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Ward identities in a general axial gauge. I. Yang-Mills theory

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Ward identities are derived in a general axial gauge by considering three distinct expressions for the gauge-breaking part of the Yang-Mills Lagrangian. The Ward identities are verified by calculating the one-loop self-energies in the appropriate axial gauge. It is shown, in particular, that one of the three gauge-breaking terms, namely $(2\alpha n^2)^{-1}(n\cdot A^a)\partial^2(n\cdot A^a)$, gives rise to a self-energy which is nontransverse. The latter gauge includes the planar gauge ($\alpha=-1$). The effect of the general axial gauge on the Yang-Mills counterterms is analyzed and the implications for quantum gravity are briefly discussed.

I. INTRODUCTION

One of the important concepts in modern field theory is the powerful notion of gauge invariance which played such a crucial role in the formulation of the Weinberg-Salam theory and which continues to be an invaluable tool in many theoretical studies, including quark confinement and supergravity. The implication of gauge invariance of a certain Lagrangian may be expressed mathematically in a variety of ways. In the context of functional integrals, for example, discussions on gauge invariance usually start from a generating functional which has to remain unaltered under a gauge transformation of the appropriate fields. This observation leads immediately to certain relations among Green's functions, called Ward identities, which express the gauge symmetry of the theory and constitute a powerful tool in the renormalization program.

Manifest gauge invariance demands *ipso facto* the introduction of a *gauge condition.* The latter imposes additional constraints on the field variables and ensures that propagators and other Green's functions are well defined. There is, of course, considerable freedom in the choice of gauge condition: it may be linear, such as the Coulomb gauge, or nonlinear; it may be covariant, like the Feynman and Landau gauges, or even noncovariant such as the axial gauge. However, regardless of the type of gauge chosen, it is mandatory (anomalies excepted) that the Green's functions of the theory respect the corresponding Ward identities and, thereby, the gauge symmetry of the original Lagrangian.

The purpose of this article is to demonstrate that the general axial gauge constitutes a perfectly self-consistent gauge. This conclusion holds not only for theories of the Yang-Mills type, but also for quantum gravity which we discuss in the sequel to this paper, henceforth referred to as paper II.

During the past few years the axial gauge has been used in many calculations involving the Yang-Mills Lagrangian,[1-5] despite the fact that the bare propagator in this gauge is substantially more complicated than in a covariant gauge, such as the Feynman gauge. The reason for this popularity is the absence of fictitious particles which lead to Green's functions that satisfy simple Ward identities. We shall show, however, that this simplicity of the Ward identities does not persist for certain generalizations of the axial gauge. It is worth pointing out here that, strictly speaking, these generalizations are *not* ghost free, since the fictitious particles do not decouple in the action. However, as ghost loops vanish by dimensional regularization, we can safely ignore them in the framework of this calculation.

The motivation behind the present Yang-Mills calculation was to help us understand a rather surprising result, obtained recently in quantum gravity: the nontransversality of the one-loop self-energy in the axial gauge.[6] Since we also obtained similar results in the Yang-Mills case, we decided for the benefit of those chiefly interested in applications to QCD to discuss the Yang-Mills computations in a separate paper.

The outline of the present paper is as follows. In Sec. II, we introduce three specific axial gauges and evaluate, with the help of the appropriate propagators, the various self-energies. We begin Sec. III with a derivation of the crucial Ward identities and then show that these are, indeed, satisfied by the respective self-energies. A complete list of integrals,[7] including those required for the evaluation of the Yang-Mills self-energy diagram, is given in the Appendix of paper II.[6]

II. THE GENERAL AXIAL GAUGE

In the axial gauge one imposes the condition

$$n_\mu A^a_\mu = 0\ , \quad n^2 \neq 0\ , \tag{2.1}$$

where n_μ is an arbitrary but constant vector, on the Yang-Mills Lagrangian[8]

$$\mathscr{L} = -\tfrac{1}{4}(F^a_{\mu\nu})^2\ ; \tag{2.2}$$

$F^a_{\mu\nu}$ is given by

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g c^{abc} A^b_\mu A^c_\nu\ , \tag{2.3}$$

where c^{abc} denotes the appropriate group structure constant and g the bare coupling constant. Condition (2.1) leads to the following *bare* propagator in momentum space:

$$G^{ab}_{\mu\nu}(p) = \frac{\delta^{ab}}{i(p^2+i\epsilon)(2\pi)^{2\omega}}\left[\delta_{\mu\nu} - \frac{(p_\mu n_\nu + p_\nu n_\mu)}{p\cdot n} + \frac{p_\mu p_\nu n^2}{(p\cdot n)^2}\right]\ , \tag{2.4}$$

which exhibits unphysical singularities at $p\cdot n = 0$. The latter may be circumvented, in the associated Feynman integrals, by means of the *principal-value prescription* which consists of the replacements[9]

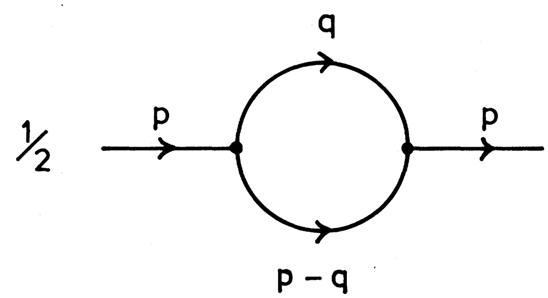

FIG. 1. The one-loop self-energy in the axial gauge. (In this diagram, and also in Fig. 2, all lines correspond to Yang-Mills fields.)

$$\frac{1}{p\cdot n} \to P\frac{1}{p\cdot n} = \tfrac{1}{2}\lim_{\eta\to 0}\left[\frac{1}{p\cdot n + i\eta} + \frac{1}{p\cdot n - i\eta}\right]$$

and (2.5)

$$\frac{1}{(p\cdot n)^2} \to P\frac{1}{(p\cdot n)^2} = \tfrac{1}{2}\lim_{\eta\to 0}\left[\frac{1}{(p\cdot n + i\eta)^2} + \frac{1}{(p\cdot n - i\eta)^2}\right]\ . \tag{2.6}$$

Our aim is to examine, in a general axial gauge, the *infinite* part of the one-loop self-energy diagram, shown in Fig. 1. This diagram has been evaluated by several authors for the special case $\alpha = 0$ and reads (in our notation)

$$\Pi^{ab}_{\mu\nu}(p) = -\tfrac{11}{3} g^2 C_{\rm YM}(p_\mu p_\nu - p^2\delta_{\mu\nu})\bar{I}\ , \tag{2.7}$$

$$c^{acd}c^{bcd} = C_{\rm YM}\delta^{ab}\ ; \tag{2.8}$$

$\bar{I}$, the divergent portion of the basic one-loop integral, is given by

$$\bar{I} \equiv \text{divergent part of} \int \frac{d^{2\omega}q}{q^2(q-p)^2} \tag{2.9}$$

$$= \text{pole part of}\left[\frac{i(-\pi)^\omega\Gamma(2-\omega)[\Gamma(\omega-1)]^2}{(p^2)^{2-\omega}\Gamma(2\omega-2)}\right] = \frac{-i\pi^2}{\omega-2}\ . \tag{2.10}$$

Condition (2.1) belongs to a general class of axial gauges, all of which turn out to be ghost free.[4,10] One of these gauges, known as the planar gauge,[11] has already been used extensively in QCD. The *three* axial gauges considered in this paper may be generated by adding to the Lagrangian density a gauge-breaking term of the type

$$\mathscr{L}_B = \frac{1}{2\alpha} n_\mu A^a_\mu f n_\nu A^a_\nu\ , \tag{2.11}$$

where α is a numerical parameter and f can have the form

$$\text{(A)}\ f = -1\ , \tag{2.12}$$

$$\text{(B)}\quad f=(n\cdot\partial)^2/(n^2)^2\,, \tag{2.13}$$

$$\text{(C)}\quad f=\partial^2/n^2\,. \tag{2.14}$$

For $\alpha\to 0$, each of these cases reduces to the familiar axial gauge (2.1) with propagator (2.4). For general values of α, however, Eqs. (2.12)–(2.14) lead to bare propagators of the form

$$\text{(A)}\quad G^{ab}_{\mu\nu}(p)=\frac{\delta^{ab}}{i(p^2+i\epsilon)(2\pi)^{2\omega}}\left[\delta_{\mu\nu}-\frac{(p_\mu n_\nu+p_\nu n_\mu)}{p\cdot n}+\frac{p_\mu p_\nu n^2}{(p\cdot n)^2}+\frac{\alpha p_\mu p_\nu p^2}{(p\cdot n)^2}\right], \tag{2.15}$$

$$\text{(B)}\quad G^{ab}_{\mu\nu}(p)=\frac{\delta^{ab}}{i(p^2+i\epsilon)(2\pi)^{2\omega}}\left[\delta_{\mu\nu}-\frac{(p_\mu n_\nu+p_\nu n_\mu)}{p\cdot n}+\frac{p_\mu p_\nu n^2}{(p\cdot n)^2}+\frac{\alpha p_\mu p_\nu p^2(n^2)^2}{(p\cdot n)^4}\right], \tag{2.16}$$

$$\text{(C)}\quad G^{ab}_{\mu\nu}(p)=\frac{\delta^{ab}}{i(p^2+i\epsilon)(2\pi)^{2\omega}}\left[\delta_{\mu\nu}-\frac{(p_\mu n_\nu+p_\nu n_\mu)}{p\cdot n}+\frac{p_\mu p_\nu n^2(1+\alpha)}{(p\cdot n)^2}\right]. \tag{2.17}$$

The next task is to calculate for each of the above propagators, the corresponding expressions for the one-loop self-energy which one would expect to be proportional to either one or both of the following transverse tensors:

$$P_{\mu\nu}\equiv p_\mu p_\nu-p^2\delta_{\mu\nu}\,, \tag{2.18a}$$

$$N_{\mu\nu}\equiv(p\cdot np_\mu-p^2n_\mu)(p\cdot np_\nu-p^2n_\nu)\,. \tag{2.18b}$$

Computation of the divergent part of the self-energy for the three cases is more challenging than for the $\alpha=0$ gauge. Since the former is the prototype of the formidable graviton calculations in paper II, we shall outline the overall strategy in some detail. The Yang-Mills vertex is the same in all three instances and is given by

$$V^{abc}_{\mu\nu\lambda}(p_1,p_2,p_3)=(2\pi)^{2\omega}gc^{abc}\left[\delta_{\mu\nu}(p_2-p_1)_\lambda+\delta_{\nu\lambda}(p_3-p_2)_\mu+\delta_{\lambda\mu}(p_1-p_3)_\nu\right].$$

Although it is possible to do the calculations by hand, it is certainly faster and more reliable to use an algebraic computer program. We used Veltman's SCHOONSCHIP program.[12] The calculation was carried out in the following stages. (i) The number of Feynman integrals required is reduced considerably by working entirely with integrands which only involve scalars. With this in mind, it is more expedient to evaluate the four scalar amplitudes $\Pi^{ab}_{\mu\mu}$, $p_\mu p_\nu\Pi^{ab}_{\mu\nu}$, $p_\mu n_\nu\Pi^{ab}_{\mu\nu}$, and $n_\mu n_\nu\Pi^{ab}_{\mu\nu}$ rather than $\Pi^{ab}_{\mu\nu}$ directly. It is also advisable, in order to minimize the number of different terms, to keep the number of Lorentz indices to a minimum. Working out $n_\mu n_\nu\Pi^{ab}_{\mu\nu}$, for example, we find it easier to start with $n_\mu V^{aef}_{\mu\alpha\beta}$ and then to multiply the resulting expression by the propagators and the remaining vertex (of the form $n_\nu V^{be'f'}_{\nu\alpha'\beta'}$), one at a time. These techniques are of crucial importance in paper II: without them, even a large computer is unable to multiply together the complete axial-gauge graviton propagators and vertices. (ii) The integrals may be simplified with the help of the identity

$$\frac{1}{q\cdot n(q-p)\cdot n}=\frac{1}{p\cdot n}\left[\frac{1}{(q-p)\cdot n}-\frac{1}{q\cdot n}\right],$$

which reduces the maximum number of factors in the denominators of the integrands from four to three (see Ref. 3). (iii) Finally, we exploit the freedom in dimensional regularization[13] which permits us to carry out momentum shifts, such as $q_\mu\to-q_\mu+p_\mu$, in the integration variable. Consequently we arrive at a certain set of integrals which are evaluated in the Appendix of paper II. Apart from massless tadpole integrals, which are zero anyway, the nonzero integrals are of the form

$$X(i,j)\equiv\int\frac{d^{2\omega}q\,(q^2)^i}{(p-q)^2(q\cdot n)^j}\,,\quad i,j=\text{integers}\,, \tag{2.19a}$$

and

$$Y(i)\equiv\int\frac{d^{2\omega}q\,(q\cdot n)^i}{q^2(p-q)^2}\,,\quad i=\text{integer}\,. \tag{2.19b}$$

Applying the procedure outlined above, we obtain the following results for the infinite components of the various self-energies:

$$\text{(A)}\quad \Pi^{ab}_{\mu\nu}(p)=C_{\rm YM}\delta^{ab}g^2\left[-\tfrac{11}{3}P_{\mu\nu}+\frac{4\alpha p^2}{3n^2}P_{\mu\nu}+\frac{4\alpha}{3(n^2)^2}N_{\mu\nu}\right]\bar{I}\ , \tag{2.20}$$

$$\text{(B)}\quad \Pi^{ab}_{\mu\nu}(p)=-\tfrac{11}{3}C_{\rm YM}\delta^{ab}g^2P_{\mu\nu}\bar{I}\ , \tag{2.21}$$

$$\text{(C)}\quad \Pi^{ab}_{\mu\nu}(p)=C_{\rm YM}\delta^{ab}g^2\left[-\tfrac{11}{3}P_{\mu\nu}-2\alpha P_{\mu\nu}-\frac{2\alpha}{n^2}[2p^2n_\mu n_\nu-p\cdot n\,(p_\mu n_\nu+p_\nu n_\mu)]\right]\bar{I}\ . \tag{2.22}$$

It is clear that all three expressions (2.20) to (2.22) reduce, for $\alpha\to 0$, to the basic self-energy (2.7).

We now consider Eqs. (2.20) to (2.22) in more detail. The structure of $\Pi^{ab}_{\mu\nu}$ in case A is essentially what one would expect. Since the α-dependent term in Eq. (2.15) is of order $O(1)$ in p^2, rather than $O(1/p^2)$, we anticipate, from power-counting arguments, that both $N_{\mu\nu}$ and $P_{\mu\nu}$ will appear in the infinite part of the self-energy. Accordingly, the counterterm in this gauge will be n_μ dependent as well as α dependent. We, therefore, conclude that this particular generalization of the axial gauge is not likely to be very convenient. The answer for case B is also reasonable: since the propagator (2.16) goes like $1/p^2$, power counting implies a contribution to $P_{\mu\nu}$ only. It is reassuring that the counterterm is α independent and independent of the noncovariant vector n_μ.

By contrast, the expression for $\Pi^{ab}_{\mu\nu}$ in Eq. (2.22) looks almost embarrassing: not only does it depend on n_μ and α, but it is also *not* transverse. One's immediate reaction to this result might well be to suspect either a computational error or a breakdown in the principal-value prescription. However, repeated checks on the calculations only confirm the correctness of this expression. We shall prove in the next section that this unorthodox result, as well as the other expressions in Eqs. (2.20) and (2.21), follow quite naturally from a careful derivation of the Ward identities.[14] Suffice it to say here that the choice $\alpha=-1$ in case C [see Eqs. (2.17) and (2.22)], leads to the planar gauge used frequently in QCD. Unfortunately, as mentioned in Ref. 15, the one-loop counterterm in this gauge is both n_μ dependent and nontransverse. We shall return to this point in the conclusion.

III. WARD IDENTITIES FOR THE GENERAL AXIAL GAUGE

It was suggested in the previous section that the structure of the self-energy in the general axial gauge could be explained by a careful analysis of the Ward identities. We shall now derive these identities. The gauge transformation

$$\delta A^a_\mu=\partial_\mu\Lambda^a+gc^{abc}A^b_\mu\Lambda^c \tag{3.1}$$

on the generating functional for Green's functions,

$$Z[j^a_\mu]=\int d[A^a_\nu]\exp\left\{i\int dz[-\tfrac{1}{4}(F^a_{\mu\nu})^2+\frac{1}{2\alpha}n\cdot A^a fn\cdot A^a+j^a_\mu A^a_\mu]\right\}, \tag{3.2}$$

gives

$$\int d[A^a_\nu][-\delta^{ac}\partial^x_\mu+gc^{abc}A^b_\mu(x)][\alpha^{-1}n_\mu fn\cdot A^a(x)+j^a_\mu(x)]\times\exp\left\{i\int dz[-\tfrac{1}{4}(F^a_{\mu\nu})^2+(2\alpha)^{-1}n\cdot A^a fn\cdot A^a+j^a_\mu A^a_\mu]\right\}=0\ . \tag{3.3}$$

As is well known, functional differentiation of Eq. (3.3) with respect to the external current $j^e_\nu(y)$ yields the required Ward identities. It is usually convenient to consider, instead of $Z[j^a_\mu]$ in Eq. (3.2), the generating functional for proper vertices. In our case it turns out that the results of the previous section can be readily understood in terms of the generating functional for Green's functions. Thus, operating with $\delta/\delta j^e_\beta(y)$ on Eq. (3.3) and then setting $j^e_\beta=0$, we obtain the identity

$$\int d[A_\nu^a][-i\alpha^{-1}A_\beta^e(y)n\cdot\partial fn\cdot A^c(x)+ig\alpha^{-1}c^{abc}A_\beta^e(y)n\cdot A^b(x)fn\cdot A^a(x) -\delta^{ec}\partial_\beta^x\delta(x-y)+gc^{ebc}A_\beta^b(x)\delta(x-y)] \times\exp\left\{i\int dz[-\tfrac{1}{4}(F_{\mu\nu}^a)^2+(2\alpha)^{-1}n\cdot A^a fn\cdot A^a]\right\}=0\ . \quad (3.4)$$

The fourth term in Eq. (3.4) can be discarded, since it corresponds to a massless tadpole which vanishes in the context of dimensional regularization.[16] Accordingly, we get the Ward identity

$$-\frac{i}{\alpha}\langle T[A_\beta^e(y)n\cdot\partial fn\cdot A^c(x)]\rangle+\frac{i}{\alpha}gc^{abc}\langle T[A_\beta^e(y)n\cdot A^b(x)fn\cdot A^a(x)]\rangle-\delta^{ec}\partial_\beta^x\delta(x-y)=0\ , \quad (3.5)$$

or, in momentum space,

$$-\frac{p\cdot n}{\alpha}\bar{f}n_\lambda D_{\lambda\beta}^{ce}(p)+\frac{i}{\alpha}gc^{abc}W_\beta^{eba}(p)+\frac{i\delta^{ec}}{(2\pi)^{2\omega}}p_\beta=0\ , \quad (3.6)$$

where

$$\text{(A)}\ \bar{f}=-1\ ,\ \text{or} \quad (3.7)$$

$$\text{(B)}\ \bar{f}=-(p\cdot n)^2/(n^2)^2\ ,\ \text{or} \quad (3.8)$$

$$\text{(C)}\ \bar{f}=-p^2/n^2\ , \quad (3.9)$$

and

$$\langle T[A_\mu^a(y)A_\nu^b(x)]\rangle=\int d^{2\omega}p\,e^{ip\cdot(y-x)}D_{\mu\nu}^{ab}(p)\ , \quad (3.10)$$

$$n^2\langle T[A_\beta^e(y)n\cdot A^b(x)fn\cdot A^a(x)]\rangle=\int d^{2\omega}p\,e^{ip\cdot(y-x)}W_\beta^{eba}(p)\ . \quad (3.11)$$

Since the second term in Eq. (3.5) does not contribute to lowest order (no loops), $D_{\lambda\beta}^{ce}(p)$ reduces to the bare propagator $G_{\lambda\beta}^{ce}(p)$; hence we find, from Eqs. (2.15) to (2.17), that

$$\text{(A)}\ n_\lambda G_{\lambda\beta}^{ce}(p)=\frac{\alpha\delta^{ce}}{(2\pi)^{2\omega}ip\cdot n}p_\beta\ , \quad (3.12)$$

$$\text{(B)}\ n_\lambda G_{\lambda\beta}^{ce}(p)=\frac{\alpha\delta^{ce}(n^2)^2}{(2\pi)^{2\omega}i(p\cdot n)^3}p_\beta\ , \quad (3.13)$$

$$\text{(C)}\ n_\lambda G_{\lambda\beta}^{ce}(p)=\frac{\alpha\delta^{ce}n^2}{(2\pi)^{2\omega}ip^2p\cdot n}p_\beta\ . \quad (3.14)$$

It is easy to check that all three $n_\lambda G_{\lambda\beta}^{ce}$ terms satisfy Eq. (3.6) to lowest order.

Let us next examine the one-loop contribution to the Ward identity (3.6). Multiplying Eq. (3.6) by the bare inverse propagator $(G_{\mu\nu}^{ab})^{-1}$ and considering only the one-loop contributions, we arrive at the identity

$$p_\beta\Pi_{\beta\gamma}^{cf}(p)=\frac{(2\pi)^{2\omega}}{\alpha n^2}gc^{abc}F_\gamma^{fba}(p)\ ; \quad (3.15)$$

F_γ^{fba} is the amputated one-loop contribution to $W_\gamma^{fba}(p)$, shown in the "pincer" diagram of Fig. 2, namely,

$$W_\beta^{eba}(p)=G_{\beta\gamma}^{ef}(p)F_\gamma^{fba}(p)\ . \quad (3.16)$$

Concerning the derivation of Eq. (3.15) from Eq. (3.6), the following two points should be stressed. First, contracting a Green's function with n_μ is, to within a factor, equivalent to contracting a momentum p_μ with the proper vertex correction. Secondly, the explicit appearance of α^{-1} in Eq. (3.6) is rather misleading, since

$$\langle T[A_\beta^e(y)n\cdot A^b(x)fn\cdot A^c(x)]\rangle$$

is itself proportional to α^2. This implies that the term $\alpha^{-1}igc^{abc}W_\beta^{eba}(p)$ is at least of order $O(\alpha)$; on the other hand, since $n_\lambda D_{\lambda\beta}^{ce}(p)$ is of order $O(\alpha)$, the first term in Eq. (3.6) is $O(1)$. Accordingly, replacement of the two explicit α's in (3.6) by $\alpha=0$ does *not* yield Ward identities of order $O(1)$ in α.

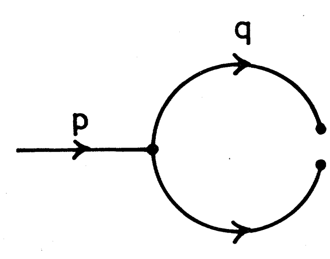

FIG. 2. The "pincer" diagram for the one-loop contribution to $F_\gamma^{fba}(p)$.

It is clear from Eq. (3.15) that the self-energy $\Pi^{cf}_{\beta\gamma}$ will be transverse only in the event that F^{fba}_{γ} vanishes identically. So the obvious question is, what does F^{fba}_{γ} really look like? To answer this question we note first that F^{fba}_{γ} depends implicitly on the operators f appearing in the gauge-breaking term (2.11). To obtain F^{fba}_{γ}, therefore, we must analyze separately the effect of the three f's in Eqs. (2.12) to (2.14).

Case A. $f=-1$. F^{fba}_{γ} vanishes identically on account of the antisymmetric nature of the structure constants c^{abc}. Consequently, the self-energy in Eq. (2.20) is transverse.

Case B. $f=(n\cdot\partial)^2/(n^2)^2$. F^{fba}_{γ} vanishes again, but for a different reason. This time the associated integrals are zero by dimensional regularization (they reduce to tadpole integrals). Hence the self-energy is transverse, as seen from Eq. (2.21).

Case C. $f=\partial^2/n^2$. An explicit computation reveals that the infinite portion of F^{fba}_{γ} is given by

$$F^{fba}_{\gamma}(p)=\frac{2ig\pi^2\alpha^2c^{bfa}p^2p\cdot n}{(2\pi)^4(\omega-2)}\left[n_\gamma-p_\gamma\frac{p\cdot n}{p^2}\right], \tag{3.17}$$

so that the self-energy $\Pi^{ab}_{\mu\nu}$ in (2.22) will *not* be transverse for general α, despite the fact that Eqs. (2.22) and (3.17) satisfy the identity (3.15).

It is interesting to ask what happens to $\Pi^{ab}_{\mu\nu}$ in case C when $\alpha\to 0$. Remembering that F^{fba}_{γ} in Eq. (3.17) is actually proportional to α^2, rather than α, we observe that the right-hand side of Eq. (3.15) is linear in α and is bound to vanish as $\alpha\to 0$. This line of reasoning explains why $\Pi^{ab}_{\mu\nu}$ in case C reduces, for $\alpha=0$, to the usual form for the self-energy, Eq. (2.7). We shall see in paper II that the situation is surprisingly different in quantum gravity, where nontransversality of the graviton self-energy occurs even for $\alpha=0$.

IV. CONCLUSION

We have examined the Ward identities in a general axial gauge by considering three distinct expressions for the gauge-breaking part of a Yang-Mills Lagrangian. These gauge-breaking terms $(2\alpha)^{-1}n\cdot A^a f n\cdot A^a$ are characterized by a differential operator f which determines, to a large degree, the structure of the self-energy in terms of the gauge parameter α and the noncovariant vector n_μ. Naturally, the properties of the individual self-energies manifest themselves in the appropriate counterterms. These counterterms display, in their respective gauges, the following characteristics.

(A) For $f=-1$, the theory leads—apart from the usual term $(F^a_{\mu\nu})^2$— to n_μ-dependent counterterms which involve fourth-order derivatives[17] such as $(\partial_\nu n_\mu F^a_{\mu\nu})^2$. Except for the choice $\alpha=0$, this gauge does not appear to be particularly useful.

(B) For $f=(n\cdot\partial)^2/(n^2)^2$ the counterterm is independent of α as well as n_μ and proportional to the usual $(F^a_{\mu\nu})^2$ expression.

(C) For $f=\partial^2/n^2$, the counterterm is a function of α and n_μ. Since the infinite part of the self-energy is nontransverse, the counterterms must contain derivatives other than those occurring in the covariant curl $F^a_{\mu\nu}$.

The choice $\alpha=-1$ in case C identifies the *planar* gauge.[11] Owing to the relative simplicity of its bare propagator,

$$G^{ab}_{\mu\nu}(p)=\frac{\delta^{ab}}{i(p^2+i\epsilon)(2\pi)^{2\omega}}\times\left[\delta_{\mu\nu}-\frac{(p_\mu n_\nu+p_\nu n_\mu)}{p\cdot n}\right],$$

this gauge has received considerable attention; yet, previous authors do not seem to have realized that the planar gauge leads to complicated n_μ-dependent counterterms.[18] The consequence of this result is significant, since it implies that the Yang-Mills theory is *not* multiplicatively renormalizable in the planar gauge. Our interest in case C, $f=\partial^2/n^2$, is essentially pedagogical, since it mimics an analogous situation in quantum gravity. The gravitational case is, of course, much more challenging, and will be treated in paper II.

Although we have not calculated some of the more complicated diagrams, for example vertex corrections, we anticipate that the conclusions reached in this paper will also apply to these. Finally, we reiterate that the above computations confirm that the principal-value prescription (2.5) to (2.6) is powerful enough to cope with the more potent singularities of the general axial-gauge propagators.

Note added. After completion of this manuscript, Professor J. C. Taylor informed us of related work by Andraši and Taylor[19] and Fadin and Milstein.[20]

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[17]There are other possible n_μ-dependent counterterms containing fourth-order derivatives. Exactly which of these occur cannot be ascertained from a self-energy calculation.

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