PHYSICAL REVIEW D VOLUME 24, NUMBER 2 15 JULY 1981

Evaluation of the time-evolution propagator for the Lee model of nuclear interaction

John T. Marshall and John L. Pell
Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803
(Received 17 November 1980)

Using functional-integration techniques, the time-dependent probabilities for all possible transitions are calculated (in the occupation-number representation) for the Lee model of a single heavy nucleon interacting with a light-particle field, without restriction on the number of light particles.

I. INTRODUCTION

The Lee model[1,2] describes a Fermi field of particles (called heavy particles) with two isotopic states (called V-particle and N-particle states) interacting with a boson field of particles (called light particles, or θ particles). The Hamiltonian describing the system is

$$\hat{H}=\sum_{\vec{p},\lambda} E_\lambda(\vec{p})\hat{b}^\dagger(\vec{p})\hat{b}_\lambda(\vec{p})+\sum_{\vec{k}}\hbar\omega_{\vec{k}}\hat{a}^\dagger_{\vec{k}}\hat{a}_{\vec{k}} -\frac{g_0}{\sqrt{V}}\sum_{\substack{\vec{p},\vec{p}',\vec{k}\\ \vec{p}'=\vec{p}+\vec{k}}}\frac{f(\omega_{\vec{k}})}{(2\hbar\omega_{\vec{k}})^{1/2}}[\hat{b}^\dagger_V(\vec{p}')\hat{b}_N(\vec{p})\hat{a}_{\vec{k}} +\hat{a}^\dagger_{\vec{k}}\hat{b}^\dagger_N(\vec{p})\hat{b}_V(\vec{p}')]\,, \tag{1}$$

where λ is an isotope index with domain (V,N), $\hat{b}^\dagger_\lambda(\vec{p})$ and $\hat{b}_\lambda(\vec{p})$ are fermionic creation and destruction operators for a heavy particle of isotopic type λ and momentum $\vec{p}$, $\hat{a}^\dagger_{\vec{k}}$ and $\hat{a}_{\vec{k}}$ are bosonic creation and destruction operators for a θ particle of momentum $\vec{k}$, $E_\lambda(\vec{p})$ is the energy of a heavy particle of type λ and momentum $\vec{p}$, $\hbar\omega_{\vec{k}}$ is the energy of a θ particle of momentum $\vec{k}$, g_0 is a coupling constant, V is the normalization volume [which ultimately is to be taken to ∞, in which limit summations over momenta are to be evaluated by the rule

$$\sum_{\vec{p}}\cdots\to(V/h^3)\int d^3p\cdots\,], \tag{2}$$

and $f(\omega_{\vec{k}})$ is a "cutoff" function which will be presumed here to have some form such that all integrals over θ-particle momenta will be finite. The first two terms in Eq. (1) represent the energies of the heavy-particle field and the light-particle field, respectively, without interaction and the third term characterizes the interaction of these two fields. This interaction induces processes consisting of successions of the basic reaction process $V\rightleftharpoons N+\theta$, which conserves not only the momentum of the system, but also the two quantities

$$Q_1\equiv\sum_{\vec{p},\lambda}\hat{b}^\dagger_\lambda(\vec{p})\hat{b}_\lambda(\vec{p}) \tag{3}$$

and

$$Q_2\equiv\sum_{\vec{p}}\hat{b}^\dagger_V(\vec{p})\hat{b}_V(\vec{p})+\sum_{\vec{k}}\hat{a}^\dagger_{\vec{k}}\hat{a}_{\vec{k}} \tag{4}$$

which represent, respectively, the number of heavy particles and the sum of the number of V particles and θ particles.

The Lee model is a simplistic, but nontrivial model for the interaction of nucleons (heavy particles) with mesons (light particles). Much of the original interest in the model pertained to renormalization methods.[3] The Lee model can be handled mathematically fairly easily only in the special case for which $Q_1=Q_2=1$, in which case a complete set of states consists only of those states for which either there is exactly one V particle present or there are present exactly one N particle and one θ particle. This special case is commonly called the $N+\theta$ sector.

There is a somewhat simplified, widely used version of the model (which retains most of the elements for which the original model is of interest) obtained by regarding the heavy particles as being equally massive and so heavy that one may neglect their momenta (which are necessarily present in the momentum-conserving original model due to their recoil upon absorption and emission of light particles). In this version

$$E_\lambda(\vec{p})=\hbar\omega=\text{constant}\,. \tag{5}$$

It is this simplified version of the Lee model which will be used in the remainder of this work.

It is the purpose of this paper to obtain a complete description of the dynamics of the Lee model for the case $Q_1=1$. In this case the number of heavy particles is restricted to one, which has only two independent states:

$$\text{the } V\text{-particle state}\equiv\begin{pmatrix}1\\0\end{pmatrix} \tag{6}$$

and

$$\text{the } N\text{-particle state} \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{7}$$

but the number of light particles is unrestricted. This case provides a simple model of a physical nucleon (consisting of a heavy bare nucleon together with its cloud of an unrestricted number of light mesons) and of meson-nucleon scattering processes involving a single nucleon. The Hamiltonian is

$$\hat{H} = \hbar\left[\left(\omega + \sum_{\vec{k}} \omega_{\vec{k}} \hat{a}^{\dagger}_{\vec{k}} \hat{a}_{\vec{k}}\right) I - \sum_{\vec{k}} f'_{\vec{k}}(\sigma_{\uparrow} a_{\vec{k}} + a^{\dagger}_{\vec{k}} \sigma_{\downarrow})\right], \tag{8}$$

where

$$I \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_{\uparrow} \equiv \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \sigma_{\downarrow} \equiv \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \tag{9}$$

and

$$f'_{\vec{k}} \equiv g_0 f(\omega_{\vec{k}})/[\hbar(2\hbar\omega_{\vec{k}} V)^{1/2}]. \tag{10}$$

In order to describe the dynamical behavior of this system, one has in essence to solve the time-dependent Schrödinger equation

$$i\hbar\frac{\partial \psi(t)}{\partial t} = \hat{H}\psi(t), \tag{11}$$

for the state $\psi(t)$ of the system at any time t, assuming that at some initial time t' the system is known to be in some initial state $\psi(t')$. If it is supposed that the initial state is specified in the occupation-number representation,

$$\psi(t') = \sum_{N} c_N(t')\,|N\rangle, \tag{12}$$

where N denotes a set of integer values, $\{N_{\vec{k}}\}$ specifying the number of θ particles with each possible momentum value $\vec{k}$, and where

$$c_N(t') = \begin{pmatrix} c_N^V(t') \\ c_N^N(t') \end{pmatrix} \tag{13}$$

are specified one-by-two coefficient matrices, then the general solution to Eq. (11) for the state of the system at any given time $t'' \geq t'$ is

$$\psi(t'') = \sum_{N''} c_{N''}(t'')\,|N''\rangle, \tag{14}$$

where

$$c_{N''}(t'') = \sum_{N'} K_{N'',N'}(t''-t')c_{N'}(t'), \tag{15}$$

wherein

$$K_{N'',N'}(t''-t') = \langle N''|\,e^{-i(t''-t')\hat{H}/\hbar}\,|N'\rangle \tag{16}$$

is the so-called time-evolution propagator in the occupation-number representation.

Since this propagator characterizes explicitly the time development of the system for all possible initial states, then a satisfactory mode of complete description is obtainable by evaluating expression (16). In Sec. II, the propagator is evaluated exactly and in general by first transforming to the coherent-state representation, performing all of the required operations, and then transforming back to the occupation-number representation. In view of its generality, the result is quite complicated. In Sec. III a simplification is obtained for the $N+\theta$ sector, and in Sec. IV the (time-dependent) survival probability for a bare V particle is expressed by a very simple formula for the special case of a dispersionless θ-particle field.

Most of the research on the Lee-model theory has been directed toward resolving questions regarding renormalization and dressing transformation procedures which arise in the determination of the energy eigenstates and in the description of scattering processes involving dressed initial and/or final states separated by infinite time intervals. (See Refs. 4–7 for such examples. Also see Ref. 8 and some of the references contained therein, for example, involving finite-time evolution of certain important production and decay processes.) Because of the complexity of the general result obtained in Sec. II, it is difficult to relate that result to the results of the types obtained in the previously mentioned references, except in a general formalistic (and therefore unenlightening) way. It is possible, however, to relate in specific details the result obtained here for the bare V-particle survival probability, for the special simplifying case of the dispersionless θ-particle field, to the well-known energy eigenstates in the $N+\theta$ sector. This relationship is developed in the last paragraph of Sec. IV.

II. EVALUATION OF THE PROPAGATOR

It is convenient to introduce single-mode coherent states defined as

$$|a_{\vec{k}}\rangle = e^{-|a_{\vec{k}}|/2} \sum_{n_{\vec{k}}=0}^{\infty} [a_{\vec{k}}^{n_{\vec{k}}}/(n_{\vec{k}}!)^{1/2}]\,|n_{\vec{k}}\rangle, \tag{17}$$

where $|n_k\rangle$ is the state with exactly $n_{\vec{k}}$ θ particles of momentum $\vec{k}$ and where $a_{\vec{k}}$ is any complex-valued function of $\vec{k}$. These coherent states are eigenstates of the bosonic creation operator $\hat{a}_{\vec{k}}$ with corresponding eigenvalue $a_{\vec{k}}$. They satisfy the completeness relation

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |a_{\vec{k}}\rangle \frac{da_{\vec{k}}^{(r)}\,da_{\vec{k}}^{(i)}}{\pi}\langle a_{\vec{k}}| = 1, \tag{18}$$

where $a_{\vec{k}}^{(r)}$ and $a_{\vec{k}}^{(i)}$ are the real and imaginary parts of $a_{\vec{k}}$. The relationship between the occupation-number representation of the propagator $K_{N'',N'}(t'',t')$ and the coherent-state representation of the same propagator $K_{a'',a'}(t'',t')$ is given by

$$K_{N'',N'}(t'',t')=\iint \langle N''|a''\rangle K_{a'',a'}(t'',t')\langle a'|N'\rangle d^B a'' d^B a' , \tag{19}$$

where

$$d^B a \equiv \prod_{\vec{k}} \frac{da_{\vec{k}}^{(r)} da_{\vec{k}}^{(i)}}{\pi} \tag{20}$$

and

$$|a\rangle \equiv \prod_{\vec{k}} |a_{\vec{k}}\rangle . \tag{21}$$

By repeated application of Eq. (18), the coherent-state propagator may be expressed as follows:

$$K_{a'',a'}(t'',t') \equiv \langle a''|e^{-i(t''-t')\hat{H}/\hbar}|a'\rangle \tag{22}$$

$$= \lim_{M\to\infty}\int\cdots\int \langle a''|e^{-i\epsilon\hat{H}/\hbar}|a(M-1)\rangle\langle a(M-1)|\ldots|a(2)\rangle\langle a(2)|e^{-i\epsilon\hat{H}/\hbar}|a(1)\rangle\langle a(1)|e^{-i\epsilon\hat{H}/\hbar}|a'\rangle \times \prod_{l=1}^{M-1} d^B a(l) , \tag{23}$$

where

$$\epsilon \equiv (t''-t')/M , \tag{24}$$

and, as a notational convenience,

$$a(M)\equiv a'', \quad a(0)\equiv a' . \tag{25}$$

Expansion of the exponential functions in Eq. (23) to first order in ϵ yields

$$K_{a'',a}(t'',t) = \lim_{M\to\infty}\int\cdots\int\left(\prod_{\underset{\leftarrow}{l=1}}\langle a(l)|a(l-1)\rangle[1-i\epsilon\langle a(l)|\hat{H}|a(l-1)\rangle/\langle\hbar a(l)|a(l-1)\rangle]\right)\prod_{l=1}^{M-1} d^B a(l) \tag{26}$$

$$= \lim_{M\to\infty} e^{-i\omega(t''-t')}\int\cdots\int P(a)e^{S(a)}D^{M-1}(a) , \tag{27}$$

where

$$D^{M-1}(a)\equiv\prod_{\vec{k}}\prod_{l=1}^{M-1} d^B a_{\vec{k}}(l) , \tag{28}$$

$$S(a)\equiv -\tfrac{1}{2}\sum_{l=1}^{M}\sum_{\vec{k}}[|a_{\vec{k}}(l)|^2+|a_{\vec{k}}(l-1)|^2-2(1-i\epsilon\omega_{\vec{k}})a^*_{\vec{k}}(l)a_{\vec{k}}(l-1)] , \tag{29}$$

and

$$P(a)\equiv\prod_{\underset{\leftarrow}{l=1}}^{M}[I+i\epsilon y^*(l)\sigma_\downarrow+i\epsilon y(l-1)\sigma_\uparrow] \tag{30}$$

$$= \sigma_V\sum_{\mu=0}^{[M/2]}\sum_{l_1>l_{2\mu}}^{M}{}'\prod_{\nu=1}^{\mu}[i\epsilon y^*(l_{2\nu})][i\epsilon y(l_{2\nu-1}-1)] + \sigma_N\sum_{\mu=0}^{[M/2]}\sum_{l_1>l_{2\mu}}^{M}{}'\prod_{\nu=1}^{\mu}[i\epsilon y^*(l_{2\nu-1})][i\epsilon y(l_{2\nu}-1)]$$
$$+\sigma_\uparrow\sum_{\mu=0}^{[(M-1)/2]}\sum_{l_1>l_{2\mu+1}}^{M}{}'\prod_{\nu=1}^{\mu}[i\epsilon y^*(l_{2\nu})]\prod_{\nu'=1}^{\mu+1}[i\epsilon y(l_{2\nu'}-1)]$$
$$+\sigma_\downarrow\sum_{\mu=0}^{[(M-1)/2]}\sum_{l_1>l_{2\mu+1}}^{M}{}'\prod_{\nu=1}^{\mu+1}[i\epsilon y^*(l_{2\nu-1})]\prod_{\nu'=1}^{\mu}[i\epsilon y(l_{2\nu'}-1)] \tag{31}$$

wherein the arrow under the product symbol indicates the direction in which the index increases in the time-ordered product, i.e.,

$$\prod_{\underset{\leftarrow}{l=1}}^{M} f(l)=f(M)f(M-1)\ldots f(2)f(1) ,$$

and wherein

$$y(l)=\sum_{\vec{k}} f_{\vec{k}}^{\dagger} a_{\vec{k}}(l) , \tag{32}$$

$$\sigma_V=\begin{pmatrix}1&0\\0&0\end{pmatrix}, \quad \sigma_N=\begin{pmatrix}0&0\\0&1\end{pmatrix}, \tag{33}$$

and

$$\sum_{l_i>l_j}^{\beta}{}' \equiv \begin{cases} \displaystyle\sum_{l_i=1}^{\beta}\sum_{l_{i+1}=1}^{l_i-1}\cdots\sum_{l_j=1}^{l_{j-1}-1}, & \text{if } j\geqslant i\geqslant 1 \\ \displaystyle\sum_{l_j=1}^{\beta}\sum_{l_{j+1}=l_j+1}^{\beta}\cdots\sum_{l_i=l_{i-1}+1}^{\beta}, & \text{if } i\geqslant j\geqslant 1\,. \end{cases} \tag{34}$$

The square brackets used in the summation limits in Eq. (31) indicate that the greatest integer less than or equal to the enclosed quantity is to be taken as the summation limit.

Equations (27)–(34) yield the coherent-state propagator in the form of a two-by-two matrix, the evaluation of whose elements still requires multifold integration. Each of these four matrix elements may be obtained by multiple differentiation of the following generating function with respect to the parameters $\lambda_1, \lambda_2, \ldots, \lambda_M$ and $\lambda'_1, \lambda'_2, \ldots, \lambda'_M$ which occur in its definition:

$$K'^{M}_{a'',a'}(t'',t') = \int e^{S_\lambda(a)} D^{M-1}(a)\,, \tag{35}$$

where

$$S_\lambda(a) = S(a) + \sum_{l=1}^{M}\left[i\epsilon y^*(l)\lambda_l + i\epsilon y(l-1)\lambda'_l\right]. \tag{36}$$

Performance of the integration indicated in Eq. (35) yields

$$\begin{aligned} K'^{M}_{a'',a'}(t'',t') = \prod_{\vec{k}} \exp\Big\{ & -\tfrac{1}{2}(|a''_{\vec{k}}|^2 + |a'_{\vec{k}}|^2) + a''^{*}_{\vec{k}} a'_{\vec{k}} (1-i\epsilon\omega_{\vec{k}})^M \\ & - i\epsilon \sum_{l=1}^{M}\left[f'_{\vec{k}} a''^{*}_{\vec{k}} (1-i\epsilon\omega_{\vec{k}})^{M-l}\lambda_l + f'_{\vec{k}} a'_{\vec{k}} (1-i\epsilon\omega_{\vec{k}})^{l-1}\lambda'_l\right] \\ & - \epsilon^2 f'^{2}_{\vec{k}} \sum_{l=1}^{M-1}\sum_{l'=l+1}^{M} (1-i\epsilon\omega_{\vec{k}})^{l'-l-1}\lambda_l\lambda'_{l'} \Big\}. \end{aligned} \tag{37}$$

This result may now be used to derive the occupation-number propagator by differentiation of $K'^{M}_{a'',a'}(t'',t')$ with respect to the λ parameters, setting the λ parameters to zero, taking the limit $M\rightarrow\infty$, using Eq. (19) with Eqs. (21) and (17), and by performing all of the integrations remaining in the resultant expression for $K_{N'',N'}(t'',t')$. The final outcome is

$$K_{N'',N'}(t'',t') = \exp\Big[-i(t''-t')\Big(\omega + \sum_{\vec{k}} N''_{\vec{k}}\omega_{\vec{k}}\Big)\Big](K'_{V\rightarrow V}\sigma_V + K'_{N\rightarrow N}\sigma_N + K'_{N\rightarrow V}\sigma_\uparrow + K'_{V\rightarrow N}\sigma_\downarrow)\,, \tag{38}$$

where

$$K'_{V\rightarrow V} = \sum_{\mu=0}^{\infty}\sum_{(\vec{k}:2\mu)}{}''\left[D_{2\mu}\sum_{(n,n';m)}^{\mu}{}'''\left(\Delta' \prod_{\substack{\nu=1\\ \nu\neq n'_i(i=1,\ldots,m)}}^{\mu} N'^{\,1/2}_{\vec{k}_{2\nu-1}}\,\delta_{N''_{\vec{k}_{2\nu-1}},N'_{\vec{k}_{2\nu-1}}-1}\right)\right], \tag{39}$$

$$K'_{N\rightarrow N} = \sum_{\mu=0}^{\infty}\sum_{(k:2\mu)}{}''\left[D^*_{2\mu}\sum_{(n,n';m)}^{\mu-1}{}'''\left(\Delta \prod_{\substack{\nu=0\\ \nu\neq n_i(i=1,\ldots,m)}}^{\mu-1} N''^{\,1/2}_{\vec{k}_{2\nu+1}}\,\delta_{N''_{\vec{k}_{2\nu+1}}-1,N'_{\vec{k}_{2\nu}}}\right)\right], \tag{40}$$

$$K'_{N\rightarrow V} = -\sum_{\mu=0}^{\infty}\sum_{(k:2\mu+1)}{}''\left[D_{2\mu+1}\sum_{(n,n';m)}^{\mu}{}'''\left(\Delta' \prod_{\substack{\nu=1\\ \nu\neq n_i(i=1,\ldots,m)}}^{\mu+1} N'^{\,1/2}_{\vec{k}_{2\nu-1}}\,\delta_{N''_{\vec{k}_{2\nu-1}},N'_{\vec{k}_{2\nu-1}}-1}\right)\right], \tag{41}$$

and

$$K'_{V\rightarrow N} = \sum_{\mu=0}^{\infty}\sum_{(\vec{k}:2\mu+1)}{}''\left[D_{2\mu+1}\sum_{(n,n';m)}^{\mu}{}'''\left(\Delta \prod_{\substack{\nu=0\\ \nu\neq n'_i(i=1,2,\ldots,m)}}^{\mu} N''^{\,1/2}_{\vec{k}_{2\nu+1}}\,\delta_{N''_{\vec{k}_{2\nu+1}}-1,N'_{\vec{k}_{2\nu+1}}}\right)\right], \tag{42}$$

with

$$\sum_{(\vec{k}:2\mu)}{}'' \equiv \sum_{\vec{k}_1}\sum_{\vec{k}_2}\cdots\sum_{\vec{k}_{2\mu}}\,, \tag{43}$$

$$D_p \equiv \Big(\prod_{j=1}^{p} f'_{\vec{k}_j}\Big)\Big(\prod_{\vec{k}}\delta_{N''_{\vec{k}},N'_{\vec{k}}}\Big)F_p\,,\quad \vec{k}\neq\vec{k}_i\ (i=1,\ldots,p) \tag{44}$$

$$\sum_{(n,n';m)}^{\mu}{}''' \equiv \sum_{m=0}^{\mu}\sum_{n_m>n_1}^{\mu}{}'\sum_{n'_1=n_1}^{\mu}\sum_{n'_2=n_2}^{\mu}\cdots\sum_{n'_m=n_m}^{\mu}\,, \tag{45}$$

$$\Delta \equiv \left(\prod_{j=1}^{m} \delta_{\vec{k}_{2n_j},\vec{k}_{2n_j+1}} \delta_{N''_{\vec{k}_{2n_j}},N'_{\vec{k}_{2n_j}}}\right)\left(\prod_{\substack{v=1\\ v\neq n_i\,(i=1,\ldots,m)}}^{\mu} N'^{\,1/2}_{\vec{k}_{2v}}\,\delta_{N''_{\vec{k}_{2v}},N'_{\vec{k}_{2v}}-1}\right), \tag{46}$$

and

$$\Delta' \equiv \left(\prod_{j=1}^{m} \delta_{\vec{k}_{2n_j-1},\vec{k}_{2n'_j}} \delta_{N''_{\vec{k}_{2n'_j}},N'_{\vec{k}_{2n'_j}}}\right)\left(\prod_{\substack{v=1\\ v\neq n'_i\,(i=1,\ldots,m)}}^{\mu} N''^{\,1/2}_{\vec{k}_{2v}}\,\delta_{N''_{\vec{k}_{2v}}-1,N'_{\vec{k}_{2v}}}\right), \tag{47}$$

wherein Eq. (34) applies and

$$F_p \equiv \sum_{n=p}^{\infty} [i(t''-t')]^n \sum_{(\gamma:p)}^{n-p}{}'' \prod_{j=1}^{p} \frac{[(-1)^j \omega_{\vec{k}_j}]^{\gamma_j}}{\gamma_j! \sum_{l=j}^{p} (\gamma_l + 1)}, \tag{48}$$

where

$$\sum_{(\gamma:\mu)}^{b}{}'' = \sum_{\gamma_1=0}^{b} \sum_{\gamma_2=0}^{b} \cdots \sum_{\gamma_\mu=0}^{b} \quad \text{with} \quad \sum_{i=1}^{\mu} \gamma_i = b. \tag{49}$$

III. $N+\theta$ SECTOR

In the $N+\theta$ sector, the propagator can be expressed much more simply by Eq. (38) together with the following results obtained by simplifying Eqs. (39)–(49) for the special case $Q_1 = Q_2 = 1$: $K'_{V\to V} = 0$ unless both the initial and final states comprise exactly one V particle, in which case

$$K'_{V\to V} = 1 + \sum_{n=2}^{\infty} \frac{[-i(t''-t')]^n}{n!} \sum_{\mu=1}^{[n/2]} \sum_{(\gamma:\mu)}^{n-2\mu} \prod_{\alpha=1}^{\mu} G(\gamma_\alpha). \tag{50}$$

$K'_{N\to N} = 0$ unless the initial state consists of one N particle and one θ particle (with some momentum $\vec{k}'$) and the final state consists of one N particle and one θ particle (with momentum $\vec{k}''$), in which case

$$K'_{N\to N} = 1 + f'_{\vec{k}'} f'_{\vec{k}''} \sum_{n=2}^{\infty} \frac{[-i(t''-t')]^n}{n!} \sum_{\mu=1}^{[n/2]} \sum_{(\gamma:\mu+1)}^{n-2\mu} (-1)^{\gamma_1} \binom{n-1}{\gamma_1} \omega_{\vec{k}''}{}^{\gamma_1} \omega_{\vec{k}'}{}^{\gamma_{\mu+1}} \prod_{\alpha=2}^{\mu} G(\gamma_\alpha). \tag{51}$$

$K'_{N\to V} = 0$ unless the initial state consists of one N particle and one θ particle (with momentum $\vec{k}'$) and the final state contains just one V particle, in which case

$$K'_{N\to V} = f'_{\vec{k}'} \sum_{n=1}^{\infty} \frac{[-i(t''-t')]^n}{n!} \sum_{\mu=0}^{[(n-1)/2]} \sum_{(\gamma:\mu+1)}^{n-2\mu-1} \omega_{\vec{k}'}{}^{\gamma_{\mu+1}} \prod_{\alpha=1}^{\mu} G(\gamma_\alpha), \tag{52}$$

and $K'_{V\to N} = 0$ unless the initial state consists of exactly one V particle and the final state comprises one N particle and one θ particle (with momentum $\vec{k}''$), in which case

$$K'_{V\to N} = f'_{\vec{k}''} \sum_{n=1}^{\infty} \frac{[-i(t''-t')]^n}{n!} \sum_{\mu=0}^{[(n-1)/2]} \sum_{(\gamma:\mu+1)}^{n-2\mu-1} (-1)^{\gamma_1} \binom{n-1}{\gamma_1} \omega_{\vec{k}''}^{\gamma_1} \prod_{\alpha=2}^{\mu+1} G(\gamma_\alpha), \tag{53}$$

where

$$G(n) = \sum_{\vec{k}} f'^2_{\vec{k}} \omega_{\vec{k}}{}^n. \tag{54}$$

IV. DISPERSIONLESS CASE

The results of the prior section are still quite complicated. They may be greatly simplified, however, for the special case for which the light-particle field is dispersionless, i.e.,

$$\omega_{\vec{k}} = \omega_0, \tag{55}$$

where ω_0 is a constant for all values of $\vec{k}$ for which the cutoff function $f(\omega_{\vec{k}})$ is nonzero. To illustrate this simplification, the bare V-particle survival probability

$$P_{V\to V}(t) = |K'_{V\to V}|^2 \tag{56}$$

will be derived as an analytically simple function of time $t \equiv t'' - t'$.

Substitution of Eq. (55) into (54) and use of the result in Eq. (50) lead to the simplified expression

$$K'_{V\to V}=1+\sum_{n=2}^{\infty}\frac{(-i\omega_0 t)^n}{n!}\sum_{\mu=1}^{[n/2]}\binom{n-\mu-1}{\mu-1}\beta^{\mu}\,, \qquad (57)$$

where β is a single dimensionless coupling constant defined by

$$\beta=\omega_0^{-2}\sum_{\vec{k}}f'^2_{\vec{k}}=\frac{4\pi^3 g_0^2}{\hbar^6\omega_0^3}\int d^3k\, f^2(\omega_{\vec{k}})\,, \qquad (58)$$

wherein Eqs. (2) and (10) have been used. Equation (57) may be reduced further to

$$K'_{V\to V}=e^{-i\omega_0 t/2}\times\{\cos[\tfrac{1}{2}(1+4\beta)^{1/2}\omega_0 t]+i\sin[\tfrac{1}{2}(1+4\beta)^{1/2}\omega_0 t]/(1+4\beta)^{1/2}\}, \qquad (59)$$

so that, from Eq. (56),

$$P_{V\to V'}(t)=1-[(1-P_{\min})/2][1-\cos(2\pi t/T)]\,, \qquad (60)$$

where

$$P_{\min}=1/(1+4\beta)^{1/2} \text{ and } T=(2\pi/\omega_0)/(1+4\beta)^{1/2}\,. \qquad (61)$$

Thus, the bare V-particle survival probability simply oscillates in time sinusoidally between its maximum value of unity and a minimum value $P_{\min}$, with a period T, where $P_{\min}$ and T are given by Eqs. (61) in terms of the frequency ω_0 of the dispersionless light-particle field and the coupling constant β, characterizing the strength of the interaction between the heavy-particle and light-particle fields.

This result may be verified by using the well-known energy spectrum in the $N+\theta$ sector.[2] The energy eigenvalues of the Hamiltonian, $\hat{H}$ given by Eq. (8), restricted to the $N+\theta$ sector, are those values of E which satisfy the secular equation

$$E=\hbar\omega-\sum_{\vec{k}}\frac{\hbar^2 f'^2_{\vec{k}}}{\hbar(\omega+\omega_{\vec{k}})-E}=\hbar\omega-\frac{\beta\hbar^2\omega_0^2}{\hbar(\omega+\omega_0)-E}\,, \qquad (62)$$

wherein definitions (55) and (58) of ω_0 and β have been used. The least such value is the nondegenerate energy E_0 of the stable dressed V-particle state V_0, for which

$$|\langle V|V_0\rangle|^2=\left\{1+\sum_{\vec{k}}\left[\frac{\hbar f'_{\vec{k}}}{\hbar(\omega+\omega_{\vec{k}})-E_0}\right]^2\right\}^{-1}, \qquad (63)$$

where V is the bare V-particle state whose survival probability amplitude is

$$K_{V\to V}\equiv e^{-i\omega t}K'_{V\to V}=\left\langle V\left|\exp\left(-\frac{i}{\hbar}t\hat{H}\right)\right|V\right\rangle\,. \qquad (64)$$

Equation (62) readily yields

$$E=\hbar\left(\omega+\frac{\omega_0}{2}[1\mp(1+4\beta)^{1/2}]\right)\equiv\begin{Bmatrix}E_0\\ E_0'\end{Bmatrix}\,. \qquad (65)$$

This result, along with Eqs. (55) and (58), allows Eq. (63) to be reduced to the following simple expression:

$$|\langle V|V_0\rangle|^2=\tfrac{1}{2}[1+1/(1+4\beta)^{1/2}]\,. \qquad (66)$$

Consequently,

$$K'_{V\to V}=e^{+i\omega t}\left[\exp\left(-\frac{i}{\hbar}tE_0\right)|\langle V|V_0\rangle|^2+\exp\left(-\frac{i}{\hbar}tE_0'\right)(1-|\langle V|V_0\rangle|^2)\right], \qquad (67)$$

which may be reduced by use of Eqs. (65) and (66) to the same result as expressed previously by Eq. (59).

[1]T. D. Lee, Phys. Rev. 95, 1329 (1954).
[2]S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Row, Peterson, Evanston, 1961), Chap. 12, Sec. 12b, p. 352.
[3]G. Källén and W. Pauli, K. Dan. Vidensk. Selsk., Mat.-Fys. Medd. 30, No. 7 (1955).
[4]L. M. Scarfone, J. Math. Phys. 9, 246 (1968).
[5]G. Dillon and M. M. Giannini, Nuovo Cimento 18, 31 (1973).
[6]V. N. Piskunov, Theor. Math. Phys. 15, 546 (1973).
[7]F. Kh. Abdullaev and A. Yu. Yumatov, Theor. Math. Phys. 22, 163 (1975).
[8]C. L. Hammer and T. A. Weber, J. Math. Phys. 10, 2067 (1969).