Brief Reports

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SU(3) × SU(2) × U(1) content of all SU(5) representations

J. Patera
Centre de Recherche de Mathématiques Appliquées, Université de Montréal, Montréal, Canada H3C 3J7

R.T. Sharp
Department of Physics, McGill University, Montréal, Canada

A rational function of eight variables is presented. Each term of its power-series expansion has a positive-integer coefficient and corresponds precisely to one irreducible representation of SU(3) × SU(2) × U(1) contained in one irreducible representation of SU(5) with multiplicity equal to the coefficient of the term.

Attempts to unify electromagnetic and weak interactions with the strong interactions by means of a local gauge theory based on a compact simple Lie group promise to be, if successful, the most interesting development of contemporary particle physics. Virtually every model put forward involves the group-subgroup pair SU(5) ⊃ SU(3) × SU(2) × U(1) and the corresponding reduction of some representations of SU(5). We are often asked about reductions of some rather high SU(5) representations to those of SU(3) × SU(2) × U(1). In this note we want to answer such questions once and for all. Namely, the reduction of the representation $(abcd)$ of SU(5) to the subgroup is given by the polynomial coefficient of the term $A^aB^bC^cD^d$ in the power expansion of the rational function

$$\frac{1}{(1-APZ)(1-AJ)(1-BQZ^2)(1-B)(1-CZ^3)(1-CPZ)(1-DJZ^3)(1-DQZ^2)} \times \left[\frac{1}{(1-BPJZ)(1-CQJZ^2)}+\frac{ACQZ^2}{(1-CQJZ^2)(1-ACQZ^2)}+\frac{ADZ^3}{(1-ACQZ^2)(1-ADZ^3)} + \frac{BDPZ^4}{(1-ADZ^3)(1-BDPZ^4)}+\frac{B^2DP^2JZ^5}{(1-BDPZ^4)(1-BPJZ)}\right]. \tag{1}$$

Each term $mP^pQ^qJ^jZ^z$ of that coefficient means that the representation $(pq)(j)(z)$ of the subgroup appears in the reduction precisely m times. Here (pq) denotes the SU(3) representation of dimension $\frac{1}{2}(p+1)(q+1)(p+q+2)$ and (j) that of SU(2) of dimension $j+1$. The label z of the one-dimensional U(1) representations is chosen to be equal to the degree of each subgroup multiplet in the basic SU(3) triplet (10). In the defining five-dimensional SU(5) representation one has (1000) ⊃ (10)(0)(1) ⊕ (00)(1)(0). Another useful labeling of the U(1) content is by the "hypercharge" y. Then (1000) ⊃ (10)(0)(2/5) ⊕ (00)(1) $(-\frac{3}{5})$. The two labels are related by

$$y=z-\tfrac{3}{5}(a+2b+3c+4d)\ ,$$

which is equivalent to the following change of variables in (1):

$$Z \to Y,\ A \to AY^{-3/5},\ B \to BY^{-6/5},$$

$$C \to CY^{-9/5},\ D \to DY^{-12/5}.$$

The function (1) was obtained as a special case of the general prescription for writing the SU $(n) \supset \mathrm{SU}(n-2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ branching-rules generating function[1] based on the integrity basis found in Ref. 2.

For actual construction of the states with definite SU(5) and SU(3) $\times$ SU(2) $\times$ U(1) transformation properties one needs the integrity basis for the problem.[1-3] To each element of the basis corresponds one denominator factor of (1) and the compatibility rules (syzygies) are implied by the structure of the term in the square brackets.

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[1]J. Patera and R. T. Sharp, J. Phys. A 13, 397 (1980). The expression for $A_k{}^2$ in Eq. (3.12) should read $A_k{}^2 = \Lambda_k M_{k-1} M Z^{k-1}$.

[2]R. T. Sharp, J. Math. Phys. 13, 183 (1972).

[3]J. Mickelsson, J. Math. Phys. 11, 2803 (1970).