Is the quantum-chromodynamic gluon a composite object?

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It is shown that the compositeness conditions required for the equivalence of quantum chromodynamics and the corresponding four-fermion theory may be demonstrated for a large class of gauges.

A unification of interparticle forces may be brought about by reducing the number of fundamental fields in interaction. One attempt in this direction is the nonlinear spinor theory of Heisenberg[1] in which the physical fields are regarded as composites of one fundamental self-coupled spinor field. In the past two decades there have been several attempts to derive both Abelian[2] and non-Abelian[3,4] gauge fields as collective modes of a quartically self-coupled spinor field. The condition for the equivalence of these four-fermion and Yukawa-type Lagrangians, as is well known, is that the effects of the bare Yukawa field vanish. This is realized by the vanishing of certain renormalization constants of the corresponding Yukawa theory.[5] Recently, it has been claimed[6,3] that for quantum chromodynamics (QCD) this happens automatically; it is, therefore, suggested that the gluon field is nonelementary and that QCD is equivalent to a four-fermion theory with a single elementary field. In this note, we reexamine the arguments of Refs. 3 and 6 wherein it has been claimed that the compositeness conditions need only be verified in the Landau gauge $[\alpha(\mu)=0]$ since it is argued therein that the renormalized gauge parameter $\alpha(\mu)$ vanishes in the limit of an infinite ultraviolet cutoff Λ. We note, however, that although $\alpha(\mu)$ vanishes when the renormalization point μ moves to infinity, it does not vanish for finite μ even when $\Lambda\to\infty$, contrary to what has been stated by these authors. As a result, the gauge independence of the compositeness conditions cannot be concluded from the arguments of Refs. 3 and 6. Our study of the gauge dependence of the renormalization constants, however, indicates that it may indeed be possible to satisfy the compositeness conditions in a wide variety of gauges. We also remark that, in spite of this, the present proofs of equivalence of four-fermion-type and Yukawa-type theories may not be complete. Alternatively, we may recognize that two theories may be equivalent with certain prescriptions for renormalizing while being not equivalent for other equally consistent renormalization schemes.

Following Eguchi,[3] we begin with the non-Abelian four-fermion Lagrangian

$$\mathcal{L}_f=\bar{\psi}_b(i\not{\partial}-m_b)\psi_b-\tfrac{1}{2}G_b(\bar{\psi}_b\gamma_\mu\tfrac{1}{2}\vec{\lambda}\psi_b)^2\,. \quad (1)$$

Here, $\vec{\lambda}$ are the generators of the local color group. By introducing auxiliary fields $\vec{A}_\mu$, $\mathcal{L}_f$ can be equivalently written as

$$\mathcal{L}_f'=\bar{\psi}_b(i\not{\partial}-m_b)\psi_b-g_b\bar{\psi}_b\gamma_\mu\tfrac{1}{2}\vec{\lambda}\psi_b\cdot\vec{A}_{\mu b}+\tfrac{1}{2}\delta\mu^2\vec{A}_{\mu b}\cdot\vec{A}_{\mu b}\,, \quad (2)$$

with $G_b=g_b^2/\delta\mu^2$. The suffix b indicates bare quantities. The Lagrangian (2) can be formally rewritten in terms of the renormalized quantities defined by

$$\sqrt{Z_2}\,\psi=\psi_b,\quad \sqrt{Z_3}\,\vec{A}_\mu=\vec{A}_{\mu b},\quad Z_3^{-3/2}Z_1 g=g_b\,, \quad (3)$$

as

$$\begin{aligned}\mathcal{L}_{fR}&=Z_2\bar{\psi}(i\not{\partial}-m)\psi-Z_{1F}g\bar{\psi}\gamma_\mu\tfrac{1}{2}\vec{\lambda}\psi\cdot\vec{A}^\mu\\&\quad+Z_2(m-m_b)\bar{\psi}\psi+\tfrac{1}{2}Z_3\delta_\mu{}^2\vec{A}_\mu\cdot\vec{A}^\mu \quad (4)\\&=\bar{\psi}(i\not{\partial}-m)\psi-g\bar{\psi}\gamma_\mu\tfrac{1}{2}\vec{\lambda}\psi\cdot\vec{A}^\mu-\tfrac{1}{4}\vec{F}^{\mu\nu}\cdot\vec{F}_{\mu\nu}\\&\quad+[(Z_2-1)\bar{\psi}(i\not{\partial}-m)\psi+Z_2(m-m_b)\bar{\psi}\psi\\&\quad-(Z_{1F}-1)g\bar{\psi}\gamma_\mu\tfrac{1}{2}\vec{\lambda}\psi\cdot\vec{A}^\mu+\tfrac{1}{4}(\partial_\mu\vec{A}_\nu-\partial_\nu\vec{A}_\mu)^2\\&\quad+\tfrac{1}{2}g(\partial_\mu\vec{A}_\nu-\partial_\nu\vec{A}_\mu)\cdot(\vec{A}^\mu\times\vec{A}^\nu)\\&\quad+\tfrac{1}{4}g^2(\vec{A}^\mu\times\vec{A}^\nu)^2+\tfrac{1}{2}Z_3\delta\mu^2\vec{A}_\mu\cdot\vec{A}^\mu]\,. \quad (5)\end{aligned}$$

with $Z_{1F}\equiv Z_3^{-1}Z_1Z_2$ and $\vec{F}_{\mu\nu}=\partial_\mu\vec{A}_\nu-\partial_\nu\vec{A}_\mu+g\vec{A}_\mu\times\vec{A}_\nu$. The separation of terms in Eq. (5) is performed to facilitate the comparison to be made later [see Eq. (7)] with the QCD Lagrangian. We also note the mass term for the field $\vec{A}_{\mu b}$ that occurred in Eq. (2) is adjusted so as to cancel that which arises in the calculation of the self-energy of the $\vec{A}_\mu$ field. This may be regarded as a specific regularization prescription.[3] It is by this means[7] that gauge invariance is introduced into the theory. Since it is not essential for our present purpose, we do not discuss this regularization procedure any further.

The steps from Eq. (2) to Eq. (5) can also be carried out for the QCD Lagrangian

$$\mathcal{L}_Q=\bar{\psi}_b(i\not{\partial}-m_b)\psi_b-g_b\bar{\psi}_b\gamma_\mu\tfrac{1}{2}\vec{\lambda}\psi_b\cdot\vec{A}_b^\mu-\tfrac{1}{4}\vec{F}_{\mu\nu b}\cdot\vec{F}_b^{\mu\nu}\,. \quad (6)$$

Again, this can be rewritten in terms of the re-

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normalized quantities defined in Eq. (3) as

$$\mathcal{L}_{QR}=\bar{\psi}(i\not\partial-m)\psi-g\bar{\psi}\gamma_\mu\tfrac{1}{2}\vec{\lambda}\psi\cdot\vec{A}^\mu-\tfrac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu}+[(Z_2-1)\bar{\psi}(i\not\partial-m)\psi+Z_2\bar{\psi}\psi(m-m_b)-(Z_{1F}-1)g\bar{\psi}\gamma_\mu\tfrac{1}{2}\vec{\lambda}\psi\cdot\vec{A}^\mu-\tfrac{1}{4}(Z_3-1)(\partial_\mu\vec{A}_\nu-\partial_\nu\vec{A}_\mu)^2-\tfrac{1}{2}(Z_1-1)g(\partial_\mu\vec{A}_\nu-\partial_\nu\vec{A}_\mu)\cdot(\vec{A}_\mu\times\vec{A}_\nu)-\tfrac{1}{4}(Z_4-1)g^2(\vec{A}_\mu\times\vec{A}_\nu)^2]. \quad (7)$$

The renormalization constants in Eq. (7) are related by virtue of gauge invariance. They satisfy[8]

$$\frac{Z_4}{Z_1}=\frac{Z_1}{Z_3}=\frac{Z_{1F}}{Z_2}. \quad (8)$$

The separation of the terms in Eq. (7) is to be understood in the sense that the terms in the square brackets are counterterms to cancel the divergent parts of the radiative corrections that occur in perturbation theory based on the renormalized QCD Lagrangian,

$$\mathcal{L}'_{QR}=\bar{\psi}(i\not\partial-m)\psi-g\bar{\psi}\gamma_\mu\tfrac{1}{2}\vec{\lambda}\psi\cdot\vec{A}^\mu-\tfrac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu}.$$

From Eqs. (5) and (7), we see that the conditions for the Lagrangians $\mathcal{L}_{fR}$ and $\mathcal{L}_{QR}$ to be identical are

$$Z_3=0, \quad (9a)$$

$$Z_1=0, \quad (9b)$$

and

$$Z_4=0. \quad (9c)$$

We note that the conditions (9b) and (9c) do not imply there is no self-coupling of the renormalized gauge fields when calculations are carried out in perturbation theory using the renormalized QCD Lagrangian, $\mathcal{L}'_{QR}$.

We now turn our attention to whether or not Eqs. (9a)–(9c) hold for QCD. We begin with the renormalization-group solution[9] for the renormalization constants Z_i which relate the bare quantities to the corresponding quantities renormalized at the Euclidean point μ. These constants have the functional dependence $Z_i=Z_i(\Lambda/\mu,\ g_b,\ \alpha_b)$, where Λ is the ultraviolet cutoff and g_b and α_b are the bare coupling constant and gauge-fixing parameter, respectively. (It is understood that the residual dependence of the finite quantities on the cutoff is eliminated by taking the limit $\Lambda\to\infty$ in these terms.[10]) This is to be distinguished from the quantity Z_i^R which is the renormalization constant that relates the corresponding quantities renormalized at two different points μ_0 and μ and has the functional dependence $Z_i^R=Z_i^R(\mu/\mu_0, g(\mu_0), \alpha(\mu_0))$. Here, $g(\mu_0)$ and $\alpha(\mu_0)$ are, respectively, the coupling constant and gauge parameter renormalized at μ_0. The relationship between the Z_i^R's and the Z_i's defined earlier is

$$Z_i^R(\mu/\mu_0, g(\mu_0), \alpha(\mu_0))=\frac{Z_i(\Lambda/\mu, g_b, \alpha_b)}{Z_i(\Lambda/\mu_0, g_b, \alpha_b)}.$$

We note that the Z_i^R's and hence the ratio of the Z_i's evaluated at different points are independent of Λ. Defining, as usual, the quantity γ_i as

$$\gamma_i\equiv\frac{\mu}{Z_i}\frac{\partial Z_i}{\partial\mu}=-\frac{\mu}{Z_i^R}\frac{\partial Z_i^R}{\partial\mu}, \quad (10)$$

we see that the corresponding integral form is

$$\int_{Z_i(\mu_0)}^{Z_i(\mu)}\frac{dZ_i}{Z_i}=-\int_{\mu_0}^{\mu}\gamma_i(\mu')\frac{d\mu'}{\mu'}=-\int_{\mu_0}^{\mu}\gamma_i(g(\mu'),\alpha(\mu'))\frac{d\mu'}{\mu'}, \quad (11)$$

where the last equality follows from the dimensionlessness of γ_i. This can be written as

$$\int_{Z_i(\mu_0)}^{Z_i(\mu)}\frac{dZ_i}{Z_i}=-\int_{g_R(\mu_0)}^{g_R(\mu)}\frac{\gamma_i(g(\mu'),\alpha(\mu'))}{\beta(g(\mu'))}dg(\mu'), \quad (12)$$

with $\beta\equiv\mu\partial g/\partial\mu$.

Expanding γ_i and β as power series in g, we have

$$\gamma_i=\tilde{\gamma}_i(\alpha)g^2+\cdots, \quad (13a)$$

$$\beta=-bg^3+\cdots, \quad (13b)$$

with[11]

$$b=\frac{1}{16\pi^2}(\tfrac{11}{3}N_c-\tfrac{2}{3}N_f), \quad (14a)$$

$$\tilde{\gamma}_3(\alpha)=\frac{1}{16\pi^2}\left[\left(\frac{13}{6}-\frac{\alpha}{2}\right)N_c-\tfrac{2}{3}N_f\right], \quad (14b)$$

$$\tilde{\gamma}_1(\alpha)=-b+\tfrac{3}{2}\tilde{\gamma}_3(\alpha), \quad (14c)$$

$$\tilde{\gamma}_2(\alpha)=\frac{-1}{16\pi^2}\alpha N_c, \quad (14d)$$

$$\tilde{\gamma}_4(\alpha)=2\tilde{\gamma}_1(\alpha)-\tilde{\gamma}_3(\alpha), \quad (14e)$$

where N_c and N_f are, respectively, the number of colors and flavors.

The solution, Eq. (12), can be written in a particularly simple form in the Landau gauge since, in this gauge, the gauge parameter α is not renormalized, i.e., $\alpha(\mu)=0$. Then

$$Z_i^R(\mu/\mu_0)=\frac{Z_i(\Lambda/\mu)}{Z_i(\Lambda/\mu_0)}=\left[\frac{g(\mu)}{g(\mu_0)}\right]^{\tilde{\gamma}_i(0)/b}. \quad (15)$$

As mentioned previously, the ratio on the left-hand side (lhs) of Eq. (15) is independent of Λ. This is in keeping with the fact that the renormalization-group equations (RGE) can only relate quantities connected by a change of the renormalization point. In other words, the RGE do not by themselves yield complete information about Z_i.

We turn now to the study of the behavior of the solution when α is different from zero. Since the bare and renormalized gauge parameters are related by $\alpha_b = Z_3\alpha$, α satisfies the RGE,

$$\frac{\partial\alpha}{\partial\ln\mu} = \gamma_3(\alpha)\alpha(\mu)\,. \tag{16}$$

This can be readily integrated to obtain[12]

$$\alpha(\mu) = \frac{[g(\mu_0)/g(\mu)]^k(\frac{13}{6}N_c - \frac{2}{3}N_f)\alpha(\mu_0)}{\frac{13}{6}N_c - \frac{2}{3}N_f - \frac{1}{2}N_c\alpha(\mu_0) + [g(\mu_0)/g(\mu)]^k\frac{1}{2}N_c\alpha(\mu_0)}\,,$$

with

$$k \equiv \frac{\tilde{\gamma}_3(0)}{b} = \frac{\frac{13}{6}N_c - \frac{2}{3}N_f}{\frac{11}{3}N_c - \frac{2}{3}N_f}\,. \tag{17a}$$

We consider only the case when the theory exhibits asymptotic freedom ($b>0$). Then[12]

$$\lim_{\mu\to\infty}\alpha(\mu) = \begin{cases} \dfrac{13}{3} - \dfrac{4}{3}\dfrac{N_f}{N_c}, & \tilde{\gamma}_3(0)>0 \\ 0, & \tilde{\gamma}_3(0)<0\,. \end{cases} \tag{17b}$$

Since, as we shall see later, it is only when $\tilde{\gamma}_3(0)<0$ that the compositeness conditions can possibly be satisfied we do not consider the other possibility any further. The second result is then the statement of the fact that the renormalized gauge parameter vanishes when $\mu\to\infty$ if $N_f > \frac{13}{4}N_c$. This is not to say that $\alpha(\mu)$ vanishes when the ultraviolet cutoff Λ goes to infinity as stated in Refs. 3 and 6.

By rewriting the solution to Eq. (16) in the form

$$\frac{\alpha}{\tilde{\gamma}_3(\alpha)} = \frac{\alpha_0}{\tilde{\gamma}_3(\alpha_0)}\left[\frac{g(\mu_0)}{g(\mu)}\right]^k \tag{17c}$$

[with $\alpha\equiv\alpha(\mu)$ and $\alpha_0\equiv\alpha(\mu_0)$], we see that the map (17c) admits one attractive and one repulsive fixed point. These are given by $\alpha=0$ and $\tilde{\gamma}_3(\alpha=\alpha_c)=0$, i.e., $\alpha_c = 32\pi^2\tilde{\gamma}_3(0)/N_c$, respectively. Any $\alpha>\alpha_c$ is obviously in the domain of attraction of the Landau gauge. On the other hand, $\alpha<\alpha_c$ is repelled therefrom. Although it appears from Eq. (17c) that in the limit $\mu\to\infty$, $\alpha(\mu)$ approaches the fixed point $\alpha=0$ through positive values[13] even for $\alpha_0 < \alpha_c$, we note that this would require a change of sign of Z_3, which is unacceptable. In what follows, we confine ourselves to $\alpha>\alpha_c$.

We now turn our attention to the study of the renormalization constants. From Eqs. (11) and (17a), it can be easily shown that

$$Z_3\left(\frac{\Lambda}{\mu}\right) = Z_3\left(\frac{\Lambda}{\mu_0}\right)\left\{\frac{1+\Delta}{1+\Delta[g(\mu_0)/g(\mu)]^k}\right\}\,, \tag{18a}$$

with $\Delta \equiv 32\pi^2\tilde{\gamma}_3(\alpha)/N_c\alpha$. The renormalization-group solutions to Z_1 and Z_4 can also be readily obtained from Eqs. (12) and (14). We find

$$Z_1\left(\frac{\Lambda}{\mu}\right) = Z_1\left(\frac{\Lambda}{\mu_0}\right)\left[\frac{Z_3(\Lambda/\mu)}{Z_3(\Lambda/\mu_0)}\right]^{3/2}\frac{g(\mu_0)}{g(\mu)} \tag{18b}$$

and

$$Z_4\left(\frac{\Lambda}{\mu}\right) = Z_4\left(\frac{\Lambda}{\mu_0}\right)\left[\frac{Z_3(\Lambda/\mu)}{Z_3(\Lambda/\mu_0)}\right]^2\left[\frac{g(\mu_0)}{g(\mu)}\right]^2. \tag{18c}$$

The quantities $Z_i(\Lambda/\mu)$, being independent of μ_0, can be conveniently evaluated by setting $\mu_0=\Lambda$. For instance, for asymptotically free theories with $N_f>\frac{13}{4}N_c$ (so that $k<0$), Eq. (18a) can be written, in the limit $\Lambda\to\infty$, as

$$Z_3\left(\frac{\Lambda}{\mu}\right) \cong Z_3(1)\left(1+\frac{1}{\Delta}\right)\left[\frac{g(\mu)}{g(\Lambda)}\right]^k. \tag{19a}$$

Since in the Landau gauge Δ diverges, we get

$$Z_3\left(\frac{\Lambda}{\mu}\right) = Z_3(1)\left[\frac{g(\mu)}{g(\Lambda)}\right]^k. \tag{19b}$$

In fact, this is the form of the solution presented in Ref. 6. From Eq. (19) it may be argued that for asymptotically free theories with $N_f>\frac{13}{4}N_c$, the right-hand side, and hence $Z_3(\Lambda/\mu)$, vanishes when $\Lambda\to\infty$. We note that this condition crucially depends on the finiteness of the undetermined constant $Z_3(1)$. Moreover, it is not entirely clear that the renormalization-group result is valid when $\Lambda/\mu\to 1$. It is amusing to note that if the Z_i's ($i=1,3,4$) can also be independently argued to be finite for values of $\mu\ll\Lambda$ (so that the RGE are valid) it follows from Eqs. (18) that the condition $N_f > \frac{13}{4}N_c$ is both necessary and sufficient to ensure the validity of the compositeness conditions. This is a consequence of the fact that $\tilde{\gamma}_3<0$ ensures $\tilde{\gamma}_1<0$ and $\tilde{\gamma}_4<0$. [See Eqs. (14).]

Although, as has already been pointed out, the RGE cannot fix the magnitudes of $Z_i(\Lambda/\mu)$, there are heuristic arguments based on perturbation theory which suggest that all renormalization constants remain finite in the limit of the renormalization point moving to infinity. The relevant parameter for the expansion is

$$x \equiv g^2(\mu)\ln\frac{\Lambda}{\mu}\,. \tag{20a}$$

The running coupling $g(\mu)$ has been computed using the RGE which are valid when

$$\frac{\Lambda}{\mu}\to\infty\,. \tag{20b}$$

Then[11] $g^2(\mu) = 1/2b\ln(\mu/M)$. Here, M is the scale at which the running coupling diverges. If we consider μ to be increased along with Λ so that Eqs. (20a) and (20b) are valid when $\Lambda\to\infty$, it may be possible to argue that the perturbation expansion for Z_i converges and, at the same time, the renormalization-group solution remains valid. A possible realization of this is to consider that $\mu \to \infty$ so that

$$\ln\frac{\mu}{M} = \ln\frac{\Lambda}{M} - \left(\ln\frac{\Lambda}{M}\right)^c , \quad 0 < c < 1 . \tag{20c}$$

Then, as $\Lambda \to \infty$, $x \sim (\ln\Lambda)^{1-c} \to 0$ and $\ln(\Lambda/\mu) = [\ln(\Lambda/M)]^c \to \infty$. The fact that $x \to 0$ in the limit considered suggests that in that case $Z_i(\Lambda/\mu) \to 1$. Admittedly, we have arrived at this conclusion for a rather unphysical choice of μ, in that we do not take the cutoff to infinity before doing so for μ. To the extent that Eqs. (18) and (19) are valid in this unconventional limit, it may be possible to argue that

$$\lim_{\Lambda \to \infty} Z_i\left(\frac{\Lambda}{\mu}\right) = 0 \text{ for finite } \mu \; (i = 1, 3, 4) .$$

This corresponds to the satisfaction of the compositeness conditions in all gauges with $\alpha > \alpha_c$.

To complete our study of the gauge dependence of the renormalization constants, we present the renormalization-group solution for Z_2. From Eqs. (12)–(14), and (17a) we find

$$Z_2\left(\frac{\Lambda}{\mu}\right) = Z_2\left(\frac{\Lambda}{\mu_0}\right)\left\{\frac{1+\Delta}{[g(\mu)/g(\mu_0)]^k + \Delta}\right\} . \tag{21}$$

We note that unlike the other Z_i's, Z_2 is not driven to zero when $\mu \to \infty$. In fact, if $Z_2(\Lambda/\mu)$ is finite and nonzero for any μ, it is so always.

The upshot of our analysis is that although we obtain the same conclusions as the authors of Ref. 6, we explicitly see from Eqs. (18), (19), and (21) that the renormalization constants are not the same as they would have been in the Landau gauge. We note also that since the proof of the compositeness conditions entailed a limit $\mu \to \infty$, and since, as pointed out earlier, our solution for $\alpha(\mu)$ was not valid in this limit for $\alpha_0 < \alpha_c$, our demonstration of the compositeness conditions excludes these gauges. This is not to say that the compositeness conditions do not hold, but merely that our renormalization-group calculation does not enable us to draw any definite conclusion.

At this state it is worth pointing out that even if the ambiguities in the demonstration of Eqs. (9a)–(9c) are ignored, the compositeness conditions, by themselves, may not be sufficient to ensure complete equivalence of the four-fermion and Yukawa-type theories. The study of the corresponding situation for the Lee model (Yukawa-type theory) and the separable-potential model (four-point contact interaction) is an explicity illustration of this point.[14] It is explicitly demonstrated that the compositeness condition ($Z = 0$) by itself is not sufficient to ensure the equivalence of the two theories. It is found that when a cutoff is introduced, the spectrum of the Lee model has, in the strong-coupling limit, an additional bound state beyond the cutoff which moves to infinity in the strong-coupling limit. There is no corresponding state for the separable potential model. It is shown that the equivalence follows if, in addition to the compositeness condition, the spectral contributions at infinity (in the Lee model) are ignored; then the Lee model is transmuted into the separable-potential model. The role of this transmutation mechanism in equivalence proofs in relativistic field theories merits investigation.

Since our study of equivalence is focused on the renormalized theories, inevitably a certain protocol for taking limits in the computations is understood. We are, therefore, not referring to a specific Hamiltonian or to a specific behavior at infinity but rather considering a family of Hamiltonians. Since we have two standard parameters, the cutoff Λ, and the renormalization point μ, the notion $\mu \to \infty$ is not uniquely defined. Our study indicates that for a wide class of protocols for such a limiting procedure, the QCD Lagrangian and the four-fermion Lagrangian lead to the same renormalized theory for a wide range of gauge parameters. For other protocols they may or may not be equivalent. While this is known to many authors, we felt it relevant to make our position as clear as possible.

In summary, we have shown that for QCD with $N_f > \frac{13}{4} N_c$, it is possible, subject to the assumptions stated in the text, to explicitly demonstrate the compositeness conditions in a wide variety of gauges. Although this suggests that the gluon field may indeed by nonelementary, it seems fair to point out that if the transmutation discussed in Ref. 14 plays a role in relativistic field theories, the present criteria for equivalence may be inadequate. Subject to the validity of the transmutation mechanism, however, the four-fermion and Yukawa-type theories may indeed be effectively equivalent.

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[8]See, for example, C. Itzykson and J. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980), p. 594.

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