PHYSICAL REVIEW D VOLUME 24, NUMBER 2 15 JULY 1981

Bose-Einstein condensation, spontaneous symmetry breaking, and gauge theories

Joseph I. Kapusta
Theoretical Division, Los Alamos National Laboratory, University of California, Los Alamos, New Mexico 87545

(Received 19 December 1979; revised manuscript received 26 March 1981)

Bosonic chemical potentials for a variety of relativistic field theories are introduced via the methods of functional integrals with the aim of studying the relationship between Bose-Einstein condensation and spontaneous symmetry breaking. The models studied include the noninteracting and the self-interacting charged scalar field, scalar electrodynamics and the Higgs model, and the Weinberg-Salam model. In general the chemical potential acts as an effective symmetry-breaking parameter although the phase diagrams for the two cases ($m^2<0$ and $m^2>0$) look very different. It is found that the symmetry-restoring temperature in the Weinberg-Salam model increases with increasing electric charge density. Finally, the analysis of Jakobsen, Kon, and Segal of a conserved isotropic total angular momentum for the cosmic background radiation is shown to be erroneous.

INTRODUCTION

There is an interesting parallel between spontaneous symmetry breaking and Bose-Einstein condensation. Namely, both types of systems possess bosonic fields which acquire a constant classical component. This parallel is not exact, however, since the physics and the mathematical details of how these constant classical components arise are different. The aim of this paper is to elucidate the similarities and differences between spontaneous symmetry breaking and Bose-Einstein condensation for a variety of relativistic-field-theory models in four dimensions. Possibly the most closely related works in the literature dealing simultaneously with bosonic chemical potentials and spontaneous symmetry breaking are those of Norton and Cornwall[1] and Campbell, Dashen, and Manassah,[2] although their stated goals are somewhat different.

Parts of this work are also closely related to the work done in a series of papers by Kirzhnits and Linde,[3] although their methods do not explicitly invoke bosonic chemical potentials. An interesting technical detail is how the chemical potentials affect the functional integration when passing from the generating functional in vacuum field theory to the partition function for the many-body system. Particularly interesting is the non-Abelian gauge theory. For the most part fermions will not be discussed in this paper.

The noninteracting charged scalar field is studied in Sec. I. This simple model cannot exhibit spontaneous symmetry breaking of course, but it does serve to introduce the functional-integral method. This method provides an especially transparent means of viewing an exactly soluble model.

The self-interacting charged scalar field is taken up in Sec. II. When the mass is imaginary the system undergoes a second-order phase transition at a temperature T_c. Below T_c there is no "conventionally" conserved charge but above T_c there is. Nevertheless μ is not constrained to be zero at any T. When the mass is real there is no phase transition if $\mu^2<m^2$, but there is a second-order phase transition if $\mu^2>m^2$ and the transition temperature increases with increasing μ^2. This behavior differs qualitatively from the noninteracting case, which was first noticed in the context of nonrelativistic systems by Bogoliubov[4] long ago.

The photon gas is considered in Sec. III. Although the photon has no conserved internal quantum numbers, there still are conserved kinematic quantities such as angular momentum with which one may associate a chemical potential if the system is large but finite. Recent measurements by Woody and Richards[5] of the cosmic background radiation have led Jakobsen, Kon, and Segal[6] to postulate that this radiation possesses a conserved total angular momentum squared. They associate a chemical potential with this quantity although the chemical potentials for the components of the angular momentum are zero. This association is shown to violate the basic assumptions of statistical mechanics.

Scalar electrodynamics and the Higgs model are treated in Sec. IV. The introduction of an Abelian vector field does not significantly alter the results obtained for the self-interacting charged scalar field, unless the Φ^4 coupling is abnormally small.

Finally the Weinberg-Salam model is studied in Sec. V. This model is interestingly different from the previous models considered in that it exhibits spontaneous symmetry breaking while at the same time allowing a nonzero chemical potential associated with electric charge. It is also technically interesting in the sense that it has a chemical potential associated with the non-Abelian vector field. The model exhibits the well-known second-order phase transition from an ordered to a disordered state, but the critical temperature increases with increasing electric charge density.

Final remarks and a conclusion are then given. It is assumed throughout this paper that all relevant couplings are small enough to permit a perturbative analysis of the models. Some expected deviations when this is not the case are mentioned in passing.

I. NONINTERACTING CHARGED SCALAR FIELD

The procedure which allows one to write the partition function as a functional integral over appropriately weighted field configurations has been discussed extensively by Bernard[7] for arbitrary relativistic renormalizable Lagrangians. This procedure is based, of course, on the original work of Feynman[8] and Feynman and Hibbs.[9] It is useful to quickly review that work, especially since Bernard did not consider bosonic chemical potentials.

In vacuum field theory the transition amplitude for going from the state $|\Phi_0\rangle$ at $t=0$ to the state $|\Phi_1\rangle$ at $t=t_1$ is

$$\langle\Phi_1|\exp(-iHt)|\Phi_0\rangle=N\int[d\pi][d\Phi]\exp\left\{i\int_0^{t_1}dt\int_V d^3x\left[\pi\frac{\partial\Phi}{\partial t}-\mathcal{H}(\pi,\Phi)\right]\right\}. \tag{1.1}$$

Here H is the Hamiltonian, $\mathcal{H}$ the Hamiltonian density, Φ represents the fields and π their associated momenta, the integration in Φ is over all paths which begin at the state $|\Phi_0\rangle$ and end at the state $|\Phi_1\rangle$, while the integration in π is unrestricted. N is an infinite normalization constant.

Next let $it_1=\beta$ and $it=\tau$. Then

$$\langle\Phi_1|\exp(-\beta H)|\Phi_0\rangle=N\int[d\pi][d\Phi]\exp\left\{\int_0^{\beta}d\tau\int_V d^3x\left[i\pi\frac{\partial\Phi}{\partial\tau}-\mathcal{H}(\pi,\Phi)\right]\right\}. \tag{1.2}$$

The partition function is $Z=\mathrm{Tr}\exp(-\beta H)$, so to get a functional integral representation let $[d\Phi]$ (but not $[d\pi]$) go over all paths periodic in τ (bosons only):

$$Z=\mathrm{Tr}\exp(-\beta H)=\sum_{\Phi}\langle\Phi|\exp(-\beta H)|\Phi\rangle=N\int[d\pi]\int_{\text{periodic}}[d\Phi]\exp\left\{\int_0^{\beta}d\tau\int_V d^3x\left[i\pi\frac{\partial\Phi}{\partial\tau}-\mathcal{H}(\pi,\Phi)\right]\right\}. \tag{1.3}$$

If the system admits a set of mutually commuting, conserved, additive observables $\vec{\mathrm{N}}$ then we may associate with them a set of chemical potentials (Lagrange multipliers) $\vec{\mu}$ Then the partition function is

$$Z=\mathrm{Tr}\exp[-\beta(H+\vec{\mu}\cdot\vec{\mathrm{N}})]=N\int[d\pi]\int_{\text{periodic}}[d\Phi]\exp\left\{\int_0^{\beta}d\tau\int_V d^3x\left[i\pi\frac{\partial\Phi}{\partial\tau}-\mathcal{H}(\pi,\Phi)+\vec{\mu}\cdot\vec{\mathfrak{N}}(\pi,\Phi)\right]\right\}. \tag{1.4}$$

For fermions $\vec{\mathfrak{N}}$ normally depends only on the fields and not on the momenta For example, $\vec{\mathfrak{N}}$ may represent baryon and lepton number densities. In that case (see, for example, Freedman and McLerran[10]),

$$Z=N'(\beta)\int_{\text{antiperiodic}}[d\psi]\exp\left\{\int_0^{\beta}d\tau\int_V d^3x[\mathcal{L}(\psi,\partial_\mu\psi)+\vec{\mu}\cdot\vec{\mathfrak{N}}(\psi)]\right\}. \tag{1.5}$$

Thus $i\pi\partial\psi/\partial\tau-\mathcal{H}$ is converted back to the Lagrangian density $\mathcal{L}$. Here $N'(\beta)$ is β dependent[7] and the integral is antiperiodic for fermion fields ψ due to the anticommutation relations. For bosons $\vec{\mathfrak{N}}$ normally does depend on π. Let us see how this works for a simple soluble model.

Consider a noninteracting charged scalar field with the Lagrangian density

$$\begin{aligned}&\mathcal{L}=\partial^\mu\Phi^*\partial_\mu\Phi-m^2\Phi^*\Phi\,,\\&\Phi=\frac{1}{\sqrt{2}}(\Phi_1+i\Phi_2)\,,\quad\Phi^*=\frac{1}{\sqrt{2}}(\Phi_1-i\Phi_2)\,.\end{aligned} \tag{1.6}$$

The metric is $g^{00}=g_{00}=1$, $g^{ij}=g_{ij}=-\delta_{ij}$, and $\partial_0=\partial/\partial t=i\partial/\partial\tau$. There is a global U(1) symmetry with the conserved current

$$J_\mu=i(\Phi^*\partial_\mu\Phi-\Phi\partial_\mu\Phi^*)\,. \tag{1.7}$$

Hence $\mathfrak{N}=J_0$.

The momenta, Hamiltonian density, and number density are given by

$$\pi_1 = \frac{\partial \Phi_1}{\partial t}, \quad \pi_2 = \frac{\partial \Phi_2}{\partial t},$$

$$\mathcal{H} = \tfrac{1}{2}[(\pi_1)^2 + (\pi_2)^2 + (\vec{\nabla}\Phi_1)^2 + (\vec{\nabla}\Phi_2)^2 + m^2(\Phi_1)^2 + m^2(\Phi_2)^2], \tag{1.8}$$

$$\mathfrak{N} = \Phi_2\pi_1 - \Phi_1\pi_2 .$$

The partition function is then

$$Z = N \int [d\pi_1][d\pi_2] \int_{\text{periodic}} [d\Phi_1][d\Phi_2] \exp\Big[\int_0^\beta d\tau \int_V d^3x \Big(i\pi_1 \frac{\partial \Phi_1}{\partial \tau} + i\pi_2 \frac{\partial \Phi_2}{\partial \tau} + \mu(\Phi_2\pi_1 - \Phi_1\pi_2) - \tfrac{1}{2}[(\pi_1)^2 + (\pi_2)^2 + (\vec{\nabla}\Phi_1)^2 + (\vec{\nabla}\Phi_2)^2 + m^2(\Phi_1)^2 + m^2(\Phi_2)^2] \Big) \Big] . \tag{1.9}$$

Following Bernard the integrations over momenta will be done first. Divide up the τ integration into a Riemann sum of n fields evaluated at τ values separated by ϵ, with $n\epsilon = \beta$, and take the limits $n \to \infty$, $\epsilon \to 0$ The space integration need not be done so carefully:

$$Z = N \lim_{n\to\infty} \prod_{i=1}^{n} \prod_{j=1}^{n} \int_{-\infty}^{\infty} d\pi_1(i) d\pi_2(j) \int_{-\infty\,\text{periodic}}^{\infty} d\Phi_1(i) d\Phi_2(j)$$
$$\times \exp\Big(\sum_{k=1}^{n} \int_V d^3x \big(i\pi_1(k)[\Phi_1(k) - \Phi_1(k-1)] + i\pi_2(k)[\Phi_2(k) - \Phi_2(k-1)] + \epsilon\mu[\Phi_2(k)\pi_1(k) - \Phi_1(k)\pi_2(k)] - \tfrac{1}{2}\epsilon\{[\pi_1(k)]^2 + [\pi_2(k)]^2 + [\vec{\nabla}\Phi_1(k)]^2 + [\vec{\nabla}\Phi_2(k)]^2 + m^2[\Phi_1(k)]^2 + m^2[\Phi_2(k)]^2\} \big) \Big) . \tag{1.10}$$

Periodicity means that $\Phi_1(0) = \Phi_1(n)$ and $\Phi_2(0) = \Phi_2(n)$. The momentum integrations may now be done to yield

$$Z = N \lim_{n\to\infty} \prod_{i=1}^{n} \prod_{j=1}^{n} \int_{-\infty\,\text{periodic}}^{\infty} \left(\frac{2\pi}{\epsilon}\right)^{1/2} d\Phi_1(i) \left(\frac{2\pi}{\epsilon}\right)^{1/2} d\Phi_2(j)$$
$$\times \exp\Big[-\tfrac{1}{2} \sum_{k=1}^{n} \int_V d^3x \Big(\frac{[\Phi_1(k) - \Phi_1(k-1)]^2}{\epsilon} + \frac{[\Phi_2(k) - \Phi_2(k-1)]^2}{\epsilon} + \epsilon([\vec{\nabla}\Phi_1(k)]^2 + [\vec{\nabla}\Phi_2(k)]^2 + (m^2 - \mu^2)[\Phi_1(k)]^2 + (m^2 - \mu^2)[\Phi_2(k)]^2) + 2i\mu\{\Phi_1(k)[\Phi_2(k) - \Phi_2(k-1)] - \Phi_2(k)[\Phi_1(k) - \Phi_1(k-1)]\} \Big) \Big] . \tag{1.11}$$

We can readily recognize this as the discrete version of

$$Z = [N'(\beta)]^2 \int_{\text{periodic}} [d\Phi_1][d\Phi_2] \exp\Big\{ -\tfrac{1}{2} \int_0^\beta d\tau \int_V d^3x \Big[\left(\frac{\partial \Phi_1}{\partial \tau}\right)^2 + \left(\frac{\partial \Phi_2}{d\tau}\right)^2 + (\vec{\nabla}\Phi_1)^2 + (\vec{\nabla}\Phi_2)^2 + m^2(\Phi_1)^2 + m^2(\Phi_2)^2 + 2i\mu\left(\Phi_1 \frac{\partial \Phi_2}{\partial \tau} - \Phi_2 \frac{\partial \Phi_1}{\partial \tau}\right) - \mu^2[(\Phi_1)^2 + (\Phi_2)^2] \Big] \Big\} . \tag{1.12}$$

Here $N'(\beta)$ is a β-dependent but μ-independent constant whose β-dependent part has been computed by Bernard. Notice that the term in the large square brackets in the exponential differs from $\mathcal{L}(\Phi_1, \Phi_2, \partial_\mu \Phi_1, \partial_\mu \Phi_2; \mu = 0) + \mu\mathfrak{N}(\Phi_1, \Phi_2, i\partial\Phi_1/\partial\tau, i\partial\Phi_2/\partial\tau)$ by the amount $\mu^2\Phi^*\Phi$. This is due to the momentum dependence of $\mathfrak{N}$.

Now consider the partition function. Since the fields are periodic in τ as well as in $\vec{x}$ we may do one integration by parts and ignore a total derivative to obtain

$$Z = [N'(\beta)]^2 \int_{\text{periodic}} [d\Phi][d\Phi^*] \exp\Big(\int_0^\beta d\tau \int_V d^3x \{ -\Phi^*[(\partial_0 + i\mu)^2 - \vec{\nabla}^2 + m^2]\Phi \} \Big) . \tag{1.13}$$

The fields may be expanded as a Fourier series,

$$\Phi = \zeta + \frac{1}{2\beta}\sum_{n=-\infty}^{\infty}\int \frac{d^3k}{(2\pi)^3}[\exp(i\vec{k}\cdot\vec{x} + i\omega_n\tau)\phi_n(\vec{k}) + \exp(-i\vec{k}\cdot\vec{x} - i\omega_n\tau)\psi_n^*(\vec{k})],$$

$$\Phi^* = \zeta + \frac{1}{2\beta}\sum_{n=-\infty}^{\infty}\int \frac{d^3k}{(2\pi)^3}[\exp(i\vec{k}\cdot\vec{x} + i\omega_n\tau)\psi_n(\vec{k}) + \exp(-i\vec{k}\cdot\vec{x} - i\omega_n\tau)\phi_n^*(\vec{k})]. \qquad (1.14)$$

Here $\omega_n = 2\pi n/\beta$ and ζ carries the full infrared character of the fields, $\phi_0(\vec{0}) = \psi_0(\vec{0}) = 0$. Without loss of generality it may be taken as real. Notice that $\phi_{-n}(-\vec{k}) = \psi_n^*(\vec{k})$.

Substituting Eq. (1.14) into Eq. (1.13) one finds

$$Z = [N'(\beta)]^2 \int_{\text{periodic}} [d\phi][d\psi]\exp[\beta V(\mu^2 - m^2)\zeta^2 - \tfrac{1}{2}(\phi, D_-\phi) - \tfrac{1}{2}(\psi, D_+\psi)], \qquad (1.15)$$

where

$$D_\pm = (\omega_n \pm i\mu)^2 + \omega^2, \quad \omega^2 = \vec{k}^2 + m^2. \qquad (1.16)$$

Thus the thermodynamic potential is given by

$$\beta\Omega = -\ln Z = \beta V(m^2 - \mu^2)\zeta^2 + \tfrac{1}{2}\ln\det D_+ + \tfrac{1}{2}\ln\det D_- - 2\ln N'(\beta). \qquad (1.17)$$

The logarithms can be written as

$$\ln\det D_+ + \ln\det D_- = V\sum_n \int \frac{d^3k}{(2\pi)^3}\{\ln[\omega_n - i\mu)^2 + \omega^2] + \ln[\omega_n + i\mu)^2 + \omega^2]\}$$

$$= V\sum_n \int \frac{d^3k}{(2\pi)^3}\{\ln[\omega_n{}^2 + (\omega + \mu)^2] + \ln[\omega_n{}^2 + (\omega - \mu)^2]\}. \qquad (1.18)$$

Bernard has shown that

$$\ln N'(\beta) = -\ln\beta V\sum_n \int \frac{d^3k}{(2\pi)^3} + \text{constant independent of } \beta. \qquad (1.19)$$

Now notice that

$$\ln[\omega_n{}^2 + (\omega \pm \mu)^2] = \int_{1/\beta^2}^{(\omega\pm\mu)^2} \frac{dx^2}{\omega_n{}^2 + x^2} + \ln(\omega_n{}^2 + 1/\beta^2), \qquad (1.20)$$

and that

$$\ln(\omega_n{}^2 + 1/\beta^2) = -2\ln\beta + \ln(4\pi^2 n^2 + 1). \qquad (1.21)$$

Putting together Eqs. (1.17)–(1.21) results in

$$\Omega = V(m^2 - \mu^2)\zeta^2 + \tfrac{1}{2}V\int \frac{d^3k}{(2\pi)^3}\frac{1}{\beta}\sum_n\left(\int_{1/\beta^2}^{(\omega+\mu)^2}\frac{dx^2}{\omega_n{}^2 + x^2} + \int_{1/\beta^2}^{(\omega-\mu)^2}\frac{dx^2}{\omega_n{}^2 + x^2}\right) + \text{vacuum contribution}. \qquad (1.22)$$

Doing the sum over n, the integral over x, and throwing away a pure vacuum contribution results in

$$\Omega = V(m^2 - \mu^2)\zeta^2 - V\int \frac{d^3k}{(2\pi)^3}\frac{k^2}{3\omega}\left(\frac{1}{\exp[\beta(\omega + \mu)] - 1} + \frac{1}{\exp[\beta(\omega - \mu)] - 1}\right). \qquad (1.23)$$

Notice that the integral in Eq. (1.23) is defined only for $\mu^2 \leqslant m^2$. The parameter ζ should be determined by minimizing Ω. This implies that

$$\zeta = 0 \text{ if } \mu^2 < m^2, \quad \zeta \text{ not determined if } \mu^2 = m^2. \qquad (1.24)$$

To determine ζ in case $\mu^2 = m^2$ recall that the number density is given by

$$\rho = -\frac{1}{V}\left(\frac{\partial\Omega}{\partial\mu}\right)_{T,V} = 2\mu\zeta^2 + \rho^*(\beta, \mu), \qquad (1.25)$$

where the contributions due to condensate and thermal particle excitation are evident,

$$\rho^*(\beta, \mu) = \int \frac{d^3k}{(2\pi)^3}\left(\frac{1}{\exp[\beta(\omega - \mu)] - 1} - \frac{1}{\exp[\beta(\omega + \mu)] - 1}\right). \qquad (1.26)$$

If the density ρ is held fixed and the temperature $T = 1/\beta$ is lowered, $|\mu|$ will increase until the point $\mu^2 = m^2$ is reached. If the temperature is lowered still further, $\rho^*(\beta, |\mu| = m)$ will be less than ρ. Therefore ζ is given by

$$\zeta^2 = \frac{\rho - \rho^*(\beta, |\mu| = m)}{2m}, \qquad (1.27)$$

when $|\mu| = m$ and $T < T_c$. T_c is the critical temperature for a second-order phase transition to a Bose-Einstein condensed state. The transition temperature is determined implicitly by the equation $\zeta^2 = 0$ or

$$\rho = \rho^*(\beta_c, |\mu| = m). \qquad (1.28)$$

Obviously T_c is a monotonically increasing function of ρ. The limiting forms are

$$T_c \sim (3\rho)^{1/3} \quad \text{as } \rho \to \infty ,$$
$$T_c \sim \frac{2\pi}{m}\left(\frac{\rho}{\zeta(3/2)}\right)^{2/3} \quad \text{as } \rho \to 0 . \tag{1.29}$$

See Fig. 1.

Of course this physics was known long ago, but it is instructive to see it in a new light especially for the analysis of the models to come.

II. SELF-INTERACTING CHARGED SCALAR FIELD

Consider a self-interacting charged scalar field with the Lagrangian density

$$\mathcal{L} = \partial^\mu \Phi^* \partial_\mu \Phi - m^2 \Phi^* \Phi - \lambda(\Phi^*\Phi)^2 ,$$
$$\Phi = \frac{1}{\sqrt{2}}(\Phi_1 + i\Phi_2) , \quad \Phi^* = \frac{1}{\sqrt{2}}(\Phi_1 - i\Phi_2) . \tag{2.1}$$

There is a global U(1) symmetry with the conserved current

$$J_\mu = i(\Phi^* \partial_\mu \Phi - \Phi \partial_\mu \Phi^*) . \tag{2.2}$$

Following the analysis of Sec. I notice that the $(\Phi^*\Phi)^2$ interaction does not affect the momentum integrations. Thus

$$Z = [N'(\beta)]^2 \int_{\text{periodic}} [d\Phi][d\Phi^*] \exp\left(\int_0^\beta d\tau \int_V d^3x \, \mathcal{L}_{\text{eff}} \right) , \tag{2.3}$$

where

$$\mathcal{L}_{\text{eff}} = (\partial^\nu - i\mu\delta^{\nu 0})\Phi^*(\partial_\nu + i\mu\delta_{\nu 0})\Phi - m^2\Phi^*\Phi - \lambda(\Phi^*\Phi)^2 . \tag{2.4}$$

If $m^2 < 0$ then the vacuum undergoes spontaneous symmetry breaking. In that case one usually writes $\Phi = \zeta + \Psi/\sqrt{2}$ and $\Phi^* = \zeta + \Psi^*/\sqrt{2}$ where $\zeta^2 = -m^2/2\lambda$. One cannot construct a conserved current via the Noether theorem applied to the field Ψ since $\Psi \to \exp(i\alpha)\Psi$ is not a symmetry of the Lagrangian. This means that one cannot build up an equilibrium state of finite charge density by successively adding, one by one, the elementary excitations Ψ and Ψ^* to the vacuum. Nevertheless the current presented in Eq. (2.2) is still conserved, since its construction depends only on the symmetry of the Lagrangian (or alternatively the equations of motion) and not on the symmetry the vacuum happens to have. Such currents we shall refer to, for lack of a better name, as "unconventionally" conserved currents. States with a definite charge can indeed be constructed even when the symmetry of the vacuum is spontaneously broken.[11,12] We may treat the cases $m^2 < 0$ and $m^2 > 0$ simultaneously by not specifying any numerical value for m^2.

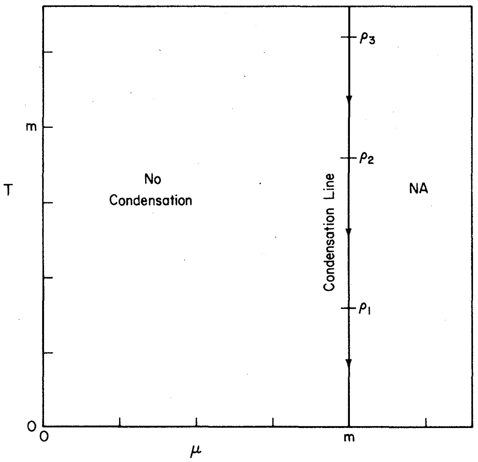

FIG. 1. Temperature vs chemical potential phase diagram for a noninteracting charged scalar field, Eq. (1.6). Bose-Einstein condensation exists only when $\mu = m$. The sample number densities satisfy $\rho_3 > \rho_2 > \rho_1 > 0$.

Since $\mathcal{L}_{\text{eff}}$ is not quadratic in the fields there is no known method of evaluating the path integral exactly. Hence one must resort to approximation techniques. There are two especially relevant techniques for relativistic many-body systems, as discussed by Linde.[3] The first technique is concerned with the evaluation of the effective potential and involves summations of infinite number of selected Feynman diagrams. See also Dolan and Jackiw[13] and Weinberg.[14] The second technique is more concerned with finding the quasiparticle spectrum of the system. Both techniques can determine the transition temperature, if there is one, and the results agree in lowest order. The quasiclassical-quasiparticle techniques will be mainly used in this paper. Some inadequacies of this approach are discussed later in this section and in the Appendix.

The Lagrange equations of motion for $\mathcal{L}_{\text{eff}}$ are

$$(\partial^2 + 2i\mu\partial_0 - \mu^2 + m^2)\Phi = -2\lambda\Phi\,\Phi^*\Phi \tag{2.5}$$

and its complex conjugate. Write

$$\Phi = \zeta + \frac{1}{\sqrt{2}}(\psi_1 + i\psi_2) ,$$
$$\Phi^* = \zeta + \frac{1}{\sqrt{2}}(\psi_1 - i\psi_2) , \tag{2.6}$$

where ζ represents the condensed part of the fields and ψ_1, ψ_2 the fluctuations. Now substitute Eq. (2.6) into Eq. (2.5) and take the Gibbs average $\langle\cdots\rangle$ to obtain

$$\zeta[\mu^2 - m^2 - 2\lambda\zeta^2 - \lambda(3\langle\psi_1{}^2\rangle + \langle\psi_2{}^2\rangle)] = 0 . \tag{2.7}$$

This has made use of the fact that the field fluctuations have been defined so that

$$\langle\psi_1\rangle = \langle\psi_2\rangle = 0\,, \tag{2.8}$$

and it has been assumed that there is no mixing,

$$\langle\psi_1\psi_2\rangle = 0\,. \tag{2.9}$$

This last statement will be verified shortly. Equation (2.7) has two possible solutions,

$$\zeta = 0 \tag{2.10}$$

and

$$\zeta^2 = \frac{1}{2\lambda}[\mu^2 - m^2 - \lambda(3\langle\psi_1^{\,2}\rangle + \langle\psi_2^{\,2}\rangle)]\,. \tag{2.11}$$

If $m^2 < 0$ and $\mu = T = 0$ then Eq. (2.11) becomes $\zeta^2 = -m^2/2\lambda$ which is the well-known result in the σ model. As the temperature rises the field fluctuations will increase until $\zeta \to 0$. At that point there is a symmetry-restoring phase transition.

In order to investigate Eq. (2.11) in more detail the quasiparticle spectrum must be determined. To this end assume that $\zeta \neq 0$, linearize the equations of motion with respect to ψ_1 and ψ_2, and take the Gibbs average:

$$[(\partial_0 + i\mu)^2 - \vec{\nabla}^2 + m^2 + 6\lambda\zeta^2 + \lambda(3\langle\psi_1^{\,2}\rangle + \langle\psi_2^{\,2}\rangle)]\langle\psi_1\rangle = 0\,, \tag{2.12}$$

$$[(\partial_0 + i\mu)^2 - \vec{\nabla}^2 + m^2 + 2\lambda\zeta^2 + \lambda(3\langle\psi_2^{\,2}\rangle + \langle\psi_1^{\,2}\rangle)]\langle\psi_2\rangle = 0\,.$$

What do the propogators for ψ_1 and ψ_2 look like?

In the notation of Eq. (1.14) we can write

$$\psi_1 = \frac{1}{2\beta}\sum_n \int \frac{d^3k}{(2\pi)^3}\left[\exp(i\vec{\mathrm{k}}\cdot\vec{\mathrm{x}} + i\omega_n\tau)\,\frac{\phi_n(\vec{\mathrm{k}}) + \psi_n(\vec{\mathrm{k}})}{\sqrt{2}} + \exp(-i\vec{\mathrm{k}}\cdot\vec{\mathrm{x}} - i\omega_n\tau)\frac{\phi_n^*(\vec{\mathrm{k}}) + \psi_n^*(\vec{\mathrm{k}})}{\sqrt{2}}\right], \tag{2.13}$$

$$\psi_2 = \frac{i}{2\beta}\sum_n \int \frac{d^3k}{(2\pi)^3}\left[\exp(i\vec{\mathrm{k}}\cdot\vec{\mathrm{x}} + i\omega_n\tau)\,\frac{\psi_n(\vec{\mathrm{k}}) - \phi_n(\vec{\mathrm{k}})}{\sqrt{2}} + \exp(-i\vec{\mathrm{k}}\cdot\vec{\mathrm{x}} - i\omega_n\tau)\frac{\phi_n^*(\vec{\mathrm{k}}) - \psi_n^*(\vec{\mathrm{k}})}{\sqrt{2}}\right].$$

Using Eq. (2.13) and the masses m_1 and m_2 implied by Eq. (2.12) one arrives at the effective quasiparticle action

$$-\frac{1}{2}\frac{1}{\beta}\sum_n \int \frac{d^3k}{(2\pi)^3}[(\omega_n^{\,2} + \vec{\mathrm{k}}^2 + m_1^{\,2} - \mu^2)\,|\phi_n(\vec{\mathrm{k}}) + \psi_n(\vec{\mathrm{k}})|^2 + (\omega_n^{\,2} + \vec{\mathrm{k}}^2 + m_2^{\,2} - \mu^2)|\phi_n(\vec{\mathrm{k}}) - \psi_n(\vec{\mathrm{k}})|^2]\,, \tag{2.14}$$

where

$$\begin{aligned} m_1^{\,2} &= m^2 + 6\lambda\zeta^2 + \lambda(3\langle\psi_1^{\,2}\rangle + \langle\psi_2^{\,2}\rangle)\,,\\ m_2^{\,2} &= m^2 + 2\lambda\zeta^2 + \lambda(3\langle\psi_2^{\,2}\rangle + \langle\psi_1^{\,2}\rangle)\,. \end{aligned} \tag{2.15}$$

Comparing with Eqs (1.15) and (1.16) one sees that the quasiparticles possess μ-dependent *effective* masses and chemical potentials given by

$$\begin{aligned} (m_1^{\mathrm{eff}})^2 &= m_1^{\,2} - \mu^2\,,\\ (m_2^{\mathrm{eff}})^2 &= m_2^{\,2} - \mu^2\,,\\ \mu_1^{\mathrm{eff}} &= \mu_2^{\mathrm{eff}} = 0\,. \end{aligned} \tag{2.16}$$

Since

$$\langle\psi_1^{\,2}\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_1}\left[\frac{1}{\exp[\beta(\omega_1 - \mu_1^{\mathrm{eff}})] - 1} + \frac{1}{\exp[\beta(\omega_1 + \mu_1^{\mathrm{eff}})] - 1}\right], \tag{2.17}$$

$$\omega_1 = [\mathrm{k}^2 + (m_1^{\mathrm{eff}})^2]^{1/2}\,,$$

and similarly for $\langle\psi_2^{\,2}\rangle$, it is evident that the self-consistent quasiparticle approach is highly nonlinear. However, it is possible to obtain a simple formula for the transition temperature.

At the transition temperature $\zeta = 0$ so that $m_1^{\,2} = \mu^2$ and also $m_2^{\,2} = \mu^2$ by symmetry. By Eqs. (2.16) and (2.17) one finds

$$\langle\psi_1^{\,2}\rangle = \langle\psi_2^{\,2}\rangle = \frac{1}{12}T_c^{\,2}\,. \tag{2.18}$$

Together with Eq. (2.11) this says that the transition temperature is given by

$$T_c^{\,2} = \frac{3}{\lambda}(\mu^2 - m^2)\,. \tag{2.19}$$

When $m^2 < 0$ and $\mu = 0$ then $T_c^{\,2} = -3m^2/\lambda$. This is an old result.[3,13,14] When $m^2 > 0$ positivity requires that $\mu^2 \geq m^2$. Thus the transition temperature increases with density, as expected in a second-order transition from an ordered to a disordered state. The T-μ phase diagrams for the cases $m^2 < 0$ and $m^2 > 0$ are shown in Figs. 2 and 3. These should be compared with each other and with Fig. 1.

To show that a second-order phase transition is indeed occurring is a boring chore in algebra. However for some specific cases it is easy. For the case $m^2 < 0$ see Linde.[3] For the case $m^2 > 0$ but $T = 0$ notice that

$$\Delta\Omega = \Omega(\zeta) - \Omega(0) = \zeta^2(m^2 - \mu^2 + \lambda\zeta^2)\,. \tag{2.20}$$

Then

$$\begin{aligned} &\Delta\Omega = -\frac{1}{4\lambda}(\mu^2 - m^2)^2\,,\\ &\frac{\partial}{\partial\mu}\Delta\Omega = 0 \quad \text{at} \quad \mu^2 = m^2\,,\\ &\frac{\partial^2}{\partial\mu^2}\Delta\Omega = -\frac{2m^2}{\lambda} \quad \text{at } \mu^2 = m^2\,. \end{aligned} \tag{2.21}$$

The condensed state is favored if $\mu^2 > m^2$ but not if $\mu^2 < m^2$. Zeroth and first-order derivatives are

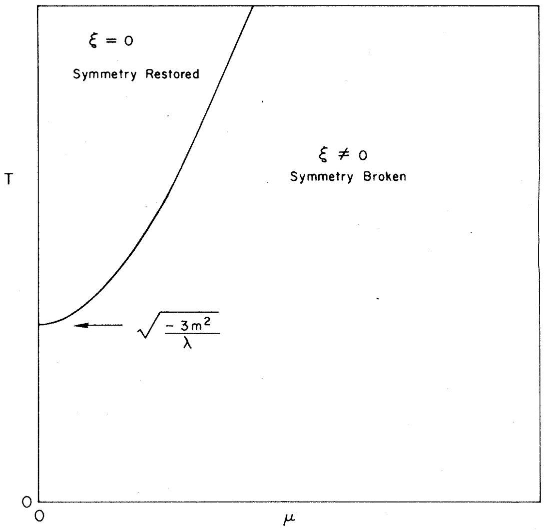

FIG. 2. Temperature vs chemical potential phase diagram for a self-interacting charged scalar field, Eq. (2.1), with $m^2 < 0$.

continuous, but the second-order derivative is discontinuous at $\mu^2 = m^2$, signaling a transition of second order.

For $T < T_c$ the effective masses in this quasiclassical approximation scheme are

$$\begin{aligned}(m_1^{\text{eff}})^2 &= 4\lambda\xi^2\,,\\(m_2^{\text{eff}})^2 &= 2\lambda(\langle\psi_2{}^2\rangle - \langle\psi_1{}^2\rangle)\,.\end{aligned} \tag{2.22}$$

One might wonder what happened to the Goldstone theorem, which requires a zero-frequency excitation at zero momentum when the symmetry is spontaneously broken ($m^2 < 0$ and $\xi \neq 0$). This is a point which is usually skimmed over in the literature. It has nothing to do with the introduction of a chemical potential since, for $T = 0$ but $\mu \neq 0$, $m_2^{\text{eff}} = 0$. In the Appendix it is shown that this violation of the Goldstone theorem at finite temperature is a consequence of retaining only some of the diagrams at a given order of loops in the meson self-energy expansion, and that it is necessary to include also the lowest-order exchange diagrams. The formula for the transition temperatures, Eq. (2.19), is unaffected by this since the quasiclassical approximation already has $m_2^{\text{eff}} = 0$ at $T = T_c$.

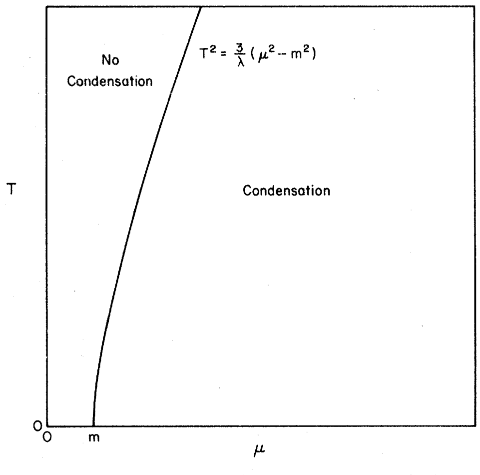

FIG. 3. Temperature vs chemical potential phase diagram for a self-interacting charged scalar field, Eq. (2.1), with $m^2 > 0$.

III. PHOTON GAS

The Feynman rules for a guage theory at finite temperature but zero bosonic chemical potentials have been extensively discussed by Bernard. The gauge freedom must be handled with care because some gauges have more than the physical number of degrees of freedom. These specious degrees of freedom never manifest themselves in the in and out states of scattering experiments; therefore, they could never come to equilibrium with a physical heat bath. The partition function in various gauges must be defined with this in mind.

The general result for the partition function is

$$Z = [N'(\beta)]^m \int_{\text{periodic}} [dA] \exp\left(\int_0^\beta d\tau \int_V d^3x \mathfrak{L} \right) \times \det\left(\frac{\partial F^b}{\partial \omega^c}\right) \prod_b \delta(F^b)\,. \tag{3.1}$$

Here m is the total number of physical particles and polarization states, the Dirac δ functions specify the gauge and the determinant relates the integration measure between different gauges. If the gauge theory is non-Abelian then it is possible that the vector fields carry a conserved quantum number and hence have an associated chemical potential. See Sec. V.

For a pure photon gas, described by

$$\begin{aligned}\mathfrak{L} &= -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}\,,\\F^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu\,,\end{aligned} \tag{3.2}$$

there are no conserved internal quantum numbers. However, there are some conserved kinematic quantities such as angular momentum $\vec{\mathrm{L}}$,

$$L^k = \int d^3x(-\tfrac{1}{2}\epsilon^{ijk}\mathfrak{M}^{oij})\,, \tag{3.3}$$

where

$$\mathfrak{M}^{\alpha\beta\delta} = T^{\alpha\beta}x^\delta - T^{\alpha\delta}x^\beta \tag{3.4}$$

and

$$T^{\alpha\beta} = F^\alpha{}_\lambda F^{\lambda\beta} + \tfrac{1}{4}g^{\alpha\beta}F_{\mu\lambda}F^{\mu\lambda} \tag{3.5}$$

is the symmetric, traceless gauge-invariant stress tensor.[15] If a heated gas of photons was placed in a container rotating about the z axis, one could describe the thermodynamics of the system by associating an effective chemical potential with L^3.

In a recent, very interesting paper, Jakobsen, Kon, and Segal[6] are led to postulate that the cosmic background radiation possesses a net total angular momentum squared, based on the measurements of Woody and Richards.[5] The partition function can always be written as

$$Z = \mathrm{Tr}\exp(-\beta H + \delta_1 K_1 + \delta_2 K_2 + \cdots), \tag{3.6}$$

where $H, K_1, K_2, \ldots$ must all be conserved, mutually commuting, and additive observables. By additive is meant that the eigenvalue of an operator for a composite system is the sum of eigenvalues of that operator for the components of the system. It is well known that these conditions on the observables are necessary in order for normal thermodynamics to hold.[16,17] For example, in a system with conserved baryon number B, one associates a chemical potential with B but never with $(B)^2$.

Likewise, under properly stated circumstances one could associate a chemical potential with L^3 but never with $(\vec{L})^2$ since it is not an additive observable. If the additivity condition was not enforced then one could also associate a chemical potential with $[(\vec{L})^2]^2$, *ad infinitum*. For the square of the square of the angular momentum one would obtain an energy (ν) distribution[6]

$$F_{(4)}(\nu) = 2\nu \int_0^\nu \frac{dx}{\exp(\beta\nu + \delta' x^2) - 1}, \tag{3.7}$$

which is not the same as one would obtain with the square of the angular momentum

$$F_{(2)}(\nu) = 2\nu \int_0^\nu \frac{dx}{\exp(\beta\nu + \delta x) - 1}. \tag{3.8}$$

In fact why not have chemical potentials for *both* the square and the square of the square of the angular momentum? This would differ from both Eqs. (3.7) and (3.8). This ambiguity is resolved by the additivity condition.

IV. SCALAR ELECTRODYNAMICS AND HIGGS MODEL

Consider the Lagrangian density

$$\begin{aligned}\mathcal{L} &= (\partial^\nu - ieA^\nu)\Phi^*(\partial_\nu + ieA_\nu)\Phi - m^2\Phi^*\Phi \\ &\quad - \lambda(\Phi^*\Phi)^2 - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}.\end{aligned} \tag{4.1}$$

Here Φ is a complex scalar field and A^ν is a vector field. If $m^2 < 0$ then spontaneous symmetry breaking occurs and there is no conventionally conserved current below a symmetry restoring temperature. (See the discussion of Sec. II.) This is the Higgs model.[18] If $m^2 > 0$ then Eq. (4.1) describes scalar electrodynamics. There is a conserved current

$$J_\nu = i[\Phi(\partial_\nu - ieA_\nu)\Phi^* - \Phi^*)\partial_\nu + ieA_\nu)\Phi]. \tag{4.2}$$

Thus a chemical potential may be associated with J_0. The possibility of Bose-Einstein condensation exists. As in Sec. II both cases $m^2 < 0$ and $m^2 > 0$ will be treated simultaneously.

The momenta conjugate to the components of the ϕ field Eq. (1.6) are

$$\begin{aligned}\pi_1 &= \partial_0\Phi_1 - eA_0\Phi_2, \\ \pi_2 &= \partial_0\Phi_2 + eA_0\Phi_1.\end{aligned} \tag{4.3}$$

The functional integrals over these moments may be done just as in Sec. I for the free field. The momenta for the vector field are given by the same formulas as in the vacuum. Thus one arrives at

$$\begin{aligned}Z = [N'(\beta)]^4 \int_{\text{periodic}} &[dA][d\Phi][d\Phi^*] \\ &\times \exp\left(\int_0^\beta d\tau \int_V d^3x\, \mathcal{L}_{\text{eff}}\right) \\ &\times \det\left(\frac{\partial F}{\partial\omega}\right)\delta(F),\end{aligned} \tag{4.4}$$

where

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= (\partial^\nu - ieA^\nu - i\mu\delta^{\nu 0})\Phi^*(\partial_\nu + ieA_\nu + i\mu\delta_{\nu 0})\Phi \\ &\quad - m^2\Phi^*\Phi - \lambda(\Phi^*\Phi)^2 - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}.\end{aligned} \tag{4.5}$$

Notice that if one would make the shift

$$A_\nu \to A_\nu - \frac{\mu}{e}\delta_{\nu 0} \tag{4.6}$$

then Eq. (4.4) for the partition function would become independent of μ. The reason for this apparent paradox is that a term $eA^\nu\mathcal{J}_\nu$ should be added to $\mathcal{L}_{\text{eff}}$, where $\mathcal{J}_\nu = \mathcal{J}_0\delta_{\nu 0}$ is a constant background charge density. Without a term of this form to offset the charge density of the scalar field the system could not be made electrically neutral (without having $\mu = 0$) and so thermodynamic equilibrium could not be established. The necessary value of the background charge density $\mathcal{J}_0$ will be determined by requiring

$$\langle A^\nu\rangle = 0. \tag{4.7}$$

The equations of motion with the $\mathcal{L}_{\text{eff}}$ of Eq. (4.5) are

$$\begin{aligned}[\partial^2 + 2i\mu\partial_0 - \mu^2 + m^2 + ie(\partial^\nu A_\nu) + 2ieA_\nu\partial^\nu \\ - e^2A^\nu A_\nu - 2e\mu A_0]\Phi = -2\lambda\Phi\Phi^*\Phi,\end{aligned} \tag{4.8}$$

and

$$-\partial^2 A^\nu = ie(\Phi\partial^\nu\Phi^* - \Phi^*\partial^\nu\Phi) + 2e\Phi(eA^\nu + \mu\delta^{\nu 0})\Phi^* + e\mathcal{J}^\nu . \qquad (4.9)$$

The procedure should be familiar by now. Write

$$\Phi = \zeta + \frac{1}{\sqrt{2}}(\psi_1 + i\psi_2),$$
$$\Phi^* = \zeta + \frac{1}{\sqrt{2}}(\psi_1 - i\psi_2),$$
$$\langle\Phi\rangle = \langle\Phi^*\rangle = \zeta ,$$
$$\langle A^\nu\rangle = 0 . \qquad (4.10)$$

Substituting these into Eqs. (4.8) and (4.9) and taking the Gibbs average, assuming no mixing among the fields, results in

$$\zeta^2 = 0$$

or

$$\zeta^2 = \frac{1}{2\lambda}[-m^2 + \mu^2 + e^2\langle A^\nu A_\nu\rangle - \lambda(3\langle\psi_1{}^2\rangle + \langle\psi_2{}^2\rangle)], \qquad (4.11)$$

and

$$\mathcal{J}^\nu = -\mu\delta^{\nu 0}(2\zeta^2 + \langle\psi_1{}^2\rangle + \langle\psi_2{}^2\rangle) . \qquad (4.12)$$

In order to determine the value of ζ^2, the pseudoparticle excitation spectra must be found. At this point it is necessary to choose a gauge. A convenient choice is the R_η gauge[18] in the limit $\eta\to\infty$. In Eq. (4.4) let

$$F = \partial^\nu A_\nu - \frac{\sqrt{2}e}{\eta}\zeta\psi_2 - f(\vec{x}, \tau) , \qquad (4.13)$$

where f is an arbitrary function. Under a gauge transformation w,

$$A^\nu \to A^\nu - \partial^\nu w ,$$
$$\Phi \to e^{iew}\Phi \approx \zeta + \frac{1}{\sqrt{2}}\psi_1 + \frac{i}{\sqrt{2}}(\psi_2 + \sqrt{2}ew\zeta) . \qquad (4.14)$$

Thus

$$\frac{\partial F}{\partial w} = -\partial^2 - \frac{2e^2\zeta^2}{\eta} \xrightarrow[\eta\to\infty]{} -\partial^2 . \qquad (4.15)$$

Furthermore, multiplying the right-hand side of Eq. (4.4) by

$$\exp\left(\frac{-\eta}{2}\int_0^\beta d\tau \int_V d^3x f^2\right) \qquad (4.16)$$

and integrating over $[df]$ gives a β and μ independent normalization correction which can be absorbed into $N'(\beta)$.[7] Hence

$$Z = \lim_{\eta\to\infty} [N'(\beta)]^4 \det(-\partial^2) \int_{\text{periodic}} [dA][d\Phi][d\Phi^*] \exp\left\{\int_0^\beta d\tau \int_V d^3x \left[\mathcal{L}_{\text{eff}} + eA^\nu J_\nu - \frac{\eta}{2}\left(\partial^\nu A_\nu - \frac{\sqrt{2}e}{\eta}\zeta\psi_2\right)^2\right]\right\} . \qquad (4.17)$$

The $\det(-\partial^2)$ cancels two specious degrees of freedom in the thermodynamic potential but otherwise is not important for the present discussion. The gauge fixing term does not affect the results of Eq. (4.10), (4.11), or (4.12).

The pseudoparticle excitation spectra are now determined by

$$[(\partial_0 + i\mu)^2 - \vec{\nabla}^2 + m^2 - e^2\langle A_\nu A^\nu\rangle + 6\lambda\zeta^2 + \lambda(3\langle\psi_1{}^2\rangle + \langle\psi_2{}^2\rangle)]\langle\psi_1\rangle = 2\sqrt{2}e\mu\zeta\langle A_0\rangle , \qquad (4.18)$$

$$[(\partial_0 + i\mu)^2 - \vec{\nabla}^2 + m^2 - e^2\langle A_\nu A^\nu\rangle + 2\lambda\zeta^2 + \lambda(3\langle\psi_2{}^2\rangle + \langle\psi_1{}^2\rangle)]\langle\psi_2\rangle = 0 , \qquad (4.19)$$

$$[\partial^2 + e^2(2\zeta^2 + \langle\psi_1{}^2\rangle + \langle\psi_2{}^2\rangle)]\langle A^\nu\rangle = -2\sqrt{2}e\mu\zeta\delta^{\nu 0}\langle\psi_1\rangle . \qquad (4.20)$$

Evidently if $e\mu\zeta \neq 0$ then there is mixing between the A^0 and ψ_1 fields. Thus it was not legitimate to set

$$\langle\psi_1 A^0\rangle = 0 \qquad (4.21)$$

in the previous equations. Nevertheless if one is only interested in locating the transition line determined by $\zeta = 0$ then it is legitimate to neglect the mixing. Following the discussion of Sec. II it is readily verified that, when $\zeta \to 0$,

$$m_1^{\text{eff}} = m_2^{\text{eff}} = 0 ,$$
$$\mu_1^{\text{eff}} = \mu_2^{\text{eff}} = 0 , \qquad (4.22)$$
$$m_A{}^2 = e^2(\langle\psi_1{}^2\rangle + \langle\psi_2{}^2\rangle) \ll T^2 .$$

This means that

$$\tfrac{1}{12}T^2 = \langle\psi_1{}^2\rangle = \langle\psi_2{}^2\rangle \cong -\tfrac{1}{3}\langle A^\nu A_\nu\rangle , \qquad (4.23)$$

so that the transition temperature from an ordered to a disordered phase is given by

$$T_c{}^2 = \frac{12(\mu^2 - m^2)}{4\lambda + 3e^2} . \qquad (4.24)$$

This formula is exactly analogous to Eq. (2.19) for a self-interacting charged scalar field. If $m^2 < 0$ then $\mu^2 \geq 0$, and if $m^2 > 0$ then $\mu^2 \geq m^2$ at the transition.

The results of this calculation should be compared with the results of Linde[3,19] who considered the Higgs model with the addition of charged fermions. One may check that our formulas for the transition temperature, Eq. (4.24), agree by eliminating the chemical potential in favor of the external electric charge density via Eq. (4.12). In higher order the formula for T_c may be different since in one case the electric charge is supplied by a fixed external source while in the other it is supplied by a dynamic source of fermions. Of course it is also possible for the symmetry-restoring temperature to be altered by the presence of some conserved fermion number even at zero electric charge density.[19,20]

At this point it is worth commenting on the connection between the methods used here and the methods used by Kirzhnits and Linde. Here we have proceeded by introducing a chemical potential, multiplying by the time component of a conserved current, and inserting it into the partition function. This is the canonical procedure in statistical mechanics. An exact result is that the gauge field is shifted by an amount $\delta_{\nu 0}\mu/e$. This is the same μ which appears for the scalar field, whose derivative is shifted to $\partial_\nu + i\mu\delta_{\nu 0}$. On the other hand Kirzhnits and Linde have essentially restricted their considerations to chemical potentials associated with conserved fermion currents only.

Actually the result Eq. (4.24) depends on a relationship between the coupling constants, $\lambda \gg e^4$. If $\lambda \lesssim e^4$ then radiative corrections in order e^4 can significantly alter the excitation spectra. In the case $m^2 < 0$ it is known that if $\lambda < 19e^4/32\pi^2$ a dynamical symmetry restoration takes place at zero temperature.[3,21] It would be interesting to examine the effects of radiative corrections when $\mu \neq 0$, but that is beyond the scope of this paper.

More generally the question of what happens in the Higgs model at finite temperature if the couplings are not perturbatively small has been studied by Banks and Rabinovici.[22] Their calculations are based on a lattice approximation. It would be very interesting to investigate the effects of bosonic chemical potentials in the context of their methods.

V. WEINBERG-SALAM MODEL

Consider the Weinberg-Salam model[23,24] of electromagnetic-weak interactions described by

$$\mathcal{L} = (D^\nu\Phi)^\dagger(D_\nu\Phi) + c^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 - \tfrac{1}{4}g^{\mu\nu}g_{\mu\nu} - \tfrac{1}{4}f^{\mu\nu}_{a}f^{a}_{\mu\nu}, \tag{5.1}$$

which is based on the gauge group SU(2)×U(1), and where

$$\begin{aligned} D_\nu &= \partial_\nu + igA^a_\nu \frac{\tau^a}{2} + i\frac{g'}{2}B_\nu, \\ g^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu, \\ f^{\mu\nu}_a &= \partial^\mu A^\nu_a - \partial^\nu A^\mu_a - g\epsilon_{abc}A^\mu_b A^\nu_c. \end{aligned} \tag{5.2}$$

Of all the models considered in this paper this model is certainly the most interesting for two reasons. Firstly, the Weinberg-Salam model is a realistic model in the sense that it provides a good description of all the relevant experimental data at the present time. Secondly, this model has the capability of exhibiting spontaneous symmetry breaking and Bose-Einstein condensation simultaneously but independently.

It is known that the standard model with one Higgs doublet undergoes symmetry restoration above some critical temperature.[3] However, it has recently been shown by Mohapatra and Senjanović[25] that an extension of the model with three Higgs doublets does not have its symmetry restored at any temperature. Hence one must be careful about making incorrect general statements about symmetry breaking in the electromagnetic-weak interactions.

The standard model has four conserved currents above the symmetry-restoring temperature, since the gauge group SU(2)×U(1) has four independent generators. Below the critical temperature there is only one conventionally conserved current, the electromagnetic current,

$$J_\nu = \frac{i}{2}[(D_\nu\Phi)^\dagger(\tau_3 + I)\Phi - \Phi^\dagger(\tau_3 + I)(D_\nu\Phi)] - \epsilon^{ab3}f^{a}_{\nu\mu}A^\mu_b + \bar{e}\gamma_\nu e\,. \tag{5.3}$$

The last term above shows the contribution of the electron to the charge current. Fermions will not be treated here, but the electron term indicates the normalization of the current and hence the charged chemical potential μ. For simplicity of discussion we will not consider chemical potentials for the three remaining currents.

Let

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2 \\ \phi_3 + i\phi_4\end{pmatrix}. \tag{5.4}$$

Then the momenta conjugate to the real fields ϕ_i are

$$\begin{aligned} \pi_1 &= \partial^0\phi_1 - \tfrac{1}{2}(g'B^0 + gA^0_3)\phi_2 + \tfrac{1}{2}gA^0_2\phi_3 - \tfrac{1}{2}gA^0_1\phi_4\,, \\ \pi_2 &= \partial^0\phi_2 + \tfrac{1}{2}(g'B^0 + gA^0_3)\phi_1 + \tfrac{1}{2}gA^0_1\phi_3 + \tfrac{1}{2}gA^0_2\phi_4\,, \\ \pi_3 &= \partial^0\phi_3 + \tfrac{1}{2}(-g'B^0 + gA^0_3)\phi_4 - \tfrac{1}{2}gA^0_2\phi_1 - \tfrac{1}{2}gA^0_1\phi_2\,, \\ \pi_4 &= \partial^0\phi_4 - \tfrac{1}{2}(-g'B^0 + gA^0_3)\phi_3 + \tfrac{1}{2}gA^0_1\phi_1 - \tfrac{1}{2}gA^0_2\phi_2\,. \end{aligned} \tag{5.5}$$

The part of J_0 involving the Φ field is

$$\phi_1\pi_2 - \phi_2\pi_1\,. \tag{5.6}$$

Now the momentum integrations over π_i can be done for the partition function. The calculation closely parallels that of Sec. I. The result, which is almost obvious, is that the only change in $\mathcal{L}$ is to replace

$$D_\nu \to \tilde{D}_\nu \equiv D_\nu + i\mu\delta_{\nu 0}\hat{Q}, \tag{5.7}$$

where

$$\hat{Q} = \tfrac{1}{2}(\tau_3 + I) \tag{5.8}$$

is the charge operator.

The Abelian vector field momentum integrals can be done just as in the vacuum. The non-Abelian vector field momentum integrals are different, since the charge density J_0 depends on the momenta. It is convenient, for the purpose of obtaining a formula for the partition function which may be generalized to other gauges, to work in the axial gauge,

$$\begin{aligned} &\partial_\nu A_a^\nu = 0, \\ &A_a^3 = 0. \end{aligned} \tag{5.9}$$

Then A_a^0 is taken as a dependent variable. The momenta are

$$\begin{aligned} P_a^1 &= \partial^0 A_a^1 - \partial^1 A_a^0 - g\epsilon_{abc}A_b^0 A_c^1 = f_a^{01}, \\ P_a^2 &= \partial^0 A_a^2 - \partial^2 A_a^0 - g\epsilon_{abc}A_b^0 A_c^2 = f_a^{02}. \end{aligned} \tag{5.10}$$

The relevant terms in the argument of the exponential are

$$\begin{aligned} -\tfrac{1}{2}P_a^1 P_a^1 &+ P_a^1(\partial^0 A_a^1 - \partial^1 A_a^0 - g\epsilon_{abc}A_b^0 A_c^1 + \mu\epsilon_{ab3}A_b^1) - \tfrac{1}{2}P_a^2 P_a^2 \\ &+ P_a^2(\partial^0 A_a^2 - \partial^2 A_a^0 - g\epsilon_{abc}A_b^0 A_c^2 + \mu\epsilon_{ab3}A_b^2) \\ &\quad + \tfrac{1}{2}(\partial^3 A_a^0)^2 - \tfrac{1}{4}f_{ij}^a f_a^{ij}. \end{aligned} \tag{5.11}$$

The result of the integration is to produce a term

$$-\tfrac{1}{4}\tilde{f}_a^{\mu\nu}\tilde{f}_{\mu\nu}^a, \tag{5.12}$$

where

$$\begin{aligned} \tilde{f}_a^{\mu\nu} = {} & \partial^\mu A_a^\nu - \partial^\nu A_a^\mu \\ & - g\epsilon_{abc}\left(A_b^\mu + \frac{\mu}{g}\delta^{\mu 0}\delta_{b3}\right)\left(A_c^\nu + \frac{\mu}{g}\delta^{\nu 0}\delta_{c3}\right). \end{aligned} \tag{5.13}$$

This is equivalent to shifting the fields by

$$A_a^\nu \to A_a^\nu + \frac{\mu}{g}\delta^{\nu 0}\delta_{a3}. \tag{5.14}$$

This shift goes to zero when μ goes to zero. The $\delta^{\nu 0}$ arises because there is a preferred reference frame, the center-of-momentum frame for the many-body medium. The δ_{a3} arises because it is the $a=3$ field which mixes with B_ν to form the photon, which is the carrier of the electromagnetic force one is focusing on by having $\mu \neq 0$ for the charge density.

The effective Lagrangian which emerges is

$$\begin{aligned} \mathcal{L}_{\text{eff}} = {} & (\tilde{D}^\nu\Phi)^\dagger(\tilde{D}_\nu\Phi) + c^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 - \tfrac{1}{4}g^{\mu\nu}g_{\mu\nu} \\ & - \tfrac{1}{4}\tilde{f}_a^{\mu\nu}\tilde{f}_{\mu\nu}^a + g'B_\nu\mathcal{J}^\nu + gA_\nu^a\mathcal{J}_a^\nu. \end{aligned} \tag{5.15}$$

The resulting equations of motion are

$$\begin{aligned} & \Big(\partial^2 + ig(\partial_\nu A_a^\nu)\frac{\tau^a}{2} + igA_a^\nu\tau^a\partial_\nu + i\frac{g'}{2}(\partial^\nu B_\nu) + ig'B_\nu\partial^\nu \\ & \quad + i\mu(\tau_3 + I)\partial_0 - \tfrac{1}{4}[gA_a^\nu\tau^a + g'B^\nu + \mu\delta^{\nu 0}(\tau_3 + I)]^2 - c^2\Big)\Phi \\ & \qquad = -2\lambda\Phi\,\Phi^*\Phi, \\ & -\partial^\mu\tilde{f}_{\mu\nu}^a = \frac{ig}{2}[(\tilde{D}_\nu\Phi)^\dagger\tau^a\Phi - \Phi^\dagger\tau^a(\tilde{D}_\nu\Phi)] \\ & \qquad + \epsilon_{abc}\tilde{f}_{\mu\nu}^b(gA_c^\mu + \mu\delta^{\mu 0}\delta_{c3}) + g\mathcal{J}_\nu^a, \\ & -\partial^\mu g_{\mu\nu} = \frac{ig'}{2}[(\tilde{D}_\nu\Phi)^\dagger\Phi - \Phi^\dagger(\tilde{D}_\nu\Phi)] + g'\mathcal{J}_\nu. \end{aligned} \tag{5.16}$$

As usual one defines

$$\begin{aligned} A^\mu &= \cos\theta B^\mu + \sin\theta A_3^\mu, \\ Z^\mu &= \sin\theta B^\mu - \cos\theta A_3^\mu, \\ W_\pm^\mu &= \frac{1}{\sqrt{2}}(A_1^\mu \mp A_2^\mu), \\ \tan\theta &= g'/g, \quad e = g\sin\theta. \end{aligned} \tag{5.17}$$

Here A^μ is the electromagnetic field, e is the electric charge, θ is the Weinberg angle, etc.

Before attacking the full model it is useful to consider the limit when $g = g' = 0$. Consider the ground state of the system. Write

$$\langle\Phi\rangle = \begin{pmatrix} \zeta\cos\alpha \\ \zeta\sin\alpha \end{pmatrix}. \tag{5.18}$$

Then the thermodynamic potential is

$$\Omega = -V\mathcal{L} = -V[(\mu^2\cos^2\alpha + c^2)\zeta^2 - \lambda\zeta^4]. \tag{5.19}$$

If $\mu = 0$ then Ω is independent of α and one usually makes the choice $\alpha = \pi/2$. When $\mu \neq 0$, however, the state with $\alpha = 0$ is energetically preferred. The reason is, of course, that the top component of Φ is charged, whereas the bottom component is electrically neutral. Thus it would be a mistake to expand about the point $\alpha = \pi/2$ if there is an external charge density present.

Next consider finite temperature. Write

$$\Phi = \begin{pmatrix} \zeta + (\psi_1 + i\psi_2)/\sqrt{2} \\ (\psi_3 + i\psi_4)/\sqrt{2} \end{pmatrix}. \tag{5.20}$$

Then the equation for ζ is

$$\zeta[\mu^2 + c^2 - \lambda(2\zeta^2 + 3\langle\psi_1^2\rangle + \langle\psi_2^2\rangle + \langle\psi_3^2\rangle + \langle\psi_4^2\rangle)] = 0. \tag{5.21}$$

When $\zeta \to 0$ one can show that all effective masses and chemical potentials are either zero or small compared to T_c. Hence the relationship between the temperature and chemical potential at the transition is

$$T_c^2 = \frac{2(c^2 + \mu^2)}{\lambda}. \tag{5.22}$$

Based on the experience gained by examining the above simpler models, it is now straightforward to show that in the full Weinberg-Salam model the transition temperature is given by the formula

$$T_c^2 = \frac{4(c^2+\mu^2)}{2\lambda + e^2(1+2\cos^2\theta)/\sin^2 2\theta}\,. \tag{5.23}$$

Notice that the critical temperature starts out at some finite value when the charge density is zero, and then increases with increasing density. See Fig. 4. This behavior is typical of a second-order phase transition from a disordered to an ordered state. This should be compared with the first-order transition from a hadron phase to a quark phase expected to occur in quantum chromodynamics (QCD). See Chin[26] and Kapusta.[27] In that theory one expects a relationship of roughly the form

$$(3T_c)^2 = (0.6\mu)^2 = \Lambda^2\,, \tag{5.24}$$

where μ is now the baryonic chemical potential and Λ is the scale-violation parameter.

In a recent paper Linde[28] has shown that it is possible for a condensation of the $W^{\pm}$ mesons to occur at sufficiently high fermion density. It turns out that it is a space component of the $W^{\pm}$ fields which develops a classical constant component. Any $W^{\pm}$ condensation which may occur at finite electric charge density but zero fermion number density is not expected to change any of the results obtained in this section, since at fixed temperature that condensation should occur at higher densities than the symmetry-restoring density. It should also be pointed out that, in this model at least, a finite density of charged fermions[19] is not equivalent to a constant background charge density because the fermions have interactions other than just electromagnetic.

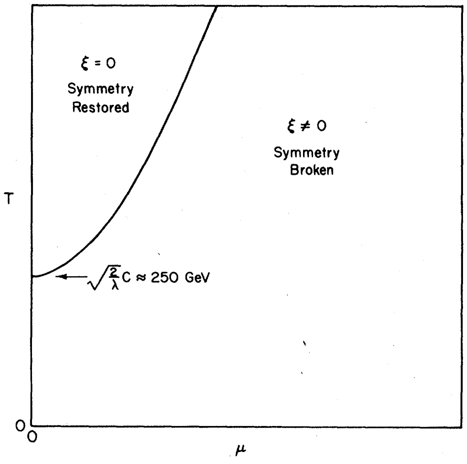

FIG. 4. Temperature vs charge chemical potential phase diagram for the Weinberg-Salam model.

CONCLUSION

The aim of this paper, to study the relationship between spontaneous symmetry breaking and Bose-Einstein condensation for relativistic renormalizable Lagrangians, especially gauge theories, in four dimensions has been accomplished in two parts.

The first part consisted of finding the effective Lagrangian when the bosonic chemical potential is nonzero. The technical means for doing this was the path or functional integral method applied to the partition function. The usual prescription for the introduction of nonzero chemical potential, $\partial_0 \to \partial_0 + i\mu$, was found to be correct for the charged scalar fields in all of the models considered. The way such a chemical potential affects a non-Abelian vector field carrying the charge is different. See Eqs. (5.12) to (5.14). As a side excursion from the goal stated above, it was shown how and how not to associate a chemical potential with the angular momentum operator for a spinning photon gas.

The second part consisted of analyzing the partition function for a variety of models and searching for the location and behavior of phase transitions. The noninteracting charged scalar field is an exactly soluble model. The functional integral solution possibly looks at an old model from a different viewpoint. The self-interacting charged scalar field, with or without an Abelian vector fields, allows some insight into the connection between spontaneous symmetry breaking and Bose-Einstein condensation. The self-consistent method of quasiparticles was used to analyze these models. Both $m^2 < 0$ and $m^2 > 0$ have second-order phase transitions, wherein an order parameter ξ corresponding to a constant part of the scalar field goes to zero. The chemical potential acts effectively as a symmetry-breaking parameter. The Weinberg-Salam model allows both spontaneous symmetry breaking and Bose-Einstein condensation for the field Φ. The critical temperature increases with electric charge density.

It would be interesting to compute the radiative corrections for gauge theories when chemical potentials are present. More generally nonperturbative lattice calculations may shed additional light on the strong-coupling limit.

I am grateful to A. Linde for stimulating comments on some aspects of this paper, and to R.

Norton for a discussion which led to a clarification of the connection between a conserved current and a noninvariant vacuum and for some of the material in the Appendix. This work was initiated at the Lawrence Berkeley Laboratory and was supported by the U. S. Department of Energy.

APPENDIX

Consider the effective Lagrangian written in Eq. (2.4). In terms of the shifted fields, Eq. (2.6), it is

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= U(\zeta) + \mathcal{L}_0 + \mathcal{L}_I\,;\\ U(\zeta) &= \zeta^2(\mu^2 - m^2 - \lambda\zeta^2)\,,\end{aligned} \tag{A1}$$

$$\begin{aligned}\mathcal{L}_0 = {}& \tfrac{1}{2}\partial_\nu\psi_1\partial^\nu\psi_1 - \tfrac{1}{2}(m^2 - \mu^2 + 6\lambda\zeta^2)\psi_1{}^2\\ &+ \tfrac{1}{2}\partial_\nu\psi_2\partial^\nu\psi_2 - \tfrac{1}{2}(m^2 - \mu^2 + 2\lambda\zeta^2)\psi_2{}^2\\ &+ \mu(\psi_1\partial_0\psi_2 - \psi_2\partial_0\psi_1)\,,\end{aligned}$$

$$\mathcal{L}_I = -2^{1/2}\lambda\zeta\psi_1(\psi_1{}^2 + \psi_2{}^2) - \tfrac{1}{4}\lambda(\psi_1{}^2 + \psi_2{}^2)^2\,.$$

Let us first focus our attention on the case of broken symmetry ($m^2 < 0$) and zero density ($\mu = 0$). Neglecting $\mathcal{L}_I$ we see that the particle masses are

$$\begin{aligned}\overline{m}_1{}^2 &= m^2 + 6\lambda\zeta^2\,,\\ \overline{m}_2{}^2 &= m^2 + 2\lambda\zeta^2\,.\end{aligned} \tag{A2}$$

Notice that these do not contain the influence of thermal fluctuations and are not the same as the effective masses given in Eq. (2.15).

Consider doing the two-loop corrections to the thermodynamic potential arising from $\mathcal{L}_I$.[1,29] These are shown in Fig. 5(a). The corresponding contributions to the self-energy of the ψ_2 field are shown in Fig. 5(b). Denoting the two-loop diagrams by $f_2(D_1, D_2)$, where D_1 and D_2 are the full propagators, we have

$$[D_2(\omega_n, \vec{k})]^{-1} - [\overline{D}_2^0(\omega_n, \vec{k})]^{-1} = 2\,\frac{\delta}{\delta D_2}f_2(D_1, D_2)\,.$$

Here (A3)

$$[\overline{D}_2^0(\omega_n, \vec{k})]^{-1} = \omega_n{}^2 + \vec{k}^2 + \overline{m}_2{}^2\,. \tag{A4}$$

The first two diagrams of Fig. 5(b) are momentum independent. Using only those in Eq. (A3) we get

$$[D_2(0, \vec{0})]^{-1} = m^2 + 2\lambda\zeta^2 + \lambda\langle\psi_1{}^2\rangle + 3\lambda\langle\psi_2{}^2\rangle\,. \tag{A5}$$

This corresponds precisely with the quasiclassical approximation used in the text which led to Eq. (2.15).

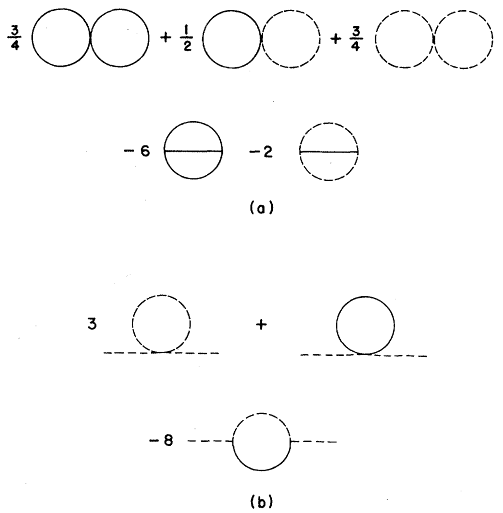

FIG. 5. (a) Two-loop contributions to the thermodynamic potential for the self-interacting charged scalar field. The solid line represents the ψ_1 field and the dashed line represents the ψ_2 field. There is a factor λ for each four-point vertex and a factor of $\lambda\zeta$ for each three-point vertex. (b) The resulting contributions to the ψ_2 field self-energy.

The last diagram of Fig. 5(b) is momentum dependent so that the self-consistent form of the meson propagator becomes more complicated. If we evaluate the diagram with propagators of the free-field form and demand consistency only at $\omega = k = 0$ we find

$$\begin{aligned}(m_2^{\text{eff}})^2 = {}& m^2 + 2\lambda\zeta^2 + \lambda\langle\psi_1{}^2\rangle + 3\lambda\langle\psi_2{}^2\rangle\\ &- 8\lambda^2\zeta^2\,\frac{\langle\psi_2{}^2\rangle - \langle\psi_1{}^2\rangle}{(m_1^{\text{eff}})^2 - (m_2^{\text{eff}})^2}\,.\end{aligned} \tag{A6}$$

Substituting Eqs. (2.11) and (2.22) into Eq. (A6) we find the solution $m_2^{\text{eff}} = 0$. Hence

$$[D_2(0, \vec{0})]^{-1} = 0 \tag{A7}$$

and so the Goldstone theorem is satisfied. The reason behind this obedience is not so mysterious. It was pointed out by Coleman and Weinberg[30] that a loop expansion is an expansion in powers of the Lagrangian. In order to insure that the symmetry of the Lagrangian is not violated by the approximations one must keep all terms through a fixed number of loops. The usual quasiclassical approximation scheme and some others do not.[31]

It is also useful to demonstrate the Goldstone theorem for finite density ($\mu \neq 0$) and in a more formal way. The inverse propagator according to $\mathcal{L}_0$ of Eq. (A1) is

$$[\overline{D}^0(\omega_n, \vec{k})]^{-1} = \begin{pmatrix} \omega_n{}^2 + k^2 + m^2 - \mu^2 + 6\lambda\zeta^2 & 2\mu\omega_n \\ -2\mu\omega_n & \omega_n{}^2 + k^2 + m^2 - \mu^2 + 2\lambda\zeta^2 \end{pmatrix}. \tag{A8}$$

It can be shown that

$$\xi^2 = \frac{1}{2\lambda}[\mu^2 - m^2 - \lambda\,\mathrm{Tr}(3D_{11} + D_{22})]\,, \tag{A9}$$

where

$$\mathrm{Tr}D_{11} = \frac{1}{\beta}\sum_n \int \frac{d^3k}{(2\pi)^3} D_{11}(\omega_n, \vec{k})\,,$$

etc. Compare with Eq. (2.11). Similarly it can be shown that

$$\begin{aligned}[D(0,\vec{0})]^{-1}{}_{22} = {} & m^2 - \mu^2 + 2\lambda\xi^2 + \lambda\,\mathrm{Tr}(D_{11} + 3D_{22}) \\ & - 4\lambda^2\xi^2\,\mathrm{Tr}(2D_{11}D_{22} + D_{12}D_{12} + D_{21}D_{21})\,.\end{aligned} \tag{A10}$$

The λTr. . . term comes from the figure-eight diagrams and the $\lambda^2\xi^2$ Tr. . . term comes from the exchange diagrams. Compare with Eqs. (A5) and (A6). Substituting Eq. (A9) into Eq. (A10) and keeping only the lowest-order terms in λ we get

$$\begin{aligned}[D(0,\vec{0})]^{-1}{}_{22} = {} & 2\lambda\,\mathrm{Tr}(D_{22} - D_{11}) \\ & - 2\lambda(\mu^2 - m^2)\mathrm{Tr}(2D_{11}D_{22} + D_{12}D_{12} + D_{21}D_{21})\,.\end{aligned} \tag{A11}$$

Furthermore

$$\det D = D_{11}D_{22} + D_{12}D_{12} = D_{11}D_{22} + D_{21}D_{21}\,, \tag{A12}$$

and

$$\mathrm{Tr}(\overline{D}{}^0_{22} - \overline{D}{}^0_{11}) = 2(\mu^2 - m^2)\,\mathrm{Tr}\,\det\overline{D}{}^0\,. \tag{A13}$$

Thus, if to lowest order we replace D with $\overline{D}{}^0$ in Eq. (A11), we find that to one loop

$$[D(0,\vec{0})]^{-1}{}_{22} = 0\,. \tag{A14}$$

One should also include the exchange corrections in the other models considered in this paper. However there is no violation of the Goldstone theorem by the quasiclassical approximation for these models and, besides, the formulas for the transition temperatures would remain unchanged.

---

[1] R. E. Norton and J. M. Cornwall, Ann. Phys. (N.Y.) 91, 106 (1975).

[2] D. K. Campbell, R. F. Dashen, and J. T. Manassah, Phys. Rev. D 12, 979 (1975); 12, 1010 (1975); 18, 1343 (1978).

[3] A. D. Linde, Rep. Prog. Phys. 42, 389 (1979), and references quoted therein.

[4] N. N. Bogoliubov, J. Phys. (Moscow) 11, 23 (1947).

[5] D. P. Woody and P. L. Richards, Phys. Rev. Lett. 42, 925 (1979).

[6] H. P. Jakobsen, M. Kon, and I. E. Segal, Phys. Rev. Lett. 42, 1788 (1979).

[7] C. W. Bernard, Phys. Rev. D 9, 3312 (1974).

[8] R. P. Feynman, Phys. Rev. 91, 1291 (1953).

[9] R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).

[10] B. A. Freedman and L. D. McLerran, Phys. Rev. D 16, 1130 (1977).

[11] J. Goldstone, Nuovo Cimento 19, 154 (1961).

[12] R. Haag, Nuovo Cimento 25, 2695 (1962).

[13] L. Dolan and R. Jackiw, Phys. Rev. D 9, 3320 (1974).

[14] S. Weinberg, Phys. Rev. D 9, 3357 (1974).

[15] J. D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, New York, 1975).

[16] L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Pergamon, London, 1958).

[17] J. E. Mayer and M. G. Mayer, *Statistical Mechanics*, 2nd ed. (Wiley, New York, 1977).

[18] E. S. Abers and B. W. Lee, Phys. Rep. 9C, 1 (1973), and references quoted therein.

[19] A. D. Linde, Phys. Rev. D 14, 3345 (1976).

[20] B. J. Harrington and A. Yildiz, Phys. Rev. Lett. 33, 324 (1974).

[21] A. D. Linde, Zh. Eksp. Teor. Fiz. Pis'ma Red. 23, 73 (1976) [JETP Lett. 23, 64 (1976)].

[22] T. Banks and E. Rabinovici, Nucl. Phys. B160, 349 (1979).

[23] S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967).

[24] A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8)*, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968).

[25] R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. 42, 1651 (1979).

[26] S. A. Chin, Phys. Lett. 78B, 552 (1978).

[27] J. I. Kapusta, Nucl. Phys. B148, 461 (1979).

[28] A. D. Linde, Phys. Lett. 86B, 39 (1979).

[29] T. D. Lee and M. Margulies, Phys. Rev. D 11, 1591 (1975).

[30] S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).

[31] G. Baym and G. Grinstein, Phys. Rev. D 15, 2897 (1977).