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Quantum field theory and nuclear structure

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We discuss recent successful calculations of the properties of nuclear matter within the context of theories exhibiting mass generation through spontaneous symmetry breaking. We start with the σ model of Gell-Mann and Lévy and introduce the nucleon mass (in a vacuum) in the usual manner. We relate the expectation value of the σ field in a vacuum to a finite value of the scalar density. If the vacuum is now filled with nucleons (nuclear matter) the scalar density is increased and the new value for the nucleon mass must be determined. We exhibit the equation whose solution determines the new mass, and we also define a perturbative scheme for the determination of this mass. This scheme involves an expansion of the various quantities of the theory in terms of matrix elements calculated with positive- *and* negative-energy spinors parametrized with the vacuum mass. Although the decrease in the mass upon going from vacuum to nuclear matter at the equilibrium density is quite large ($\sim$400 MeV), we are still able to exhibit a small parameter which allows for a perturbative expansion of the binding energy and other observables. The leading term in such an expansion reproduces the approximation widely used in other calculations of the properties of nuclear matter. The truncation of the expansion at the leading term is inadequate and this fact accounts for the lack of success in previous calculations using the standard formalism. We proceed to make a transformation to the Weinberg Lagrangian retaining the fluctuating parts of the σ field. We further make a small-oscillation approximation, dropping the nonlinear terms in this Lagrangian. If one adds terms which describe ρ and ω meson fields interacting with the nucleons one can identify the Lagrangian used in the one-boson-exchange model of nuclear forces. We discuss some aspects of the calculation of the properties of nuclear matter using the one-boson-exchange Lagrangian. We stress the inadequacy of the mean-field approximation and point out the importance of the effects of exchange and short-range correlations. We comment on the nature of the vacuum state in the presence of nuclear matter and exhibit a wave function for nuclear matter in a relativistic independent-pair approximation.

I. INTRODUCTION

Recent calculations have shown that it is possible to provide a good description of the binding energy and saturation density of nuclear matter if one treats nuclear matter as a *relativistic system*.[1] We believe it is important to explain why the nonrelativistic theories that have been used for about 25 years have failed to provide a satisfactory description of the properties of nuclear matter.[2]

In a recent paper it has been shown that the ground state in the nonrelativistic theory has not been made stable against the addition of zero-momentum pairs composed of a nucleon and an antinucleon.[3] (In Bogoliubov's terminology, one has not eliminated "dangerous diagrams.") In this work we wish to discuss this matter from another point of view. In particular we wish to consider mass generation through spontaneous symmetry breaking, for example, as this takes place in the σ model,[4,5] and then note the modification required when we consider the presence of nuclear matter. Following Lee and Wick[6] we note that we can generate mass through a symmetry-breaking mechanism if there is a finite value for the scalar density in the vacuum. (We assume that this scalar density acts as the source for the finite expectation value of the σ field.) We next observe that if nuclear matter is added to the vacuum, the scalar density changes significantly. (As we will see the additional scalar density differs only by a few percent from the baryon density.) If we pursue these ideas we find that the nuclear mass m in a region of space where the baryon density has the value usually associated with nuclear matter ($\rho_B = 0.17\ \mathrm{fm}^{-3}$) is significantly reduced from the vacuum value. A value of $m = 540$ MeV is typical of the values obtained in current relativistic calculations.[7] As we will see, if we neglect this major modification of the mass in nuclear matter and assume that we can use the vacuum nucleon mass, we make large errors in determining the binding energy and saturation density of nuclear matter.[1] In this work we elaborate upon these ideas and discuss several other matters.

The plan of our work is as follows. In Sec. II we introduce Dirac spinors, which describe particles of arbitrary mass. We show how these spinors may be related to those of zero mass by a unitary transformation. We also introduce operators which create and destroy particles with arbitrary mass and also show how these may be related to the operators for zero-mass particles. We also discuss an infinite class of orthogonal vacuum states labeled by different mass parameters. These transformations have been discussed previously by Nambu and Jona-Lasinio[8] who stressed a close analogy to the theory of superconductivity. We believe our introduction of these ideas into the theory of nuclear matter is novel. (Our ideas have some relation to those presented by Lee and Wick[6] in their discussion of abnormal phases of nuclear matter; however, on a technical level, our approach is quite different. In particular, we stress that the mean-field approximation is inadequate. Further there are no adjustable parameters in the calculations we describe.)

In Sec. III we review the σ model of Gell-Mann and Lévy[4] and estimate the scalar density of the vacuum. In Sec. IV we show how this scalar density is modified by the addition of a uniform medium of baryons to the vacuum. In Sec. V we introduce the Weinberg Lagrangian which exhibits pseudovector coupling for the pion-nucleon vertex. If we neglect various higher-order terms in the Weinberg Lagrangian (small-oscillation approximation) *and* include the fluctuations of the σ field, we can begin to identify the Lagrangian used in the one-boson-exchange (OBE) model of nuclear forces.[9,10] It is necessary to explicitly introduce the ρ and ω meson fields to complete the specification of the OBE Lagrangian (see Sec. VI).

In Sec. VII we discuss the perturbative approach to the construction of spinors parametrized by the self-consistent mass in nuclear matter. We point out that when the self-consistent spinor solution is expanded in a basis of positive- *and* negative-energy spinors with a mass parameter appropriate to the vacuum, the calculation of various observables involves a systematic treatment of "pair-current" corrections to the nonrelativistic theory. (If one avoids the expansion in an arbitrary spinor basis and constructs the self-consistent spinor states directly, one can avoid all reference to pair currents.)

In Sec. VIII we review methods for the calculation of the self-energy in nuclear matter using the expansion discussed in Sec. VII. We summarize some results of calculations reported elsewhere.[1,11]

In Sec. IX we use some of the results reported in Sec. VIII to demonstrate the limitations of the mean-field or Hartree approximation for the calculation of the self-energy. We emphasize the role of quantum fluctuations; in particular we discuss the role of exchange and short-range correlations. In Sec. X we refine our treatment of the vacuum state in the presence of nuclear matter and discuss the wave function of the interacting system. Section XI contains some concluding remarks.

II. SPINORS, OPERATORS, AND VACUUMS WITH CONTINUOUS MASS PARAMETER

One of the essential characteristics of the theory considered here is the modification of the nucleon mass in nuclear matter from the value it has in a vacuum. To discuss this question it is useful to introduce Dirac spinors where the mass parameter is a continuous variable. In addition we will consider creation and destruction operators which create and destroy particles of arbitrary mass. For definiteness we will denote the nucleon mass in a vacuum as m_N and we will use m to represent a continous mass parameter. Finally, the mass parameter in nuclear matter at the equilibrium density will be $\bar{m}$. We will use the Bjorken-Drell[12] conventions for the Dirac matrices and therefore $\vec{\gamma}$ will be anti-Hermitian.

We start with spinors describing particles of $m=0$ ($[E_0(\vec{p})]^{1/2}=|\vec{p}|$),

$$u(\vec{p}, s, 0)=[E_0(\vec{p})]^{1/2}\begin{bmatrix}\chi_s\\ \vec{\sigma}\cdot\hat{p}\chi_s\end{bmatrix}, \tag{2.1}$$

$$w(\vec{p}, s, 0)=\gamma_5\gamma_0 u(\vec{p}, s, 0)=v(-\vec{p}, -s, 0) \tag{2.2}$$

$$=[E_0(\vec{p})]^{1/2}\begin{bmatrix}-\vec{\sigma}\cdot\hat{p}\chi_s\\ \chi_s\end{bmatrix}. \tag{2.3}$$

Here χ_s is a Pauli spinor and $\hat{p}$ is a unit vector in the direction of the momentum. These spinors are normalized as follows:

$$u^\dagger(\vec{p}, s, 0)u(\vec{p}, s', 0)=w^\dagger(\vec{p}, s, 0)w(\vec{p}, s', 0)=2E_0(\vec{p}), \tag{2.4}$$

$$\bar{u}(\vec{p}, s, 0)u(\vec{p}, s', 0)=\bar{w}(\vec{p}, s, 0)w(\vec{p}, s', 0)=0. \tag{2.5}$$

From these spinors we can construct spinors of mass m:

$$u(\vec{p}, s, m)=Ne^{-\theta_0^m\vec{\gamma}\cdot\hat{p}}\,u(\vec{p}, s, 0), \tag{2.6}$$

$$w(\vec{p}, s, m)=Ne^{-\theta_0^m\vec{\gamma}\cdot\hat{p}}\,w(\vec{p}, s, 0) \tag{2.7}$$

$$=\gamma_5\gamma_0 u(\vec{p}, s, m). \tag{2.8}$$

In these equations N is a normalization constant and the angle θ_0^m is given by

$$\tan\theta_0^m=-\left[\frac{E_m(\vec{p})+m-|\vec{p}|}{E_m(\vec{p})+m+|\vec{p}|}\right], \tag{2.9}$$

where $E_m(\vec{p})=(\vec{p}^2+m^2)^{1/2}$. The expression for $2\theta_0^m$ is particularly simple:

$$\tan(2\theta_0^m)=-m/|\vec{p}|. \tag{2.10}$$

For some purposes it is useful to write

$$u(\vec{p}, s, m)=N(\cos\theta_0^m-\vec{\gamma}\cdot\hat{p}\sin\theta_0^m)u(\vec{p}, s, 0) \tag{2.11}$$

$$=N\left[\cos\theta_0^m u(\vec{p}, s, 0)+\sin\theta_0^m\sum_{s'}\langle s'|\vec{\sigma}\cdot\hat{p}|s\rangle w(\vec{p}, s', 0)\right], \tag{2.12}$$

$$w(\vec{p}, s, m) = N(\cos\theta_0^m - \vec{\gamma}\cdot\hat{p}\sin\theta_0^m)w(\vec{p}, s, 0) \tag{2.13}$$

$$= N\left[\cos\theta_0^m w(\vec{p}, s, 0) - \sin\theta_0^m \sum_{s'} \langle s'|\vec{\sigma}\cdot\hat{p}|s\rangle u(\vec{p}, s', 0)\right]. \tag{2.14}$$

These relations are particularly simple for states of definite helicity.

As we will see it sometimes will be useful to consider the construction of spinors of mass m in two steps. First, one can introduce spinors of mass m_N and then by means of a further rotation through an angle $\theta_{m_N}^m$ produce spinors of mass m. Thus

$$u(\vec{p}, s, m) = Ne^{-\theta_{m_N}^m \vec{\gamma}\cdot\hat{p}}u(\vec{p}, s, m_N) \tag{2.15}$$

$$= Ne^{-(\theta_{m_N}^m + \theta_0^{m_N})\vec{\gamma}\cdot\hat{p}}u(\vec{p}, s, 0), \tag{2.16}$$

with a similar relation for $w(\vec{p}, s, m) = \gamma_5\gamma_0 u(\vec{p}, s, m)$. We find

$$\tan\theta_{m_N}^m = -|\vec{p}|\left(\frac{E_m(\vec{p}) + m - E_N(\vec{p}) - m_N}{[E_m(\vec{p}) + m][E_N(\vec{p}) + m_N] + |\vec{p}|^2}\right) \tag{2.17}$$

and

$$\tan(2\theta_{m_N}^m) = -|\vec{p}|(m - m_N)/(|\vec{p}|^2 + mm_N). \tag{2.18}$$

Here $E_N(\vec{p}) = (|\vec{p}|^2 + m_N^2)^{1/2}$. We remark that if we place $m_N = 0$ in Eq. (2.17), we reproduce the result given in Eq. (2.9) for the angle that takes one from spinors of mass 0 to those of mass m.

If we take

$$N = [E_m(\vec{p})/E_0(\vec{p})]^{1/2} \tag{2.19}$$

we have

$$u^\dagger(\vec{p}, s, m)u(\vec{p}, s, m) = 2E_m(\vec{p}), \tag{2.20}$$

$$w^\dagger(\vec{p}, s, m)w(\vec{p}, s, m) = 2E_m(\vec{p}), \tag{2.21}$$

which differs from the normalization of the Bjorken-Drell spinors. Those spinors would have $E_m(\vec{p})/m$ on the right in Eqs. (2.20) and (2.21).

The reader may note some similarity between the transformations introduced here and the Foldy-Wouthuysen transformation. This relationship is discussed in the Appendix.

Now we may expand the Dirac field operator $\Psi(\vec{x}) = \Psi(\vec{x}, t=0)$ in terms of spinors with any mass parameter; however, the appropriate choice will depend on the problem at hand. (The standard expansion using the parameter m_N is appropriate for problems involving the interaction of a few particles in a vacuum.) We start with an expansion with $m = 0$:

$$\Psi(\vec{x}) = \sum_s \int \frac{d\vec{p}}{(2\pi)^3}\frac{1}{[2E_0(\vec{p})]^{1/2}}[u(\vec{p}, s, 0)b(\vec{p}, s, 0) + w(\vec{p}, s, 0)d^\dagger(-\vec{p}, -s, 0)]e^{i\vec{p}\cdot\vec{x}}, \tag{2.22}$$

where $b(\vec{p}, s, 0)$ destroys a massless fermion of momentum $\vec{p}$ and spin projection s. The operator $d^\dagger(-\vec{p}, -s, 0)$ creates a massless antiparticle of momentum $-\vec{p}$ and spin projection $-s$.

Now we may invert Eqs. (2.11)–(2.14) to find

$$u(\vec{p}, s, 0) = \left[\frac{E_0(\vec{p})}{E_m(\vec{p})}\right]^{1/2}\left[\cos\theta_0^m u(\vec{p}, s, m) - \sin\theta_0^m \sum_{s'}\langle s'|\vec{\sigma}\cdot\hat{p}|s\rangle w(\vec{p}, s', m)\right], \tag{2.23}$$

$$w(\vec{p}, s, 0) = \left[\frac{E_0(\vec{p})}{E_m(\vec{p})}\right]^{1/2}\left[\cos\theta_0^m w(\vec{p}, s, m) + \sin\theta_0^m \sum_{s'}\langle s'|\vec{\sigma}\cdot\hat{p}|s\rangle u(\vec{p}, s', m)\right]. \tag{2.24}$$

These forms may be substituted into Eq. (2.22). We then obtain

$$\Psi(\vec{x}) = \sum_s \int \frac{d\vec{p}}{(2\pi)^{3/2}}\frac{1}{[2E_m(\vec{p})]^{1/2}}[u(\vec{p}, s, m)b(\vec{p}, s, m) + w(\vec{p}, s, m)d^\dagger(-\vec{p}, -s, m)]e^{i\vec{p}\cdot\vec{x}}, \tag{2.25}$$

where we have introduced

$$b(\vec{p}, s, m) = \cos\theta_0^m b(\vec{p}, s, 0) + \sin\theta_0^m \sum_{s'}\langle s'|\vec{\sigma}\cdot\hat{p}|s'\rangle d^\dagger(-\vec{p}, -s', 0), \tag{2.26}$$

$$d^\dagger(-\vec{p}, -s, 0) = \cos\theta_0^m d^\dagger(-\vec{p}, -s, 0) - \sin\theta_0^m \sum_{s'}\langle s|\vec{\sigma}\cdot\hat{p}|s'\rangle b(\vec{p}, s', 0). \tag{2.27}$$

These operators will clearly be the creation and destruction operators of particles of mass m. We may exhibit the operator that generates the transformation given in Eqs. (2.26) and (2.27):

$$U(m)b(\vec{p}, s, 0)U^{-1}(m) = b(\vec{p}, s, m), \tag{2.28}$$

$$U(m)d^\dagger(-\vec{p}, -s, 0)U^{-1}(m) = d^\dagger(-\vec{p}, -s, m), \text{ etc.} \tag{2.29}$$

We have

$$U(m)=\exp\left[-\int d\vec{k}\,\theta_0^m(\vec{k})A(\vec{k})\right] \tag{2.30}$$

with

$$A(\vec{k})=\sum_{ss'}[b^\dagger(\vec{k},s,0)\langle s|\vec{\sigma}\cdot\hat{k}|s'\rangle d^\dagger(-\vec{k},s',0) - d(-\vec{k},-s',0)\langle s'|\vec{\sigma}\cdot\hat{k}|s\rangle b(\vec{k},s,0)]. \tag{2.31}$$

We also note the commutation relations

$$[A(\vec{k}),b(\vec{k}',s,0)] =-\sum_{s''}\langle s|\vec{\sigma}\cdot\hat{k}|s''\rangle d^\dagger(-\vec{k},-s'',0)\delta(\vec{k}-\vec{k}'), \tag{2.32}$$

$$[A(\vec{k}),d^\dagger(-k',-s,0)] =\sum_{s'}\langle s|\vec{\sigma}\cdot\hat{k}|s''\rangle b(\vec{k},s'',0)\delta(\vec{k}-\vec{k}'). \tag{2.33}$$

The relations given in Eqs. (2.28)–(2.33) are also valid if $m=0$ is replaced by m_N and θ_0^m is replaced by $\theta_{m_N}^m$.

At this point we can introduce various vacuum states:

$$b(\vec{p},s,0)\,|\,\text{vac};0\rangle=d(\vec{p},s,0)\,|\,\text{vac};0\rangle=0, \tag{2.34}$$

$$b(\text{p},s,m)\,|\,\text{vac};m\rangle=d(\vec{p},s,m)\,|\,\text{vac};m\rangle=0, \tag{2.35}$$

$$b(\vec{p},s,m_N)\,|\,\text{vac},m_N\rangle=d(\vec{p},s,m_N)\,|\,\text{vac};m_N\rangle=0. \tag{2.36}$$

As discussed by Nambu and Jona-Lasinio,[8] these vacuum states define *different Hilbert spaces* and the relation

$$U(m)\,|\,\text{vac};0\rangle=|\,\text{vac};m\rangle \tag{2.37}$$

is only a formal statement. Indeed

$$\langle\text{vac};m\,|\,\text{vac};m'\rangle=0\quad\text{for } m\neq m'. \tag{2.38}$$

We note that

$$|\,\text{vac};m\rangle =\prod_{\vec{p},s}\left[\cos\theta_0^m-\sin\theta_0^m\sum_{s'}\langle s|\vec{\sigma}\cdot\hat{p}|s'\rangle b^\dagger(\vec{p},s',0) \times d^\dagger(-\vec{p},-s,0)\right]|\,\text{vac};0\rangle. \tag{2.39}$$

For states of definite helicity this becomes

$$|\,\text{vac};m\rangle =\prod_{\vec{p}\lambda}[\cos\theta_0^m-\sin\theta_0^m b^\dagger(\vec{p},\lambda,0)d^\dagger(-\vec{p},\lambda,0)]\,|\,\text{vac};0\rangle. \tag{2.40}$$

Of course, the analogy to the BCS theory of superconductivity is manifest.

We conclude this section with the following pedagogical remark: Consider the Hamiltonian describing zero-mass fermions interacting with an externally imposed c-number scalar field ϕ_0:

$$H=H_0+H_{\text{int}}, \tag{2.41}$$

$$H_0=\int\Psi^\dagger(\vec{x})\left(\vec{\alpha}\cdot\frac{1}{i}\vec{\nabla}\right)\Psi(\vec{x})d\vec{x}, \tag{2.42}$$

$$H_{\text{int}}=g\phi_0\int\overline{\Psi}(\vec{x})\Psi(\vec{x})d\vec{x}. \tag{2.43}$$

We see that, rather than expanding in the eigenfunctions of H_0, it is appropriate to pass immediately to Eq. (2.25). The choice for the mass parameter of $m_0=g\phi_0$ immediately puts H into diagonal form with eigenvalues $\pm(|\vec{p}|^2+m_0^2)^{1/2}$ and ground state $|\,\text{vac};m_0\rangle$. This mechanism for introducing the mass of the nucleon appears in the σ model of Gell-Mann and Lévy which we review in the next section.

III. THE σ MODEL

We consider the Lagrangian density for the Gell-Mann–Lévy σ model[4,5] $\mathcal{L}=\mathcal{L}_0+\mathcal{L}_1$:

$$\mathcal{L}_0=\overline{\psi}[i\gamma^\mu\partial_\mu-g(\sigma+i\vec{\pi}\cdot\vec{\tau}\gamma_5)]\psi +\tfrac{1}{2}[(\partial_\mu\sigma)^2+(\partial_\mu\vec{\pi})^2]-\tfrac{1}{2}\mu^2(\sigma^2+\vec{\pi}^2)-\tfrac{1}{4}\lambda^2(\sigma^2+\vec{\pi}^2)^2, \tag{3.1}$$

$$\mathcal{L}_1=c\sigma, \tag{3.2}$$

where the presence of $\mathcal{L}_1$ breaks, in an explicit fashion, the chiral symmetry of $\mathcal{L}_0$. This Lagrangian may be written as

$$\mathcal{L}_0=\overline{\psi}[i\gamma^\mu\partial_\mu-g(\sigma+i\vec{\pi}\cdot\vec{\tau}_5)]\psi -\tfrac{1}{4}\lambda^2(\sigma^2+\vec{\pi}^2-f_\pi^2)^2+\tfrac{1}{2}[(\partial_\mu\sigma)^2+(\partial_\mu\vec{\pi})^2], \tag{3.3}$$

where we have dropped an unimportant constant term. The fundamental idea is to note that because of the form of the potential term (with $f_\pi>0$), it is useful to write $\sigma(x)=f_\pi+\phi_\sigma(x)$ where $\phi_\sigma(x)$ is the fluctuating part of the field. The mass parameter of the theory is then $m_N=gf_\pi$, and one may identify

$$f_\pi=\langle\text{vac};m_N\,|\sigma(x)|\,\text{vac};m_N\rangle. \tag{3.4}$$

The appropriate expansion of the nucleon field operators is then in terms of spinors of mass m_N. In the σ model, f_π is the pion decay constant ($f_\pi\sim 93$ MeV) and $g\simeq 13.7$, so that m_N comes out about 30% too large. Mainly this is due to the fact that in this model the axial-vector coupling constant $g_A=1$, so that the Goldberger-Treiman relation $m_N=gf_\pi/g_A$ becomes $m_N=gf_\pi$. We will somewhat arbitrarily replace f_π by $f_\pi^{\rm eff}=68.6$ MeV.[13] (With $g=13.7$, this gives $m_N=940$ MeV.) It is useful to associate the development of an expectation value of the σ field in the vacuum with a corresponding finite scalar density.[6] Let us write

$$\rho_s^{\rm vac}=\langle {\rm vac}; m_N\,|\bar\Psi(x)\Psi(x)|\,{\rm vac}; m_N\rangle\,. \tag{3.5}$$

We further assume that the expectation values of $\sigma(x)$ and $\bar\Psi(x)\Psi(x)$ are proportional. We write

$$\begin{aligned}\langle {\rm vac}; m_N\,|\sigma(x)|\,{\rm vac}; m_N\rangle &= f_\pi^{\rm eff}\\ &= -\frac{g}{m_\sigma^2}\langle {\rm vac}; m_N\,|\bar\Psi(x)\Psi(x)|\,{\rm vac}; m_N\rangle\,,\end{aligned} \tag{3.6}$$

where m_σ is a parameter. For example, if we take $m_\sigma=700$ MeV, we have $\rho_s^{\rm vac}=-0.32$ fm^{-3}. The value assigned to the scalar density in the vacuum does not play a significant role in the theory, but it is of interest to compare this value to the increment of the scalar density due to the presence of nuclear matter. It is worth noting that the theory of nuclear matter that we have developed[1] does not require that the relation given in Eq. (3.6) be correct. If this is indeed a correct relation, we can see that a rather interesting physical picture emerges [see Secs. IV and V]. Finally we remark that Eq. (3.6) is the analog of the linear relation that exists between the gap function $\Delta(\vec x)$ and the anomalous expectation value of the electron-field operators in the BCS theory of superconductivity:

$$\Delta(\vec x)=-G\langle\psi_\uparrow(\vec x)\psi_\downarrow(\vec x)\rangle\,. \tag{3.7}$$

IV. THE SCALAR DENSITY IN NUCLEAR MATTER

The baryon density in nuclear matter is

$$\rho_B^{\rm NM}=4\int_0^{k_F}\frac{d\vec p}{(2\pi)^3}=\frac{2}{3\pi^2}k_F^{\,3}\,, \tag{4.1}$$

while the scalar density is[14]

$$\rho_s^{\rm NM}=4\int_0^{k_F}\frac{d\vec p}{(2\pi)^3}\frac{m}{(p^2+m^2)^{1/2}} \tag{4.2}$$

$$=\tfrac{3}{2}\rho_B^{\rm NM}\left\{\frac{1}{y^2}(y^2+1)^{1/2}-\frac{1}{y^3}\ln[y+(y^2+1)^{1/2}]\right\}, \tag{4.3}$$

where $y=(k_F/m)$. As we will see at the equilibrium density ($\rho_B^{\rm NM}=0.17$ fm^{-3}) $m=\bar m\simeq 540$ MeV. Therefore, $y=k_F/\bar m=0.5$ and $\rho_s^{\rm NM}=0.93\rho_B^{\rm NM}=0.16$ fm^{-3}. In the previous section we gave the scalar density in a vacuum as $\rho_s^{\rm vac}=-0.32$ fm^{-3}. Thus the *total* scalar density is

$$\rho_s=\rho_s^{\rm vac}+\rho_s^{\rm NM}=-0.16\ {\rm fm}^{-3}\,. \tag{4.4}$$

Of course our value for $\rho_s^{\rm vac}$ is quite model dependent [see Eq. (3.6)], so that the value given in Eq. (4.4) for ρ_s and the value quoted for $\rho_s^{\rm vac}$ at the end of the last section are subject to modification.

V. THE WEINBERG LAGRANGIAN

In this section we review a procedure which may be used to go from the pseudoscalar pion coupling of the σ model to pseudovector coupling. (We follow the review of Brown and that article may be consulted for a more detailed discussion.[5]) We begin by recalling the σ-model Lagrangian of Eqs. (3.1)–(3.3). As a first step we again write $\sigma=f_\pi+\phi_\sigma$, where ϕ_σ represents the fluctuating part of the σ field. These fluctuations are an essential feature of the description of the nucleon-nucleon interaction in the OBE model of nuclear forces.[9,10] *For the moment*, however, we drop ϕ_σ and consider the σ and $\vec\pi$ fields to be constrained through the relation

$$\sigma^2+\vec\pi^2=(f_\pi^{\rm eff})^2\,. \tag{5.1}$$

One then writes

$$\sigma(x)=f_\pi^{\rm eff}\cos\theta(x)\,, \tag{5.2}$$

$$\vec\pi(x)=f_\pi^{\rm eff}\hat\pi(x)\sin\theta(x)\,, \tag{5.3}$$

and defines

$$\vec\phi_\pi(x)=2f_\pi^{\rm eff}\tan[\tfrac{1}{2}\theta(x)]\hat\pi(x)\,. \tag{5.4}$$

Further with the transformation

$$\psi_W(x)=e^{i\vec\tau\cdot\hat\pi(x)\gamma_5\theta(x)/2}\psi(x)\,, \tag{5.5}$$

one finally obtains the Weinberg Lagrangian

$$\begin{aligned}\mathfrak{L}_W=\bar\psi(x)(i\gamma^\mu D_\mu-m_N)\psi(x)-\frac{f}{m_\pi}\bar\psi(x)\gamma_5\gamma^\mu\vec\tau\cdot\psi(x)D_\mu\vec\phi_\pi(x)\\ -\tfrac{1}{2}D_\mu\vec\phi_\pi(x)\cdot D^\mu\vec\phi_\pi(x)-\tfrac{1}{2}m_\pi^{\,2}\left[1+\frac{\vec\phi_\pi^{\,2}(x)}{4f_\pi^{\,2}}\right]\vec\phi_\pi^{\,2}(x)\,.\end{aligned} \tag{5.6}$$

In Eq. (5.6) we have replaced $2f_\pi^{\rm eff}$ by $(f/m_\pi)^{-1}$. Here f is the pseudovector π-nucleon coupling constant, which is related to g of Eq. (3.1) by $(f/m_\pi)=g/2m_N$. Further, the covariant derivatives are

$$D_\mu\vec\phi_\pi=\left(1+\frac{\vec\phi_\pi^{\,2}}{4f_\pi^{\,2}}\right)^{-1}\partial_\mu\vec\phi_\pi\,, \tag{5.7}$$

$$D_\mu\psi=[\partial_\mu+i(4f_\pi^{\,2}+\vec\phi_\pi^{\,2})^{-1}\vec\tau\cdot(\vec\phi_\pi\times\partial_\mu\vec\phi_\pi)]\psi\,. \tag{5.8}$$

As a next step we assume that the complicated nonlinear terms are small and replace the covariant derivatives by the ordinary derivatives. Secondly, we restore those terms that describe the small fluctuations of the σ field about the equilibrium point $\sigma = f_\pi^{\text{eff}}$. With these modifications we have

$$\begin{aligned}\mathcal{L}_W(x) \simeq\ & \bar\psi_W(x)(i\gamma^\mu\partial_\mu - m_N)\psi(x) \\ & - \frac{f}{m_\pi}\bar\psi_W(x)\gamma_5\gamma^\mu\vec\tau\cdot\psi_W(x)\partial_\mu\vec\phi_\pi(x) \\ & - \tfrac{1}{2}[\partial_\mu\vec\phi_\pi(x)\cdot\partial^\mu\vec\phi_\pi(x) + m_\pi^2\phi_\pi^2(x)] \\ & - \tfrac{1}{2}[\partial_\mu\phi_\sigma(x)\partial^\mu\phi_\sigma(x) + m_\sigma^2\phi_\sigma^2(x)] \\ & - g\bar\psi_W(x)\psi_W(x)\phi_\sigma(x)\,, \end{aligned} \tag{5.9}$$

where we have dropped all terms that are of higher power than quadratic in the meson fields.

If we use Eq. (5.9) we find the equation of motion for the field $\phi_\sigma(x)$:

$$\Box\phi_\sigma(x) + m_\sigma^2\phi_\sigma(x) = -g\left[\bar\psi_W(x)\psi_W(x) - \frac{m_\sigma^2}{g}f_\pi^{\text{eff}}\right]. \tag{5.10}$$

In the vacuum we have

$$\langle \text{vac}; m_N|\phi_\sigma(x)|\text{vac}; m_N\rangle = 0\,, \tag{5.11}$$

while in nuclear matter, *in the Hartree approximation*, one has

$$\langle \text{NM}|\phi_\sigma(x)|\text{NM}\rangle = -\frac{g}{m_\sigma^2}\rho_s^{\text{NM}} \tag{5.12}$$

and

$$\langle \text{NM}|\sigma(x)|\text{NM}\rangle = f_\pi^{\text{eff}} - \frac{g}{m_\sigma^2}\rho_s^{\text{NM}}\,. \tag{5.13}$$

(We note that in the Hartree approximation *only* σ-meson exchange contributes to the modification of the expectation value of the σ field.)

We remark that with $g = 13.7$, $m_\sigma = 760$ MeV and $\rho_s^{\text{NM}} = 0.16\ \text{fm}^{-3}$, we would obtain $\langle\text{NM}|\sigma(x)|\text{NM}\rangle = 39.4$ MeV in nuclear matter corresponding to a mass $\tilde m = 540$ MeV. In our calculations, however, we do not use Eq. (5.13) since quantum fluctuations are quite important in calculating the *effective* σ field in the nuclear matter (see Sec. IX). [Presumably, quantum fluctuations will also modify the relation given in Eq. (3.6); however, we will not attempt to discuss that question here.]

We will discuss the calculation of the nucleon self-energy in nuclear matter in some detail in Secs. VIII and IX. However, we will note here that our equation which determines the mass in nuclear matter is

$$m(\vec p) = m_N + \tfrac{1}{4}\text{tr}\Sigma(\vec p, k_F, m(\vec p))\,, \tag{5.14}$$

where $\Sigma(\vec p, k_F, m(\vec p))$ is the self-energy of the nucleon that depends upon the Fermi momentum k_F and the mass in nuclear matter $m(\vec p)$. In general, Eq. (5.14) must be solved in a self-consistent manner [see Eqs. (7.5), (7.8), and (7.9)].

VI. THE ONE-BOSON-EXCHANGE MODEL OF NUCLEAR FORCES

The Weinberg Lagrangian considered in the small-oscillation approximation may be compared to the Lagrangian used in the one-boson-exchange model of nuclear forces. The essential ingredients of the OBE model are the σ, ω, π, and ρ mesons.[9,10] The Lagrangian of Eq. (5.9) agrees with the Lagrangian of the OBE model except for the absence of any reference to the ω and ρ fields. [The last term of Eq. (5.8) gives rise to a ρ-nucleon interaction.[5]] As the ρ and ω fields play an essential role in the description of the nucleon-nucleon interaction, we add their Lagrangians to $\mathcal{L}_W$.

In this way we obtain

$$\begin{aligned}\mathcal{L}_{\text{OBE}} = & \mathcal{L}_N + \mathcal{L}_\pi + \mathcal{L}_\sigma + \mathcal{L}_\rho + \mathcal{L}_\omega \\ & + \mathcal{L}_{\pi N} + \mathcal{L}_{\sigma N} + \mathcal{L}_{\rho N} + \mathcal{L}_{\omega N}\,,\end{aligned} \tag{6.1}$$

where the first five terms of Eq. (6.1) are the Lagrangian of the free fields and the last four represent interaction terms that are *linear* in the meson fields. For the vector mesons we have

$$\mathcal{L}_{\omega N} = -g_\omega\bar\psi_W(x)\gamma^\mu\psi_W(x)\omega_\mu(x) - \frac{f_\omega}{4m_N}\bar\psi_W(x)\sigma^{\mu\nu}\psi_W(x)G_{\mu\nu}(x) \tag{6.2}$$

with

$$G_{\mu\nu}(x) = \partial_\mu\omega_\nu(x) - \partial_\nu\omega_\mu(x) \tag{6.3}$$

and

$$\begin{aligned}\mathcal{L}_{\rho N} = & -g_\rho\bar\psi_W(x)\gamma^\mu\vec\tau\cdot\psi_W(x)\vec\rho_\mu(x) \\ & - \frac{f_\rho}{4m_N}\bar\psi_W(x)\sigma^{\mu\nu}\vec\tau\cdot\psi_W(x)\vec F_{\mu\nu}(x)\,,\end{aligned} \tag{6.4}$$

with

$$\vec F_{\mu\nu}(x) = \partial_\mu\vec\rho_\nu(x) - \partial_\nu\vec\rho_\mu(x)\,. \tag{6.5}$$

Studies of nucleon-nucleon scattering, and other theoretical considerations, favor $f_\omega \simeq 0$, $g_\rho^2/4\pi \simeq 0.5$, and $f_\rho/g_\rho \sim 6$. The strong ρ tensor coupling seems to give a more consistent picture of photodisintegration of the deuteron[15] and is favored on other theoretical grounds.[16] Strong ρ tensor coupling also leads to a somewhat better result for the binding of nuclear matter if calculations are made using the *relativistic* theory described here.[1]

The introduction of the ρ and ω fields has been somewhat *ad hoc*; however, once one accepts the Lagrangian of Eq. (6.1)–(6.5), one is able to provide highly successful fits to nucleon-nucleon scattering.[9] (In application of the one-boson-exchange model of nuclear forces, one often introduces additional mesons of higher mass such as the ϕ, η, and δ.[9,10] These play a relatively minor role in the theory and we need not enter into a detailed discussion of these mesons.) Since one does not usually discuss chiral symmetry when introducing the OBE model, the coupling constant of the σ fields is treated as a free parameter. In addition, the σ mass is a free parameter which is often chosen to be about 500 Mev.[9,10] By increasing this σ mass it is possible to obtain good fits to the nucleon-nucleon data with the σNN coupling constant of the σ model, $g \simeq 13.7$. Fits of that character have been reported in the literature (see Ref. 17, Table 7.3).

We are now able to go on to discuss recent calculations of the properties of nuclear matter that are based upon the OBE Lagrangian. We will discuss these calculations within the general framework developed in the previous sections.

VII. PERTURBATIVE EXPANSIONS AND PAIR CURRENTS

We consider the situation in which the modification of the vacuum mass m_N is not so large as to invalidate a perturbative approach. Specifically we can consider the expansion

$$f(\vec{p}, s, m) = \frac{1}{\sqrt{2m}}\Big[\cos\theta^m_{m_N} u(\vec{p}, s, m_N) + \sin\theta^m_{m_N} \sum_{s'} \langle s'|\vec{\sigma}\cdot\hat{p}|s\rangle w(\vec{p}, s', m_N)\Big], \tag{7.1}$$

$$\begin{aligned} h(\vec{p}, s, m) &\equiv \gamma_5\gamma_0 f(\vec{p}, s, m) \\ &= \frac{1}{\sqrt{2m}}\Big[\cos\theta^m_{m_N} w(\vec{p}, s, m_N) - \sin\theta^m_{m_N} \sum_{s'} \langle s'|\vec{\sigma}\cdot\hat{p}|s\rangle u(\vec{p}, s', m_N)\Big]. \end{aligned} \tag{7.2}$$

These spinors are normalized such that

$$f^\dagger(\vec{p}, s, m) f(\vec{p}, s, m) = \delta_{ss'} E_m(\vec{p})/m\,, \tag{7.3}$$

$$h^\dagger(\vec{p}, s, m) h(\vec{p}, s, m) = \delta_{ss'} E_m(\vec{p})/m\,. \tag{7.4}$$

The parameter m is to be determined via the self-consistent solution of the Dirac equation

$$[\gamma^0 p^0 - \vec{\gamma}\cdot\vec{p} - g f^{\rm eff}_\pi - \Sigma(\vec{p}, k_F)] f(\vec{p}, s, m) = 0\,, \tag{7.5}$$

where $\Sigma(\vec{p}, k_F)$ is the addition to the nucleon self-energy due to the presence of nuclear matter. Note that $\Sigma(\vec{p}, k_F)$ vanishes as $k_F \to 0$. In that case $f(\vec{p}, s, m) \to u(\vec{p}, s, m_N)/\sqrt{2m_N}$ and $h(\vec{p}, s, m) \to w(\vec{p}, s, m_N)/\sqrt{2m_N}$, corresponding to setting $\theta^m_{m_N} = 0$.

The expansion given in Eqs. (7.1) and (7.2) is useful if one can identify a small parameter. It turns out that for $\bar{m} \sim 540$ MeV, $\sin\theta^m_{m_N}$ is quite small ($\theta^m_{m_N} \sim 6°$), so that we may write

$$f(\vec{p}, s, m) \simeq \frac{1}{\sqrt{2m_N}}\Big\{\Big[1 - \frac{(\theta^m_{m_N})^2}{2}\Big] u(\vec{p}, s, m_N) + \theta^m_{m_N} \sum_{s'} \langle s'|\vec{\sigma}\cdot\hat{p}|s\rangle w(\vec{p}, s', m_N)\Big\}, \tag{7.6}$$

$$h(\vec{p}, s, m) \simeq \frac{1}{\sqrt{2m_N}}\Big\{\Big[1 - \frac{(\theta^m_{m_N})^2}{2}\Big] w(\vec{p}, s, m_N) - \theta^m_{m_N} \sum_{s'} \langle s'|\vec{\sigma}\cdot\hat{p}|s\rangle u(\vec{p}, s', m_N)\Big\}. \tag{7.7}$$

One sees that the corrections to the theory proportional to $\theta^m_{m_N}$ have the appearance of pair-current corrections to the standard expressions of nuclear physics. Therefore, we will call the expansions given in Eqs. (7.1) and (7.2) and Eqs. (7.6) and (7.7) the pair-current expansion. This expansion is useful in several respects. For example, it allows for a specific description of the contributions of the negative-energy states. The approximation $f(\vec{p}, s, m) = u(\vec{p}, s, m_N)/\sqrt{2m_N}$ will generally reproduce the results of a *low-density theory*. For example, it has been shown in previous work that the expression for the energy of nuclear matter in a relativistic theory is[3,18]

$$\begin{aligned} E(m, k_F) = {} & \sum_s \int \frac{d\vec{p}}{(2\pi)^3} \frac{m_N}{E_N(\vec{p})} \bar{f}(\vec{p}, s, m)[\vec{\gamma}\cdot\vec{p} + m_N] f(\vec{p}, s, m) \\ & + \tfrac{1}{2}\sum_{ss'} \iint \frac{d\vec{p}}{(2\pi)^3} \frac{m_N}{E_N(\vec{p})} \frac{d\vec{q}}{(2\pi)^3} \frac{m_N}{E_N(\vec{q})} \langle \bar{f}(\vec{p}, s, m)\bar{f}(\vec{q}, s', m)|\hat{M}(1 - P_{12})|f(\vec{p}, s, m) f(\vec{q}, s', m)\rangle\,. \end{aligned} \tag{7.8}$$

Further, $\hat{M}$ is the nucleon-nucleon scattering amplitude in the nuclear medium, appropriately modified to include Pauli-principle corrections, etc.[18] In Eq. (7.8) we have written the energy as a function of the mass parameter m and the Fermi momentum k_F. These are not really independent variables since m is determined as a function of k_F *via* a *self-consistent* solution of Eq. (7.5) [see Eq. (7.10)].

If we now drop the self-consistency requirement and replace $f(\vec{p}, s, m)$ by $u(\vec{p}, s, m_N)/\sqrt{2m_N}$, we have the energy expression of the Brueckner theory, for example, as used by the Bonn group[9,10]:

$$E(k_F)=\sum_s \int \frac{d\vec{p}}{(2\pi)^3}\frac{1}{2E_N(\vec{p})}\bar{u}(\vec{p},s,m_N)(\vec{\gamma}\cdot\vec{p}+m_N)u(\vec{p},s,m_N)$$
$$+\tfrac{1}{2}\sum_{ss'}\iint \frac{d\vec{p}}{(2\pi)^3}\frac{d\vec{q}}{(2\pi)^3}\frac{1}{2E_N(\vec{p})}\frac{1}{2E_N(\vec{q})}\langle\bar{u}(\vec{p},s,m_N)\bar{u}(\vec{q},s',m_N)|\hat{M}(1-P_{12})|u(\vec{p},s,m_N)u(\vec{q},s',m_N)\rangle . \tag{7.9}$$

If one uses Eqs. (7.6) and (7.7) in the evaluation of Eq. (7.8), one finds corrections to the energy that are of order $2m_N(\theta^m_{m_N})^2$.[1,11] Note that for the calculation of the *energy* it is important to keep the normalization correction term in Eqs. (7.6) and (7.7). (However, if one is interested in corrections to the *wave function*, it is sufficient to consider only terms of order $\theta^m_{m_N}$. This is analogous to the procedure used in the standard calculations of "pair-current" corrections to inelastic electron scattering from finite nuclei.) As has been shown in previous works,[1,11] the approximation given in Eq. (7.9) is only accurate at low densities, $k_F<1.0\ \mathrm{fm}^{-1}$, while nuclear matter saturates at about $k_F=1.36\ \mathrm{fm}^{-1}$. For higher densities ($k_F>1.0\ \mathrm{fm}^{-1}$), there are major modifications of the saturation curve and the non-self-consistent calculation represented by Eq. (7.9) is inadequate.

If we make the dependence of $\Sigma(\vec{p},k_F)$ on m explicit it is clear that the self-consistency condition is

$$m(\vec{p})=gf_\pi^{\mathrm{eff}}+\tfrac{1}{4}\mathrm{tr}\Sigma(\vec{p},k_F,m(\vec{p})). \tag{7.10}$$

As we will see, the dependence of $\mathrm{tr}\Sigma(\vec{p},k_F,m(\vec{p}))$ on momentum is weak and it is useful for the purposes of this discussion to write Eq. (7.10) as

$$m=gf_\pi^{\mathrm{eff}}+\tfrac{1}{4}\mathrm{tr}\Sigma(0,k_F,m). \tag{7.11}$$

(This approximation would be exact in a Hartree calculation but, as we will see, the Hartree approximation is not sufficiently accurate for the calculation of the properties of nuclear matter.) We will return to a discussion of this equation in the next section where we discuss the self-energy in some detail.

We close this section with a few general remarks. As is well known in many-body physics, the use of a self-consistent representation leads to a ground state that is stable against the addition of one-particle, one-hole excitations. If diagrams are used to represent elements of a perturbation theory, one can say that the introduction of a self-consistent basis leads to cancellation of "tadpole" diagrams (see Fig. 1). In the problem discussed here, as a consequence of introduction of nuclear matter into the vacuum, the $u(\vec{p},s,m_N)$ are no longer self-consistent spinor solutions of the Dirac equation. If the spinors $u(\vec{p},s,m_N)$ and $w(\vec{p},s,m_N)$ are used to expand the self-consistent solutions of the Dirac equations, corrections to the calculation of various observables contain the diagrammatic elements shown in Fig. 1(b). Use of the self-consistent spinors $f(\vec{p},s,\tilde{m})$ would eliminate such tadpole effects from the diagrammatic representation and eliminate *explicit* reference to pair currents in the theory.

VIII. THE SELF-ENERGY OF A NUCLEON IN NUCLEAR MATTER

In this section we will discuss the quantity $\Sigma(\vec{p},k_F)$. To this end we note that a general form for the self-energy is[11]

$$\Sigma(\vec{p})=A(\vec{p})+\gamma^0B(\vec{p})+\frac{\vec{\gamma}\cdot\vec{p}}{m_N}C(\vec{p}), \tag{8.1}$$

where we have suppressed reference to the density dependence for simplicity. Previous calculations[11] have shown that $C(\vec{p})$ is small. Therefore, $C(\vec{p})$ may be neglected for the purpose of this discussion. Further, we find that $A(\vec{p})$ evaluated at $k_F=1.36\ \mathrm{fm}^{-1}$ is approximately given by

$$A(\vec{p})\simeq[-400+25(|\vec{p}|/k_F)^2]\ \mathrm{MeV}. \tag{8.2}$$

The value for $A(\vec{p})$ given here is the result of the relativistic calculations described in Ref. 11 (see Fig. 24 of Ref. 11). These calculations include the effect of correlations and are obtained using a one-boson-exchange potential which fits NN scat-

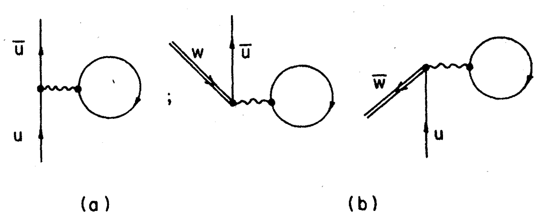

FIG. 1. (a) Tadpole diagram which is included in a nonrelativistic many-body theory. (Only the direct diagram contributing to the particle self-energy is shown.) (b) Tadpole diagrams which appear in a relativistic many-body theory. These effects may be treated perturbatively as in the "pair-current" expansion introduced in this work. Alternatively, the effects of these diagrams may be summed completely by solving the Dirac equation in a self-consistent manner as discussed in the text.

tering data and which gives a good account of the properties of nuclear matter. The values given in Eq. (8.2) result from the first iteration of what could be made into a fully consistent calculation.[11] Thus, for our qualitative considerations, we may take $A(\vec{p}) = A$, a constant of the order of -400 MeV. In this approximation we obtain from Eq. (7.11)

$$m = g f_{\pi}^{\text{eff}} + A = m_N + A\,. \tag{8.3}$$

Since A depends on m, Eq. (8.3) should be solved in a self-consistent manner.

It is useful to recall Eq. (2.17). We see that we may put

$$\tan\theta_{m_N}^{m} \simeq -\frac{|\vec{p}|}{2m_N}\left(\frac{A}{m_N + A}\right), \tag{8.4}$$

so that for $|\vec{p}| = k_F = 268$ MeV/c and $A = -400$ MeV we have $\tan\theta_{m_N}^{m} \simeq \theta_{m_N}^{m}$ and $\theta_{m_N}^{m} \sim 6°$. For example, if we put $|\vec{p}| = k_F$ in Eq. (2.10) we find that $\theta_0^{m_N} = -37°$. Thus we have $(\theta_0^{m_N} + \theta_{m_N}^{m}) = -31°$ at $p = k_F$. The significance of these angles may be seen in Eqs. (2.15) and (2.16). We may say that $\theta_0^{m_N}$ provides a measure of the mass generation in the vacuum through spontaneous symmetry breaking, while $\theta_{m_N}^{m}$ provides a measure of the reduction of this mass in the presence of matter.

It is useful to introduce the matrix elements of the self-energy using the basis introduced in the last section,[11]

$$\Sigma_{ss'}^{++}(\vec{p}) = \frac{1}{2m_N}\bar{u}(\vec{p}, s, m_N)\Sigma(\vec{p})u(\vec{p}, s, m_N)\delta_{ss'}\,, \tag{8.5}$$

$$\Sigma_{ss'}^{--}(\vec{p}) = \frac{1}{2m_N}\bar{w}(\vec{p}, s, m_N)\Sigma(\vec{p})w(\vec{p}, s, m_N)\delta_{ss'}\,, \tag{8.6}$$

$$\Sigma_{ss'}^{+-}(\vec{p}) = \frac{1}{2m_N}\bar{u}(\vec{p}, s, m_N)\Sigma(\vec{p})w(\vec{p}, s', m_N)\,, \tag{8.7}$$

and

$$\Sigma_{ss'}^{-+}(\vec{p}) = \frac{1}{2m_N}\bar{w}(\vec{p}, s, m_N)\Sigma(\vec{p})u(\vec{p}, s', m_N)\,. \tag{8.8}$$

We find [see Eq. (8.1) or Ref. 11]

$$\Sigma_{ss'}^{+-}(\vec{p}) = -\frac{|\vec{p}|}{m_N}\langle s|\vec{\sigma}\cdot\hat{p}|s'\rangle[A(\vec{p}) - C(\vec{p})]\,. \tag{8.9}$$

If we take $\hat{p}$ along the positive z axis and drop $C(\vec{p})$ [$C(p) \sim 10$ MeV], we have

$$\Sigma_{ss'}^{+-}(\vec{p}) \simeq \delta_{ss'}\left[-\frac{|\vec{p}|}{m_N}A(\vec{p})(-1)^{(1/2)-s}\right]. \tag{8.10}$$

We find $\Sigma^{+-} \simeq 114$ MeV for $A = -400$ MeV and $|\vec{p}| = k_F = 1.36$ fm^{-1}. Comparing Eqs. (8.4) and (8.3), we see that knowledge of the "transition potential" Σ^{+-}, which one calculates when the pair-current expansion of Sec. VII is used, gives a direct measure of the reduction of the nucleon mass from its vacuum value.

While short-range correlations effects are quite important in the calculation of the energy of the system, the value of A is only changed by a few percent when such correlation effects are included.[11] It is interesting to discuss the role of each meson as it contributes to the determination of A. In Figs. 2–4 we present the results of detailed calculation of $\Sigma_{(1/2)(1/2)}^{-+}$.[11] In these figures the solid lines refer to the results of a calculation based on the relativistic Hartree-Fock approximation, while the dashed lines include the effects of correlations in modifying the contribution of each meson to $\Sigma_{(1/2)(1/2)}^{-+}$. The decrease in the σ and ω contribution in the presence of correlations is compensated by the *increase* in the pion contribution leading to only small correlation effects in the *sum* of the various contributions (see Fig. 2). From these figures we see that in the presence of correlations the σ meson contributes only about 72 MeV to the value of Σ^{-+} (see Figs. 2 and 3). Using Eq. (8.10) in conjunction with Fig. 3 we can note that *the contribution of the σ field alone would give $A \simeq -260$ MeV, which is only 65% of the total mass reduction upon going from vacuum to nuclear matter.* This is a manifestation of the inadequacy of the mean-field (or Hartree) approximation for this problem. Corrections to the mean-field approximation are usually classed as "quantum fluctuations." We continue our discussion of these corrections to the mean-field approximation in the next section.

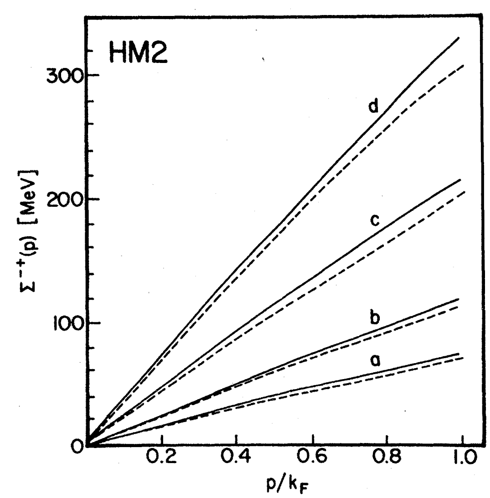

FIG. 2. Calculated values of $\Sigma_{(1/2)(1/2)}^{-+}(\vec{p})$ for the interaction of Ref. 19. The various curves represent calculations made for various values of k_F. The solid lines denote the results for calculations carried out in a relativistic Hartree-Fock approximation. The dashed lines are results obtained after correlations effects are included. (a) $k_F = 1.2$ fm^{-1}, (b) $k_F = 1.36$ fm^{-1}, (c) $k_F = 1.6$ fm^{-1}, (d) $k_F = 1.8$ fm^{-1}.

IX. QUANTUM FLUCTUATIONS

At this point is is useful to discuss the calculation of the self-energy in more detail. We recall Eqs. (8.5)–(8.8) and remark that in the (relativistic) independent-pair approximation, we have

$$\Sigma^{++}(\vec{p}) = \sum_{s'} \int \frac{d\vec{q}}{(2\pi)^3} \frac{m_N}{E_N(\vec{q})} \frac{1}{2m_N} \langle \bar{u}(\vec{p}, s, m_N) \bar{f}(\vec{q}, s') \,|\, \hat{M}(1 - P_{12}) \,|\, u(\vec{p}, s, m_N) f(\vec{q}, s') \rangle , \tag{9.1}$$

$$\Sigma^{--}(\vec{p}) = \sum_{s'} \int \frac{d\vec{q}}{(2\pi)^3} \frac{m_N}{E_N(\vec{q})} \frac{1}{2m_N} \langle \bar{w}(\vec{p}, s, m_N) \bar{f}(\vec{q}, s') \,|\, \hat{M}(1 - P_{12}) \,|\, w(\vec{p}, s, m_N) f(\vec{q}, s') \rangle , \tag{9.2}$$

and

$$\Sigma_{ss'}^{-+} = \sum_{s''} \int \frac{d\vec{q}}{(2\pi)^3} \frac{m_N}{E_N(\vec{q})} \frac{1}{2m_N} \langle \bar{w}(\vec{p}, s, m_N) \bar{f}(\vec{q}, s'') \,|\, \hat{M}(1 - P_{12}) \,|\, u(\vec{p}, s', m_N) f(\vec{q}, s'') \rangle . \tag{9.3}$$

Note that Σ^{++} and Σ^{--} do not depend upon the spin label s. In these equations $\hat{M}$ is the scattering amplitude in the medium and satisfies a Bethe-Salpeter equation of the form[1,11]

$$\hat{M} = U + U\hat{g}^{++}\hat{M} . \tag{9.4}$$

In Eq. (9.4), $\hat{g}^{++}$ is a propagator that restricts the intermediate particles to positive-energy states and includes Pauli-principle restrictions for the intermediate-state scattering. The quasipotential U describes the exchange of the various mesons of the theory ($\sigma, \pi, \rho, \omega, \ldots$).

The calculation of the matrix elements of the self-energy requires the knowledge of the self-consistent spinor $f(\vec{q}, s)$. In what is the first iteration of a fully self-consistent calculation, one can approximate $f(\vec{q}, s, m_N)$ by $(2m_N)^{-1/2} u(\vec{q}, s, m_N)$. Thus one has, in this approximation,

$$\Sigma^{++}(\vec{p}) \simeq \sum_{s'} \int \frac{d\vec{q}}{(2\pi)^3} \frac{1}{2E_N(\vec{p})} \frac{1}{2E_N(\vec{q})} \langle \bar{u}(\vec{p}, s, m_N) \bar{u}(\vec{q}, s', m_N) \,|\, \hat{M}(1 - P_{12}) \,|\, u(\vec{p}, s, m_N) u(\vec{q}, s', m_N) \rangle , \tag{9.5}$$

$$\Sigma^{--}(\vec{p}) \simeq \sum_{s'} \int \frac{d\vec{q}}{(2\pi)^3} \frac{1}{2E_N(\vec{p})} \frac{1}{2E_N(\vec{q})} \langle \bar{w}(\vec{p}, s, m_N) \bar{u}(\vec{q}, s', m_N) \,|\, \hat{M}(1 - P_{12}) \,|\, w(\vec{p}, s, m_N) u(\vec{q}, s', m_N) \rangle , \tag{9.6}$$

and

$$\Sigma_{ss'}^{-+}(\vec{p}) \simeq \sum_{s''} \int \frac{d\vec{q}}{(2\pi)^3} \frac{1}{2E_N(\vec{q})} \frac{1}{2E_N(\vec{p})} \langle \bar{w}(\vec{p}, s, m_N) \bar{u}(\vec{q}, s'', m_N) \,|\, \hat{M}(1 - P_{12}) \,|\, u(\vec{p}, s', m_N) u(\vec{q}, s'', m_N) \rangle . \tag{9.7}$$

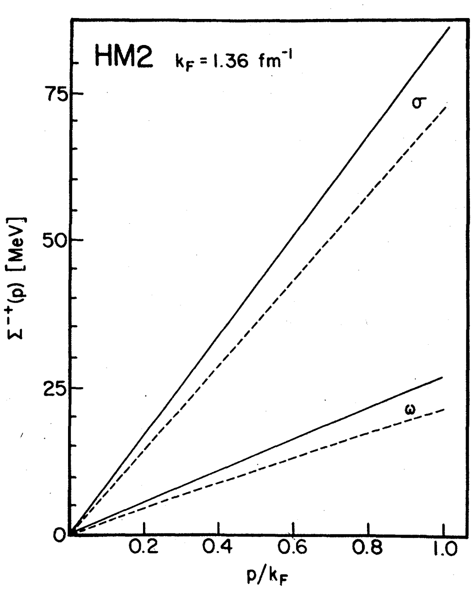

FIG. 3. Contributions to $\Sigma^{-+}_{(1/2)(1/2)}(\vec{p})$ from the exchange of σ and ω mesons for the interaction of Ref. 19.

The Hartree approximation is obtained by replacing $\hat{M}(1 - P_{12})$ by U, while the Hartree-Fock approximation is obtained when $\hat{M}(1 - P_{12})$ is replaced by $U(1 - P_{12})$. The calculations reported previous-

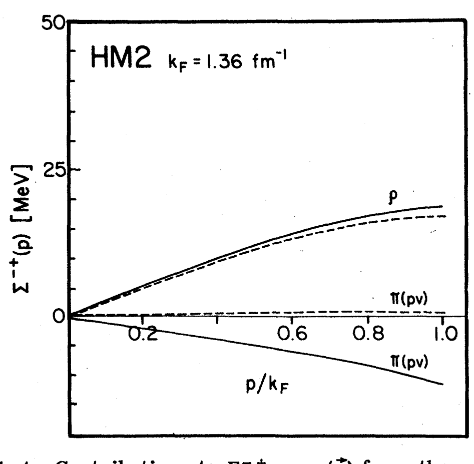

FIG. 4. Contributions to $\Sigma^{-+}_{(1/2)(1/2)}(\vec{p})$ from the exchange of π and ρ mesons for the interaction of Ref. 19. (Pseudovector coupling is used for the πNN vertex.)

ly[1,11] were performed using the approximation for Σ given in Eqs. (9.5)–(9.7). This approximation is sufficiently accurate at low densities and in the vicinity of the nuclear saturation point, $k_F = 1.36$ fm^{-1}. For higher densities ($k_F > 1.5$ fm^{-1}) a fully self-consistent calculation is required.

Since the quantity $A(\vec{p})$ provides a measure of the mass modification and $A(\vec{p})$ and $B(\vec{p})$ provide a simple parametrization of the quasiparticle energy, we can profitably discuss the results of our calculation in terms of the values of these quantities.

We note that if we continue to neglect the small quantity $C(\vec{p})$, we find

$$\Sigma_{ss'}^{++}(\vec{p}) \simeq \left[A(\vec{p}) + \frac{E_N(\vec{p})}{m_N} B(\vec{p})\right]\delta_{ss'} \tag{9.8}$$

and

$$\Sigma_{ss'}^{--}(\vec{p}) \simeq \left[-A(\vec{p}) + \frac{E_N(\vec{p})}{m_N} B(\vec{p})\right]\delta_{ss'} . \tag{9.9}$$

The energy of a particle in the medium (with $p^0 > 0$) is given by

$$p^0(\vec{p}) = B(\vec{p}) + [\vec{p}^2 + (m_N + A(\vec{p}))^2]^{1/2} \tag{9.10}$$

$$= E_N(\vec{p}) + V_{\rm eff}(\vec{p}) , \tag{9.11}$$

where

$$V_{\rm eff}(\vec{p}) \simeq \frac{m_N}{E_N(\vec{p})} A(\vec{p}) + B(\vec{p}) + \left[\frac{|\vec{p}|}{E_N(\vec{p})}\right]^2 \frac{A^2(\vec{p})}{2[E_N(\vec{p}) + A(\vec{p})]} \tag{9.12}$$

is the effective potential energy for a nucleon in nuclear matter in this theory. In terms of the quantity given in Eq. (9.10), one may write for the energy of nuclear matter[3]

$$E = \tfrac{1}{2} \int \frac{d\vec{p}}{(2\pi)^3} \{p^0(\vec{p}) + E_N(\vec{p})[1 - \sin^2\theta_{m_N}^{\tilde{m}}(\vec{p})]\} . \tag{9.13}$$

It is clear from Eq. (9.12) that an accurate calculation of the energy requires a very accurate calculation of A and B. Since $A \simeq -400$ MeV and $B \simeq 300$ MeV,[11] even small errors in either of these quantities would lead to large errors in $V_{\rm eff}(\vec{p})$ of Eq. (9.12) and an even larger percentage error in $p^0(\vec{p})$.

We can consider a sequence of approximations which would describe systematic improvement in the calculation of A and B. As a first step one might work in the relativistic Hartree or mean-field approximation. Introduction of the relativistic Hartree-Fock approximation typically reduces the Hartree matrix elements of the interaction by about 25%. At the level of the Hartree-Fock approximation one has $A(0) \simeq -405$ MeV, $B(0) \simeq 375$ MeV, and $A(0) + B(0) \simeq -30$ MeV. In this approximation nuclear matter is unbound. Inclusion of short-range correlation effects yields $A(0) \simeq -396$ MeV, $B(0) \simeq 300$ MeV, and $A(0) + B(0) \simeq -96$ MeV.[11] (These numbers refer to calculations made at $k_F = 1.36$ fm^{-1}.)

As can be seen from these comments, extremely accurate calculations of exchange effects and short-range correlation effects are required if one is to obtain accurate values for the binding energy of nuclear matter.

X. THE WAVE FUNCTION OF NUCLEAR MATTER

In the mean-field or Hartree approximation the mass $\tilde{m}$ would be a constant and therefore the wave function for nuclear matter would be

$$|\mathrm{NM}\rangle = \prod_{\vec{p},s} b^\dagger(\vec{p}, s, \tilde{m}) \,|\, \mathrm{vac}; \tilde{m}\rangle . \tag{10.1}$$

This wave function is somewhat unrealistic since it implies that all negative-energy states, including those of very large momentum have their mass parameter modified from m_N to $\tilde{m}$. To address this question we should consider the momentum dependence of the solutions of the equation which determines the mass of a state of momentum $\vec{p}$,

$$m(\vec{p}) = m_N + \tfrac{1}{4}\mathrm{tr}\Sigma(\vec{p}, k_F, m(\vec{p})) . \tag{10.2}$$

For a relativistic quasiparticle with momentum $\vec{p}$ we have, for $p^0 < 0$,

$$p^0(\vec{p}) = B(\vec{p}) - [\vec{p}^{\,2} + m^2(\vec{p})]^{1/2} . \tag{10.3}$$

The solution of Eq. (10.2) for the mass parameter of states of negative energy is to be constructed such that Eq. (10.3) is satisfied. It is reasonable to assume that $m(\vec{p}) \to m_N$ as $|\vec{p}| \to \infty$. Once we accept the momentum dependence of $m(\vec{p})$, we can introduce a vacuum state which is more reasonable (on physical grounds) than that of Eq. (2.40). We generalize the vacuum state given there to

$$|\,\mathrm{vac}; m(\vec{p})\rangle = \prod_{\vec{p},\lambda} [\cos\theta_0^{m(\vec{p})} - \sin\theta_0^{m(\vec{p})} b^\dagger(\vec{p}, \lambda, 0) d^\dagger(-\vec{p}, \lambda, 0)] \times |\,\mathrm{vac}; 0\rangle , \tag{10.4}$$

where $\theta_0^{m(\vec{p})} \to 0$ as $|\vec{p}| \to \infty$. Equation (10.1) is generalized to read

$$|\mathrm{NM}\rangle = \prod_{\vec{p},s} b^\dagger(\vec{p}, s, \tilde{m}) \,|\, \mathrm{vac}; \tilde{m}(\vec{p})\rangle . \tag{10.5}$$

In this equation we have continued to neglect the weak momentum dependence $m(\vec{p})$ for *positive*-energy states with $0 \leq |\vec{p}| \leq k_F$. For such states we have $\tilde{m}(\vec{p}) \simeq [540 + 25\,(|\vec{p}|/k_F)^2$ MeV] [see Eq.

(8.2)].

As a further refinement we note that Eq. (10.5) is only valid in a theory without short-range correlations. If we include the effects of short-range correlations using the independent-pair approximation, we have[20]

$$|\mathrm{NM}\rangle = \mathfrak{N}\exp[S]\prod_{\vec{p},s} b^\dagger(\vec{p}, s, \tilde{m})\,|\,\mathrm{vac}; \tilde{m}(\vec{p})\rangle \tag{10.6}$$

with

$$S=\sum b^\dagger(\vec{p}, s_1, \tilde{m})b^\dagger(\vec{p}, s_2, \tilde{m})\left\langle\Phi(\vec{p}, s_1, \tilde{m})\Phi(\vec{p}_2, s_2, \tilde{m})\left|\frac{Q}{e}\hat{M}(1-P_{12})\right|\phi(\vec{p}_3, s_3, \tilde{m})\phi(\vec{p}_4, s_4, \tilde{m})\right\rangle b(\vec{p}_4, s_4, \tilde{m})b(\vec{p}_3, s_3, \tilde{m})\,. \tag{10.7}$$

In Eq. (10.7) the operator Q restricts the values of $\vec{p}_1$ and $\vec{p}_2$ to be outside of the Fermi sea of positive-energy particles; $\vec{p}_4$ and $\vec{p}_3$ represent states inside this Fermi sea and e is an energy denominator appropriate to the relativistic self-consistent theory. Further, $\mathfrak{N}$ is a normalization factor and

$$\phi(\vec{p}, s, \tilde{m}) \equiv [m_N/E_N(\vec{p})]^{1\;2} f(\vec{p}, s, \tilde{m})\,.$$

XI. CONCLUSION AND SUMMARY

In this work we have attempted to show how the introduction of some concepts of quantum field theory in the study of nuclear structure leads to a unified description. We are able to bring together a series of ideas which include the Nambu–Jona-Lasinio analogy to superconductivity, the σ model, the Weinberg effective Lagrangian, the one-boson-exchange model of nuclear forces, and a (relativistic) independent-pair approximation for the study of nuclear matter. To this end we have considered three vacuum states: $|\,\mathrm{vac}; 0\rangle$, in which the quasiparticle spectrum is $\epsilon_\pm(\vec{p}) = \pm\,|\vec{p}\,|$; $\mathrm{vac}; m_N\rangle$ in which the quasiparticle spectrum is $\epsilon_\pm(\vec{p}) = \pm(\vec{p}^{\,2} + m_N^2)^{1/2}$; and $|\,\mathrm{vac}; \tilde{m}(\vec{p})\rangle$, in which the spectrum is $\epsilon_\pm(\vec{p}) \simeq B(\vec{p}) \pm [\vec{p}^{\,2} + \tilde{m}^2(p)]^{1/2}$. [In the latter case $\epsilon_+(0) \simeq 840$ MeV and $\epsilon_-(0) \simeq -240$ MeV.] [We may also define a gap parameter $\Delta = \epsilon_+(0) - \epsilon_-(0)$. For $|\,\mathrm{vac}; 0\rangle$, $\Delta = 0$; for $|\,\mathrm{vac}; m_N\rangle$, $\Delta = 2m_N$; and for $|\,\mathrm{vac}; \tilde{m}(p)\rangle$, $\Delta = 2\tilde{m}(0) \simeq 1080$ MeV. It may also be useful to consider this parameter to be a function of the baryon density $\Delta(k_F)$. Then $\Delta(k_F)$ can take on various values between $\Delta(k_F = 0) = 2m_N$ and $\Delta(k_F = 1.36\ \mathrm{fm}^{-1}) = 2\tilde{m}(0)$.]

Our discussion of mass generation through symmetry breaking, which makes use of the σ model, is schematic and neglects quantum fluctuations. Within the context of an effective Lagrangian approach we stress the great importance of a detailed treatment of quantum-fluctuation effects in a successful calculation of the properties of nuclear matter. To our knowledge all previous applications of effective Lagrangians to the study of *relativistic* nuclear matter[7] have used the mean field, Hartree, or Hartree-Fock approximation. The use of such approximations requires the introduction of two or more parameters that play the role of effective coupling constants. Many calculations are limited to a study of only the σ and ω fields, while we have seen that the pion field can be quite important in detailed calculations.[11]

In order to perform our calculations we have introduced what we have termed "the pair-current expansion." In this case the self-consistent spinor (with mass parameter $\tilde{m}$) is expanded in terms of spinors of positive *and* negative energy and of mass parameter m_N. We remind the reader that the self-energy parameter A, which governs the mass change on going from vacuum to nuclear matter ($\tilde{m} = m_N + A$), also determines the parameter that measures the size of the pair-current effects, $\tan\theta_{m_N}^{\tilde{m}} \simeq (-\,|\vec{p}\,|/m_N)[A/2(m_N + A)]$ [see Eq. (8.4)], if one uses the pair-current expansion, expanding the self-consistent spinor [Eq. (7.1)] in terms of spinors of positive *and* negative energy and of mass parameter m_N. We see that working with a spinor basis that refers to the symmetry breaking of the vacuum, while treating nuclear matter, leads to systematic corrections which can be interpreted as pair-current effects. (This feature is well known in the theory of electron scattering in which one attempts to correct the nonrelativistic density matrix of the target nucleus through the introduction of pair currents. The appearance of pair currents in the analysis of electron scattering is a reflection of the use of nonrelativistic wave functions which do not provide a self-consistent basis.[21]) One useful aspect of our pair-current expansion is that the leading term can be seen to reproduce conventional nuclear structure calculations. Except for a small number of works that have made use of the Walecka model,[7,22] standard nuclear physics calculations have used positive-energy spinor states with mass parameter m_N. For this reason, the standard calculations have not been able to give a good account of the saturation properties of nuclear matter. Inclusion of pair-current effects when calculating other nuclear properties lead to more subtle effects. One of the more interesting of these effects is the introduction of a strong density dependence of the effective interaction between relativistic quasiparticles in nuclei.[23] This density dependence is in accord with that found in phenomenological studies of the effective force in nuclei.

In future works we plan to extend the ideas developed here to a study of finite nuclei. Nonrelativisitic theories have failed to provide a satisfactory account of the size and binding energy of finite nuclei without the introduction of several phenomenological parameters and density-dependent interactions. We hope to demonstrate that, as in the case of nuclear matter, we can understand theproperties of finite nuclei in a theory that introduces no parameters in addition to those determined in studies of nucleon-nucleon scattering using the one-boson-exchange model of nuclear forces.

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APPENDIX

The spinor transformations we have been using have some formal similarity to the well known Foldy-Wouthuysen (FW) transformation. We wish to emphasize that the manipulations involving the FW transformation have little to do with the analysis given in our work. In our work we have used unitary transformations of the spinors and the creation and destruction operators of the theory in order to express the Dirac field in various normal-mode expansions. The Dirac field and the Hamiltonian are invariant under the transformations we have considered. However, it may be of some interest to contrast the unitary FW transformation and the unitary transformation we have introduced in this work when we consider a *one-body* Hamiltonian.

Consider the Hamiltonian, $H=\vec{\alpha}\cdot\vec{p}+\beta m$ and the unitary transformation $\exp(\theta\vec{\gamma}\cdot\vec{p})$, where θ is an arbitrary (real) angle. Now

$$H'=e^{\theta\vec{\gamma}\cdot\vec{p}}He^{-\theta\vec{\gamma}\cdot\vec{p}} \tag{A1}$$

$$=\vec{\alpha}\cdot\vec{p}\left(\cos 2\theta-\frac{m}{|\vec{p}|}\sin 2\theta\right)+\beta(m\cos 2\theta+|\vec{p}|\sin 2\theta)\,. \tag{A2}$$

In the case of the FW transformation one chooses to eliminate the *odd* operators. This condition requires that one chooses $\theta=\theta_{\rm FW}$, where $\tan(2\theta_{\rm FW})=|\vec{p}|/m$. Thus one finds

$$H'=\beta(m^2+|\vec{p}|^2)^{1/2}=\beta E_m(\vec{p})\,. \tag{A3}$$

One may also choose to eliminate the even operators. This requires that we take $\theta=\theta_0^m$, where $\tan(2\theta_0^m)=-m/|\vec{p}|$ [see Eq. (2.10)]. In this case we find

$$H''=\frac{\vec{\alpha}\cdot\vec{p}}{|\vec{p}|}E_m(\vec{p})\,. \tag{A4}$$

We remark that in this appendix we are discussing a unitary transformation of the Hamiltonian. Therefore H, H', and H'' all have the same spectrum. The various eigenfunctions are different. For example, the positive-energy eigenfunctions of H are $u(\vec{p},s,m)$, corresponding to the eigenvalue $E_m(\vec{p})$. The eigenfunctions of H' corresponding to the same eigenvalue $E_m(\vec{p})$ are $u(\vec{p},s,0)$.

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