Electroweak production of heavy quarks in e^+e^- annihilation

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We study the production of heavy quarks in e^+e^- annihilation via γ and Z exchange. The gluon radiative corrections to the total cross section and to the angular distributions of the quark jets are calculated. If heavy quarks with masses around 20 GeV or above exist, the mass corrections as well as the radiative gluon corrections are important even for e^+e^- energies in the Z resonance region.

I. INTRODUCTION

With e^+e^- colliding-beam facilities now operating in the center-of-mass energy range $\sqrt{s} > 30$ GeV, weak-interaction effects in e^+e^- annihilation become more and more important. The standard unified Glashow-Salam-Weinberg theory[1] makes detailed predictions for these effects from lowest energies throughout the Z resonance region (see Ref. 2 and references quoted therein). Apart from purely leptonic reactions, such as $e^+e^- \to e^+e^-$ and $\mu^+\mu^-$, the total cross section for hadron production and forward-backward asymmetries of hadron jets due to vector–axial-vector interference are of particular importance. Their measurement determines the electromagnetic and weak quark charges.

The predictions derived from the standard unified theory for these quantities are generally based on the parton model for $e^+e^- \to \gamma, Z \to q\bar{q}$ (Fig. 1A). We first discuss these parton-model predictions for heavy quarks. Finite masses give rise to a threshold factor $\sim(1-4m_q^2/s)^{1/2}$ in the vector part of the cross section; this threshold factor rapidly approaches its asymptotic value 1. In contrast to this, the threshold factor of the axial-vector part is $\sim(1-4m_q^2/s)^{3/2}$; this part of the cross section approaches its asymptotic behavior only slowly.

Quantum chromodynamics (QCD),[3] however,

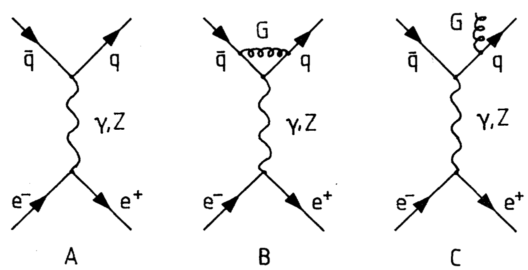

FIG. 1. Diagrams contributing to e^+e^- annihilation into quark and gluon final states. (Gluon bremsstrahlung from the antiquark and quark mass correction diagrams are to be added.)

modifies the parton cross section of e^+e^- annihilation into hadrons. For the electromagnetic total cross section the modification in first order of α_s ($= g_s^2/4\pi$ where g_s is the quark-gluon coupling constant) is well known to be[4]

$$\sigma_q = \left(1 + \frac{\alpha_s}{\pi}\right)\sigma_q^{\text{parton}} \qquad (1)$$

if quark masses can be neglected. This radiative correction may be derived either by means of the renormalization-group equation for the vacuum-polarization tensor or calculated as a sum of vertex and gluon-bremsstrahlung corrections to the parton-model cross section (Figs. 1B and 1C).

Here we use the second method to determine the QCD radiative corrections to the total cross section when axial-vector contributions (due to Z exchange in the standard theory) are included and the quark masses are kept finite. Finite quark masses render the theory γ_5 noninvariant so that the QCD correction to the axial-vector part of the cross section differs from the vector part. This will be of importance if new heavy quarks are produced in e^+e^- annihilation above 36 GeV c.m. energy available now. We expand on previous work which discussed the purely electromagnetic reaction,[5,6] assumed vanishing quark masses,[7] or treated hard-gluon bremsstrahlung only.[8-10]

In the same way we determine the first-order QCD radiative corrections to the differential cross section $d\sigma_q/d\cos\vartheta$ for quark jets produced at the polar angle ϑ (the angle between the quark jet and e^- momentum). The angular distribution includes a vector–axial-vector interference term which can be isolated by measuring the difference between forward and backward cross sections,

$$\Delta_q(\vartheta) = [d\sigma_q(\vartheta) - d\sigma_q(\pi-\vartheta)]/d\cos\vartheta\,, \qquad (2a)$$

or the forward-backward asymmetry

$$A_q(\vartheta) = \frac{d\sigma_q(\vartheta) - d\sigma_q(\pi-\vartheta)}{d\sigma_q(\vartheta) + d\sigma_q(\pi-\vartheta)}\,. \qquad (2b)$$

This requires the identification of the charge of the quark which develops into the jet. Concentrating on heavy-quark production has two implications. Firstly, as long as the ratio of quark mass to beam energy[11] $\mu = 2m_q/\sqrt{s} \geq 10^{-1}$, one expects one single heavy-quark–antiquark pair to be produced (directly coupled to γ or Z) whereas associated production via strong interactions in the jet development should be negligible.[12] Thus the decay products of the hadron which carries the flavor quantum number of the heavy quark, reveal the flight direction ϑ of the quark. Secondly, the higher-order perturbation theory yields logarithmic mass terms $\ln\mu^{-1}$ for processes in which quark charges are measured. They invalidate the perturbative expansion for $\mu \to 0$ (Ref. 13) but bear no problem for heavy quarks if $\ln\mu^{-1} \sim 1$. Neither are large logarithms expected to occur when light quarks are included in higher-order calculations, as their charges are not held fixed. The detailed analysis will show that in first order the QCD corrections to $\Delta_q(\vartheta)$ are $O(\alpha_s m_q/\sqrt{s})$. The VA interference term of the cross section is not strongly affected by gluon radiative corrections.

The material of this paper is presented as follows. The next section gives the definitions of coupling constants, and the basic cross sections for pair production of heavy quarks in e^+e^- annihilation are discussed. In the third section we derive the quark form factors from the vertex correction for vector and axial-vector currents (diagram 1B). The infrared part of diagram 1C is added to find the properly defined cross section $\sigma(e^+e^- \to q\bar{q} + q\bar{q}\,G_{\text{soft}})$. Subsequently we present the hard-gluon cross section $\sigma(e^+e^- \to q\bar{q}\,G_{\text{hard}})$. Finally the various parts are added to obtain the total cross section and the angular distributions of the heavy-quark jets.

II. ELECTROWEAK PRODUCTION OF HEAVY-QUARK–ANTIQUARK PAIRS

The cross section for the production of quarks q in e^+e^- annihilation via γ and Z exchange $e^+e^- \to \gamma, Z \to q + \cdots$, is a binomial in $\cos\vartheta$ [$\vartheta = \sphericalangle(e^-, q)$],

$$\frac{d\sigma_q}{d\cos\vartheta} = \tfrac{3}{8}(1+\cos^2\vartheta)\sigma_U + \tfrac{3}{4}\sin^2\vartheta\,\sigma_L + \tfrac{3}{4}\cos\vartheta\,\sigma_F . \tag{3}$$

The various parts of the cross section correspond to the following γ and Z spin components along the flight direction of the quark:

U = unpolarized transverse;

L = longitudinally polarized;

F = difference between right and left polarizations.

A nonzero σ_F (after higher-order QED effects have been subtracted) ensues from parity-violating vector–axial-vector interference.

The total cross section as obtained from Eq. (3) is

$$\sigma_q = \sigma_U + \sigma_L \tag{4}$$

while the differences between forward and backward cross sections at the angle ϑ and the asymmetry are given as

$$\Delta_q(\vartheta) = \tfrac{3}{2}\cos\vartheta\,\sigma_F \tag{5a}$$

and

$$A_q(\vartheta) = \frac{2\cos\vartheta}{1+\alpha_q\cos^2\vartheta}\,\beta_q . \tag{5b}$$

The parameter α_q is, as usual, defined by

$$\alpha_q = \frac{\sigma_u - 2\sigma_L}{\sigma_U + 2\sigma_L} \tag{5c}$$

and the asymmetry parameter β_q reads

$$\beta_q = \frac{\sigma_F}{\sigma_U + 2\sigma_L} . \tag{5d}$$

To evaluate the cross sections we specify the electromagnetic and weak coupling constants of the vertices in Fig. 1 as follows:[14]

(i) *Electromagnetic current coupled to the photon*

$e_0 Q_j \gamma_\mu$ with $Q_j = -1$ for electrons,

$= +\tfrac{2}{3}$ and $-\tfrac{1}{3}$ for quarks,

(ii) *Weak current coupled to Z*

$$-\frac{1}{\sqrt{2}}\left(\frac{G_F}{\sqrt{2}}\right)^{1/2} m_Z(v_j\gamma_\mu - a_j\gamma_\mu\gamma_5) ;$$

vector and axial-vector coefficients acquire the following values in the standard theory:

$$v_j = \mp 1 - 4\,Q_j \sin^2\vartheta_W$$

$$a_j = \mp 1$$

– for electrons and quarks with charge $-\tfrac{1}{3}$, + for quarks with charge $+\tfrac{2}{3}$. In terms of these parameters the contributions of the parton diagram (Fig. 1A) to the helicity cross sections of $e^+e^- \to \gamma, Z \to q\bar{q}$ are given as

$$\sigma_U^0 = v\sigma_{VV} + v^3\sigma_{AA},$$
$$\sigma_L^0 = \tfrac{1}{2}\mu^2 v\sigma_{VV} , \tag{6}$$
$$\sigma_F^0 = v^2\sigma_{VA} .$$

σ_{VV}, σ_{AA}, and σ_{VA} correspond to the VV, AA, and VA quark current products, respectively; for the quark species q,

$$\sigma_{VV} = \frac{4\pi\alpha^2 Q_e^2 Q_q^2}{s} - \frac{G_F \alpha}{\sqrt{2}} Q_e Q_q v_e v_q \left(1 - \frac{s}{m_Z^2}\right) D^{-1} + \frac{G_F^2}{32\pi} v_q^2 (v_e^2 + a_e^2) s D^{-1},$$

$$\sigma_{AA} = \frac{G_F^2}{32\pi} a_q^2 (v_e^2 + a_e^2) s D^{-1}, \tag{7}$$

$$\sigma_{VA} = -\frac{G_F \alpha}{\sqrt{2}} Q_e Q_q a_e a_q \left(1 - \frac{s}{m_Z^2}\right) D^{-1} + \frac{G_F^2}{8\pi} v_e v_q a_e a_q s D^{-1},$$

where

$$D^{-1} = \left[\left(1 - \frac{s}{m_Z^2}\right)^2 + \left(\frac{\Gamma_Z}{m_Z}\right)^2\right]^{-1},$$

s denotes the square of the c.m. energy, $\mu = 2m_q/\sqrt{s}$ is the quark mass in units of the c.m. beam energy, and $v = (1-\mu^2)^{1/2}$ is the velocity of the quark. Notice the different threshold behavior of the VV and AA terms in the helicity cross sections. The overall factor v in the cross sections is due to phase space. The matrix element of the transverse vector current is ~1 whereas that of the transverse axial-vector current is proportional to the quark velocity (this is related to the fact that a $q\bar{q}$ pair at rest cannot be in a $J^P = 1^+$ state). The matrix element of the longitudinal axial-vector current is $\sim u^+(\vec{p})(\vec{\sigma}\vec{p})v(-\vec{p}) \equiv 0$; $\gamma_\mu\gamma_5$ axial-vector currents do not contribute to longitudinal cross sections in e^+e^- annihilation, unlike longitudinal vector currents which vanish only asymptotically for $\mu \to 0$. (Some of these features can also be read off the lowest-order spectral functions of the currents, e.g., in Ref. 15).

To show the importance of mass effects in the helicity cross sections we present the R value of the total cross section (σ_q in units of $\sigma(e^+e^- \to \gamma \to \mu^+\mu^-)$ and the parameters α_q and β_q in Figs. 2(a) to 2(c), for charge $+\frac{2}{3}$ and $-\frac{1}{3}$ quarks in the standard theory. We have taken the parameters $\sin^2\vartheta_W = 0.23$ and $m_Z = 89$ GeV. The quark mass value is chosen[16] as $m_q = 20$ GeV and $\Gamma_Z = 2.5$ GeV. Note that the asymptotic values for $m_q/\sqrt{s} \to 0$ are reached only slowly (in particular on the Z resonance); this is due to the fact that the axial-vector matrix elements induce a strong threshold suppression $\sim v^3$ in the cross sections. On the Z resonance this suppression is still of the order of 2 for a quark mass of 20 GeV. To show this in more detail we present in Fig. 2(d) the ratio of the parton cross section for quark mass m_q to the massless case on the Z resonance.

III. QUARK FORM FACTORS

The gluonic vertex correction in diagram 1B changes the pointlike vertices γ_μ and $\gamma_\mu\gamma_5$ to

$$\gamma_\mu \to \gamma_\mu(1+f_1) + i\sigma_{\mu\nu}Q^\nu f_2/2m_q,$$
$$\gamma_\mu\gamma_5 \to \gamma_\mu\gamma_5(1+f_A). \tag{8}$$

(Additional terms proportional to Q_μ, the four-momentum of γ and Z, vanish if multiplied with the lepton current; $\sigma_{\mu\nu}Q^\nu\gamma_5$ cannot arise as a second-class current.) We have derived the form factors and the soft-gluon-bremsstrahlung cross section by employing dimensional regularization. For γ_5 we have used the prescription of Refs. 17 which takes γ_5 anticommuting with all γ_μ matrices in D dimensions.[18] The lowest-order QCD diagrams 1B and 1C can then be evaluated in the same way as the vertex correction and photon bremsstrahlung in QED.[19]

The form factors read in the limit $D \to 4$ as follows:

Vector form factors

$$\mathrm{Re} f_1 = \frac{4}{3}\frac{\alpha_s}{2\pi}\left\{h_{\mathrm{I}} - 2 - \frac{1+2v^2}{2v}\ln\frac{1-v}{1+v} + \frac{1+v^2}{v}\left[\mathrm{Li}_2\left(\frac{1-v}{1+v}\right) + \frac{\pi^2}{3} - \frac{1}{4}\ln^2\frac{1-v}{1+v} + \ln\frac{1-v}{1+v}\ln\frac{2v}{1+v}\right]\right\}, \tag{9}$$

$$\mathrm{Re} f_2 = \frac{4}{3}\frac{\alpha_s}{2\pi}\frac{1-v^2}{2v}\ln\frac{1-v}{1+v},$$

$$\mathrm{Im} f_1 = \frac{4}{3}\frac{\alpha_s}{2\pi}\left(h_{\mathrm{II}} - \frac{1+2v^2}{2v} - \frac{1+v^2}{2v}\ln\frac{1-v^2}{4v^2}\right),$$
$$\mathrm{Im} f_2 = \frac{4}{3}\frac{\alpha_s}{2\pi}\frac{1-v^2}{2v}\pi. \tag{10}$$

Axial-vector form factors

$$\mathrm{Re} f_A = \mathrm{Re} f_1 - \frac{4}{3}\frac{\alpha_s}{2\pi}\frac{1-v^2}{2v}\ln\frac{1-v}{1+v},$$
$$\mathrm{Im} f_A = \mathrm{Im} f_1 - \frac{4}{3}\frac{\alpha_s}{2\pi}\frac{1-v^2}{2v}\pi. \tag{11}$$

The $h_{\mathrm{I,II}}$ denote the infrared-divergent parts of the form factors

$$h_{\mathrm{I}} = \lim_{D\to 4}\frac{1}{D-4}\left(2 + \frac{1+v^2}{v}\ln\frac{1-v}{1+v}\right),$$
$$h_{\mathrm{II}} = \lim_{D\to 4}\frac{1}{D-4}\frac{1+v^2}{v} \tag{12}$$

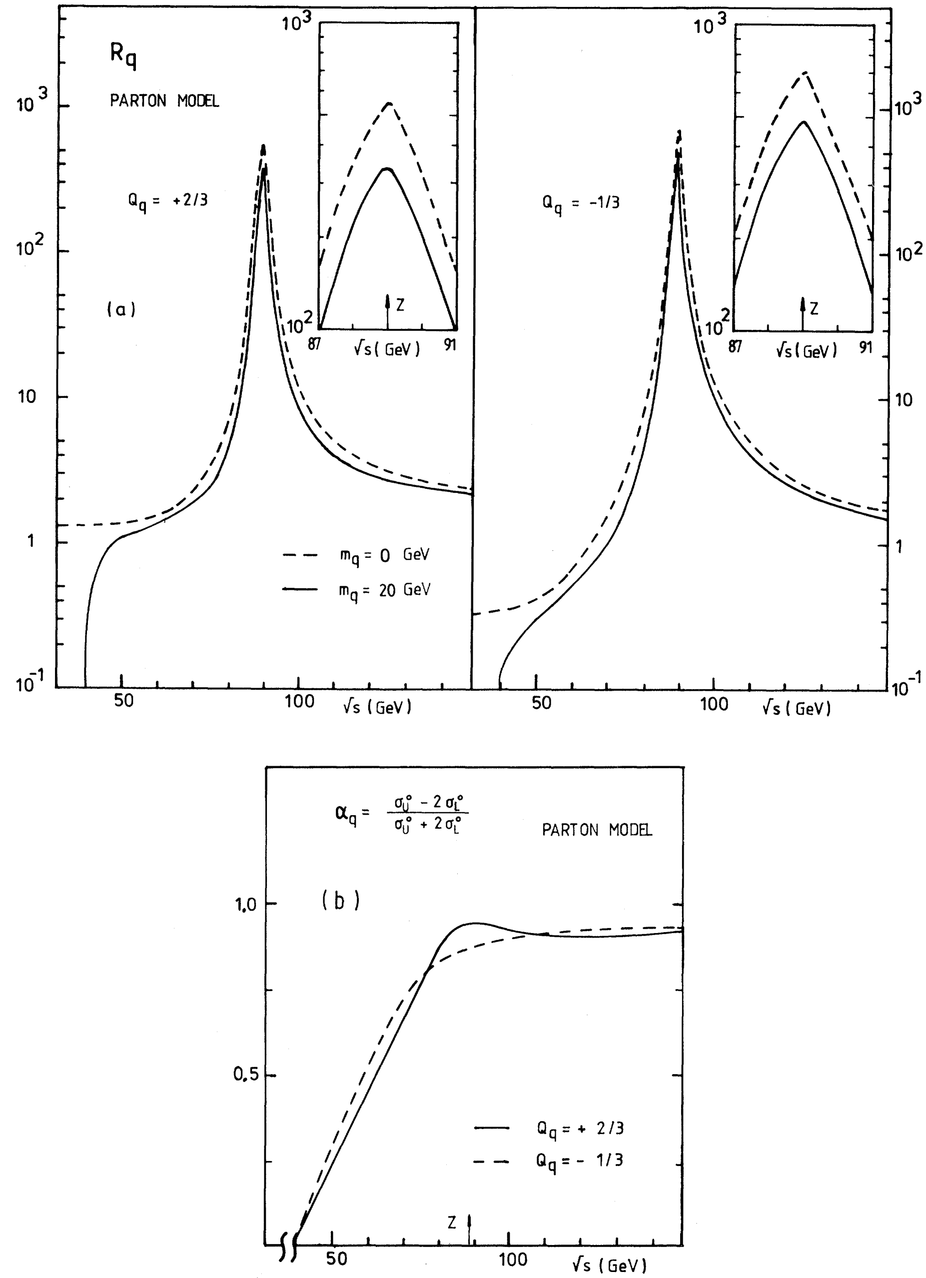

FIG. 2. (a) $R_q = \sigma(e^+e^- \to \gamma, Z \to q\bar{q})/\sigma(e^+e^- \to \gamma \to \mu^+\mu^-)$, for the production of massless and heavy quarks in the parton model; (b) α_q parameter, $\alpha_q = (\sigma_U - 2\sigma_L)/(\sigma_U + 2\sigma_L)$ with σ_U and σ_L being the unpolarized transverse and longitudinal cross section, respectively ($\alpha_q = 1$ for massless quarks in the parton model); (c) forward-backward asymmetry parameter β_q; (d) R_q value for quark-pair production on the Z resonance as a function of the quark mass m_q.

which get canceled by adding to the cross section the corresponding term of the soft-gluon bremsstrahlung.[20]

Two points are worth emphasizing. (i) In the high-energy limit $v \to 1$ the vertex correction of the vector current modifies the γ_μ term only and does not induce a $\sigma_{\mu\nu}$ term. (ii) The axial-vector-current correction and the vector-current correction become asymptotically equal. This is a consequence of asymptotic γ_5 invariance of the theory emerging directly from the γ_5 prescription we used.

The interference between the parton diagram 1A and the vertex correction 1B adds the following

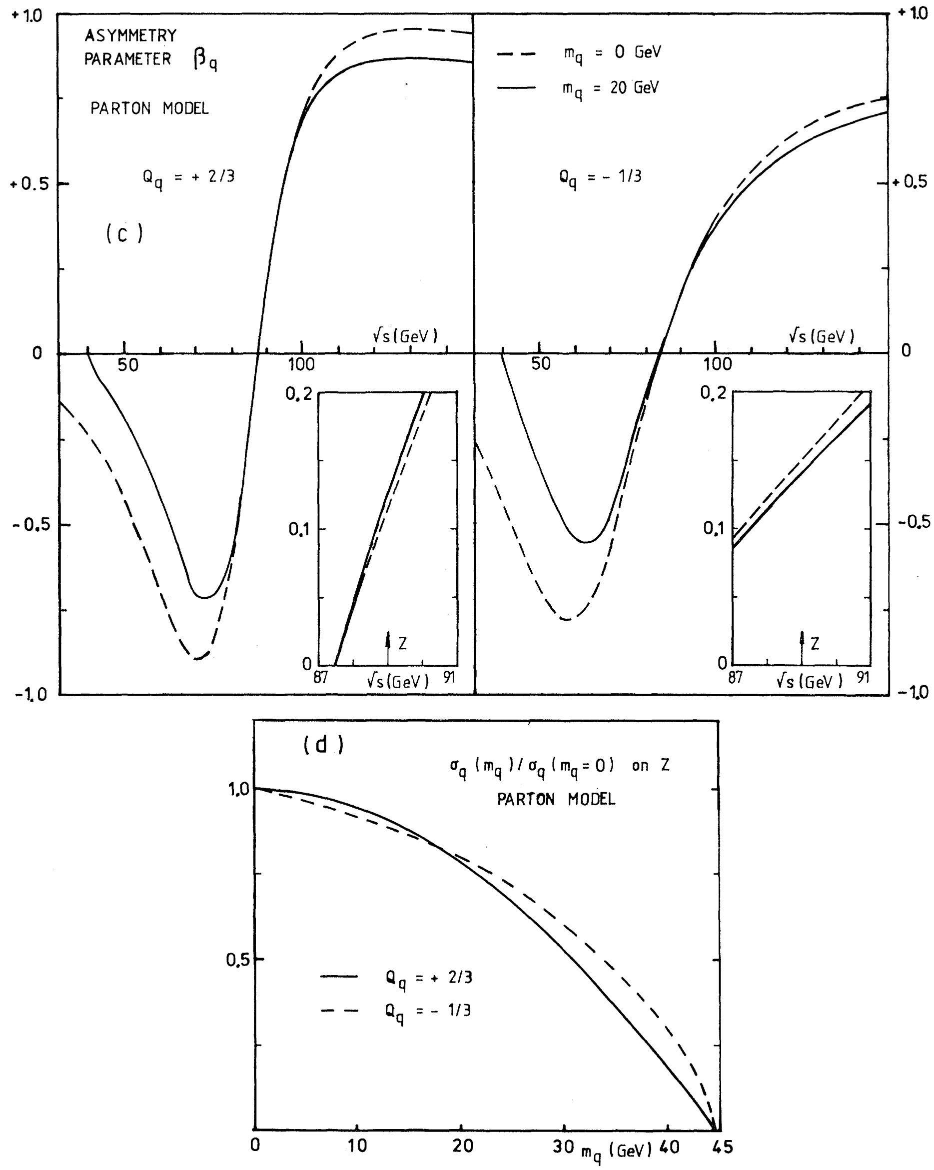

FIG. 2. *(Continued.)*

terms to the parton cross sections:

$$\begin{aligned}\delta\sigma_U &= v\sigma_{VV} 2\,\mathrm{Re}(f_1+f_2) + v^3\sigma_{AA} 2\,\mathrm{Re} f_A,\\ \delta\sigma_L &= v\sigma_{VV}\,\mathrm{Re}(\mu^2 f_1 + f_2),\\ \delta\sigma_F &= v^2\sigma_{VA}\,\mathrm{Re}(f_1+f_2+f_A)\\ &\quad - v^2\sigma'_{VA}\,\mathrm{Im}(f_1+f_2-f_A).\end{aligned} \tag{13}$$

The additional second term in $\delta\sigma_F$ is proportional to the imaginary part of the Z propagator,

$$\sigma'_{VA} = \frac{G_F\alpha}{\sqrt{2}} Q_e Q_q a_e a_q \frac{\Gamma_Z}{m_Z} D^{-1}. \tag{14}$$

The phase-space factor v renders all these contributions finite at threshold.

We finally add the cross section for soft-gluon emission up to a maximal gluon energy λ in the e^+e^- c.m. frame. This part factorizes in the parton cross sections and a universal function δ_{SB} which depends logarithmically on the cutoff parameter λ for $\lambda \to 0$. It is the same function as in purely electromagnetic processes,[21]

$$\sigma_i(\text{soft bremsstrahlung}) = \sigma_i^0\,\delta_{SB}\left(v, \frac{\lambda}{m_q}\right); \quad i = U, L, F, \tag{15}$$

with

TABLE I. Coefficients of the QCD helicity cross sections in Eq. (20); $\kappa = 1/\bar{\kappa} = (1-x_q)/(1-x_{\bar{q}})$ and $\lambda_\pm = \kappa + \bar{\kappa} \pm 2$.

i	S_i	N_i	$\bar{N}_i - N_i$
U	$\frac{8}{3}(x_q^2 + x_{\bar{q}}^2 - \frac{1}{2}p_\perp^2)$	$-\frac{4}{3}(\lambda_+ - \frac{1}{2}\kappa p_\perp^2)$	$\frac{8}{3}\left[\frac{1}{2}p_\perp^2 - 2(1-x_G) + \frac{\mu^2}{2}\lambda_+\right]$
L	$\frac{4}{3}p_\perp^2$	$\frac{2}{3}(\bar{\kappa}p_q^2 + \kappa p_{\bar{q}}^2 - 2p_q p_{\bar{q}} - \lambda_-)$	$\frac{4}{3}\left[x_G^2 - 2(1-x_G) - p_\perp^2 + \frac{\mu^2}{2}\lambda_+\right]$
F	$\frac{4}{3}(x_q p_q - x_{\bar{q}} p_{\bar{q}})$	$\frac{2}{3}x_G\left(\frac{p_{\bar{q}}}{1-x_{\bar{q}}} - \frac{p_q}{1-x_q}\right)$	

$$\delta_{SB} = -\frac{4}{3}\frac{\alpha_s}{2\pi}\left\{2h_I + 2\ln\frac{2\lambda}{m_q}\left(2 + \frac{1+v^2}{v}\ln\frac{1-v}{1+v}\right) + \frac{2}{v}\ln\frac{1-v}{1+v} + \frac{1+v^2}{v}\left[2\,\mathrm{Li}_2\left(\frac{2v}{1+v}\right) + \frac{1}{2}\ln^2\frac{1-v}{1+v}\right]\right\}. \tag{16}$$

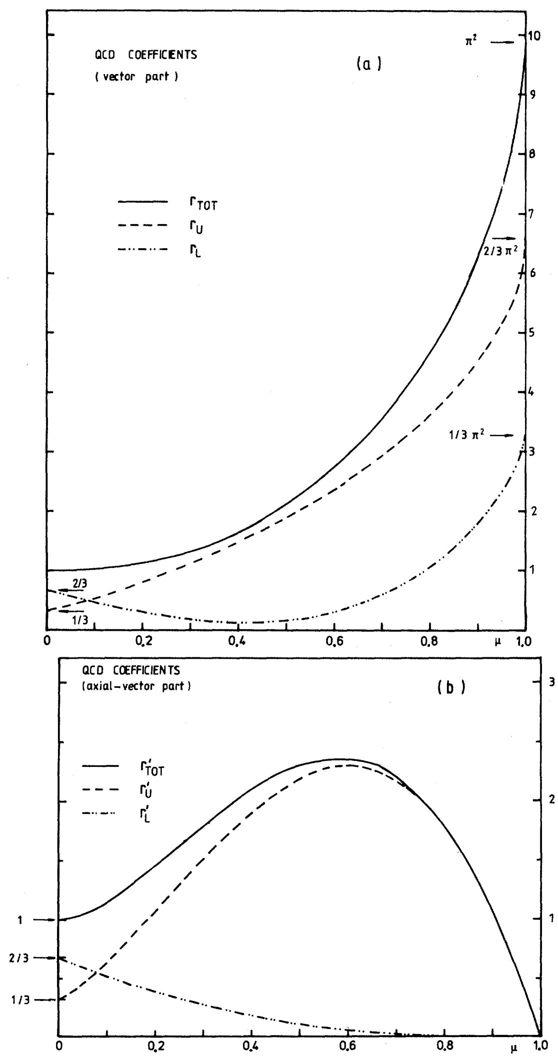

FIG. 3. Coefficients of the QCD corrections to the transverse and longitudinal cross sections and their sum; (a) vector terms, (b) axial-vector terms.

The infrared-singular parts h_I of the vertex correction and the soft-gluon emission obviously cancel each other. [The imaginary part h_{II} already disappears in the combination of form factors occuring in Eq. (13).]

Up to terms of the order μ the sum of the vertex correction and the soft-gluon emission are the same for all cross sections $\sigma_{U,L,F}$,

$$\sigma_i^0 \to \sigma_i = \sigma_i^0[1 + \delta_R + O_i(\mu)] \tag{17}$$

with

$$\delta_R = \frac{4}{3}\frac{\alpha_s}{2\pi}\left[4\ln\frac{2\lambda}{m_q}\left(\ln\frac{s}{m_q^2} - 1\right) - 2\ln^2\frac{s}{m_q^2} + 5\ln\frac{s}{m_q^2} + \frac{2\pi^2}{3} - 4\right]. \tag{18}$$

δ_R develops a logarithmic singularity if the quark mass $m_q \to 0$.[13] This singularity for light quarks disappears once the cross section for collinear configurations of the quark and gluon quanta is added.

IV. HARD-GLUON BREMSSTRAHLUNG

The cross section for the angular distribution of the quark q in the hard gluon process $e^+e^- \to \gamma, Z \to q\bar{q}G$ is conveniently parametrized as

$$\frac{d\sigma(q\bar{q}G)}{dx_q dx_{\bar{q}}\, d\cos\vartheta} = \frac{3}{8}(1+\cos^2\vartheta)\frac{d\sigma_U}{dx_q dx_{\bar{q}}} + \frac{3}{4}\sin^2\vartheta\frac{d\sigma_L}{dx_q dx_{\bar{q}}} + \frac{3}{4}\cos\vartheta\frac{d\sigma_F}{dx_q dx_{\bar{q}}}. \tag{19}$$

Here $d\sigma_{U,L,F}$ correspond to the same γ and Z spins as the respective parton cross sections presented in Sec. II; x_q and $x_{\bar{q}}$ denote the energy of the quark and antiquark in units of the beam energy. We write the cross sections $d\sigma_i$ as

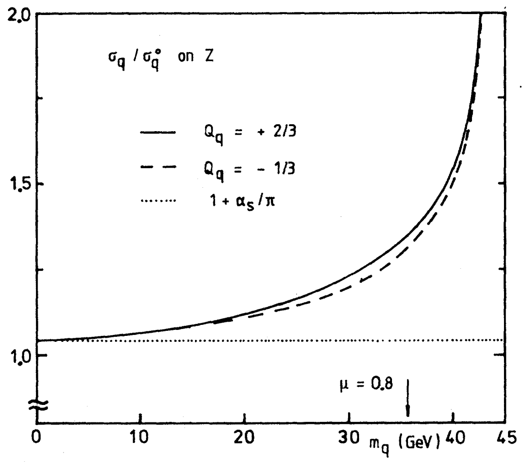

FIG. 4. QCD corrections to the quark-pair production cross section σ_q in units of the parton term. The dotted line shows the QCD correction in the massless-quark case for comparison.

$$\frac{d\sigma_i}{dx_q dx_{\bar q}} = \frac{\alpha_s}{4\pi}\,\frac{1}{(1-x_q)(1-x_{\bar q})}\left[\sigma_{VV}(S_i+\mu^2 N_i) + \sigma_{AA}(S_i+\mu^2\bar N_i)\right]_{i=U,L}$$

$$\frac{d\sigma_F}{dx_q dx_{\bar q}} = \frac{\alpha_s}{4\pi}\,\frac{1}{(1-x_q)(1-x_{\bar q})}\,\sigma_{VA}2(S_F+\mu^2 N_F). \tag{20}$$

σ'_{VA} cannot contribute to $d\sigma_F$ since the amplitudes $\mathfrak{M}(\text{current}\to q\bar q G)$ are real. The functions S_U, etc., are collected in Table I.[23] Note that the mass-dependent terms of the vector and axial-vector contributions N_i and $\bar N_i$ are different. p_q and $p_{\bar q}$ are the components of the q and $\bar q$ momenta along the flight direction of the quark, $p_\perp$ is the component of the $\bar q$ momentum perpendicular to this direction; $x_G = 2 - x_q - x_{\bar q}$ is the energy of the gluon (all quantities in units of the beam energy).

V. CROSS SECTIONS AND ANGULAR DISTRIBUTIONS

All ingredients are now available to evaluate the first-order QCD corrections to the parton cross sections and the angular distributions of the quark jets.[24] The final cross sections are found by adding the cross section for hard-gluon emission to the parton terms, corrected for soft-gluon bremsstrahlung and virtual gluon exchange, $\sigma_i^0(1+\delta_{SB})+\delta\sigma_i$. By combining Eqs. (13) and (15) with Eq. (19) this can be done for any experimental cut on the particle flow emerging from the fragmenting quarks and gluons. Here we restrict ourselves to the discussion of the hard-gluon-bremsstrahlung cross section integrated over the entire Dalitz plot (the infrared region excluded). This integration has been carried out analytically and checked numerically. Since the expressions are rather lengthy, we do not give any details. We merely present the final numerical results for all the gluon corrections and discuss their implication on the cross sections and the parameters α_q and β_q.

Cross section and α parameter

Adding the vertex correction and the gluon-bremsstrahlung contribution to the parton terms we can parametrize the QCD corrections to the transverse and longitudinal cross sections by four coefficients r_i, r'_i:

$$\sigma_U = \left(v + \frac{\alpha_s}{\pi} r_U\right)\sigma_{VV} + \left(v^3 + \frac{\alpha_s}{\pi} r'_U\right)\sigma_{AA},$$
$$\sigma_L = \left(\tfrac{1}{2}v\mu^2 + \frac{\alpha_s}{\pi} r_L\right)\sigma_{VV} + \frac{\alpha_s}{\pi} r'_L\sigma_{AA}. \tag{21}$$

The coefficients $r_{U,L}$ and $r'_{U,L}$ depend merely on

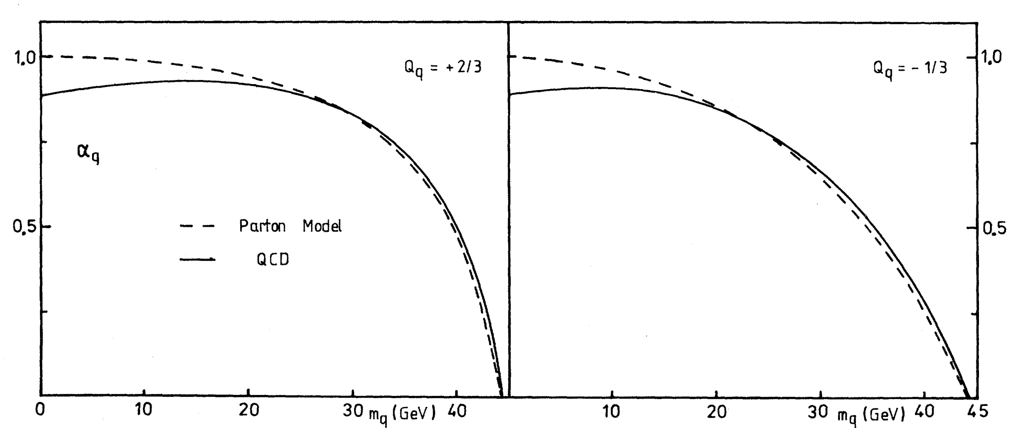

FIG. 5. QCD corrections to the α_q parameter on the Z, compared with the parton-model value.

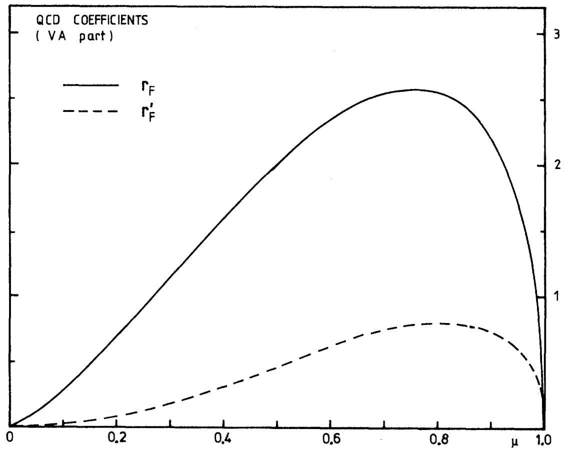

FIG. 6. Coefficients of the QCD corrections to the difference between forward and backward scattering cross sections σ_F.

the quark mass via $\mu = 2m_q/\sqrt{s}$ but they are independent of the flavor quantum numbers. Their μ dependence is shown in Figs. 3(a) and 3(b). In the high-energy limit ($\mu \to 0$) r_U and r'_U both approach $\frac{1}{3}$ while r_L and $r'_L \to \frac{2}{3}$. For $\mu \to 1$, $r_U \to 2\pi^2/3$ and $r_L \to \pi^2/3$; r'_U and r'_L vanish in this limit.

Adding the transverse to the longitudinal cross section we obtain the total cross section for the production of a quark species q,

$$\sigma(q\bar{q} + q\bar{q}G) = \left[v\left(1 + \frac{\mu^2}{2}\right) + \frac{\alpha_s}{\pi} r_{\text{tot}}\right]\sigma_{VV} + \left(v^3 + \frac{\alpha_s}{\pi} r'_{\text{tot}}\right)\sigma_{AA}, \qquad (22)$$

with $r_{\text{tot}} = r_U + r_L$, $r'_{\text{tot}} = r'_U + r'_L$. Since r_{tot} and $r'_{\text{tot}} \to 1$ in the high-energy limit, one easily recognizes the QCD correction to the total cross section for massless quarks in Eq. (1). For intermediate values of μ the coefficients are presented in Fig. 3. The result for vector currents is well known from quantum electrodynamics.[5]

Of course, Eqs. (2) and (22) cannot be valid down to the $q\bar{q}$ threshold where resonance formation invalidates perturbation theory. However, one might expect the perturbative QCD calculation to be applicable once the cross sections are smeared over sufficiently large energy bins (see, e.g., Ref. 25). Assuming the effective expansion parameter $\alpha_s/v \lesssim \frac{1}{4}$ and $\alpha_s \sim 0.15$, one derives an upper limit of $\mu \sim 0.8$ for the range of validity of the first-order correction formula. In this range of μ the gluon radiative corrections are considerably larger than the asymptotic value α_s/π. This is visualized in greater detail in Fig. 4 which shows the ratio of the QCD-corrected cross section to the parton cross section on the Z resonance as a function of the quark mass m_q. (The quark-gluon coupling constant is taken to be $\alpha_s = 12\pi/25 \ln Q^2/\Lambda^2$ with $\Lambda = 0.5$ GeV.) Figure 5, on the other hand, demonstrates that the gluon radiative corrections to the α_q parameter (which measures the strength of the longitudinal versus the transverse cross section) are small; a similar observation was made for vector currents in Ref. 6. In the high-energy limit $\mu \to 0$, the α_q parameter approaches the value $(1 - \alpha_s/\pi)/(1 + \frac{5}{3}\alpha_s/\pi) \approx 1 - \frac{8}{3}\alpha_s/\pi$ independent of the flavor quantum numbers of the quarks.

Forward-backward asymmetry

Let us now discuss the forward-backward asymmetry of (heavy-) quark jets in the process $e^+e^- \to \gamma, Z \to q + \cdots$ (for $q\bar{q}$ and $q\bar{q}G$ final states). Integrating $d\sigma_F/dx_q dx_{\bar{q}}$ over the Dalitz plot one finds that the $\ln s/m_q^2$ terms and the mass-independent terms just add up to $-\delta_R \sigma_F^0$ with the same δ_R as given by the vertex and soft-gluon corrections. Thus the difference between the forward and backward cross sections of a quark jet $\Delta_q(\vartheta)$ is only modified to the order $O(\alpha_s m_q/\sqrt{s})$ in first-order perturbation theory.[26] Higher-order corrections

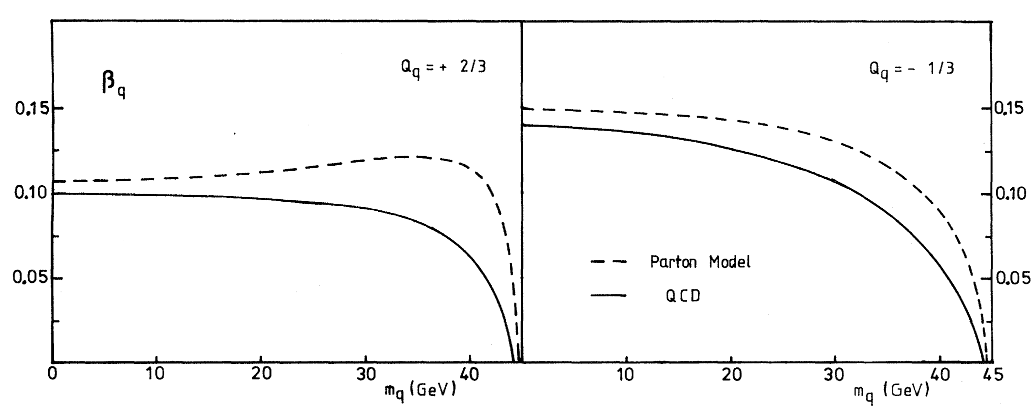

FIG. 7. QCD corrections to the asymmetry parameter β_q on the Z, compared with the parton-model value.

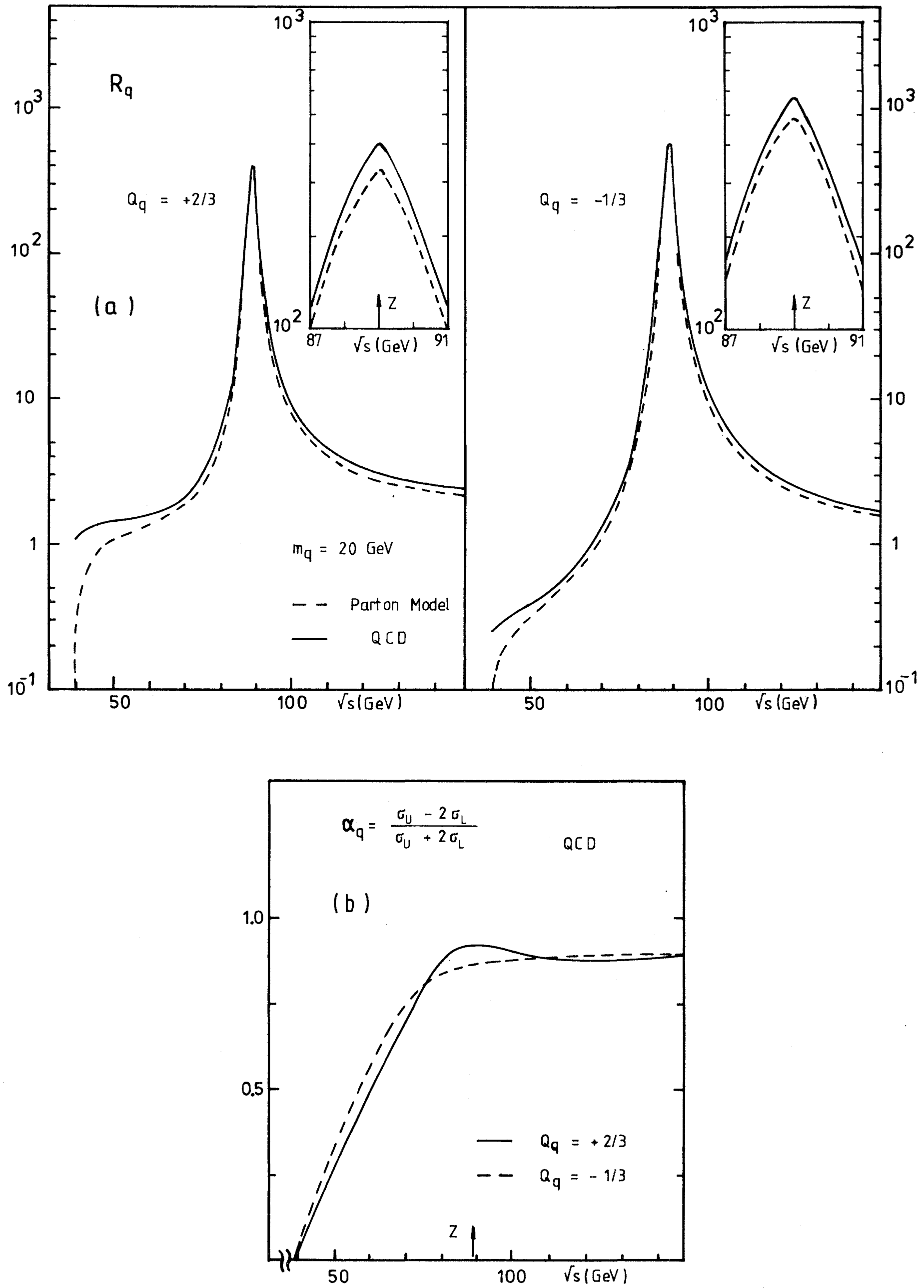

FIG. 8. The energy dependence of the parameters R_q, α_q, and β_q in (a), (b), and (c), respectively, taking full account of quark masses and QCD corrections ($m_q = 20$ GeV). Radiative QED corrections are not included.

which include light quarks should not spoil this result. Since the light-quark charges are not held fixed, all states of gluons and light quarks with degenerate energies are summed up and no large logarithms are expected to arise.[27]

Including mass effects and all QCD corrections in $\sigma_F(e^+e^- \to q\bar{q} + q\bar{q}G)$ one can parametrize σ_F as

$$\sigma_F = \left(v^2 + \frac{\alpha_s}{\pi} r_F\right)\sigma_{VA} + \frac{\alpha_s}{\pi} r'_F \sigma'_{VA}. \tag{23}$$

The coefficients r_F and r'_F depend on $\mu = 2m_q/\sqrt{s}$ only; they are shown in Fig. 6. r_F rises up to ~ 2.6 while r'_F stays below 1 over the whole μ range. As discussed before, this perturbative

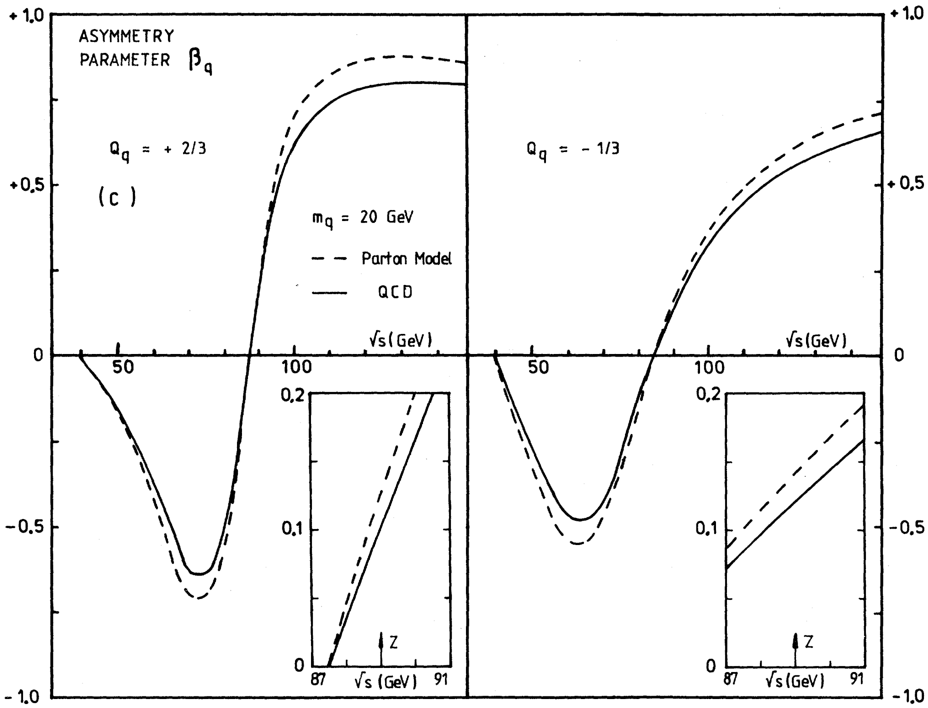

FIG. 8. *(Continued.)*

calculation is not expected to be valid at the edges of the μ interval. For μ close to 1 nonperturbative resonance production invalidates perturbation theory while for small μ ($\ln\mu^{-1} > 1$) mass singularities restrict the validity of the perturbative expansion. Even though r_F and r_F' vanish for $\mu \to 0$, these singularities should appear in higher-order calculations.[13] But in principle, the confinement mechanism could provide a cutoff which eliminates the singularities and justifies the application of the first-order result to light-quark jets, too.

In Fig. 7 we show how the asymmetry parameter β_q is affected by QCD corrections. β_q is plotted as a function of the quark mass m_q on the Z resonance. The deviations from the parton-model values are $\geqslant 20\%$ for $m_q \gtrsim 20$ GeV, and they rise with increasing mass. This is mainly due to the normalization factor $(\sigma_U + 2\sigma_L)$ in β_q [recall Eq. (5d)]; σ_F itself remains less affected by gluon corrections.[28]

The energy dependence of R_q, α_q, and β_q is finally illustrated in Fig. 8, again for quark masses of 20 GeV. Mass effects as well as QCD corrections are taken account of. These figures supplement the results presented in Fig. 2 where gluon corrections had not yet been incorporated. The QCD corrections reduce the threshold suppression in the parton model considerably.

VI. SUMMARY

We have investigated the production of quarks in e^+e^- annihilation. The total cross section and the angular distributions of the quark jets have been analyzed, including first-order gluon radiative corrections to these quantities. This was done for heavy quarks in the e^+e^- c.m. range from 30 GeV upwards throughout the Z resonance region. The measurement of these quantities is one of the means to determine the charge and the vector and axial-vector coupling constants of heavy quarks to Z.

The results can be divided into two parts. (i) If a t quark with mass $m_q \gtrsim 20$ GeV exists, mass effects in the cross section $\sigma(e^+e^- \to t\bar{t})$ will be important up to e^+e^- c.m. energies in the range of the Z mass. This is mainly due to the fact that the axial-vector part of the cross section has a strong threshold suppression $\sim v^3$, unlike the vector part which grows as v (the velocity of the quark). (ii) Gluon radiative corrections to the heavy-quark pair production cross section are much larger than the classical correction factor $\alpha_s/\pi \sim 5\%$ for massless quarks. This is due to the fact that radiative corrections grow proportionally to the inverse of the quark velocity near threshold. In particular, $t\bar{t}$ production on the Z resonance is increased by $\sim 20\%$ if the t mass is of the order of 20 GeV or above. On the other hand, the ratio of longitudinal to transverse cross sections of t quarks is less severely affected by QCD corrections. The difference between the forward and backward cross section for t-quark jets is not strongly affected by QCD corrections either. This implies, however, that the forward-backward asymmetry of these jets is subject to large corrections arising from the normalization in Eq. 2(b). These gluon corrections would be applicable to light-quark jets, too, if the problem of mass singularities (occurring in higher-order perturbative calculations when jet charges are fixed) could be solved. This, however, is related to nonperturbative aspects of QCD which are not yet under control.

ACKNOWLEDGMENT

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