Resonances and decay of the D^0 meson

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A model of D^0 nonleptonic decay is studied in which the D^0 meson is mixed by the weak Hamiltonian to $0^{\pm}$ resonances which then decay.

I. INTRODUCTION

At present there exists no entirely satisfactory theoretical description of charmed-meson decays. However, the importance of final-state interactions and/or dominance of s-channel $I=\frac{1}{2}$ states has been stressed.[1] There are several reasons for this: Final-state mesons emerging from D decay have energies at which resonant interactions are known to occur, data on two-body D^0 modes are consistent with such a picture, and aspects of three-body D^0 modes can be understood on this basis.[2]

While this emphasis on final-state interactions provides an attractive way to analyze the data, it is nonetheless disappointing that the underlying quark dynamics plays a less than central role. In an attempt to address this matter, we consider a hybrid approach containing elements of both the S-matrix and quark descriptions. We use the quark model explicitly to compute the mixing induced by the weak Hamiltonian of D^0 into noncharmed quark configurations of either parity which then propagate and decay (see Fig. 1). Such amplitudes are not present in D^+ decay if the noncharmed resonance carries isospin one-half.

A presumably reliable method for estimating weak transitions involving the D^0 meson is to employ the quark wave functions of the spherical bag model. The *spherical* nature of the bag for a heavy, flavored meson like D^0 is in accord with a picture of a light, relativistic quark moving near a massive nonrelativistic quark. Moreover, because only resonances whose mass difference with D^0 is not substantial are considered, a *static* description of the meson bags is appropriate. Crossed processes, where the weak transition occurs on either final-state leg in Fig. 1, are ignored because the large propagator denominators effectively suppress their contributions.

It is useful to briefly review data pertinent to this analysis. For S-wave 0^-0^- final states, we employ decay amplitudes A_S obeying $|A_S|^2 = 8\pi\Gamma M_D{}^2/p$, and for P-wave 0^-1^- final states, we use decay amplitudes A_P such that $|A_P|^2 = 8\pi\Gamma m_V{}^2/p^3$. Amplitudes A_S have units of energy, whereas the A_P are dimensionless. Our notation is standard: p is the decay momentum, m_V is the decay vector-meson mass, and Γ is the decay width into some mode, $\Gamma=\hbar B/\tau$ where τ is the total D^0 lifetime, and B is the branching ratio for the particular mode. We take[3] $\tau=1.01^{+0.43}_{-0.27}\times 10^{-13}$ sec, $B(K^-\pi^+)=0.030\pm0.006$, $B(\bar{K}^0\pi^0)=0.022\pm0.011$, $B(K^-\rho^+)=0.072\pm0.025$, $B(K^{*-}\pi^+)=0.032\pm0.01$, $B(\bar{K}^{*0}\pi^0)=0.014^{+0.019}_{-0.014}$, and $B(K^0\rho^0)=0.001^{+0.004}_{-0.001}$ in this paper. The range of D^0 total lifetimes implies a range in the decay amplitudes $|A_S| = 1.0\text{–}1.9\times10^{-5}B^{1/2}M_D$ for the S-wave $K\pi$ decays, $|A_P|=1.5\text{–}2.1\times10^{-5}B^{1/2}M_D$ for the P-wave $K\rho$ decays, and $|A_P|=1.6\text{–}2.2\times10^{-5}B^{1/2}$ for the P-wave $K^*\pi$ decays. These values provide benchmarks for our theoretical computations.

II. WEAK TRANSITION AMPLITUDES

For $|\Delta C|=1$ Cabibbo-favored transitions, the nonleptonic weak Hamiltonian is

$$H_W=\frac{G_F}{2\sqrt{2}}\cos^2\theta_C(c_+H_+ + c_-H_-)\,, \tag{1}$$

where

$$H_\pm=\bar{s}_i\gamma^\mu(1+\gamma_5)c_i\bar{u}_j\gamma_\mu(1+\gamma_5)d_j \pm \bar{s}_i\gamma^\mu(1+\gamma_5)c_j\bar{u}_j\gamma_\mu(1+\gamma_5)d_i\,. \tag{2}$$

The operators in Eq. (2) are understood to be normal-ordered and $i,j=1,2,3$ are color indices. We employ the simple Cabibbo prescription for mixing angles.[4] The enhancement and suppression coefficients c_- and c_+ are given by[5]

$$c_-=\left(\frac{\alpha_S(m_c{}^2)}{\alpha_S(M_W{}^2)}\right)^{0.57}\left(1+1.59\,\frac{\alpha_S(m_c{}^2)-\alpha_S(M_W{}^2)}{\pi}\right),$$
$$c_+=\left(\frac{\alpha_S(m_c{}^2)}{\alpha_S(M_W)}\right)^{-0.29}\left(1-0.41\,\frac{\alpha_S(m_c{}^2)-\alpha_S(M_W{}^2)}{\pi}\right), \tag{3}$$

where $\alpha_S(\mu^2)$ is the quantum-chromodynamics (QCD) running coupling strength evaluated at scale μ^2. For the value $\Lambda=0.25$ GeV of the QCD scale factor, we find $c_-=2.3$, $c_+=0.6$. These $c_\pm$ coefficients contain complete one-loop and leading-logarithmic two-loop contributions. Also, observe that H_W contains only left-handed operators.

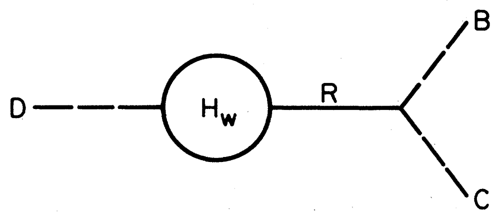

FIG. 1. Weak mixing of D^0 into meson R which then decays into mesons B and C.

There are no "penguin" contributions which would introduce right-handed components to H_W.

The amplitude of Fig. 1 can be written in the form

$$\langle R|H_W|D^0\rangle[M_D{}^2 - M_R{}^2 - i\Gamma_R(M_DM_R)^{1/2}]^{-1}g(R\to AB)\,, \tag{4}$$

where R is the intermediate-state meson resonance of mass M_R, width Γ_R, and coupling strength to final state $g(R\to AB)$. For definiteness consider a parity-violating transition [see Fig. 2(a)] $D^0 \to \bar{K}^0$, where $\bar{K}^0$ is a 0^+ meson containing the $q\bar{q}$ P-wave configuration $s\bar{d}$. Evaluation in the bag model yields the expression

$$\langle \bar{K}^0|H_W|D^0\rangle = G_F\cos^2\theta_C(c_- - 2c_+)(4M_DM_K)^{1/2} \times R^{-3}NN''(N'\tilde{N}I_1 - \tilde{N}'NI_2)\,, \tag{5}$$

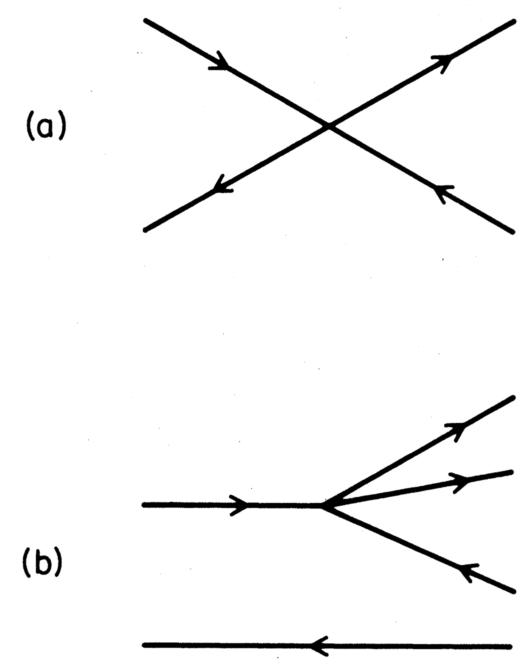

FIG. 2. Two mechanisms for the weak mixing of the D^0 meson. The initial quark configuration $c\bar{u}$ is converted by the weak Hamiltonian into (a) a $q\bar{q}$ meson, or (b) a $q^2\bar{q}^2$ "primitive" (Ref. 10).

where unprimed, primed, and doubly primed quantities refer to u-, s-, and c-quark kinematics, respectively, R and N are the bag radius and normalization factors, a tilde implies occupation in a bag P-wave mode, and

$$I_1 = \frac{1}{4\pi}\int_0^1 u^2du[(f''f - g''g)(-f'\tilde{f} + g'\tilde{g}) + (fg''+f''g)(f'\tilde{g}+g'\tilde{f})]\,, \tag{6a}$$

$$I_2 = \frac{1}{4\pi}\int_0^1 u^2du[f''f - g''g)(-\tilde{f}'f + \tilde{g}'g) + (fg''+f''g)(\tilde{f}'g+\tilde{g}'f)]\,. \tag{6b}$$

In Eqs. (6a) and (6b), each quantity in the brackets has argument consisting of the appropriate mode momentum times the integration variable u, e.g., $f'' \equiv f(p''u)$, etc. Our notation is $f = j_0$, $g = [(\omega - mR)/(\omega + mR)]^{1/2}j_1$, $\tilde{g} = [(\omega + mR)/(\omega - mR)]^{1/2}j_1$, where m is the quark mass, $\omega = (p^2 + m^2R^2)^{1/2}$, and $j_{0,1}$ are spherical Bessel functions.

The combination $c_- - 2c_+$ in Eq. (5) occurs because the color content of the second operator in H_W contributes three times that of the first operator. Incidentally, the combination $c_- - 2c_+$ has the numerical value 1.1, thus hardly affecting the calculation. Also, in the SU(3)-invariant limit of equal strange and nonstrange kinematics, the quantity $N'\tilde{N}I_1 - \tilde{N}'NI_2$ vanishes. This can be traced to (i) the quark structure of $\bar{K}^0$ (it belongs to an octet of positive charge conjugation and thus is s, d flavor antisymmetric in the bag-model wave function), and (ii) the flavor content of H_W which, via a Fierz transformation, can be cast in a form in which the s and d operators appear symmetrically.

Next consider the parity-conserving transition $D^0 \to \bar{K}^{0\prime}$, where $\bar{K}^{0\prime}$ is a pseudoscalar state. We obtain an amplitude analogous to that of Eq. (5) except with the replacement of $N'\tilde{N}I_1 - \tilde{N}'NI_2$ by $N'\bar{N}J_1 + \bar{N}'NJ_2$, where

$$J_1 = \frac{1}{4\pi}\int_0^1 u^2du[(f'f'' - g'g'')(fF - gG) - (f'g''+g'f'')(fG+gF)]\,, \tag{7a}$$

$$J_2 = \frac{1}{4\pi}\int_0^1 u^2du[(F'f'' - G'g'')(f^2 - g^2) - 2fg(F'g''+G'f'')]\,. \tag{7b}$$

In the bag model the state $\bar{K}^{0\prime}$ contains a $q\bar{q}$ pair both in an S-wave bag mode but with one of the pair excited. We denote such an excited mode by a overbar for the normalization factors and a capital letter for the quark wave functions in Eqs. (7a) and (7b).

Now we perform a numerical study of our formulas.[6] Assume that $D^0 \to \pi^+K^-$, $\pi^0\bar{K}^0$ proceeds specifically via the 0^+ intermediate state $\kappa(1500)$. From the total width $\Gamma(\kappa(1500)) \simeq 0.25$ GeV and dominance of final states by the $K\pi$ channel, we deduce $g(\bar{\kappa}^0K^-\pi^+) \simeq 3.8$ GeV and $g(\bar{\kappa}^0\bar{K}^0\pi^0) \simeq 2.7$ GeV. This difference in coupling strengths, in qualitative accord with the observed $K^-\pi^+$, $\bar{K}^0\pi^0$ branching ratios, is of course one of the motivations for considering $I=\frac{1}{2}$ final-state dominance. The $\kappa(1500)$ propagator contributes 0.8 GeV^{-2}, most of the contribution arising from the real part, i.e., the D-κ squared-mass difference. Inserting these values into Eq. (5) for, e.g., the $K^-\pi^+$ mode, we have

$$|A_S(D^0K^-\pi^+)| = 0.11 \times 10^{-6}(R/R_\pi)^{-3}F(R/R_\pi)M_D\,, \tag{8}$$

where F is the ratio of bag-related quantities in Eq. (5) evaluated at radii R, R_π (see Table I. The work of Ref. 7 has placed in doubt previous spectroscopic determinations of R for charmed mesons, while not affecting the bag-wave functions. Observation of the $D^\pm$ leptonic mode would provide an estimate for R.[8] In lieu of information about the bag radii of charmed mesons, we consider a range of values. For $0.2 \le R^{-1} \le 0.6$ GeV, we find $|A_S(D^0K^-\pi^+)|$ to vary over the range (0.05–0.4) $\times 10^{-6}M_D$.

The only 0^- resonance accessible to a parity-conserving mixing with D^0 is $\kappa'(1400)$.[6] Unfortunately this state has been analyzed in just one experiment,[9] where it was found that $\Gamma(\kappa'(1400)) \simeq 0.25$ GeV with $\epsilon(700)K$ predominant among the decay modes. Although the lack of data curtails our analysis of $D^0 \to K\pi\pi$ transitions, we can make several observations. Clearly, if $\kappa'(1400)$ plays a significant role in $D^0 \to K\pi\pi$ decays, the $\pi\pi$ data should be examined for an $I=0$ enhancement near 0.7 GeV. The apparent large $\Delta I=\frac{3}{2}$ signal in $D^0 \to \rho K$ is consistent with the apparent lack of coupling between $\kappa'(1400)$ and ρK.[2] If further study of $\bar{\kappa}^{0\prime}(1400)$ observes a branching ratio B_{-+} into, say, the $K^{*-}\pi^+$ channel, then analogous to Eq. (8) we can write

$$|A_P(D^0 \to K^{*-}\pi^+)| = 0.015 \times 10^{-6}(R/R_\pi)^{-3}G(R/R_\pi)B_{+-}{}^{1/2}\,, \tag{9}$$

where G is the ratio of bag-related quantities (see Table I) appropriate to this transition. The magnitude of $A_P(K^{*-}\pi^+)$ rises sharply with R^{-1}, equaling $1.24 \times 10^{-6}B_{+-}{}^{1/2}$ for $R^{-1} = 0.6$ GeV. The sharp variation of $G(R/R_\pi)$ evidenced in Table I is due to the fact that $G(R/R_\pi)$ has a zero near our normalization point of $R=R_\pi$.

TABLE I. $F(R/R_\pi)$ and $G(R/R_\pi)$ as a function of R. These quantities appear in Eqs. (8) and (9).

R^{-1} (MeV)	$F(R/R_\pi)$	$G(R/R_\pi)$
200	1.60	−8.96
300	1.00	1.00
400	0.71	5.90
500	0.53	8.71
600	0.42	10.5

Data tables of $q\bar{q}$ mesons may not exhaust the spectrum of multiquark configurations. Perhaps, as suggested in Ref. 10, $q^2\bar{q}^2$ composites occur in the energy range of interest for charm decay. Mass estimates exist only for positive-parity $J=0$ $q^2\bar{q}^2$ states, so we discuss just two-body D^0 decay here. Three $q^2\bar{q}^2$ multiplets (called 9*, 36, 36*) have states of possible significance. The recoupling coefficients of the 9* and 36* multiplets to 0^-0^- color-singlet pairs are so small (0.178, 0.041, respectively) as to render these multiplets useless for explaining D^0 decay, but the substantial recoupling coefficient (0.644) of 36 motivates further study of this multiplet. It contains three mesons which carry strangeness −1, an octet state $C_K^s(1750)$, and two 27-plet states $E_{\pi K}(1350)$, $C_K(1350)$. Observe the near degeneracy of the isospin $C_K^s(1750)$ with $D^0(1863)$. However, SU(3) mixing results in an $s\bar{s}$ pair in the $C_K^s(1750)$ wave function. Such a state cannot be mixed with D^0 by means of the weak Hamiltonian of Eq. (2). Even if some operator could induce such a mixing, the $C_K^s(1750)$ would decay predominantly to the ηK channel. There appears to be no experimental evidence for this mode at present. As for the states at 1.35 GeV, the $I=\frac{3}{2}$ $E_{\pi K}(1350)$ has the larger amplitude which, however, is proportional to the suppression coefficient C_+, and is further decreased by the smaller propagation function. As a result, we can see why a strong $\Delta I=\frac{3}{2}$ effect, at least from this source, is lacking in $D^0 \to \bar{K}\pi$ decays.

III. CONCLUSION

It is a natural extension within the framework of final-state interactions to explore whether *specific* quark configurations play a dynamic role in D^0 decay. Our study yields several insights and also motivations for future study.

For example, we have seen that existing data suggest $q^2\bar{q}^2$ states have little influence on $\pi\bar{K}$ decay modes of D^0. However, an effort should be made to obtain an experimental limit on the reaction $D^0 \to \eta\bar{K}$. This mode can arise from weak mixing between D^0 and the nearly degenerate $q^2\bar{q}^2$

level $C_K^s(1750)$. If observed, this would constitute evidence for a penguin-type $s\bar{s}$ pair term in the weak Hamiltonian, a rather important finding. The lack of hard data on the $\kappa'(1400)$ meson frustrates an analysis of the 0^-1^- component of $K\pi\pi$ decay modes. Yet the $D^0 \to K^*\pi$ amplitude can be roughly fit provided that the $\kappa'(1400)$ branching ratio into $K^*\pi$ is not too small, say $B_{+-} > 0.2$. It would be interesting therefore to have additional experimental effort on $\kappa'(1400)$ decay systematics and, conversely, to determine whether there is a $K\epsilon(700)$ signal in $D^0 \to K\pi\pi$ data.

On the theoretical side, our model can begin to reproduce the experimental amplitudes only if the heavy D^0 and κ mesons are rather smaller than the light K and π mesons. This would be the case if mesons were to mimic the decrease in size found in going from positronium to the hydrogen atom, i.e., as the reduced mass increases. Is there any evidence for this tendency in mesons? A hadron bag with two relativistic quarks rattling around within is likely to be larger than a bag with only one light quark plus a heavy quark which presumably would tend to stay near the bag center. This picture is borne out to some extent by considering the contribution of a given quark to the charge radius, $\langle r^2 \rangle_q = R^2 A(m_q R)/6$ as a function of quark mass for fixed R.[11] We find $A(0)/A(\infty) \simeq 2$ for the limiting cases of zero and infinite quark mass, implying that the heavy quark indeed stays away from the bag walls. Or one can use the harmonic-oscillator relation $\langle r^2 \rangle \propto (mE_0)^{-1}$, where E_0 is the ground-state energy and m is the reduced mass. In this case, the D^0 meson is more than a factor of 2 smaller than the ρ meson. It should be noted that in the bag model, even with $R/R_\pi = 0.5$, our $D^0 \to K\pi$ amplitude is somewhat too small [see Eq. (8)]. Perhaps the bag-model wave functions are inadequate, although for nonleptonic decays of light hadrons there is general agreement between bag and harmonic-oscillator fits.[12] The resonance picture itself is not above criticism. Effects of helicity suppression are expected in 0^--to-0^- matrix elements of purely left-handed operators such as H_W, raising the possibility that mixing amplitudes might be too small in any specific quark model. However, in view of uncertainties in the D^0 lifetime, it is premature to insist on exact agreement between theory and experiment. More knowledge is first needed about the size of heavy-flavored mesons. Work on this subject has begun.

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