quarks. In the first step, the quarks are taken to be static, and, to lowest order in the coupling constant, numerical solutions for the shape and the energy of the bag as a function of the $q\bar{q}$ separation are obtained. This energy serves as the potential energy $V_{q\bar{q}}$ between quarks in the second step of the Born-Oppenheimer approximation, which is the solution of the Schrödinger equation. In Ref. 1 a running coupling constant was used which satisfies the requirements of asymptotic freedom in the limit of small separation; but the same values of the bag constant B and zero-point-energy constant Z_0 were used as were obtained in the original bag-model fit[5] to the spectrum of hadrons composed of light quarks.[6] The solutions of the Schrödinger equation[1] are in fair agreement with the experimental data on the $c\bar{c}$ and $b\bar{b}$ spectra and their leptonic decay widths.

Given the success of the numerical solution of the bag-model equations for the $q\bar{q}$ problem, one may be inclined to consider a similar approach to the q^3 system. For a fixed coupling constant this has been done.[7] The situation is somewhat different, however, for a running coupling constant. In the $q\bar{q}$ problem there was a transparent way to incorporate such a coupling constant: for each value of the $q\bar{q}$ separation the bag equations were solved using the value of the coupling constant at that separation.[1] There is no analogous procedure in the q^3 problem since there are three distances instead of the one $q\bar{q}$ separation. Therefore, a numerical solution will not contribute as much to our understanding of the q^3 problem as it has done for the $q\bar{q}$ system.

In Ref. 8 the present authors proposed an analytic approach to solving the Born-Oppenheimer approximation for the heavy $q\bar{q}$ and q^3 systems, which exploits the fact that the bag model allows one to treat these systems on the same footing. Using the simple bag shapes appropriate to small and large separations, and simply joining the corresponding results in the middle region, we obtained a global approximation to $V_{q\bar{q}}$ in good agreement with the result of the numerical treatment which involves more complicated bag shapes.[1] One therefore feels confident that the analogous analytic treatment of V_{q^3} is as well founded as that of $V_{q\bar{q}}$. All potentials derived in this way are flavor-independent.

In Ref. 8 we derived in this manner an approximation for V_{q^3} using a fixed coupling constant. In this paper we will discuss a way to include the variation of α_s with distance in V_{q^3}, and study its consequences for the heavy q^3 spectroscopy. The fact that the potential separates into two-body and three-body terms of specified analytic form suggests how this should be done. The two-body color

Coulomb terms are due to single-gluon exchange, and differ from the $q\bar{q}$ case only by a color factor $F_q \cdot F_{q'}$, instead of $F_q \cdot F_{\bar{q}}$. The same one-loop correction plus renormalization-group argument should apply, therefore, and give the very same variation of α_s with $|\bar{r}_q - \bar{r}_{q'}|$. (Note that to this order one does not get three-body forces; the three-gluon vertex is proportional to $f_{abc} F^a_{q_1} F^b_{q_2} F^c_{q_3}$ which vanishes between two color singlets.) For the three-body (confining) term the situation is less clear. When all quark separations are small, which is the most important region for the low-lying states, this term depends only on the hyperspherical radius ρ. Therefore we make α_s in this term also depend on ρ; but this still leaves some ambiguity which is discussed in Sec. II. For large separations it is less clear what to do; but it is also less important since α_s has become almost constant in this region, and, in addition, the probability of finding quark configurations with these large separations is small.

Besides the energy contributed by the static, longitudinal glue fields generated by the quarks, and the volume-energy term, the potential energy also contains a term E_0 coming from the zero-point energies of the transverse glue fields and the (light) quarks. This term depends on the shape of the bag and thus on the configuration of the system as a whole, rather than of any particular subsystem.

Once the three-quark Hamiltonian is specified, one can proceed to solve the Schrödinger equation. This is greatly facilitated by the fact that the three-body part of the potential is a function only of the hyperradius for the very important small quark separations. One finds that a hyperspherical expansion[9] of the wave function converges very fast[7,8]; this is discussed in Sec. III, where we calculate the b^3, c^3, and s^3 excitation spectra. Some results for mixed systems such as $c^2 s$ are also given there. A brief discussion of the hyperspherical method is given in the Appendix.

The experimental situation does not look as good for the heavy q^3 system as for the heavy $q\bar{q}$ systems. The analog of the $c\bar{c}$ and $b\bar{b}$ systems would be the c^3 and b^3 systems, respectively: baryons composed of three identical heavy quarks. Such systems could tell us a lot about the static interaction between (three) heavy quarks, just as quarkonium has done it for the static $q\bar{q}$ interaction. Unfortunately, there is no c^3 spectroscopy to compare the predictions of the theory to. Moreover, the chances that such baryons will be found in the near future look rather slim.

Fortunately, the study of the three-heavy-quark system is not quite as academic as it may seem. One can use the s^3 system as a first guide to gain some understanding of the static interactions between quarks. The s^3 spectroscopy is not yet well explored, with only one state, the Ω^-, known at present. There are, however, negative-hyperon beams in operation at Fermilab and the CERN SPS which contain sizable fraction of Ω^-, and which have already been used to establish the static properites of the Ω^- and its main branching ratios.[10] The exploration of its excitation spectrum then may be also be within reach.

A word of caution is needed for the s^3 system, however, since the strange quark generally is not quite as nonrelativistic as one might wish. Nevertheless, when one applies the identical potential (which was used for the $c\bar{c}$ and $b\bar{b}$ systems) to mesons containing the s quark, consistency is obtained between the masses of the $\phi(s\bar{s})$ and the $F^*(c\bar{s})$.[1,11] One finds[1] the rather small quark mass, $m_s = 0.641$ GeV, which indeed makes these systems somewhat relativistic: $\langle \vec{p}^2/m^2 \rangle = 0.56$ for the ϕ.

In Sec. IV we comment on relativistic and spin-dependent corrections, which have not been included here. Next, a comparison between the quark and the nuclear few-body systems is made. We then emphasize the lack of theoretical foundation for the use of a confinement term consisting solely of the sum of two-body interactions, within the framework of QCD, and point out some of the differences which arise were one to substitute such a potential term for the three-body confining potential.

II. THE MODEL

In Ref. 8 we studied the potential energies of the heavy-quark-antiquark and the three-heavy-quark systems, in the Born-Oppenheimer approximation to the MIT bag model. In the limit of small $q\bar{q}$ separations, $r = |\bar{x}_q - \bar{x}_{\bar{q}}|$, the following expression for $V_{q\bar{q}}(r)$, obtained by approximating the bag shape by a sphere of radius $R = (\alpha_s r^2/\pi B)^{1/6}$, and keeping only the dipole term from the homogeneous part of the potential, accurately represents the numerical calculations[1]

$$V_{q\bar{q}}(r) = -\frac{4\alpha_s}{3r} + (\tfrac{2}{3})^{1/2} kr \ . \tag{2.1}$$

In the limit of large $q\bar{q}$ separations, the bag shape is accurately approximated by a tube of radius R_∞, and $V_{q\bar{q}}(r)$ becomes

$$V_{q\bar{q}}(r)=-\frac{4\alpha_s}{3r}+kr+V^0_{q\bar{q}}\ , \tag{2.2}$$

where $R_\infty\equiv(8\alpha_s/3\pi B)^{1/4}$ and $k\equiv(32\pi B\alpha_s/3)^{1/2}$, in terms of the quark-gluon coupling constant $\alpha_s=g^2/4\pi$ and the bag constant B. $V^0_{q\bar{q}}$ is a constant, and (2.2) becomes accurate when r exceeds R_∞. By comparison with the numerical solution,[1] we found that a rather accurate description could be obtained by simply taking expression (2.1) for separations $r\le R_\infty$, and (2.2) for $r\ge R_\infty$. $V^0_{q\bar{q}}$ is fixed by the requirement that $V_{q\bar{q}}$ be continuous at $r=R_\infty$.

Insofar as the potential energy is concerned, it is R_∞ (the radius of the flux tube for large separations) which sets the distance scale in the problem and determines the transition region between small and large separations. For simplicity we have chosen to match (2.1) to (2.2) right at $r=R_\infty$, although one could do a somewhat better job of fitting any given set of heavy-meson energy levels by choosing a slightly different value for the matching radius, tailored to the range of distances spanned by the wave functions under consideration. As the quark mass is increased this entire question becomes moot since the wave functions are all confined to $r<R_\infty$.

Following the identical steps for the heavy q^3 systems, one arrives at[8]

$$V_{q^3}(\bar{x}_1,\bar{x}_2,\bar{x}_3)=\begin{cases} -\frac{2}{3}\sum_{i>j}\dfrac{\alpha_s}{|\bar{x}_i-\bar{x}_j|}+k\rho, & \rho\le R_\infty, \quad (2.3a)\\ -\frac{2}{3}\sum_{i>j}\dfrac{\alpha_s}{|\bar{x}_i-\bar{x}_j|}+kr_M+V^0_{q^3}, & \rho\ge R_\infty \quad (2.3b)\end{cases}$$

in terms of the hyperspherical radius

$$\rho=\left[\frac{1}{3}\sum_{i>j}(\bar{x}_i-\bar{x}_j)^2\right]^{1/2}.$$

The term kr_M is the generalization to three tubes of the one-tube result kr in (2.2). r_M is the minimized sum[7,8,12] of the lengths of the three tubes in a Y-shaped bag. With $x_{ij}\equiv|\bar{x}_i-\bar{x}_j|$,

$$r_M=\left[\frac{3}{2}\right]^{1/2}\rho\left\{1+\sqrt{3}\left[4\frac{x_{12}^2x_{13}^2+x_{13}^2x_{23}^2+x_{12}^2x_{23}^2}{(x_{12}^2+x_{23}^2+x_{13}^2)^2}-1\right]^{1/2}\right\}^{1/2}, \tag{2.4a}$$

if none of the angles of the quark triangle exceeds 120°, and

$$r_M=|\bar{x}_i-\bar{x}_j|+|\bar{x}_i-\bar{x}_k| \quad \text{for } \theta_i\ge 120^\circ\ . \tag{2.4b}$$

$V^0_{q^3}$ depends on the geometry, and is also chosen to make the potential continuous.

So far we have looked at the fixed coupling-constant case. But in the Born-Oppenheimer approximation the quarks enter the equations as fixed-point sources, and consequently the variations of coupling constant with distance as required by QCD should be incorporated. The leading behavior of the coupling constant, including one-loop corrections, for a heavy $q\bar{q}$ pair at very small separations is found to be[13]

$$\alpha_s(r)\to\frac{12\pi}{33-2n_f}\left[\ln\left[\frac{1}{\Lambda^2r^2}\right]\right]^{-1} \tag{2.5}$$

in terms of the number of quark flavors n_f and the QCD scale parameter Λ. [This is to be contrasted with the case of light, relativistic quarks which spread out over the entire bag. For such quarks one can only speak of an average separation $\bar{r}$, and the coupling constant then has to be interpreted as $\alpha_s(\bar{r})$, for which the original MIT fit[5] gave a value 2.2 for $\bar{r}\cong 1$ fm.]

Rather than repeating the one-loop calculation in

the bag model, the correct effective coupling was incorporated in Ref. 1 by replacing the quark color charges $Q_i^a = gF_i^a$ by effective charges $Q_i^a(r) = g(r)F_i^a$ in the bag equations of motion for the glue fields. With $\alpha_s(r) = g^2(r)/4\pi$, α_s was parametrized as

$$\alpha_s(r) = \frac{12\pi}{33-2n_f} \frac{1}{\ln[(1/\Lambda^2 r^2)+\gamma]} , \tag{2.6}$$

where Λ and γ were treated as free parameters. For $n_f = 3$, $\gamma = 3.36$, and $\Lambda = 0.24$ GeV [and consequently $\alpha_s(1 \text{ fm}) = 1.0$] were found to give a good fit to the $c\bar{c}$ and $b\bar{b}$ spectra, using the value of the bag constant B from Ref. 5: $B^{1/4} = 145$ MeV.[14] For these values of the parameters $\alpha_s(\infty) = 1.15$.

In the $q\bar{q}$ problem proceeding in this manner simply makes α_s in (2.1), (2.2), and in the definitions of k and R_∞, become $\alpha_s(r)$. In other words, (2.1) and (2.2) become

$$V_{q\bar{q}}(r) = \begin{cases} -\frac{4}{3}\frac{\alpha_s(r)}{r} + (\frac{2}{3})^{1/2} k(\alpha_s(r))r, & r \le R_\infty , \quad (2.7a) \\ -\frac{4}{3}\frac{\alpha_s(r)}{r} + k(\alpha_s(r))r + V^0_{q\bar{q}}, & r \ge R_\infty , \quad (2.7b) \end{cases}$$

where $k(\alpha_s)$ is the function defined following (2.2), and we now take R_∞ to be $R_\infty(\alpha_s(\infty))$. Equation (2.7) is referred to as the global approximation to the potential, V_G. Figure 1 shows a comparison between (2.7) and the potential from Ref. 1. Just as for the fixed-coupling-constant case[8] the discrepancy between the two curves is maximal, yet only 40 MeV, at $r = R_\infty = 1.35$ fm.

Examination of the wave functions plotted in Fig. 9 of Ref. 1 shows that, with the exception of the highly excited states of charmonium, they are all essentially confined within R_∞. If one uses (2.7a) for *all* distances, the energy of the ground state of charmonium changes by less than 1 MeV. The $3S$-state energy comes down only 5 MeV.

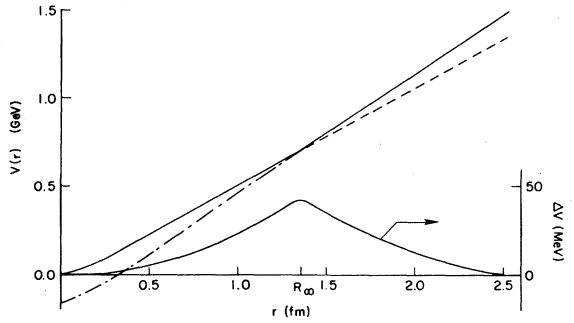

FIG. 1. The global approximation to the heavy $q\bar{q}$ potential, V_G. The solid curve, which represents V_G, is defined in (2.7). For $r \le R_\infty$ it is the dipole approximation (Ref. 8), (2.7a), the continuation of which for $r > R_\infty$ is shown as a dashed curve. For $r \ge R_\infty$, V_G is given by (2.7b), and its continuation for $r < R_\infty$ is shown as the dash-dot curve. All these approximations use the same running coupling constant, (2.6), with $\Lambda = 240$ MeV and $\gamma = 3.36$. The Coulomb term of the potential has been omitted from all curves. The difference $\Delta V = V_H - V_G$ is shown on the right-hand scale, where V_H is the numerical result from Ref. 1.

In the q^3 system where there are three different quark separations it is not obvious how to include the variation of α_s by replacing the fixed quark charges Q_i^a with effective r-dependent ones. But the analytic approximation to the potential given in (2.3) suggests a way to proceed. The most important and also the least ambiguous α_s dependence is found in the Coulomb-type two-body terms. As indicated in the Introduction, it seems quite reasonable to let the coupling constant in each of these terms depend on the appropriate two-particle separation, thus retaining its two-body character. It also seems reasonable that the α_s which appears in the three-body term of the potential (2.3) should develop a dependence on a collective, three-body variable since that term is a direct consequence of the confinement mechanism which affects all three quarks at the same time. We know, from the discussion of the $q\bar{q}$ potential presented above, that the most important part of V_{q^3} will also be the one for the small distances, $\rho \le R_\infty$. From Eq. (2.3a) one sees that the three-body term in the potential depends exclusively on ρ, and therefore one is led to expect the α_s which occurs there to also be a function of ρ. We therefore propose

$$V_{q^3}(\bar{x}_1,\bar{x}_2,\bar{x}_3)=-\tfrac{2}{3}\sum_{i<j}\frac{\alpha_s(|\bar{x}_i-\bar{x}_j|)}{|\bar{x}_i-\bar{x}_j|}$$

$$+k(\alpha_s(\lambda\rho))\rho,\quad \rho\le R_\infty\ . \qquad (2.8a)$$

The constant λ can be interpreted as a measure of the effective distance at which a quark sees the other two quarks. As such it can range from $(\frac{3}{2})^{1/2}$, for a quark-diquark configuration, to $(\frac{1}{3})^{1/2}$ for an equilateral-triangle configuration. Comparing the probabilities for the occurrence of the various configurations, one expects λ to be closer to the lower limit.

When dealing with states for which the wave function extends beyond R_∞, we make use of the fact that the coupling constant has essentially reached its asymptotic value and write

$$V_{q^3}(\bar{x}_1,\bar{x}_2,\bar{x}_3)=-\tfrac{2}{3}\sum_{i<j}\frac{\alpha_s(|\bar{x}_i-\bar{x}_j|)}{|\bar{x}_i-\bar{x}_j|}$$

$$+k_\infty r_M+V^0_{q^3},\quad \rho\ge R_\infty\ , \qquad (2.8b)$$

where $k_\infty=k(\alpha_s(\infty))$.

III. THE SPECTRUM

The potentials proposed in (2.7) and (2.8) are the major parts of the potential input to the Schrödinger equation. The actual Hamiltonians, which are given by

$$H_{q\bar{q}}=\sum_i^2\frac{p_i^2}{2m_i}+V_{q\bar{q}}(r)+E_0 \qquad (3.1)$$

and

$$H_{q^3}=\sum_i^3\frac{p_i^2}{2m_i}+V_{q^3}(\bar{x}_1,\bar{x}_2,\bar{x}_3)+E_0\ , \qquad (3.2)$$

also include the MIT-bag-model zero-point energy E_0 due to the presence of confined quantum fields. This term was parametrized in the original MIT calculation[5] as

$$E_0=-\frac{Z_0}{R}\ , \qquad (3.3)$$

where R is the radius of the spherical bag. From Ref. 8 it is seen that the bag radius depends on the same coupling which appears in k in (2.8a) and consequently we take

$$R_{q^3}=\left[\frac{3\alpha_s(\lambda\rho)}{2\pi B}\rho^2\right]^{1/6}\ . \qquad (3.4)$$

The corresponding formula for the $q\bar{q}$ system is[1,8]

$$R_{q\bar{q}}=\left[\frac{\alpha_s(r)}{\pi B}r^2\right]^{1/6}\ . \qquad (3.5)$$

Even though our bags become spherical only in the limit of small separations, we use (3.3) and (3.4) or (3.5) for all separations. The MIT-fit value $Z_0=1.84$ is taken.

To see how well (3.1) along with (3.3) and (3.5) does in the $c\bar{c}$ and $b\bar{b}$ systems we have listed in Table I both the theoretical and experimental spectra. The agreement is seen to be good. For comparison, the spectrum of Ref. 1 has also been included in the table.[18]

In describing the three-body system there is the additional parameter λ in (2.8), which represents some ignorance about the correct way to incorporate a running coupling constant into the three-quark problem. Our procedure for assigning a value to this parameter (within the range discussed in Sec. II) is the following. Using (2.7), (3.3), and (3.5) we find that $m_s=0.649$ GeV is needed to fit the mass of the $\phi(1020)$. We would like to choose λ so that this same quark mass, when used with (2.8), (3.3), and (3.4), would give the mass of the $\Omega^-(1672)$. Anticipating some of the results below, we were not able to achieve this except by pushing λ down to an unacceptably small value of about $\frac{1}{7}$. In the next section we will discuss some of the physics which is missing from the present calculation and which might account for this discrepancy, but for now we shall simply choose λ to be at the lower limit of the acceptable range, $\lambda=1/\sqrt{3}$. With this value, $m_s=0.571$ GeV puts the Ω^- at its physical mass of 1.672 GeV.

A method for obtaining the energy spectrum of the three-body Hamiltonian (3.2) is suggested by the particular form of the confinement term in (2.8a) and also by the zero-point energy (3.3) with (3.4): they only depend on the hyperspherical radius ρ [defined following (2.3)]. One therefore expands the wave function in terms of hyperspherical basis functions (see Appendix) and expects rapid convergence.[9] Since the Coulomb-type two-body forces in (2.8), and the remaining three-body terms in (2.8b) have angle dependence, and thus are not diagonal in the hyperspherical basis, this expansion leads to a set of coupled radial Schrödinger equa-

TABLE I. Spectra of the $c\bar{c}$ and $b\bar{b}$ systems. For each system a comparison of the spectrum from the present work is made with the experimental spectrum and with that from Ref. 1. The last column lists the expectation value of the $q\bar{q}$ separation for each state.

		Calculated energy (GeV)		
$c\bar{c}$ state	Expt.[a] (GeV)	This work	Ref. 1	$\langle r \rangle$ (fm)
1^3S_1	3.095	3.095	3.095	0.34
1^3P^c	3.522	3.506	3.525	0.57
2^3S_1	3.685	3.669	3.686	0.72
1^3D_1	3.770	3.782	3.809	0.77
3^3S_1	4.030	4.054	4.089	1.03
2^3D_1	4.160	4.129	4.171	1.06
4^3S_1	4.415	4.379	4.418	1.29
$b\bar{b}$ state	Expt.[b] (GeV)	This work	Ref. 1	$\langle r \rangle$ (fm)
1^3S_1	9.462	9.460	9.460	0.20
1^3P^c		9.886	9.946	0.36
2^3S_1	10.015	10.005	10.048	0.45
1^3D_1		10.133	10.198	0.50
3^3S_1	10.351	10.333	10.393	0.68
2^3D_1		10.415	10.489	0.71
4^3S_1	10.576	10.582	10.660	0.88

[a]See Ref. 15.
[b]See Ref. 16.
[c]See Ref. 17.

tions. Already one term in the expansion gives an accurate value for the energy eigenvalue.[19] In the case of the s^3 ground state, for example, the inclusion of a second term only shifts the value of the ground-state energy by an amount $\Delta E \sim 0.35$ MeV. This shift becomes larger with increasing quark mass, since then the ground state becomes more deeply bound and the nondiagonal Coulomb terms are more important there. Nevertheless, the coupling of this highly localized state to higher states is sufficiently limited by the strong angular momentum barriers, and the convergence of the hyperspherical series remains rapid. For the b^3 ground state one finds $\Delta E \cong 1.3$ MeV. The numerical results in this paper were all obtained using the hyperspherical method.

The lowest six levels in the energy spectrum of Hamiltonian (3.2) for $\lambda=(\frac{1}{3})^{1/2}$ and $m_s=0.571$ GeV are shown in Fig. 2. The grouping of the levels in this particular spectrum is close to that of a weakly perturbed six-dimensional, isotropic harmonic oscillator. Indeed, the assumption that the perturbation consists of the sum of two-body potentials (Ref. 20), $\sum_{i>j} U(r_{ij})$, plus a three-body

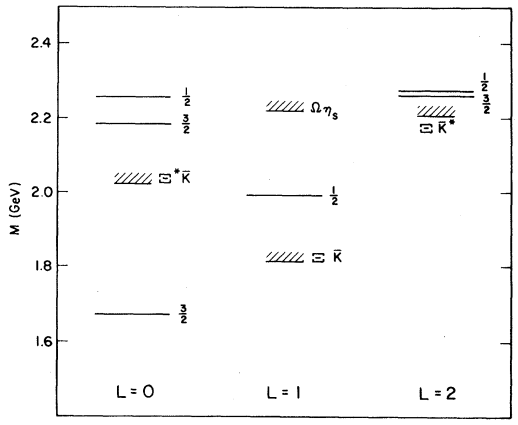

FIG. 2. The lowest few eigenstates for the s^3 system. For each state the total quark spin S, and the total orbital angular momentum L have been given. The thresholds for strong decay through quark-antiquark pair creation have been inserted at convenient places and are labeled by the two-body final state. The parameters used are $m_s = 0.571$ GeV and $\lambda = (\frac{1}{3})^{1/2}$.

potential which is a function only of ρ, leads to a three-parameter description of the five energy-level differences in Fig. 2. This information is summarized in Table II, where it is seen that a rather good fit to the energies can be obtained by choosing the parameters to have the indicated values. One also sees that the two- and the three-body corrections are comparable, and can be regarded as perturbations. However, the harmonic oscillator leads to expectation values for the hyperradius ρ, which, for the excited s^3 states, are about 10% smaller than those obtained from the Hamiltonian (3.2) (see Table III); the ground-state expectation value is used to set the scale.

For comparison the splitting obtained by Chao, Isgur, and Karl[21] for the $S=-3$ excitation spectrum (before the inclusion of the hyperfine splittings) have also been given in Table II. These authors assume a harmonic-oscillator potential perturbed by two-body forces. Their level splittings can be described by two parameters whose values were fixed by simply extrapolating from $N\Delta$ and $\Sigma\Lambda$ spectroscopy. Note how large the two-body splitting turns out to be. The two sets of separations are sufficiently different to remain recognizable, even after the application of spin-dependent corrections. Especially the position of the first radial excitation of the ground state relative to the other states, is sensitive to the structure of the actual potential being used.

The resemblance of the s^3 mass spectrum to that of a weakly perturbed harmonic osillator is accidental. In Table III the level splittings we obtain for the s^3, c^3, and b^3 systems are listed. The separations for these different systems become smaller, and differently arranged, with increasing quark mass. As m_q continues to increase beyond m_b the spacings become larger again, as appropriate for Coulomb-type bound states. At this point, the approximation of the potential, to zeroth order, by a harmonic oscillator obviously has become bad.

We now want to give some indication of the sensitivity of the spectrum to variations in λ. If λ increases the three-body part of the potential becomes more positive, since the coupling constant, which appears in the slope k of the confining term increases. (In addition, the zero-point energy becomes less negative.) The effect of this is to raise the entire spectrum, while the spacings of the lowest few excited states are hardly affected. Changes in these splittings are typically of the order of 5 MeV, and at most 12 MeV, when λ is increased from $(\frac{1}{3})$ to $(\frac{3}{2})^{1/2}$.

All Ω^- states, except the $\Omega^-(1672)$ ground state, in Fig. 2 are seen to lie above the $\Xi\bar{K}$ threshold and can decay strongly. A crude estimate for the lowest threshold for decay through the creation of a nonstrange-quark-antiquark pair for the c^3 system yields $M(ccn)+M(c\bar{n})\simeq 5.72$ GeV. Since the c^3 ground state is expected at $M(c^3)=5.04$ GeV,

TABLE II. The s^3 system, showing energy-level differences of the lowest excited states from the ground state (in MeV). The total quark spin S, the total angular momentum L, and the total spin and parity J^P are given for each state L_π. The symmetry label $\pi=S$ (for symmetric) or M (for mixed) stands for the permutation symmetry of the space (or quark spin) wave function. The column labeled "Hamiltonian (3.2)" refers to the present calculations. The column labeled "Perturbed harmonic oscillator" gives the differences for an isotropic six-dimensional harmonic oscillator which is perturbed by two- and three-body potentials, with the following values for the parameters: the effective harmonic-oscillator quantum $h\Omega=320$ MeV, the two-body-potential-induced splitting $\Delta = 65$ MeV, and the three-body-potential-induced splitting $\delta = 75$ MeV. The results in the last column are from Ref. 21 and correspond approximately to taking $h\Omega\simeq\Delta\simeq 350$ MeV and $\delta=0$ MeV.

State	S	L	J^P	Hamiltonian (3.2)	Perturbed harmonic oscillator (two- and three-body potentials)	Ref. 21
S_S	$\frac{3}{2}$	0	$\frac{3}{2}^+$	0	0	0
P_M	$\frac{1}{2}$	1	$\frac{1}{2}^-,\frac{3}{2}^-$	320	$h\Omega = 320$	370
S_S'	$\frac{3}{2}$	0	$\frac{3}{2}^+$	510	$2h\Omega-\delta-\Delta = 500$	370
S_M	$\frac{1}{2}$	0	$\frac{1}{2}^+$	585	$2h\Omega-\delta/4-\Delta/2 = 585$	535
D_S	$\frac{3}{2}$	2	$\frac{1}{2}^+,\frac{3}{2}^+,\frac{5}{2}^+,\frac{7}{2}^+$	595	$2h\Omega-\delta/4-2\Delta/5 = 595$	565
D_M	$\frac{1}{2}$	2	$\frac{3}{2}^+,\frac{5}{2}^+$	605	$2h\Omega-\delta/4-\Delta/5 = 605$	630

TABLE III. Properties of the s^3, c^3, and b^3 systems. The states are labeled as in Table II. For each state the energy difference ΔE (in MeV) from the ground state and the expectation value of the hyperradius ρ (in fm) is given. $\lambda=(\frac{1}{3})^{1/2}$, $m_s = 0.571$ GeV, $m_c = 1.872$ GeV, and $m_b = 5.237$ GeV. The ground-state energies are $M(s^3)=M(\Omega^-) = 1.672$ GeV, $M(c^3) = 5.04$ GeV, and $M(b^3) = 14.72$ GeV.

	s^3		c^3		b^3	
State	ΔE	$\langle\rho\rangle$	ΔE	$\langle\rho\rangle$	ΔE	$\langle\rho\rangle$
S_S	0	0.75	0	0.46	0	0.28
P_M	320	0.95	265	0.59	255	0.38
S'_S	510	1.06	390	0.68	340	0.43
S_M	585	1.12	465	0.72	425	0.46
D_S	595	1.12	470	0.72	435	0.46
D_M	605	1.11	485	0.72	450	0.47

all the c^3 states listed in Table III are presumably rather stable. A similar estimate for the b^3 states yields a decay threshold at 15.7 GeV, to be compared with a ground-state mass of $M(b^3) = 14.7$ GeV.

IV. COMMENTS

(A) In the previous section we noted that it was not possible for us to fit the masses of the ϕ and the Ω^- with a common value for the strange-quark mass, indicating some defect in the Hamiltonians (3.1) and (3.2). Examination of Table IV reveals one source of the difficulty, which is that the quark motion is moderately relativistic in both these systems. It is also seen that the corresponding systems composed of the heavier c and b quarks are much less relativistic. In addition, there are spin-dependent interactions which have not been included in our calculations. Since these terms are inversely proportional to $m_q{}^2$, and contribute 115 MeV to the $\psi-\eta_c$ splittings[15] they presumably are also not negligible for the strange-quark system.[23]

TABLE IV. $\langle \vec{p}^2/m^2\rangle$ for a single quark in the ground state of heavy $q\bar{q}$ and heavy q^3 systems, for s ($m_s = 0.649$ GeV) (Ref. 22), c ($m_c = 1.872$ GeV), and b ($m_b = 5.237$ GeV) quarks. The q^3 potential has $\lambda=(\frac{1}{3})^{1/2}$.

	s	c	b
$q\bar{q}$	0.565	0.187	0.078
q^3	0.495	0.147	0.053

For the heavier quark flavors $c,b,\ldots$, where relativistic effects are smaller, we expect that it will be possible to choose a value for the parameter λ such that the q^3 systems will be fit with the same values of the quark masses as are needed for the corresponding $q\bar{q}$ systems.

(B) In Fig. 3 we have summarized some of the properties of the ground states of a number of three-heavy-quark systems, using $\lambda = (\frac{1}{3})^{1/2}$. Note the great variation in size of these systems, which is accompanied by a comparable variation in the values of the coupling constant $\alpha_s(\lambda\rho)$ at the expectation value of the hyperradius, $\bar{\rho}$, in each ground state. Three-quark states are generally more extended than the corresponding quark-antiquark states. This is opposite to the situation encountered in the few-nucleon systems where the three-body states (triton, ^{3}He) are more compact than the two-body state (deuteron). The q^3 potential is somewhat weaker than the $q\bar{q}$ one, which manifests itself also in the reduction of the binding energy per quark: 140 MeV for $s\bar{s}$, compared to 35 MeV for s^3 (Ref. 22); 325 MeV for $c\bar{c}$ compared to 190 MeV for c^3; and 510 MeV for $b\bar{b}$ compared to 330 MeV for b^3; i.e., a reduction of 35 to 75% in going from $q\bar{q}$ to q^3.

(C) In Refs. 7 and 8 the confining part of the tential energy for the three-heavy-quark system was shown to be a pure three-body term. Although this result was obtained from the MIT bag model, we know of no mechanism for confinement, within the context of QCD, which would lead to a sum of two-body potentials. The QCD Lagrangian contains explicit three- and four-gluon interactions,

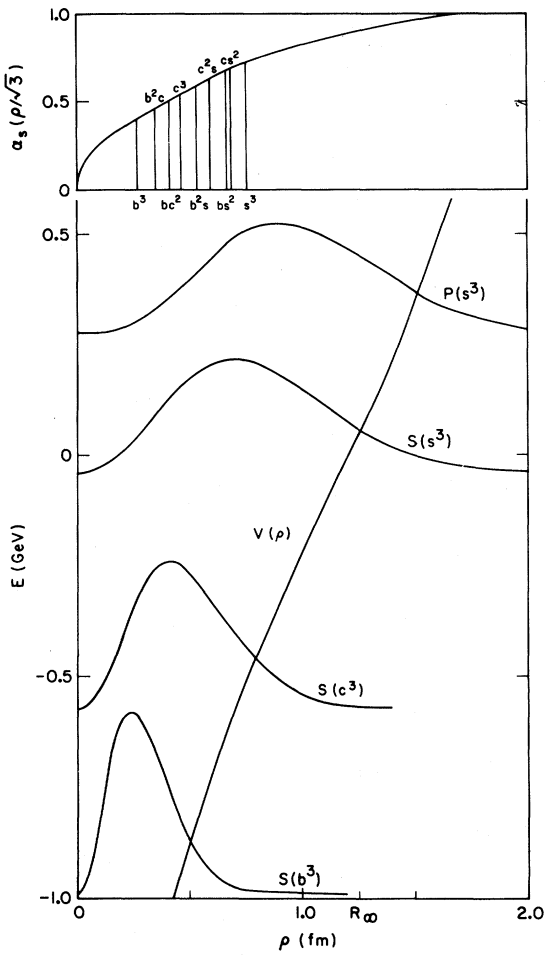

FIG. 3. The calculated ground-state hyperradial wave function and energy eigenvalue are shown for the s^3, c^3, and b^3 ground states (S) and the first s^3 excited state (P). The three-heavy-quark potential averaged over angles, $V(\rho)$, is also shown for $\lambda=(\frac{1}{3})^{1/2}$. In the top part of the graph, the running coupling constant, as present in the three-body terms of $V(\rho)$, has been plotted. The expectation value of the hyperspherical radius ρ for various three-heavy-quark systems, including mixed systems such as c^2s, is indicated by a vertical line.

which in a q^3 system must give three quark interactions (and, in multiquark systems, through successive gluon fissioning, all orders of multiquark interactions). In addition, once the coupling constant α_s becomes large enough (at large distances) multiple exchange of single gluons will create multibody interactions.

Nevertheless, some authors have tried to describe the three-heavy-quark system with a sum of two-body potentials.[24] Their prescription is to take the heavy-quark-antiquark potential, and multiply it by the ratio $\langle F_q\cdot F_q'\rangle/\langle F_q\cdot F_{\bar{q}}\rangle=\frac{1}{2}$ and sum over pairs, as if the q-$\bar{q}$ and the hypothetical q-q potential were proportional to the color factors $F_q\cdot F_{\bar{q}}$ and $F_q\cdot F_{q'}$, respectively. From (2.7) and (2.8) it is seen that the color Coulomb terms (and in fact all terms originating from single-gluon exchange[25]) in the two potentials are related by this factor of $\frac{1}{2}$; but *the confining terms are not simply proportional to the color charges.*

It is interesting to see how different the effective confining term arising from this two-body prescription is from our three-body confining term. For simplicity, take the coupling constant to be fixed and form the ratio of the averge over the angles of the two terms, f. For small separations, using Eqs. (2.7a) and (2.8a) this becomes

$$f_s\equiv\left\langle \tfrac{1}{2}(\tfrac{2}{3})^{1/2}k\sum_{i>j}\left|\bar{x}_i-\bar{x}_j\right|\right\rangle/\langle k\rho\rangle\ . \qquad (4.1)$$

Note that the denominator has no angle dependence. From considering extreme geometric configurations for the quarks, namely a straight line and an equilateral triangle, one expects $1\le f_s\le(\frac{3}{2})^{1/2}$, with the larger value corresponding to the triangle. We compute $f_s=1.18$. This implies that taking a sum of two-body terms amounts to overestimating the slope of the confinement term by 18%. (It also shows that the quarks in a q^3 ground state are more likely to be found in the triangle configuration. Roughly the same conclusion holds for all states listed in Table III.)

The corresponding result for large separations is obtained from (2.7b) and (2.8b) to give the ratio

$$f_1\equiv\left\langle \tfrac{1}{2}k\sum_{i>j}\left|\bar{x}_i-\bar{x}_j\right|\right\rangle/\langle kr_M\rangle \qquad (4.2)$$

and this time[12] $f_1=0.91$. The simple two-body potential now under estimates the slope. Therefore, the prescription of taking a heavy-quark-antiquark potential multiplied by a factor of $\frac{1}{2}$ as the heavy-quark-quark interaction, and summing over pairs, would only give a good description for states which extend far enough to experience the compensating effects of the two regions—yielding an effective ratio for f of about 1. This is clearly not the case for the low-lying q^3 states we have considered so far.

In Fig. 4 we compare the energy spectra resulting from Hamiltonian (3.2) (already shown in Fig. 2) and from one in which the potential (2.8) has been replaced, according to the above-mentioned

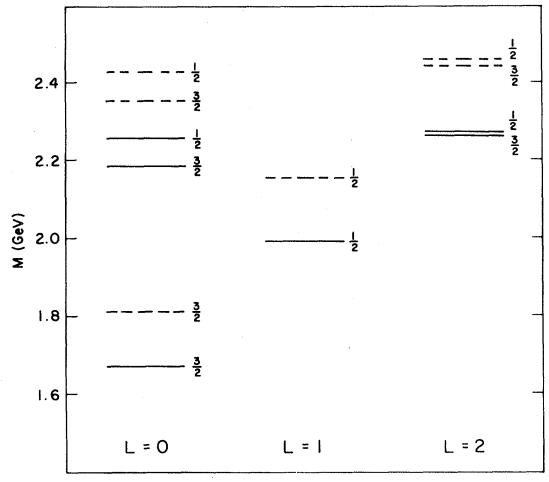

FIG. 4. Comparison of the lowest eigenstates of H_{q^3} (as in Fig. 2, solid levels) to those of a Hamiltonian obtained from H_{q^3} by replacing the three-body confining term by a sum of two-body terms (dashed levels).

recipe, by a pure two-body potential (which we based on the $q\bar{q}$ potential (2.7) but which still contains the three-body zero-point energy E_0). Such a potential is still fairly diagonal in the hyperspherical basis; however, the convergence of the expansion in this case in not rapid as before because the two-body confinement terms generate a coupling between the channels which grows linearly with distance. As expected, the two-body spectrum lies too high.

V. SUMMARY

We have presented a simple analytic formula for the potential energy of a system of three heavy quarks, including the effect of asymptotic freedom,

$$V_{q^3}(\bar{x}_1,\bar{x}_2,\bar{x}_3)=-\tfrac{2}{3}\sum_{i<j}\frac{\alpha_s(|\bar{x}_i-\bar{x}_j|)}{|\bar{x}_i-\bar{x}_j|}$$

$$+k(\alpha_s(\lambda\rho))\rho\ . \qquad (2.8a)$$

$\alpha_s(r)$ is the same running coupling constant that was previously employed in the study of heavy $q\bar{q}$ systems.[1] In the two-body Coulomb terms the argument of α_s is the q-q separation; but in the three-body confining term the argument is the hyperspherical radius scaled by a parameter λ. For systems which are not sufficiently compact, (2.8a) must be supplemented with (2.8b).

We expect Eq. (2.8a) to give as accurate a description of the potential energy of the b^3 and c^3 systems as Eq. (2.7a) does for the $b\bar{b}$ and $c\bar{c}$ systems, for the values of the parameters. We do not expect the same accuracy for sytems containing strange quarks, since the treatment of such quarks in the Born-Oppenheimer approximation has only marginal validity, because of relativistic effects. Also spin-dependent effects will modify the results. Still, we have studied the Ω^- (s^3) system, since this is the only available baryon on which to apply the method. With λ in Eq. (2.8a) assigned the lower limit of its expected range of values, $(\frac{1}{3})^{1/2}$, it is found that the mass of the Ω^- comes out too large. To bring it down to the experimental value it is necessary to reduce the mass of the strange quark from $m_s=0.65$ GeV [the value needed to fit the mass of the $\phi(s\bar{s})$ meson] to $m_s=0.57$ GeV. This change hardly affects the splittings between the low-lying levels of the s^3 mass spectrum. These splittings are the most reliably calculated feature of the s^3, and also the b^3 and c^3 mass spectra.

We have argued on theoretical grounds against the idea that confinement can be represented by a sum of two-body potentials. Nevertheless, we have compared the spectrum from our potential energy (which contains a three-body confining term) with the spectra from two models[21,24] which assume two-body potentials. One of these was used recently by Chao *et al.*[21] and assumes two-body harmonic-oscillator forces between the quarks, and the other[24] was obtained from the prescription that $V_{qq'}(|\bar{r}_q-\bar{r}_{q'}|)=\frac{1}{2}V_{q\bar{q}}(|\bar{r}_q-\bar{r}_{q'}|)$. We find that our spectrum differs distinctly from both spectra which come from two-body potentials, a feature that will not be obscured by the application of spin-dependent corrections.[26]

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APPENDIX

The Schrödinger equation for three free heavy quarks of mass in the center-of-mass frame is

$$-\frac{h^2}{2m}(\Delta_x+\Delta_y)\psi(\vec{x},\vec{y})=E\psi(\vec{x},\vec{y}) \tag{A1}$$

in terms of the Jacobi coordinates

$$\vec{x}=(\tfrac{1}{2})^{1/2}(\vec{r}_1-\vec{r}_2)\ , \tag{A2a}$$

$$\vec{y}=(\tfrac{2}{3})^{1/2}\left[\frac{\vec{r}_1+\vec{r}_2}{2}-\vec{r}_3\right]\ . \tag{A2b}$$

This equation (A1) can be separated into six one-variable differential equations by going over to spherical coordinates $(\vec{x};\vec{y})\rightarrow(x,\theta_x,\phi_x;y,\theta_y,\phi_y)$ and introducing the hyperradius ρ and another variable θ:

$$\begin{aligned} x&=\rho\cos\theta,\quad 0\le\rho<\infty\ ,\\ y&=\rho\sin\theta,\quad 0\le\theta\le\pi/2\ . \end{aligned} \tag{A3}$$

The eigensolutions to the equations for the angles can be combined into hyperspherical harmonics:

$$Y^{Kl\lambda}_{m\mu}(\Omega)=Y^{Kl\lambda}_{m\mu}(\theta,\hat{x},\hat{y})=N^{l\lambda}_n\cos^l\theta\sin^\lambda\theta P^{l+1/2,\lambda+1/2}_n(-\cos2\theta)Y^l_m(\hat{x})Y^\lambda_\mu(\hat{y}) \tag{A4}$$

which are defined in terms of the spherical harmonics Y^l_m and Y^λ_μ and the Jacobi polynomials of the first kind $P^{a,b}_n$. $K=2n+l+\lambda$ and $N^{l\lambda}_n$ is a normalization factor chosen to make the $Y^{kl\lambda}_{m\mu}$ an orthonomal set with the volume element $d\Omega=\cos^2\theta\sin^2\theta\,d\theta\,d\theta\,d\Omega_x d\Omega_y$.

Including a permutation- and translation-invariant potential, Eq. (A1) becomes

$$\left[-\frac{h^2}{2m}(\Delta_x+\Delta_y)+V(\vec{x},\vec{y})\right]\psi(\vec{x},\vec{y})=E\psi(\vec{x},\vec{y})\ . \tag{A5}$$

If V is almost exclusively a function of ρ, it is efficient to expand ψ in terms of hyperspherical harmonics. Ignoring the quark spin for the moment one has

$$\psi(\vec{x},y)=\sum_{\gamma=[k,l,\lambda]}\chi^{L\gamma}(\rho)Y^{L\gamma}_M(\Omega) \tag{A6}$$

in terms of the radial wave function $\chi^{L\gamma}$, and the eigenfunctions

$$Y^{L\gamma}_m=\sum_{m,\mu}C^{l\lambda L}_{m\mu M}Y^{Kl\lambda}_{m\mu}$$

of the total orbital angular momentum $\vec{L}=\vec{l}+\vec{\lambda}$. Substituting (A6) into (A5) then leads to a coupled set of radial equations of the form

$$\left[-\frac{h^2}{2m}\left[\frac{\partial^2}{\partial\rho^2}+\frac{5}{\rho}\frac{\partial}{\partial\rho}-\frac{K(K+4)}{\rho^2}\right]+V_{\gamma\gamma}(\rho)-E\right]\chi^{L\gamma}(\rho)=-\sum_{\gamma'\neq\gamma}V_{\gamma\gamma'}(\rho)\chi^{L\gamma'}(\rho) \tag{A7}$$

with

$$V_{\gamma\gamma'}(\rho)=\int d\Omega\,Y^{L\gamma*}_M(\Omega)V(\vec{x},\vec{y})Y^{L\gamma'}_M(\Omega)\ .$$

In our case one- or two-component approximations to ψ already give good results. It is straightforward to include the quark spin.

An alternate technique for obtaining the energy spectrum of the three-body Hamiltonian (3.2) is to separate the Schrödinger equation into coupled Faddeev equations, and then make a partial-wave expansion of the wave function.[27] This method amounts to solving a partial differential equation in the two remaining variables. We have checked our results obtained with the hyperspherical method, using two coupled channels for the ground states of the s^3, c^3, and b^3 systems, and their first radial excitations, with the Faddeev method. We find that the energy eigenvalues agree to within 0.5 MeV, and the expectation values for $(\rho^2/3)^{1/2}$, the mass radius, agree to within 0.5%.

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[18]The fact that the spectrum of Ref. 1 and our spectrum do not agree as well as might be expected from Fig. 1 arises because the radius R appearing in the zero-point energy was determined differently, yielding values for R in Ref. 1 which are noticeably larger than those obtained from Eq. (3.5) for values of $r\geq 0.5$ fm.

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