Brief Reports

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Self-forces and atoms in gravitational fields

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We consider the effect on an atom of the non-Coulomb terms in the electromagnetic field of a point charge and the corresponding self-forces and self-torques which arise in a gravitational field such as that of a black hole.

A solution of Maxwell's equations for the electric potential of a point charge held at rest outside a Schwarzschild black hole was given by Copson.[1] It was shown by Linet[2] that Copson's result required an additional correction term in order to satisfy the proper boundary condition at infinity on the electric field of a point charge. The electrostatic self-force related to that correction term was studied by Smith and Will.[3] Recently, Léauté and Linet[4] have rewritten the electrostatic potential in a small neighborhood of the charge in terms of coordinates y^α in which the Schwarzschild gravitational field appears locally homogeneous. In such coordinates, they find that the electrostatic potential has the same form as found by Whittaker[5] for a point charge at rest in a uniform gravitational field, except for the additional term

$$-qMr_S^{-3}y^1, \quad (1)$$

where M is the mass of the black hole, and r_S is the *fixed* Schwarzschild radial coordinate of the charge q. The spatial coordinates (y^1, y^2, y^3) have their origin at the position of the point charge and are oriented so that the gravitational acceleration in the small neighborhood of the charge is in the negative y^1 direction. Although the Whittaker term in the potential exerts no self-force on the charge, the additional term in Eq. (1) does exert a force on the charge in the direction away from the black hole, and of magnitude

$$q^2Mr_S^{-3}, \quad (2)$$

in agreement with Ref. 3. This also agrees with the large-r_S result of Vilenkin[6] and with the general weak-field expression of DeWitt and DeWitt.[7]

Recently, we considered the effect of spacetime curvature on a one-electron atom.[8,9] We found the Hamiltonian of the Dirac equation to first order in the Riemann curvature tensor of an arbitrary gravitational field at the position of the atom and calculated the perturbations of the atomic energy levels. Although the leading terms in the result do not depend on the curvature corrections to the Coulomb field of the central charge, we calculated those corrections to the electromagnetic field so as to obtain *all* terms in the Hamiltonian which were of first order in the Riemann tensor. The question naturally arises, in a spacetime in which terms analogous to Eq. (1) are present, how would they enter into our calculation of the A_μ of a point charge, and what effect would they and their corresponding self-forces have on the spectrum of the atom? In addition, do the curvature corrections we found in the electromagnetic field give rise to further self-forces? In our case, the atom was not held at rest, so that one would also expect self-torques to be exerted on the magnetic moments of the electrons and nucleus.

The effect of such terms on the spectrum would in any event be small with respect to the leading curvature contributions already found, but it is interesting that the effect of such terms is actually null for a neutral atom (assuming the radius of the atom is small with respect to its distance from the center of the mass distribution). We find that in a neutral atom the field term from the proton at the position of the electron cancels the corresponding self-force and self-torque of the electron. Furthermore, the curvature correction terms in the electromagnetic field do not give rise to an additional self-force or self-torque on a point charge. Therefore, the Hamiltonian and energy-level perturbations we obtained for a neutral atom are not modified by the non-Coulomb field terms which give rise to self-forces and self-torques. For ions there is a net effect of those terms which is small with respect to the leading term in the Hamiltonian, and which depends only on the total charge and magnetic moment of the ion, not on its internal structure.

In Ref. 9, the atom was in free fall and we worked in a coordinate system, known as Fermi normal coordinates, which is normal (locally inertial) on the world line of the atom. The electromagnetic field of a geodesically moving point charge of charge q was found to have the vector potential

$$A_0 = -qr^{-1} + A_0^{(1)}, \quad A_k = A_k^{(1)}, \tag{3}$$

where $\nabla_\mu A^\mu = 0$ (Lorentz gauge) and

$$\delta^{ij}\partial_i\partial_j A_0^{(1)} + \tfrac{1}{3}qr^{-3}(3R^0{}_{l0m} - R_{lm})x^l x^m = 0, \tag{4}$$

and

$$\delta^{ij}\partial_i\partial_j A_k^{(1)} + \tfrac{2}{3}qR^0{}_k r^{-1} + \tfrac{2}{3}qR^0{}_{lkm}x^l x^m r^{-3} = 0. \tag{5}$$

Here the x^i are the spatial Fermi normal coordinates, $r = (\delta_{ij}x^i x^j)^{1/2}$, and the components of the external Riemann tensor $R^\alpha{}_{\beta\gamma\delta}$, are evaluated at the position of the point charge at $x^i = 0$. The $R^\alpha{}_{\beta\gamma\delta}$ are regarded as essentially constant on time scales of interest for atomic processes. We have assumed that A_0 has the same singularity at the origin as in flat spacetime. The potentials $A_\mu^{(1)}$ are required to vanish at the origin of these normal coordinates, as they do in flat spacetime. Adding solutions of the homogeneous equation $\delta^{ij}\partial_i\partial_j A_\mu^{(1)} = 0$ to the solution found in Ref. 9, we obtain, in a neighborhood of the charge,

$$A_0^{(1)} = qK_i x^i + \tfrac{1}{12}q(R + 4R_{00})r + \tfrac{1}{12}q(3R^0{}_{l0m} - R_{lm})x^l x^m r^{-1} \tag{6}$$

and

$$A_k^{(1)} = qL_{kj}x^j + \tfrac{1}{2}qR_{0k}r + \tfrac{1}{6}qR^0{}_{lkm}x^l x^m r^{-1}. \tag{7}$$

The K_i and L_{kj} are constants, and one can require that $L_{kj} = -L_{jk}$ without altering the $F_{\mu\nu}$ or the Lorentz-gauge condition. The values of the K_i and L_{kj} depend on the global nature of the spacetime. In the Schwarzschild black-hole spacetime, Léauté and Linet[4] showed that for a point charge supported at rest the correction term to the electrostatic potential ($\phi = -A_0^{(1)}$) has the form of Eq. (1). That term is of the form $-qK_i y^i$ with $K_1 = Mr_S{}^{-3}$ and $K_2 = K_3 = 0$. It is analogous to the terms involving K_i and L_{kj} in Eqs. (6) and (7). In the neighborhood of a geodesically moving charge in an arbitrary spacetime, we expect that K_i and L_{kj} in Eqs. (6) and (7) will also be independent of q and will be no larger in order of magnitude than the largest component of $|R_{\alpha\beta\gamma\delta}|$.

The electric field of the charge is $E_i = F_{i0} = \partial_i A_0$. We define the self-force f_i on the charge as the limit of qE_i as $r \to 0$, averaged over all directions of approach to the origin. That is, $f_i = \lim_{r\to 0}(4\pi)^{-1}\int qE_i(\Omega)d\Omega$, where $E_i(\Omega)$ denotes the value of E_i on a line through the origin in the $\Omega = (\theta, \phi)$ direction. We find that the directional averages of the terms not involving K_i vanish, and thus do not contribute additional self-forces. The electric self-force on the charge is then

$$f_i = q^2 K_i. \tag{8}$$

The magnetic field is $B_i = \epsilon_{ijk}\partial_j A_k$. We define the self-magnetic field at the origin by a similar averaging process: $B_i(\text{self}) = \lim_{r\to 0}(4\pi)^{-1}\int B_i(\Omega) \times d\Omega$. The only nonvanishing contribution comes from the L_{kj} term in A_k and gives

$$B_i(\text{self}) = q\epsilon_{ijk}L_{kj}. \tag{9}$$

This will produce a self-torque

$$\tau_j = 2q\mu_k L_{kj} \tag{10}$$

on a spinning charge such as an electron through the coupling to its magnetic dipole moment $\vec{\mu}$.

Now consider a one-electron atom having a geodesic as its world line. (The general case of an atom on an arbitrary world line is presently being investigated with L.O. Pimentel.) The electric field produced at the position of the electron (regarded classically as a point charge) by the term involving K_i in Eq. (6) is $E_i = ZeK_i$, where Ze is the nuclear charge. This produces a force $f_i(\text{nuc}) = -Ze^2K_i$ on the electron. From Eq. (8) there is also a self-force acting on the electron and given by $f_i(\text{self}) = e^2K_i$. Therefore, the net force on the electron from the K_i terms in the field of the nucleus and of the electron is

$$f_i = (1 - Z)e^2K_i. \tag{11}$$

More generally, for an atom having Z' electrons one finds by taking into account the fields of the other electrons as well as the nucleus that the net force on one of the electrons from the K_i terms is $(Z' - Z)e^2K_i$. It follows that the effect of those terms vanishes for a neutral atom, and they should not be included in the Hamiltonian of such an atom. One might expect this result for a neutral atom near a black hole because the K_i terms were needed to ensure that the monopole contribution to the electric field of a charge had the correct form at infinity, but for a neutral atom the monopole term vanishes. It is also possible to show in the same way that if one includes the magnetic field of Eq. (9) produced by the electron itself, then the magnetic field produced by the L_{kj} terms vanishes in a neutral atom and such terms should be omitted from the Hamiltonian. If the terms involving K_i or L_{kj} did not cancel, then neutral particles placed in a gravitational field would behave differently, depending on their internal structures. Similarly, one finds that the net force and torque produced by the K_i and L_{kj} terms on each of the charges in an ion is the same as the self-force and torque on a point charge having the same net charge as the ion. Then the effects of the self-force and self-torque on the ion do not depend on its internal structure.

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