Nonleptonic weak decays of mesons

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A unified view for nonleptonic decays of mesons is presented on the basis of a model introduced for interpreting the enhancement of D- and K-meson decay. We show that inclusive and exclusive nonleptonic decays of mesons can be described in a unified picture and the enhancements in inclusive decays are closely related to two-body decays (0^-0^- decays). The model is applied to the decays of K, charmed, and B mesons and predicts the values $[\Gamma(K^+\to\pi^+\pi^0)]^{-1}\simeq 6.2\times10^{-8}$ sec (expt. 5.9×10^{-8} sec), $\tau^+/\tau^0\simeq 3$–6, $\tau^F/\tau^0\simeq 1.3$–1.5, $B(B^0\to e\nu X)\simeq 7$–12%, $B(B^-\to e\nu X)\simeq(10$–$12)\%$, where τ^0 (τ^+,τ^F) is the lifetime of the D^0 (D^+,F) meson and $B(A\to e\nu X)$ stands for the semileptonic branching ratio. The large difference between the ratios, $\Gamma(D^0\to\bar K^0\pi^0)/\Gamma(D^0\to K^-\pi^+)\simeq 0.7^{+0.7}_{-0.4}$ and $\Gamma(D^0\to\bar K^0\rho^0)/\Gamma(D^0\to K^-\rho^+)\simeq 0.01^{+0.10}_{-0.01}$, is naturally understood as the difference between the s-wave decays and the p-wave decays. We also point out that the model can include many of the models proposed up to this point, e.g., W-exchange model, quark-number-conservation model, penguin diagram, etc. We comment that the $\Delta I = 1/2$ rule for hyperon decays is easily understood in the model.

I. INTRODUCTION

A new approach to nonleptonic decays of hadrons proposed by the author and Yamazaki[1,2] gives us a simple view for the enhancement of D-meson decays and K-meson decays. It has also been shown that the approach can be consistently applied to decays of B mesons with b-quark flavor.[3] Up to this point this model was applied only to inclusive decays of mesons such as semileptonic branching ratios. In order to see the consistency of the model, however, we must study exclusive decays. A few unknown parameters introduced in the analysis of the inclusive decays[1-3] may be determined in the analysis of exclusive decays. Fortunately, two-body decays of D mesons are going to be clear in experiments[4-6] and some interesting results are found; for example, the ratios[5,6]

$$\begin{aligned} r(K\pi) &\equiv \frac{\Gamma(D^0\to\bar K^0\pi^0)}{\Gamma(D^0\to K^-\pi^+)} = 0.7^{+0.7}_{-0.4}\,, \\ r(K^*\pi) &\equiv \frac{\Gamma(D^0\to\bar K^{*0}\pi^0)}{\Gamma(D^0\to K^{*-}\pi^+)} = 0.4^{+1.1}_{-0.4}\,, \\ r(K\rho) &\equiv \frac{\Gamma(D^0\to\bar K^0\rho^0)}{\Gamma(D^0\to K^-\rho^+)} = 0.01^{+0.10}_{-0.01} \end{aligned} \tag{1.1}$$

seem to indicate that decay mechanisms for $D^0\to\bar K\pi$ and $D^0\to\bar K\rho$ are quite different. In fact, the very small value of $r(K\rho)$ contradicts the prediction $[r(K\pi)=r(K^*\pi)=]\ r(K\rho)=\frac12$ derived from the assumption of the dominance of amplitudes with $I=\frac12$ final states, which is predicted by W exchange[7] and the quark-number-conservation ($\Delta n_q=0$) rule.[8] We may expect that the analysis of two-body decays exposes the characteristic features of the models.

In this paper we shall investigate inclusive and exclusive nonleptonic decays of mesons, e.g., K mesons, charmed mesons, and B mesons, in a unified view based on the model proposed in Refs. 1–3. In particular, we shall see that the introduction of symmetry breaking among quark flavors ignored in Refs. 1–3 is very important and also that the contribution of nonexotic intermediate states to the s-wave cross section (σ_R^s) is determined in the analysis of two-body decays. The importance of symmetry breaking is already well known from the fact that the prediction of the SU(3) symmetry $\Gamma(D^0\to K^-K^+)\simeq\Gamma(D^0\to\pi^-\pi^+)$ is badly broken in the observations.[5,6] In decays of B mesons we must take account of symmetry breaking among c, b, t, and light quarks (u, d, and s). Of course, it is impossible to estimate all of these symmetry-breaking effects from the presently available experimental data. For the decays of charmed mesons, however, we shall point out a simple method for describing symmetry breakings (in Sec. III). Another important parameter σ_R^s, which was estimated to be 1–2 mb from the quark-counting ansatz,[1] will be shown to be closely connected with the imaginary part of amplitudes for two-body decays and we show that a smaller value ($\sigma_R^s\simeq 0.6$ mb) is derived from $D\to\bar K\pi$ decays (in Sec. III). We shall also discuss the enhancement of p-wave decays and show that the large difference between $r(K\pi)$ and $r(K\rho)$ is due to no enhancement in the p-wave decays such as $D\to\bar K\rho$.

In Sec. II we shall investigate our model in an extended version and derive general formulas for inclusive and exclusive (two-body) decays. The charmed-meson decays will be discussed in Sec. III, where we shall study a simple method for introducing symmetry breaking and derive $\sigma_R^s\sim 0.6$ mb from two-body decays. Inclusive decays of charmed mesons will also be reanalyzed by using $\sigma_R^s \simeq 0.6$ mb. In Sec. IV, inclusive and two-body decays of B mesons will be discussed and we shall obtain values a little bit different from those given in Ref. 3 for semileptonic branching ratios. Remarks on K-meson decays into two pions will be given in Sec. V. In Sec. VI, we shall show that other models, e.g., W exchange, the $\Delta n_q = 0$ rule, etc., can be represented in our approach, and an essential difference of nonleptonic decays of baryons from those of mesons and the $\Delta I = \frac{1}{2}$ rule for hyperon decays will also be discussed. Note that throughout the paper we shall discuss on the basis of the assumption of no weak flavor-changing neutral current.

II. A MODEL FOR NONLEPTONIC DECAYS OF PSEUDOSCALAR MESONS

Nonleptonic decay amplitudes may be written down in terms of the amplitudes with and without final-state interactions, which are illustrated by quark diagrams presented in Fig. 1, where the solid lines denote quarks or antiquarks and the shadowed regions with quark-line edges mean the color-singlet states described by one or some hadrons. In Fig. 1 the diagrams 1(a)–1(c), respectively, represent the amplitudes with no final-state interaction, with nonexotic final-state interactions, and with exotic final-state interactions. All of the exclusive nonleptonic decays can be written by one or a sum of some diagrams given in Fig. 1. For example, the decay amplitude for the process $D^0 \to K^- \pi^+$ is represented by

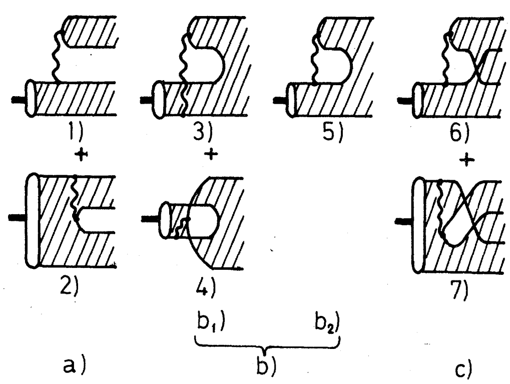

FIG. 1. Quark diagrams for nonleptonic decays of mesons, where solid lines and wavy lines, respectively, stand for quarks (or antiquarks) and weak bosons. The diagrams (a), (b) [(b_1) and (b_2)], and (c), respectively, represent the diagrams with no final-state interaction, with nonexotic-final-state interactions and with exotic-final-state interactions. Diagrams with topologically same structures for strong interactions are omitted here.

the sum of four diagrams shown in Fig. 2.

The total nonleptonic decay width is expressed in terms of the imaginary parts of the diagrams derived from the unitarity sum of the quark diagrams given in Fig. 1. In general discussions we find a lot of complicated diagrams derived from the cross terms. In the case of heavy-quark decays (heavy-meson decays), however, we may expect that the imaginary parts of the cross terms (the off-diagonal terms) between two different diagrams, i.e., (a)$\times$($b_{1,2}$), (a)$\times$(c), and ($b_{1,2}$)$\times$(c), will be small compared with the diagonal terms [(a)$\times$(a), ($b_{1,2}$)$\times$($b_{1,2}$), and (c)$\times$(c)] because phase differences between physical states represented by the different diagrams reduce the imaginary parts of the off-diagonal terms. The main contribution for heavy-quark decay may, therefore, be represented by the imaginary parts of the diagrams given in the left-hand side (LHS) of Fig. 3, where the dashed lines stand for the sum over physical states. When the decay-quark mass (the meson mass) is heavy enough, we may rewrite the diagrams in the LHS of Fig. 3 by the diagrams in the right-hand side (RHS) of Fig. 3. That is to say, the diagram (1) and (2) in the LHS may be reduced to the free-quark-decay diagrams[9] given by diagram (1) in the RHS, diagrams (3) and (4) in the LHS, respectively, to diagrams (2) and (3) in the RHS (the free-quark-exchange decay diagrams),[1,2] diagrams (5), (6), and (7), respectively, to the Regge-pole-exchange diagrams with nonexotic intermediate states[1,3] represented by diagrams (4), (5), and (6) in the RHS, diagrams (8) and (9) in the LHS, respectively, the Pomeron-exchange diagrams described by diagrams (7) and (8) in the RHS, and the last diagrams in the LHS to the Regge-pole-exchange diagrams with exotic intermediate states shown by the last one in the RHS. We know that the last diagram (the

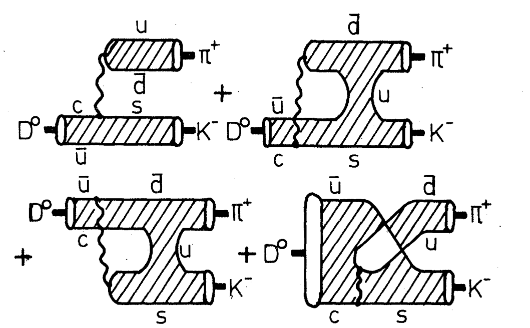

FIG. 2. Decay diagrams for $D^0 \to K^- \pi^+$. We shall call the second and third diagrams the u-quark annihilation diagram and the c-quark annihilation diagram, respectively.

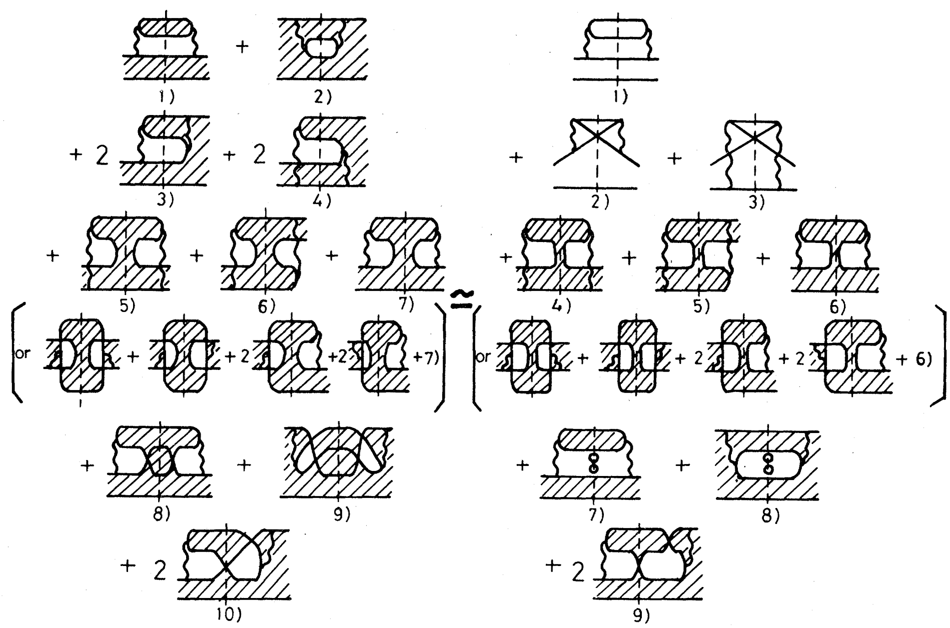

FIG. 3. Unitarity diagrams contributing to decays of heavy mesons, where dashed lines denote the sum over physical states and diagrams with topologically same structures for strong interactions are omitted. In the right-hand side diagram (1) stands for the free-quark-decay diagrams, diagrams (2) and (3) the free-quark-exchange-decay diagrams, (4), (5), and (6) (also those in brackets) the Regge-pole-exchange diagrams with nonexotic intermediate states, diagrams (7) and (8) the Pomeron-exchange diagrams, and diagram (9) the Regge-pole- exchange diagrams with exotic intermediate states.

exotic contribution) has only a small imaginary part. Then we may drop the diagram from the evaluation of the total nonleptonic decay width. Since the contribution of the Pomeron-exchange diagrams was shown to be very small in comparison with those of the first through the sixth diagrams in the RHS,[1] the diagrams may also be neglected. The diagrams left in the RHS[10] are nothing but the diagrams given in the previous works.[1-3]

Following Ref. 1, the contribution of one of the Regge-pole-exchange diagrams with the nonexotic intermediate states to the nonleptonic decay width of a meson (P) with the mass M is approximated by

$$\tilde{\Gamma}_R \simeq \frac{G_F{}^2 \prod_{i=1}^{4} \Theta_i}{2} M M_0^4 f_{P_1}{}^2 |f_+^{PP_2}(0) - f_-^{PP_2}(0)|^2 \sigma_R^s(P_1P_2)|I_R|^2 \tilde{N}_R K_a K_b , \qquad (2.1)$$

where G_F is the Fermi-coupling constant, $\prod_{i=1}^4 \Theta_i$ represents the product of the quark mixing parameters (e.g., the Cabibbo mixing parameters $\cos\theta_C$ and $\sin\theta_C$) appearing in the four weak vertices, M_0 the mass parameter in the form factors defined by $f_\pm^{PP_2}(q^2) = f_\pm^{PP_2}(0)/(1 - q^2/M_0^2)$, f_{P_1} the decay constant of the P_1 meson, $\sigma_R^s(P_1P_2)$ the nonexotic contribution to the s-wave cross section in the P_1P_2 scattering at $E_{\text{c.m.}} = M$, I_R (the loop integral) $\simeq 1/8\pi^2$, $\tilde{N}_R$ (the sum over possible scattering processes) = 16, and the short-distance enhancement factors K_a and K_b for two weak decay processes in the diagram should be chosen to be $K_1 = \frac{1}{2}[(1+k)c_+ + (1-k)c_-]$ (Ref. 11) for the weak vertices appearing in the diagrams (1), (3), (5), and (6) of Fig. 1 and $K_2 = \frac{1}{2}[(1+k)c_+ - (1-k)c_-]$ for the vertices in the diagrams (2), (4), and (7) of Fig. 1. [For details, see Ref. 1.] In Ref. 1 the approximation $f_+(0) - f_-(0) \simeq f_+(0) \simeq 1$ and $\Gamma_R = 4\tilde{\Gamma}_R$, which stands for the sum of possible diagrams corresponding to the diagrams (4) and (5) in the RHS of Fig. 3, are used. For the numerical discussion of (2.1) we still need some parameters which must be determined in each decay process. We shall do it for charmed-meson decays and B-meson decays in Secs. III and IV.

Now let us discuss the exclusive nonleptonic decays. For the decays of a pseudoscalar meson (P) into two pseudoscalar mesons (P_1P_2) (PP decays), we can write the decay amplitudes represented by the diagrams in Fig. 1 as follows:

$$B = \frac{G_F}{\sqrt{2}} \Theta_1 \Theta_2 K_a f_{P_1} [(M^2 - m_2{}^2) f_+^{PP_2}(m_1{}^2) + m_1{}^2 f_-^{PP_2}(m_1{}^2)] \qquad (2.2)$$

for one of the diagrams 1(a) (the Born amplitude),

$$R = \frac{G_F}{\sqrt{2}}\,\Theta_1\Theta_2 K_a \tilde{T}_R \tag{2.3}$$

for one of the diagrams 1(b) (the enhanced amplitude), and

$$E = \frac{G_F}{\sqrt{2}}\,\Theta_1\Theta_2 K_a \tilde{T}_E \tag{2.4}$$

for one of the diagrams 1(c), where M, m_1, and m_2 are, respectively, the masses of P, P_1, and P_2 mesons. Following the same approximation used in the derivation of (2.1), we can write $\tilde{T}_R$ ($\tilde{T}_E$) as

$$\tilde{T}_{R(E)} \simeq (N_{R(E)})^{1/2}[f_-^{PP_2'}(0) - f_+^{PP_2'}(0)]I_{R(E)}M_0^2 f_{P_1'} \times T^s_{R(E)}(P_1'P_2' \to P_1P_2), \tag{2.5}$$

where $T^s_{R(E)}(P_1'P_2' \to P_1P_2)$ denotes the nonexotic (exotic) s-wave scattering amplitude in the $P_1'P_2' \to P_1P_2$ scattering. Here $P_1'P_2'$ indicates the two-meson intermediate state to be independently determined for each decay diagram. (Explicit forms of T_R for charmed-meson decays are given in Appendix A.) We should notice that among the three amplitudes only R has a considerably large imaginary part. We can rewrite the imaginary part of R as follows:

$$\begin{aligned}\mathrm{Im}R &\simeq \frac{G_F}{\sqrt{2}}\,4M^2\sum_i \Theta_1^i\Theta_2^i K_a^i f_{P_1^i}[f_-^{PP_2^i}(0) - f_+^{PP_2^i}(0)] \times I_R^i[M_0^i]^2\,\sigma_R^s(P_1^iP_2^i \to P_1P_2)\\ &\simeq \frac{G_F}{\sqrt{2}}\,8M^2\Theta_1\Theta_2K_a f_{P_1}[f_-^{PP_2}(0) - f_+^{PP_2}(0)]I_R M_0^2 \times \sigma_R^s(P_1P_2 \to P_1P_2),\end{aligned} \tag{2.6}$$

where the summation should be done over the possible diagrams represented by diagram (b) in Fig. 1, and for the last equality we must assume the symmetry among quarks appearing in the diagrams. (Symmetry breakings will be discussed in Secs. III–V.) From the comparison of (2.1) with (2.6) we see that there is a very important relation between the contribution of the nonexotic diagrams to the total nonleptonic decay width and the imaginary part of two-body decay amplitudes. In the application of (2.1) (Refs. 1 and 3) we always encountered an unknown parameter σ_R^s, which is difficult to determine from presently available experimental data for meson-meson scattering. We can, however, fix the parameter from data for two-body decays. We shall show it for the charmed-meson decays in the next section.

We would like to comment on I_E in (2.5). Since the contribution of the exotic amplitudes is important in rather high energies, we estimate I_E for heavy-quark decays. The ratio of diagram (8) to diagram (5) in the LHS of Fig. 3 may approximately be described by the ratio of the Pomeron-exchange diagram to the Regge-pole-exchange one in the RHS of Fig. 3 as follows:

$$\frac{I_E^2\sum|T_E^s|^2}{I_R^2\sum|T_R^s|^2} \simeq \frac{I_P^2\sigma_P^s}{I_R^2\sigma_R^s},$$

where the summations in the LHS stand for the sum over possible physical states. When $\sum|T_E^s|^2/\sum|T_R^s|^2 \propto \sigma_P^s/\sigma_R^s$ is taken, we obtain

$$I_E \simeq I_P \simeq \tfrac{2}{5} I_R, \tag{2.7}$$

where the last relation was estimated in Ref. 1. In the following discussion we shall use the relation (2.7). If we take into account that there is no diagram in the exotic contribution corresponding to the heavy-quark annihilation-diagram (see the note in Fig. 2) in the nonexotic contribution, the contribution of the exotic states becomes $\frac{1}{2}I_E/I_R \sim 1/5$ of the nonexotic contribution and not very important.

Up to now we ignored the p-wave contribution in the evaluation of the total nonleptonic decay width. The estimation of the enhancement factor for the p wave is not so easy as that of the s wave. We may, however, expect that the contribution of the p wave will be much smaller than that of the s wave. The suppression ~1/2 arises from the number of possible scattering ($\tilde{N}_R \simeq 9$) for the p wave, because the 0^-0^- intermediate states do not contribute in the p wave even in the off-shell region. We also find another suppression in the loop integration I_R^p, because in general the p-wave scattering amplitudes have momentum-transfer dependence and can possibly change the sign of the imaginary part as the momentum transfer changes.[12] Hereafter we assume that the enhancement in the p wave is negligible compared with that in the s wave. In the next section we shall see that this assumption is right in the analysis of p-wave decays of D mesons (PV decays).

III. NONLEPTONIC DECAYS OF CHARMED MESONS

In order to determine the parameter σ_R^s, we shall start by studying two-body decays.

A. Two-body decays

Here we investigate the charmed-meson decays into two pseudoscalar mesons (PP decays) and those into a pseudoscalar meson + a vector meson (PV decays).

1. PP decays

We can write these decay amplitudes by B,[13] R, and E given by (2.2), (2.3), and (2.4). In these equations, however, many parameters to be determined are contained, e.g., the decay constant (f_P), the form factors $[f_\pm(0)]$, and the scattering amplitudes (T_R^s, T_E^s). Furthermore, it is known that the SU(3)-symmetry prediction $B(D^0\to K^-K^+)=B(D^0\to\pi^-\pi^+)$ is badly broken in experiment.[5,6] We must take account of symmetry breakings for the parameters. The introduction of the symmetry breakings via f_P and $f_\pm(0)$ for the Born amplitude is trivial, but the treatment of symmetry breakings in R and E is somewhat complicated.

Now we discuss the symmetry breakings in R for the processes observed in the experiments,[4,5,6] i.e., $D^0\to K^-\pi^+$, $\bar K^0\pi^0$, K^-K^+, $\pi^-\pi^+$, and $D^+\to\bar K^0K^+$. The formulas required in the evaluation of the amplitudes R are given in Appendix A. We introduce SU(4)-symmetry-breaking parameters as follows:

$$\begin{aligned}
&f_F F^D T_8(FD\to K\pi) = -\sqrt{\tfrac{3}{5}}\, f_K F^\pi T_8(\bar K\pi\to\bar K\pi)(1-\delta),\\
&f_D F^D T_8(\bar D D\to\pi\pi) = -\sqrt{\tfrac{3}{5}}\, f_\pi F^\pi T_8(\pi\pi\to\pi\pi)(1-\delta'),\\
&f_F F^F T_8(\bar F F\to\bar K K) = -\sqrt{\tfrac{3}{5}}\, f_K F^K T_8(\bar K K\to\bar K K)(1-\delta''),
\end{aligned} \tag{3.1}$$

where $F^A\equiv f_+^{DA}(0)-f_-^{DA}(0)$, the parameters δ, δ', and δ'' stand for the SU(4)-symmetry breakings in each process, the factor $(-\sqrt{\frac{3}{5}})$ is derived from the 16-plet symmetry like the nonet symmetry in SU(3) (see Appendix A), and $T_8(AB\to CD)$ denotes the s-channel octet amplitude for the scattering $AB\to CD$. We still have too many parameters. We, therefore, assume $\delta=\delta'=\delta''$. When we take account of the symmetry breakings in the first order, we can write the amplitudes R as follows:

$$\begin{aligned}
&R(D^0\to K^-\pi^+) = -\sqrt{2}R(D^0\to\bar K^0\pi^0) = K_1R^0,\\
&R(D^0\to K^-K^+) = (3\alpha-\beta)K_1R_S^0/C,\\
&R(D^0\to\pi^-\pi^+) = -(3\beta-\alpha)K_1R_S^0/C,\\
&R(D^+\to\bar K^0K^+) = -[(\beta-\alpha)K_1+2\beta K_2]R_S^0/C,
\end{aligned} \tag{3.2}$$

where $R^0=-8(G_F/\sqrt{2})\cos^2\theta_C I_R M_0^2 f_\pi F^K \frac{6}{5} T_8(\bar K\pi\to\bar K\pi)(1-\frac{1}{2}\delta)$, $R_S^0=(\tan\theta_C)R^0$, $\alpha=f_K/f_\pi$, $\beta=F^\pi/F^K$, $C=(1+\alpha\beta)$, and $f_\pm^{D(d\bar d)}(0)=f_\pm^{D\pi}(0)$ is assumed for $D^+\to\bar K^0K^+$. In (3.2) we introduced two symmetry-breaking parameters α and β, but the parameter $\alpha\simeq1.28$ with $f_\pi\simeq0.135$ GeV is known in experiments. The unknown parameters in R are three, that is, β and the real and the imaginary parts of R^0. We can perform the same procedure for the amplitudes E. Including the decays of the F meson, we derive the following decay amplitudes.

Cabibbo-allowed decays:

$$\begin{aligned}
&M(D^0\to K^-\pi^+) = K_1(B^0+R^0)+\alpha\beta K_2E^0,\\
&M(D^0\to\bar K^0\pi^0) = \frac{1}{\sqrt{2}}(\alpha\gamma K_2B^0-K_1R^0+K_1E^0),\\
&M(D^0\to\bar K^0\eta) = \frac{1}{2}\alpha\gamma_\eta K_2B^0+\left(\frac{1}{2}-\frac{1}{\sqrt{2}}\right)K_1R^0-\frac{1}{2}K_1E^0,\\
&M(D^+\to\bar K^0\pi^+) = (K_1+\alpha\gamma K_2)B^0+(K_1+\alpha\beta K_2)E^0,\\
&M(F^+\to K^+\bar K^0) = \alpha K_2B^0+2\alpha K_2R^0/C+\beta_{(s\bar s)}K_1E^0,\\
&M(F^+\to\eta\pi^+) = -\frac{1}{\sqrt{2}}\gamma_\eta K_1B^0+(\alpha+\beta)K_2R^0/C\\
&\qquad -\frac{1}{\sqrt{2}}\alpha K_2E^0,
\end{aligned} \tag{3.3}$$

Cabibbo-suppressed decays:[14]

$$\begin{aligned}
&M(D^0\to K^-K^+) = \alpha K_1B_S^0+R(D^0\to K^-K^+)+\alpha_{(s\bar s)}\beta K_2E_S^0,\\
&M(D^0\to\pi^-\pi^+) = -\gamma K_1B_S^0+R(D^0\to\pi^-\pi^+)-\alpha\beta K_2E_S^0,\\
&M(D^0\to K^0\bar K^0) = 2(\alpha-\beta)K_1R_S^0/C,\\
&M(D^0\to\pi^0\pi^0) = -\frac{1}{\sqrt{2}}\gamma K_2B_S^0+\frac{1}{\sqrt{2}}(3\beta-\alpha)K_1R_S^0/C\\
&\qquad -\frac{1}{\sqrt{2}}\beta K_1E_S^0,\\
&M(D^+\to\bar K^0K^+) = \alpha K_1B_S^0+R(D^+\to\bar K^0K^+)+\alpha_{(s\bar s)}\beta K_2E_S^0,\\
&M(D^+\to\pi^+\pi^0) = -\frac{1}{\sqrt{2}}(K_1+K_2)(\gamma B_S^0+\beta E_S^0),\\
&M(F^+\to K^0\pi^+) = K_1B_S^0+(K_2+\alpha K_1-K_1)R_S^0-K_2E_S^0,\\
&M(F^+\to K^+\pi^0) = \frac{1}{\sqrt{2}}\{K_2B_S^0-(K_2+\alpha K_1-K_1)R_S^0+K_1E_S^0\},
\end{aligned} \tag{3.4}$$

where $E^0\simeq-4(G_F/\sqrt{2})\cos^2\theta_C I_E M_0^2 f_\pi F^K T_E^s$, $B_S^0(E_S^0)=(\tan\theta_C)B^0(E^0)$, $\alpha_a\equiv f_a/f_\pi$, $\beta_a\equiv F^a/F^K$, $\gamma_a\equiv f_+^{Da}(0)/f_+^{DK}(0)$, $\gamma\equiv\gamma_\pi$, and an additional relation $f_DF^DT_8(\bar DD\to\bar KK)=-\sqrt{(3/5)}f_KF^KT_8(\bar KK\to\bar KK)\times(1-\delta)$ is used for the F decays. The mass differences between π, K, and η mesons are neglected in (3.3) and (3.4). In order to decrease the parameters, we may postulate the real part dominance for T_E^s and $|T_E^s|\simeq|T_R^s|$, which is known in the high-energy region. This assumption, however, is not serious in numerical analysis, because the contribution of E^0 is nearly $\sim\frac{1}{5}$ of that of R^0 as noted in the last section. We still have six unknown parameters in B and R even for the $\bar K\pi$, $\pi\pi$, and $\bar KK$ decays, that is, $f_\pm^{D\pi}(0)$, $f_\pm^{DK}(0)$, and the real and the imaginary parts of R^0. Only for the purpose of simplification of numerical analysis do we put relations $\gamma=\beta$ and $F^K\simeq1$.[15] The numerical results for the choice of the parameters $f_+^{DK}(0)\simeq0.95$, $\beta\simeq0.87$, $\mathrm{Re}[f_+^{DK}(0)R^0]/B^0\simeq-0.78$, and $|\mathrm{Im}[f_+^{DK}(0)R^0]/$

TABLE I. Predicted branching ratios (in %) for *PP* decays. The predictions in column (a) are based on the choice of parameters $(f_+^{DK}(0)\mathrm{Re}R^0)/B^0 \simeq -0.78$, $|(f_+^{DK}(0)\mathrm{Im}R^0)/B^0| \simeq 0.70$, $f_+^{DK}(0) \simeq 0.95$, $\gamma = \beta \simeq 0.87$. The predictions in column (b) are based on the experimental values for lifetimes $\tau^0 = (1.0^{+0.52}_{-0.31})\times 10^{-13}$, $\tau^+ = (10.3^{+10.5}_{-4.1})\times 10^{-13}$, $\tau^F = (2.24^{+2.78}_{-1.05})\times 10^{-13}$ sec.[17]

	(a)	(b)	Experiments LGW[4]	Experiments Mark II[5,6]
(Cabbibo allowed)				
$D^0 \to K^-\pi^+$	$2.7\times(\tau^0/10^{-13}\,\mathrm{sec})$	4.1–1.8	2.2 ± 0.6	3.0 ± 0.6
$\to \bar{K}^0\pi^0$	$1.5\times(\tau^0/10^{-13}\,\mathrm{sec})$	2.3–1.0		2.2 ± 1.1
$\to \bar{K}^0\eta$	$1.0\times(\tau^0/10^{-13}\,\mathrm{sec})$	1.5–0.7		
$D^+ \to \bar{K}^0\pi^+$	$0.19\times(\tau^+/10^{-13}\,\mathrm{sec})$	4.0–1.2	1.5 ± 0.6	2.3 ± 0.7
$F^+ \to \bar{K}^0K^+$	$2.6\times(\tau^F/10^{-13}\,\mathrm{sec})$	13–3		
$\to \eta\pi^+$	$1.6\times(\tau^F/10^{-13}\,\mathrm{sec})$	8–2		
(Cabbibo suppressed)				
$D^0 \to K^-K^+$	$0.28\times(\tau^0/10^{-13}\,\mathrm{sec})$	0.43–0.20		0.3 ± 0.09
$\to \bar{K}^0K^0$	$0.05\times(\tau^0/10^{-13}\,\mathrm{sec})$	0.07–0.03		
$\to \pi^-\pi^+$	$0.07\times(\tau^0/10^{-13}\,\mathrm{sec})$	0.10–0.05		0.09 ± 0.04
$\to \pi^0\pi^0$	$0.01\times(\tau^0/10^{-13}\,\mathrm{sec})$	0.02–0.01		
$D^+ \to \bar{K}^0K^+$	$0.14\times(\tau^+/10^{-13}\,\mathrm{sec})$	3.0–0.8		0.5 ± 0.27
$\to \pi^0\pi^+$	$0.01\times(\tau^+/10^{-13}\,\mathrm{sec})$	0.23–0.07		
$F^+ \to K^0\pi^+$	$0.06\times(\tau^F/10^{-13}\,\mathrm{sec})$	0.31–0.07		
$\to K^+\pi^0$	$0.02\times(\tau^F/10^{-13}\,\mathrm{sec})$	0.12–0.03		

$B^0| \simeq 0.70$, are presented in Table I,[16] where $K_1 = 1.66$, $K_2 = -1.21$,[1] $\alpha_\eta = \alpha_{(s\bar{s})} \simeq \alpha$, and $\beta_\eta = \beta_{(s\bar{s})} \simeq 1$ are used. Note that $|\mathrm{Im}R^0/B^0|$ is determined so as to reproduce $B(D^0 \to K^-\pi^+) \simeq 3\%$ for $\tau^0 = 1.1\times10^{-13}$ sec on the assumption $\mathrm{Re}M(D^0 \to K^-\pi^+) = 0$, and the value may be considered to be nearly the upper bound for $\mathrm{Im}R^0$. Using the experimental values for the lifetimes (τ^0, τ^+ and τ^F) for D^0, D^+, and F mesons[17]

$$\begin{aligned} \tau^0 &= 1.0^{+0.52}_{-0.21}\times10^{-13}\ \mathrm{sec}\,, \\ \tau^+ &= 10.3^{+10.5}_{-4.1}\times10^{-13}\ \mathrm{sec}\,, \\ \tau^F &= 2.24^{+2.78}_{-1.05}\times10^{-13}\ \mathrm{sec}\,, \end{aligned} \tag{3.5}$$

we obtain the results in column (b) in Table I. They are satisfactory. The above values for β and $f_\pm^{DK}(0)$, which will be determined from experiments for the semileptonic processes $D \to Kl\nu$ and $\pi l\nu$, are consistent with the estimation based on a quark model[18] and the comparable value for ReR and ImR are also reasonable because $E_{\mathrm{c.m.}} = M_D$ is in the intermediate energy region between the resonance-dominance region and the high-energy (Regge-pole-dominance) region. One of the characteristic features of this analysis appears in the phase difference between two amplitudes for $D^0 \to K^-\pi^+$ and $D^+ \to \bar{K}^0\pi^+$, which is evaluated to be ~90° (or ~270°). We can calculate the phase difference from the data for $D^0 \to K^-\pi^+$, $\bar{K}^0\pi^0$, and $D^+ \to \bar{K}^0\pi^+$ because these processes can be written by two independent amplitudes. At present, however, the data are too ambiguous to determine them. We are eager to find the precise values for them. The symmetry-breaking effect for the ratio $B(D^0 \to K^-K^+)/B(D^0 \to \pi^-\pi^+) \simeq 4$ is two times larger than the values $|\alpha\beta^{-1}|^2 \simeq 2$ predicted from the Born amplitude dominance. The good test for the value $\beta \simeq 0.87$ will be done in the decay $D^0 \to \bar{K}^0K^0$, of which amplitude is proportional to $(\alpha - \beta)$.

The above numerical analysis would possibly change when more precise data is given. We also remark that the relation $R_S^0(E_S^0) = (\tan\theta_C)R^0(E^0)$ cannot be derived in general because the intermediate states for the suppressed decays are not equal to those of the allowed ones.

2. PV decays

These decays occur in the *p* wave. As noted in Sec. II, the enhancement mechanism predicted in the *s* wave will not work there and the Born amplitude will dominate these decay processes. As the first step we, therefore, study *PV* decays on the assumption of $R = E = 0$.

The decay widths for the diagrams (a) and (b) in Fig. 4 may be written down as follows:

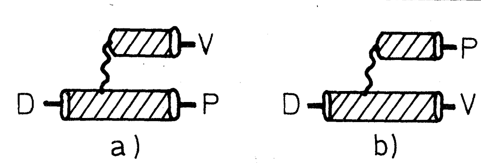

FIG. 4. Two different types of Born diagrams for *PV* decays.

$$\Gamma_{a(b)}(D\to PV)=\frac{(G_F\cos^2\theta_C)^2}{32\pi M}[\Delta(M,m_P,m_V)]^3 \times|F_{a(b)}(m_{a(b)}{}^2)|^2 f_{a(b)}{}^2 M^4 C_{a(b)}{}^2\,, \quad (3.6)$$

where the phase-space correction is

$$\Delta(M,m_P,m_V)\equiv\{[1-(m_P-m_V)^2/M^2]\times[1-(m_P+m_V)^2/M^2]\}^{1/2}\,,$$

the form factors $[F_a(m_a{}^2)\equiv f_+^{DP}(m_V{}^2)$ and $F_b(m_b{}^2)\equiv f^{DV}(m_P{}^2)]$ are assumed to be described by a simple pole, the decay constants and $f_a=f_V$, $f_b=f_P$, and $C_{a(b)}$ denote factors including a Clebsch-Gordan coefficient and a short-distance-enhancement factor. For $f_+^{DK}(0)\simeq 0.95$, $\beta=0.87$, $f_\rho=0.21$ GeV, $f_{K^*}=(f_\rho+f_\phi)/2\simeq 0.22$ GeV, and $\tau^0=1\times 10^{-13}$ sec, we obtain

$$B(D^0\to K^-\rho^+)\simeq 6.6\%\,,\qquad B(D^0\to \bar K^{*0}\pi^0)\simeq 2.0\%\,, \quad (3.7)$$

which are consistent with the data[6] $(7.2\pm2.5)\%$ and $(1.4^{+1.9}_{-1.4})\%$, respectively. In the simplest case, i.e., on the assumption $f^{D\rho}(0)=f^{DK^*}(0)=f_+^{DK}(0)$, our predicted values are listed in Table II, where the experimental upper limit for $D^+\to\bar K^0\rho^+$ is taken from the data for $D^+\to\bar K^0\pi^0\pi^+$. Except for $B(D^0\to\bar K^0\rho^0)$, the predictions are quite satisfactory compared with the data. We can easily check that the introduction of a symmetry breaking such as $f^{D\rho}(0)/f_+^{DK}(0)<1$ so as to reproduce $B(D^0\to\bar K^0\rho^0)\lesssim 0.5\%$ pushes out other values from the experimentally allowed region. It is, however, very interesting that $r(K\rho)\simeq 0.15$ in this analysis is much better than the value $\frac{1}{2}$ predicted by the $I=\frac{1}{2}$ final-state dominance. We may say that the gross features of the PV decays can be reproduced by the Born amplitudes.

Now we analyze the PV decay in the case with $R\neq 0$, but we shall neglect E. In general, the amplitude R is written by two different types of couplings, i.e., F and D types. In our model only the F-type coupling is available, because 0^--nonexotic $\bar q q$ states dominate in the intermediate states. The decompositions of the amplitudes into B and R are done in Appendix B. The predictions for the choice of $R^0/B_1^0=-0.1$, $f^{D\rho}(0)/f_+^{DK}(0)=0.9$ and $f^{DK^*}(0)/f_+^{DK}(0)=0.95$ are also presented in Table II. The values are consistent with the data[6] within the errors. Note that our predictions, particularly those for $D^0\to\bar K^0\rho^0$ and $D^+\to\bar K^{*0}\pi^+$, are quite sensitive to the variation of the parameters and $\mathrm{Re}R^0/B_1^0$ must be negative to reproduce the small value for $B(D^0\to\bar K^0\rho^0)$. We can also estimate the upper limit for the ratio $|\mathrm{Re}R^0/B_1^0|$ to be $\lesssim 0.16$ from the data for $B(D^0\to K^-\rho^+)$. Such a small

TABLE II. Predicted branching ratios (in %) for PV decays. Here we use the lifetimes τ^0, τ^+, and τ^F given in Ref. 17.

R^0/B^0 $f^{D\rho}(0)$ $f^{DK^*}(0)$	0 $f_+^{DK}(0)$ $f_+^{DK}(0)$	-0.1 $0.9f_+^{DK}(0)$ $0.95f_+^{DK}(0)$	Experiment[6]
$D^0\to K^-\rho^+$	10–4.5	8.0–3.6	7.2 ± 2.5
$\to\bar K^0\rho^0$	1.5–0.7	0.8–0.3	$0.1^{+0.4}_{-0.1}$
$\to K^{*-}\pi^+$	3.6–1.6	4.7–2.1	3.2 ± 1.0
$\to\bar K^{*0}\pi^0$	2.9–1.3	4.0–1.8	$1.4^{+1.9}_{-1.4}$
$\to\bar K^0\omega$	1.5–0.7	0.8–0.3	
$\to\bar K^{*0}\eta$	1.1–0.5	1.9–0.8	
$D^+\to\bar K^0\rho^+$	26–7.8	34–10	$<12.9\pm8.4$
$\to\bar K^{*0}\pi^+$	3.1–0.9	5.5–1.6	<3.7
$F^+\to K^{*+}\bar K^0$	13–3	16–4	
$\to K^+\bar K^{*0}$	28–7	22–5	
$\to\rho^+\eta$	24–6	18–4	
$\to\pi^+\phi$	14–3	13–3	
$\to\pi^+\rho^0$	0	1.2–0.3	
$\to\rho^+\pi^0$	0	1.2–0.3	
$\to\pi^+\omega$	0	0	

value for the ratio is consistent with our expectation for the p-wave decays. In the above analysis we put $\mathrm{Im}R^0\simeq 0$, but $|\mathrm{Im}R^0/B_1^0|<0.4$ is derived from $B(D^0\to\bar K^0\rho^0)\lesssim 0.5$.

We would like to comment that the absolute value of R in the p wave can experimentally determine from the decay $F^+\to\pi^+\rho^0$ or $\rho^+\pi^0$, to which only R can contribute. The contribution of the exotic intermediate state (E) neglected here can also be seen in the decay $F^+\to\pi^+\omega$, because the Born and the nonexotic amplitude do not contribute to the decay.

B. Inclusive decays

In the analysis of III A we obtained $|\mathrm{Im}R^0/B^0|\simeq 0.74$ in the s-wave decays and may put it to be zero in the p-wave decays. As the value $|\mathrm{Im}R^0/B^0|\simeq 0.74$ gives $\sigma_R^s\simeq 0.6$ mb, we can estimate the ratios of nonleptonic decay width to semileptonic decay width as follows:

$$\Gamma_{NL}(D^0)/\Gamma_{SL}(D\to e\nu X)\simeq 5.8+\frac{21}{g(\epsilon)}\sigma_R^s \simeq 5.8+12.6/g(\epsilon)\,,$$

$$\Gamma_{NL}(D^+)/\Gamma_{SL}(D\to e\nu X)\simeq 5.8-3.7\xi+\frac{0.55}{g(\epsilon)}\sigma_R^s \simeq 5.8-3.7\xi+0.33/g(\epsilon)\,, \quad (3.8)$$

$$\Gamma_{NL}(F)/\Gamma_{SL}(F\to e\nu X)\simeq 5.8+\frac{11}{g(\epsilon)}\sigma_R^s \simeq 5.8+6.6/g(\epsilon)\,,$$

where $m_c=1.5$ GeV is used, σ_R^s is in mb, 5.8 stands for the contribution of the free quark decay, -3.7ξ is due to the $\bar{d}$-quark-exchange diagram in the D^+ decay,[1,2] and $g(\epsilon)$ is the correction factor by the final quark masses.[19] We have the following ratios for the lifetimes of charmed mesons:

$$\begin{aligned} &\tau^+/\tau^0 \simeq 4-8\,, \\ &\tau^F/\tau^0 \simeq 1.4-1.6 \end{aligned} \tag{3.9}$$

for the variations of $g(\epsilon)=1-0.5$ and $\xi\simeq 0.8-1.2$.[2] For the choice of $m_s/m_c \simeq 0.1$ [$g(\epsilon)\simeq 0.925$ (Ref. 19)] and $\xi=1$, we obtain $\tau^+/\tau^0\simeq 4.7$, $\tau^F/\tau^0\simeq 1.4$, $B(D^0\to e\nu X)\simeq 4.7$, $B(D^+\to e\nu X)\simeq 22\%$, and $B(F\to e\nu X)=6.7\%$.[20] We should pay attention that the value $\sigma_R^s\simeq 0.6$ mb derived from $|\mathrm{Im}R^0/B^0|\simeq 0.74$ may be considered to be nearly the upper bound for σ_R^s. The ratios of lifetimes given here, therefore, are possibly a little bit overestimated. Then we predict $\tau^+/\tau^0\simeq 3-6$ and $\tau^F/\tau^0\simeq 1.3-1.5$.

IV. NONLEPTONIC DECAYS OF B MESON

In the B-meson decays we are caught by a problem of large symmetry breakings among c, b, t, and light quarks. Considering that the effect of symmetry breakings is not small even in the charmed-meson decays as shown in Sec. III, we cannot discuss the B decays without introducing the symmetry breakings. Unfortunately, we do not have experimental data enough to determine many parameters describing the symmetry breakings. We shall, however, see that we can derive the upper and the lower limits for the semileptonic branching ratios and some relations among two-body decays. Here we study the B decays in some limited cases for the symmetry breakings from the viewpoint of the six-quark model.[21]

A. Inclusive decays

The amplitude (b_1) in Fig. 1, which contributes to the decays of B^0 $(b\bar{d})$ and $B_s^0(b\bar{s})$, is the sum of two diagrams with the annihilation of the d quark and that of the b quark (R_1). On the other hand, the amplitude (b_2) contributing to all B decays is described by the sum of two diagrams with the annihilation of the c quark and that of the t quark[22] (R_2). It should be remarked that in the standpoint of the six-quark model,[21] the c- and t-quark annihilation diagrams are expected to contribute destructively, while the contributions of the d- and b-quark annihilation diagrams work constructively. We may also note that R_1 and R_2, respectively, have the $c\bar{u}$ $(c\bar{c})$ final state for the B^0 (B_s^0) decay and the $s\bar{d}$ $(s\bar{u}, s\bar{s})$ final state for the B^0 (B^-, B_s^0) decay. The ratios of nonleptonic decay width to semileptonic decay width are given for $M_B=5.3$ GeV,[23] $m_b=5$ GeV, and M_0 $(\simeq m_b+m_c)\simeq 6.5$ GeV as follows:

$$\begin{aligned} \Gamma_{\rm NL}(B^0\ (B_s^0))/\Gamma_{\rm SL}(B\to e\nu X) &\simeq 6.4+19.2\sigma_R^s(c\bar{u}(c\bar{c}))\,(f_D/f_\pi)^2(F_B^{D(F)})^2\left|\frac{1+\beta_b}{2}\right|^2 \\ &\quad +19.2\sigma_R^s(s\bar{d}(s\bar{s}))\,(f_F/f_\pi)^2(F_B^{D(F)})^2\left|\frac{1-\beta_t}{2}\right|^2, \\ \Gamma_{\rm NL}(B^-)/\Gamma_{\rm SL}(B\to e\nu X) &\simeq 6.4-0.24\xi_B+19.2\sigma_R^s(s\bar{u})(f_F/f_\pi)^2(F_B^D)^2\left|\frac{1-\beta_t}{2}\right|^2, \end{aligned} \tag{4.1}$$

where σ_R^s is in mb, $F_B^A\equiv f_+^{BA}(0)-f_-^{BA}(0)$, the short-distance-enhancement factors $K_1\simeq 1.52$ and $K_2\simeq -1.04$ (Ref. 3) are used, 6.4 are the contribution of the free-quarks decays, for which the corrections for the final quark and lepton masses are done,[3] the complex numbers,[24] β_b and β_t, respectively, represent the difference of the b-quark annihilation diagram from the d-quark one and that of the t-quark annihilation diagram from the c-quark one, and the contribution of the free-quark-exchange diagram in the B^- decays, $-0.24\xi_B$ with $\xi_B\simeq(f_B/f_D)^2\xi$, is negligible. [For details, see (Ref. 3).]

In the symmetry limit $\sigma_R^s(c\bar{u})=\sigma_R^s(c\bar{c})=\sigma_R^s(s\bar{d})=\sigma_R^s(s\bar{u})=\sigma_R^s(s\bar{s})$, $f_F=f_D$, $F_B^D=F_B^F$, and $\beta_b=\beta_t=1$, we obtain

$$\begin{aligned} &B(B^0\ (B_s^0)\to e\nu X)\simeq 8\%\,, \\ &B(B^-\to e\nu X)\simeq 12\%\,, \end{aligned} \tag{4.2}$$

where $F_B^D\simeq f_D/f_\pi\simeq f_F/f_\pi\simeq 1$ and $\xi_B\simeq 1$ are used, and $\sigma_R^s\simeq 0.2$ mb is derived from the simple Regge-pole analysis, that is, $\sigma_R^s(E_{\rm c.m.}=M_B)\simeq\sigma_R^s(E_{\rm c.m.}=M_D)(M_D/M_B)\simeq 0.2$ mb by using the value $\sigma_R^s(E_{\rm c.m.}=M_D)\simeq 0.6$ mb obtained in the last section. These values are considerably different from the predictions given in Ref. 3, where $B(B^0(B_s^0)\to e\nu X)\simeq 5\%$ and $B(B^-\to e\nu X)\simeq 9\%$ are derived for the choice of $\sigma_R^s(E_{\rm c.m.}=M_B)\simeq 0.53$ mb and $\beta_b=1$ and $\beta_t=0$. Our estimations are sensitive to the value of σ_R^s, β_b, and β_t. If we consider that $\sigma_R^s(c\bar{c})\lesssim\sigma_R^s(c\bar{u})\lesssim\sigma_R^s(s\bar{s})\lesssim\sigma_R^s(s\bar{d})\approx\sigma_R^s(s\bar{u})$,

$\beta_b<1$, and $\beta_t<1$, we can estimate the lower limit for the semileptonic branching ratios by taking $\sigma_R^s(q\bar q)=\sigma_R^s(s\bar u)\simeq 0.2$ mb, $\beta_b\simeq 1$, and $\beta_t=0$, whereas the upper limit is given in the case with no enhancement, that is, the free-quark limit. We predict

$$\begin{aligned} &B(B^0(B_s^0)\to e\nu X)\simeq(7\text{–}12)\%\,,\\ &B(B^-\to e\nu X)\simeq(10\text{–}12)\%\,. \end{aligned}\tag{4.3}$$

These values are consistent with the data of the CLEO collabration, i.e., $B(B\to eX)\simeq(16\pm4\pm7)\%$ and $(18\pm9)\%$ in independent searches for electrons and $B(B\to\mu X)\simeq(7.5\pm3.1)\%$ for muons.[25] We should also note that the enhancement of the uncharmed final states derived in Ref. 3 is strongly depressed for $\beta_t\simeq 1$.

B. *PP* decays

The decomposition of PP decay amplitudes into B, R, and E are given in Appendix C. The B-meson mass is heavy enough, so we may use the knowledge for high-energy scattering, that is, if $\mathrm{Re}T_R\simeq\mathrm{Im}T_E\simeq 0$ and $|T_E|\simeq|T_R|$, then $|E^0|\simeq\frac{1}{5}|R^0|$. It is interesting that the ratio $|R_1^0/B^0|$ described as

$$|R_1^0/B^0|\simeq\frac{8I_R M_B{}^2(M_0{}^2-m_2{}^2)}{M_B{}^2-m_1{}^2}[2.57\sigma_R^s(\text{mb})]\times\frac{F_B^D}{f_+^{BD}(0)}\left|\frac{1+\beta_b}{2}\right|\tag{4.4}$$

increases as the meson mass (M_B) increases, even if σ_R^s decreases as M_B^{-1}.[26] That is to say, even though the enhancement mechanism represented by the amplitude R decreases as the meson mass increases, the enhancement mechanism is important in PP decays. In fact, we can obtain

$$|R_1^0/B^0|\simeq(2-3)\frac{F_B^D}{f_+^{BD}(0)}\left|\frac{1+\beta_b}{2}\right|>1\,,\tag{4.5}$$

while the ratio is ~1 for the D^0 decay. The branching ratios for uncharmed final states represented by the amplitude R_2 are given as follows:

$$\begin{aligned} B(B^{0,-}\to K^-\pi^+)&\simeq 2B(B^{0,-}\to\bar K^0\pi^0)\simeq 23B(B^{0,-}\to\bar K^0\eta)\simeq 30(F_B^D)^2\left(\frac{f_F}{f_\pi}\sigma_R^s(s\bar u)\right)^2\left|\frac{1-\beta_t}{2}\right|^2 B(B^{0,-}\to e\nu X)\\ &\simeq 0.3|1-\beta_t|^2 B(B^{0,-}\to e\nu X)\,,\\ B(B_s^0\to K^-K^+)&\simeq B(B_s^0\to\bar K^0K^0)\simeq 4B(B_s^0\to\eta\eta)\simeq 0.3|1-\beta_t|^2B(B_s^0\to e\nu X)\,, \end{aligned}\tag{4.6}$$

where the mass differences between π, K, and η are neglected, $\eta=\frac{1}{2}(u\bar u+d\bar d)-1/\sqrt2(s\bar s)$ is taken, and in the derivation of the last equalities in two equations $F_B^D\simeq F_B^F\simeq f_F/f_\pi\simeq 1$ and $\sigma_R^s(q\bar q)=0.2$ mb are used. We can say that these branching ratios are less than 3% when $0\lesssim\beta_t\lesssim 2$ is right. For the branching ratios of charmed final states we obtain the following relations:

$$\begin{aligned} &B(B^0\to D^+\pi^-)[\simeq B(B^0\to F^+K^-)]\\ &\quad\simeq 0.2\left[\left(1+\frac{K_2}{K_1}\frac{E_2^0}{B_1^0}\right)^2+(R_1^0/B_1^0)^2\right]\times|f_+^{BD(F)}(0)|^2B(B^0\to e\nu X)\,,\\ &B(B^0\to D^0\pi^0)\\ &\quad\simeq 0.1\left[\left(\frac{K_2}{K_1}+\frac{E_1^0}{B_2^0}\right)^2+(R_1^0/B_2^0)^2\right]\left(\frac{f_D}{f_\pi}\right)^2\times|f_+^{B\pi}(0)|^2B(B^0\to e\nu X)\,,\\ &B(B^-\to D^0\pi^-)\\ &\quad\simeq 0.2\left[\left(1+\frac{E_1^0}{B_1^0}\right)f_+^{BD}(0)+\frac{K_2B_2^0}{K_1B_1^0}\left(1+\frac{E_2^0}{B_2^0}\right)\frac{f_D}{f_\pi}f_+^{B\pi}(0)\right]^2\times B(B^-\to e\nu X)\,. \end{aligned}\tag{4.7}$$

In (4.7) we used the equation

$$\frac{\Gamma_{\text{Born}}(B^0\to D^+\pi^-)}{\Gamma_{\text{SL}}(B\to e\nu X)}\simeq 6\pi^2\frac{|f_+^{BD}(0)|^2}{g(\epsilon)}\frac{f_\pi^2}{m_b{}^5}M_B{}^3K_1{}^2\simeq 0.10K_1{}^2|f_+^{BD}(0)|^2\,,\tag{4.8}$$

where $g(\epsilon)\simeq 0.52$ and $\Gamma_{\text{Born}}(B\to D\pi)$ stands for the decay width evaluated on the assumption $R=E=0$. We can also derive similar equations for the decays $B\to D\bar D$, $D\bar F$, $F\bar D$, and so on (see Appendix C).

For the p-wave decays, such as $B\to\psi K$, $\psi\pi$, and ψD, etc., we may write the decay amplitudes on the restriction of the Born-amplitude dominance. Detections of these decay modes, therefore, are very good observations to determine the form factors $f_+^{BA}(0)$ in experiments.

V. REMARKS ON *K*-MESON DECAYS

Here we shall discuss the $\pi\pi$ decays of K mesons (K_S^0 and K^+) in our model. The $\pi\pi$ decay amplitudes are written as follows:

$$M(K_S^0\to\pi^-\pi^+)\simeq\sqrt2K_1(B^0+R^0)\,,\tag{5.1}$$

$$M(K_S^0\to\pi^0\pi^0)\simeq -K_2B^0+K_1R^0\,,\tag{5.2}$$

$$M(K^+ \to \pi^0\pi^+) = \frac{1}{\sqrt{2}}\,(K_1 + K_2)B^0\,, \tag{5.3}$$

where $f_+^{K\pi}(0) \simeq 1$, $R^0 = R_1^0 + R_2^0$ stands for the sum of two diagrams (b_1) and (b_2) in Fig. 1, and the exotic amplitude E is ignored because E has the suppression factor ~1/5 and the exotic diagram may be not important at such a small energy $E_{\text{c.m.}} = m_K$. We may note that the so-called penguin diagram[27] is included in R_2. By use of $K_1 \simeq 1.87$ and $K_2 \simeq -1.46$, which are derived from the usual choice of the short-distance correction factors $c_- \simeq 2.5$ and $c_+ \simeq 0.63$, we obtain from (5.3)

$$[\Gamma(K^+ \to \pi^0\pi^+)]^{-1} \simeq 6.2\times 10^{-8}\ \text{sec}\,, \tag{5.4}$$

which is quite satisfactory in comparison with the experimental value 5.9×10^{-8} sec. From the data for $\Gamma(K_S^0 \to \pi^-\pi^+)/\Gamma(K^+ \to \pi^0\pi^+) \simeq 450$ and $\Gamma(K_S^0 \to \pi^-\pi^+)/\Gamma(K_S^0 \to \pi^0\pi^0) \simeq 2.19$, we can estimate the values

$$\text{Re}R^0/B^0 \simeq 0.34\,,\quad |\text{Im}R^0/B^0| \simeq 1.95\,. \tag{5.5}$$

It is very interesting that R^0 and B^0 are comparable in our model. If we postulate the SU(3) symmetry for u, d, and s quarks, R^0 is given by

$$R^0 = (3-\beta_c)R_1^0/2\,, \tag{5.6}$$

where β_c stands for the ratio of the contribution of the c-quark annihilation diagram to that of the u, d, or s-quark one and is written as follows:

$$\beta_c \simeq \left(\frac{M_{D^*}}{M_{K^*}}\right)^2 \frac{I_R^c}{I_R^u}\,\frac{f_D}{f_\pi}\,\frac{F_K^D}{F_K^\pi}\,\frac{T_R^s(D^-D^+ \to \pi^-\pi^+; E_{\text{c.m.}} = m_K)}{T_R^s(\pi^-\pi^+ \to \pi^-\pi^+; E_{\text{c.m.}} = m_K)}\,, \tag{5.7}$$

where the CP-violating phase in the six-quark model[21] is ignored. Even if we put $I_R^c \simeq I_R^u$ and $f_D \simeq f_\pi$, the estimation of the off-shell scattering amplitude $T_R^s(D^-D^+ \to \pi^-\pi^+)$ is difficult. We cannot say even either $|\beta_c| < 1$ or $|\beta_c| > 1$. We can, however, derive the relation $|\text{Im}R_1^0/B^0| \simeq 0.22\sigma_R^s$ ($u\bar{u}$ or $d\bar{d}$) from the optical theorem and obtain, by assuming β_c = real,

$$\sigma_R^s(u\bar{u}\ \text{or}\ d\bar{d}; E_{\text{c.m.}} \simeq m_K) \simeq |18/(3-\beta_c)|\,\text{mb}\,. \tag{5.8}$$

For $\beta_c \simeq 1$, $\sigma_R^s \simeq 10$ mb, which was derived in Ref. 1. On the contrary, we can determine β_c, if the s-wave $\pi^-\pi^+$ scattering amplitude at $E_{\text{c.m.}} = m_K$ is experimentally given. To determine β_c will be very interesting in understanding the symmetry breaking and also to evaluate the CP violation in the six-quark model.

VI. SUMMARY AND DISCUSSIONS

We described the nonleptonic decays of mesons in terms of the diagrams given in Fig. 1. The description by these diagrams does not lose any generality. In fact, most of the models proposed up to this point can be represented by some of them, e.g., the W-exchange dominance is represented by the dominance of diagram (b_1), the quark-number-conservation rule by the selection of (b_1) and (b_2), and the so-called penguin diagram is included in diagram (b_2). What we did in this paper was to give a method of how to calculate these diagrams. We actually showed how the enhancement mechanism represented by the nonexotic intermediate states[1] and also the suppression mechanism described by the free-quark exchange[2] decrease as meson masses increase. We also showed that all of these diagrams are important to describe all of the meson-nonleptonic-decay phenomena; in other words, any assumptions of the dominance of one or two of these diagrams are not applicable to describe all of them. In particular, we saw that diagram (a) (the Born amplitude) and diagrams (b_1) and (b_2) (the enhanced diagrams) are comparable in PP decays of K, charmed, and B mesons, while the dominance of diagram (a) is naturally understood in PV decays. We may say that the greatest advantage of our model is to give a unified view for the nonleptonic decays of all mesons. On the other hand, the disadvantage of our model is that many parameters, e.g., σ_R^s, f_P, $f_\pm(0)$, and so on, must be determined from data. However, this must be done if the model is correct. We consider that such a complicated structure of the nonleptonic decay mechanism has prevented us from understanding the nonleptonic-decay phenomena correctly. At present, experimental data are not enough to determine all the parameters, but in the near future experiments will allow us to understand the nonleptonic decays well.

Analyses for the PP decays of charmed mesons, in which the final-state interaction is taken into account, have been done.[28] These analyses are similar to ours on the point that the amplitudes B and R are comparable. The difference between those analyses and ours is quite clear, because they take account of the final-state interaction only on-shell or nearly on-shell, while in our model the off-shell contribution is important to interprete the enhancement in D^0 and K^0 decays.[1]

Finally, we would like to comment on the difference between baryon decays and meson decays. The main difference arises from the difference between transition form factors for the two processes. That is to say, the form factor for baryon decays should be described by a dipole for transitions between ground-state baryons with $\frac{1}{2}^+$, while that for mesons is represented by a simple pole. This difference is very important. That is, the integration in nonexotic diagrams (I_R) will become comparable with that in exotic diagrams (I_P) in the case of a dipole form factor, because the off-shell (far from on-shell) contribution becomes unimportant, while it is important for a simple pole. Of course, we must not forget the contribution of the heavy-quark annihilation diagrams like the c-quark one in K and charmed-meson decays. In the case of hyperon decays, however, we need not worry about the derivation of the $\Delta I=\frac{1}{2}$ rule, because the contribution of the Born amplitudes which can violate the rule is known to be small, e.g., $\Gamma_{\text{Born}}(\Lambda\to p\pi)/\Gamma_{\text{exp}}(\Lambda\to p\pi)\simeq\frac{1}{30}$, there is no nearby decuplet resonance with $J^P=\frac{1}{2}^{\pm}$, the contribution of which can also violate the rule in the nonexotic amplitudes R, and the exotic amplitudes E are not important at such small energies. We, of course, encounter the same difficulty for the estimation of the c-quark annihilation diagrams as encountered in K decays, but we will be able to determine the parameter by using the rich data for hyperon decays. Numerical discussions will be done elsewhere.

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APPENDIX A: DECOMPOSITION OF *PP* DECAY AMPLITUDES

The decay amplitude (M) is written in terms of the enhanced amplitude (R) and the Born amplitude (B). [The exotic amplitude (E) is neglected here]. For example,

$$M(D^0\to K^-\pi^+)=\frac{G_F}{\sqrt{2}}\cos^2\theta_C K_1[R^0(K^-\pi^+)+B^0(K^-\pi^+)]\,,$$
$$R^0(K^-\pi^+)=A^0(K^-\pi^+\to K^-\pi^+)-A^0(D^+F^-\to K^-\pi^+)\,; \tag{A1}$$

$$M(D^0\to K^-K^+)=\frac{G_F\cos\theta_C\sin\theta_C}{\sqrt{2}}\times K_1[R^0(K^-K^+)+B^0(K^-K^+)]\,, \tag{A2}$$

$$R^0(K^-K^+)=A^0(K^-K^+\to K^-K^+)-A^0(\pi^-\pi^+\to K^-K^+)-A^0(F^-F^+\to K^-K^+)\,;$$

where amplitudes which violate the Okubo-Zweig-Iizuka rule are omitted from R and $B^0(P_1P_2)=f_{P_1}F_{P_2}(M_D{}^2-m_{P_1}{}^2)$. The amplitudes $A(P_1P_2\to P_3P_4)$ described by nonexotic intermediate states are decomposed into singlet and octect representations of SU(3) in s channel as follows:

$$A(K^-\pi^+\to K^-\pi^+)=-4I_R M_0{}^2 f_\pi F^K\tfrac{3}{5}T_8(\bar K\pi\to\bar K\pi)\,, \tag{A3}$$

$$A(D^+F^-\to K^-\pi^+)=-4I_R M_0{}^2 f_F F^D\sqrt{\tfrac{3}{5}}\,T_8^c(D\bar F\to\bar K\pi)\,, \tag{A4}$$

$$A(K^-K^+\to K^-K^+)=-4I_R M_0{}^2 f_K F^K[\tfrac{1}{4}T_1(\bar KK\to\bar KK)+\tfrac{2}{5}T_8(\bar KK\to\bar KK)]\,, \tag{A5}$$

$$A(\pi^-\pi^+\to K^-K^+)=-4I_R M_0{}^2 f_\pi F^\pi[\tfrac{1}{4}T_1(\pi\pi\to\bar KK)-\tfrac{1}{5}T_8(\pi\pi\to\bar KK)]\,, \tag{A6}$$

$$A(F^-F^+\to K^-K^+)=-4I_R M_0{}^2 f_F F^F\left[\frac{1}{2\sqrt{3}}\,T_1^c(\bar FF\to\bar KK)-\frac{1}{\sqrt{15}}T_8^c(\bar FF\to\bar KK)\right]\,, \tag{A7}$$

where we use the notations $F^A\equiv f_+^{DA}(0)-f_-^{DA}(0)$ and $F_A\equiv f_+^{DA}(0)$, and T_8 (T_8^c) and T_1 (T_1^c) are, respectively, the scattering amplitudes transforming as octet and singlet representations without (with) a charm-quark annihilation, and the factor 4 is multiplied because of the number of processes corresponding to PP, PV, VP, and VV scatterings (P: pseudoscalar meson, V: vector meson).

On the 16-plet symmetry assumption, like the nonet symmetry in SU(3), we obtain

$$T_1=\tfrac{16}{5}T_8\,,\quad T_8^c=-\sqrt{\tfrac{3}{5}}\,T_8\,,\quad T_1^c=-\tfrac{8}{5}\sqrt{3}\,T_8\,. \tag{A8}$$

Using Eqs. (A1)–(A8), the decay amplitudes $M(D^0\to K^-\pi^+)$ and $M(D^0\to K^-K^+)$ are rewritten as

$$M(D^0\to K^-\pi^+)=\frac{G_F}{\sqrt{2}}\cos^2\theta_c K_1[(f_\pi F^K+f_D F^F)T+f_\pi F_K(M_D{}^2-m_K{}^2)]\,,$$
$$M(D^0\to K^-K^+)=\frac{G_F}{\sqrt{2}}\sin\theta_C\cos\theta_C K_1[(2f_K F^K-f_\pi F^\pi+f_F F^F)T+f_K F_K(M_D{}^2-m_K{}^2)]\,, \tag{A9}$$

where $T=-4I_R M_0{}^2\tfrac{3}{5}T_8$. In the same way the other decay amplitudes are obtained as follows:

(i) Cabibbo-allowed decays [the factor $(G_F/\sqrt{2})\cos^2\theta_C$ is neglected]:

$$\begin{aligned}
&M(D^0\to\pi^0\bar{K}^0)=\frac{1}{\sqrt{2}}[-K_1(f_\pi F^K+f_F F^D)T+K_2 f_K F_\pi(M_D{}^2-m_\pi{}^2)],\\
&M(D^0\to\eta\bar{K}^0)=K_1\left(\frac{1}{2}-\frac{1}{\sqrt{2}}\right)(f_\pi F^K+f_F F^D)T+\tfrac{1}{2}K_2 f_K F_\eta(M_D{}^2-m_\eta{}^2),\\
&M(D^+\to\bar{K}^0\pi^+)=[K_1 f_\pi F_K+K_2 f_K F_\pi](M_D{}^2-m_K{}^2),\\
&M(F^+\to K^+\bar{K}^0)=K_2(f_K F^K+f_D F^D)T+K_2 f_K F_K'(M_D{}^2-m_K{}^2),\\
&M(F^+\to\eta\pi^+)=K_2(f_K F^K+f_D F^D)T-\frac{K_1}{\sqrt{2}}f_\pi F_\eta'(M_D{}^2-m_\eta{}^2),\\
&M(F^+\to\pi^0\pi^+)=0.
\end{aligned}\tag{A10}$$

(ii) Cabibbo-suppressed decays [the factor $(G_F/\sqrt{2})\sin\theta_C\cos\theta_C$ is neglected]:

$$\begin{aligned}
&M(D^0\to\pi^-\pi^+)=K_1(f_K F^K-2f_\pi F^\pi-f_D F^D)T-K_1 f_\pi F_\pi(M_D{}^2-m_\pi{}^2),\\
&M(D^0\to\pi^0\pi^0)=\frac{K_1}{\sqrt{2}}(f_K F^K-2f_\pi F^\pi-f_D F^D)T+\frac{K_2}{\sqrt{2}}f_\pi F_\pi(M_D{}^2-m_\pi{}^2),\\
&M(D^0\to\bar{K}^0K^0)=K_1(f_K F^K-f_\pi F_\pi-f_D F^D+f_F F^F)T,\\
&M(D^0\to\pi^0\eta)=\frac{-1}{\sqrt{2}}K_1(f_K F^K+f_\pi F^\pi)T+K_2\left[\left(\frac{1}{2\sqrt{2}}+\frac{1}{2}\right)f_\pi F_\pi(M_D{}^2-m_\pi{}^2)-\frac{f_\eta}{2\sqrt{2}}F_\pi(M_D{}^2-m_\eta{}^2)\right],\\
&M(D^0\to\eta\eta)=K_1\frac{1}{\sqrt{2}}(+\tfrac{3}{2}f_K F^K-f_\pi F^\pi-\tfrac{1}{2}f_D F^D+f_F F^F)T-K_2\tfrac{1}{2}\left(1+\frac{1}{\sqrt{2}}\right)f_\eta F_\eta(M_D{}^2-m_\eta{}^2),\\
&M(D^+\to\bar{K}^0K^+)=[-(K_1+K_2)f_\pi F^{(d\bar{d})}+K_1 f_K F^K-K_2 f_D F^D]T+f_K F_K(M_D{}^2-m_K{}^2),\\
&M(D^+\to\pi^0\pi^+)=-\frac{(K_1+K_2)}{\sqrt{2}}f_\pi F_\pi(M_D{}^2-m_\pi{}^2),\\
&M(D^+\to\eta\pi^+)=[K_1(f_K F^K-f_\pi F^\pi)-K_2(f_{(d\bar{d})}F^\pi+f_D F^D)]T-\tfrac{1}{2}K_1 f_\pi F_\eta(M_D{}^2-m_\pi{}^2)-K_2\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\right)f_\eta F_\pi(M_D{}^2-m_\eta{}^2),\\
&M(F^+\to K^0\pi^+)=[K_2(f_{(s\bar{s})}F^K+f_D F^F)+K_1(f_K F^{(s\bar{s})}-f_\pi F^K)]T+K_1 f_\pi F_K'(M_F{}^2-m_\pi{}^2),\\
&M(F^+\to K^+\pi^0)=\left(K_1\frac{1}{\sqrt{2}}(-f_K F^{(s\bar{s})}+f_\pi F^K)+\frac{K_2}{\sqrt{2}}(f_{(s\bar{s})}F^K+f_F F^F)\right)T-\frac{K_2}{\sqrt{2}}f_K F_\pi'(M_F{}^2-m_K{}^2),\\
&M(F^+\to K^+\eta)=[K_1(f_K F^{(s\bar{s})}-f_\pi F^K)+K_2(f_{(s\bar{s})}F^K+f_F F^F)]\left(\frac{1}{2}-\frac{1}{\sqrt{2}}\right)T+\left(-\frac{K_1}{\sqrt{2}}-\frac{K_2}{2}-\frac{K_2}{\sqrt{2}}\right)f_K F_\eta'(M_F{}^2-m_\pi{}^2),
\end{aligned}\tag{A11}$$

where $\eta=-1/\sqrt{2}\,|s\bar{s}\rangle+|u\bar{u}\rangle+|d\bar{d}\rangle/2$ is assumed and F_P' is a form factor describing transition of the F meson into a pseudoscalar meson P.

APPENDIX B: DECOMPOSITION OF *PV* DECAY AMPLITUDES

Cabibbo-allowed PV decay amplitudes (M) are decomposed into nonexotic scattering amplitudes R and Born amplitudes B_1 and B_2 which, respectively, stands for the amplitudes corresponding to two diagrams in Fig. 4:

$$\begin{aligned}
&M(D^0\to K^-\rho^+)=K_1[R^0(\bar{K}\rho)+B_1^0(\bar{K}\rho)],\\
&M(D^0\to\bar{K}^0\rho^0)=\frac{1}{\sqrt{2}}[-K_1R^0(\bar{K}\rho)+K_2B_2^0(\bar{K}\rho)],\\
&M(D^0\to K^{*-}\pi^+)=K_1[-R^0(\bar{K}^*\pi)+B_2^0(\bar{K}^*\pi)],\\
&M(D^0\to\bar{K}^{*0}\pi^0)=\frac{1}{\sqrt{2}}[K_1R^0(\bar{K}^*\pi)+K_2B_1^0(\bar{K}^*\pi)],
\end{aligned}$$

$$M(D^0 \to \bar{K}^0\omega) = -\frac{1}{\sqrt{2}} K_1 R^0(\bar{K}\omega) + \frac{K_2}{\sqrt{2}} B_2^0(\bar{K}\omega),$$

$$M(D^0 \to \bar{K}^{*0}\eta) = \left(\frac{1}{2} + \frac{1}{\sqrt{2}}\right) K_1 R^0(\bar{K}^*\eta) + \frac{K_2}{2} B_1^0(\bar{K}^*\eta),$$

$$M(D^+ \to \bar{K}^0\rho^+) = K_1 B_1^+(\bar{K}\rho) + K_2 B_2^+(\bar{K}\rho),$$

$$M(D^+ \to \bar{K}^{*0}\pi^+) = K_2 B_1^+(\bar{K}^*\pi) + K_1 B_2^+(\bar{K}^*\pi),$$

$$M(F^+ \to K^{*+}\bar{K}^0) = K_2[-R^F(K^*\bar{K}) + B_1^F(K^*\bar{K})],$$

$$M(F^+ \to K^+\bar{K}^{*0}) = K_2[R^F(K\bar{K}^*) + B_2^F(K\bar{K}^*)],$$

$$M(F^+ \to \rho^+\eta) = K_2 R^F(\rho\eta) - \frac{1}{\sqrt{2}} K_1 B_2^F(\rho\eta),$$

$$M(F^+ \to \pi^+\phi) = K_1 B_2^F(\pi\phi),$$

$$M(F^+ \to \pi^+\omega) = 0,$$

$$M(F^+ \to \pi^+\rho^0) = -M(F^+ \to \rho^+\pi^0) = -\sqrt{2} K_2 R^F(\pi\rho),$$

where indices $(0, +, F)$ denote the amplitudes for (D^0, D^+, F) meson decays, respectively, and $R^F(PV) = 2(f_K/f_\pi)R^0(PV)/[1 + (f_K/f_\pi)(F^\pi/F^K)]$. In the limit of 16-plet symmetry all the nonexotic amplitudes coincide.

APPENDIX C: DECOMPOSITION OF *PP* DECAY AMPLITUDES OF *B* MESONS

PP decay amplitudes of *B* mesons are decomposed into Born, nonexotic, and exotic amplitudes as follows:

$$\begin{aligned} M(B^0 \to K^-\pi^+) &= -\sqrt{2}\, M(B^0 \to \bar{K}^0\pi^0) \\ &= -\sqrt{2}\, M(B^- \to K^-\pi^0) \\ &= M(B^- \to K^0\pi^-) \simeq M(B_s^0 \to K^-K^+) \\ &= -M(B_s^0 \to \bar{K}^0K^0) \\ &\simeq -2(1+\sqrt{2}) M(B^0 \to \bar{K}^0\eta) \\ &= 2(1+\sqrt{2})\, M(B^- \to K^-\eta) \\ &\simeq -2M(B_s^0 \to \eta\eta) = -K_1 R_2, \end{aligned}$$

$$M(B^0 \to D^+\pi^-) \simeq M(B^0 \to F^+K^-) = -K_1(B + R_1) - K_2 E_2,$$

$$M(B^0 \to D^0\pi^0) = \frac{1}{\sqrt{2}}[-K_2B_2 + K_1R_1 - K_1E_1],$$

$$M(B^0 \to D^+F^-) = K_1(B_1 + R_2) + K_2E_2,$$

$$M(B^0 \to \eta_c\pi^0) = \frac{1}{\sqrt{2}} K_2B_2,$$

$$M(B^0 \to \eta_c\eta) = \frac{1}{2} K_2B_2,$$

$$M(B^- \to D^0F^-) = K_1(B_1 + R_2) + K_2E_2,$$

$$M(B^- \to \eta_c\pi^-) = -K_2B_2,$$

$$M(B_s^0 \to D^+D^-) = M(B_s^0 \to D^0\bar{D}^0) = K_1(B_1 + R_1) + K_2E_2,$$

$$M(B_s^0 \to \eta_c\eta) = -\frac{1}{\sqrt{2}}[K_2B_2 + K_1E_1],$$

$$M(B_s^0 \to F^+F^-) = K_1(B_1 + R_1 + R_2) + K_2E_2,$$

$$M(B_s^0 \to \eta_c\eta) = \frac{1}{\sqrt{2}}[K_2B_2 + K_1E_1],$$

where B_1 and B_2 (E_1 and E_2) correspond to two different diagrams in Fig. 1. Amplitudes for *PV* decays can be obtained by the replacement of one of two pseudoscalar mesons with a vector meson.

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$$c_- = \left(1 + \frac{13 - 2N_f}{12}\alpha_s \ln\frac{M_W^2}{\mu^2}\right)^{2/(33-2N_f)} \text{ and } c_+ = c_-^{-1/2},$$

where N_f stands for the number of flavors and α_s the running coupling constant of quantum chromodynamics.

Following Ref. 1, we take the color-matching factor $k=-1/3$.

[12]If we may write the p-wave amplitude as $T_R^p \sim f(k_0, \vec{k}^2)\cos\theta$ (θ the angle between $\vec{k}$ and the momentum of a final meson), we can evaluate the integration as $I_R^p \propto \int d^4k\, g(K_0, \vec{k}^2) f(k_0, \vec{k}^2)\cos\theta \simeq 0$, where $g(k_0, \vec{k}^2)$ is given in Ref. 1.

[13]We may ignore the term with $f_-(0)$ in B because this term is proportional to the square of a final pseudoscalar meson mass (m_1^2).

[14]In (3.4) we omit the decays with η, which are discussed in Appendix A.

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[26]In general, I_R can depend on M_B as $(M_B^2/s_0)^a$ with $a<0$. Though $s_0 \simeq 1$ (GeV)^2 is taken in ordinary Regge-pole analysis, we do not know if s_0 is a universal constant even for heavy-particle scattering or increase with heavy-particle masses. On the discussions in this section we assume $I_R \simeq 1/8\pi^2$, i.e., independent of M_B.

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