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Can we make sense out of the measurement process in relativistic quantum mechanics?

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We study the instantaneity of the state-reduction process in relativistic quantum mechanics. The conclusion of various authors that this instantaneity will restrict the set of relativistic observables to purely local ones (i.e., that the measurement of any nonlocal property of a system at a well-defined time would give rise to violations of relativistic causality) is found to be erroneous, and experiments (of a kind not encountered before in measurement theory) are described whereby certain nonlocal properties of some simple physical systems can be measured at a well-defined time without violating causality. The attempts of certain authors to reconcile the reduction process with the covariance of the relativistic quantum state are considered and found wanting, and it is argued that the covariance of relativistic quantum theories resides exclusively in the experimental probabilities, and not in the underlying quantum states. The problem of nonlocal measurement is considered in general: distinctions (which are not to be met with in the nonrelativistic case) arise in relativistic quantum mechanics between what can be measured for fermions and what can be measured for bosons, between what can be measured for individual systems and what can be measured for ensembles, and between what kinds of states can be verified by measurement and what kinds of states can be prepared by measurement; and these pose difficult questions about the nature of measurement itself.

I. INTRODUCTION

By now it is well known that the change of state associated with the measurement process, particularly the *instantaneity* of this change, will produce novelties and difficulties in relativistic quantum mechanics which are not to be met with in the nonrelativistic theory. Two of these are in the form of paradoxes which we have begun to reexamine in a recent paper.[1] They arise roughly as follows:

(1) Suppose that a particle is initially localized[2] to within some finite region of space-time A, and that at some well-defined time t_1 a measurement of the momentum of the particle is carried out. Whatever value is obtained for the momentum, the measurement will instantaneously redistribute the probability uniformly throughout all space, since it will certainly cause the wave function to collapse onto *some* eigenfunction of the momentum (Fig. 1). Thus, if the position of the particle is measured at time $t_1+\epsilon$, a nonzero probability exists that the particle may be found in a region entirely spacelike separated from A. Apparently the momentum measurement process is capable of moving the particle around at superluminal velocities. Thereby a paradox arises: on the one hand we may formally attribute a given momentum at a well-defined time to a particle in relativistic quantum theories; on the other hand the possibility of actually *measuring* the momentum at a well-defined time seems to give rise to violations of causality.

(2) Suppose that at time $t=-\infty$ a free particle has been prepared in a momentum eigenstate and that at time $t=0$ the same particle is found at the origin by means of a detector which has been positioned there, and which interacts locally with the particle. The wave function associated with the particle will change instantaneously at $t=0$, according to the reduction postulate, from an eigenfunction of momentum to an eigenfunction of position (Fig. 2). What is paradoxical in this for the relativistic theory is that the $t=0$ hypersurface across which the state changes will not be an equal-time hypersurface in any other frame. Thus if the change of state is, say, stipulated to occur instantaneously in some particular frame, then it will not occur instantaneously in any other (Fig. 3). On the other hand the statement that the de-

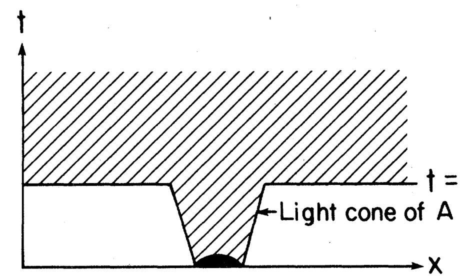

FIG. 1. A particle is first localized to within a region A (solid region), and subsequently the momentum of the particle is measured at time $T=t_1$. The shaded regions are those in which the amplitude of the wave function will be nonzero.

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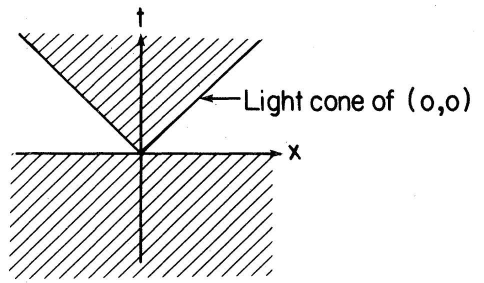

FIG. 2. A particle is prepared in a momentum eigenstate at $t = -\infty$, and is subsequently located at the origin at $t = 0$. As in Fig. 1, the shaded regions are those in which the amplitude will be nonzero.

tector has located the particle and thereby measured its position is apparently an entirely covariant one, so that the position measurement cannot be said to be attached to any particular frame. The prescription that the change of state be *covariantly* instantaneous, however, is as we have seen a contradiction in terms.

The first of these two paradoxes was considered half a century ago in a paper by Landau and Peierls,[3] wherein they concluded that the momentum at a given time and indeed all nonlocal properties of the wave function at a given time cannot be observables for relativistic quantum-mechanical systems.[4] Upon a more careful examination (which we shall undertake in the present paper), however, this conclusion turns out to be erroneous.

One must take care, in arriving at such conclusions, to have considered the most general class of experiments which can measure these properties, and to prove that *all* such experiments will give rise to violations of causality. However, the argument of Landau and Peierls assumes not only that the momentum measurement is carried out at a well-defined time, but also, as will become clear, that it is carried out *in a particular way*. In fact it is possible to measure a variety of nonlocal properties of the wave function at a well-defined time without any violation of causality. We shall have occasion in the course of this paper to construct a number of explicit examples of such procedures, which are of a kind not encountered before in measurement theory.

The question of what can and what cannot be measured at a well-defined time, which, therefore, needs to be re-opened, gives rise to a hoard of difficulties and anomalies (which are peculiar to the relativistic case); and a substantial part of our paper will be devoted to discussions of these.

The problem of the second paradox[5] is that the reduction postulate is not, as it stands, Lorentz covariant. Apparently we must design a new prescription for the relativistic case, and to this end it has been proposed[6] that the relativistic reduction process be taken to occur not instantaneously but rather along the backward light cone of the measurement event (Fig. 4). This process is first of all manifestly covariant (since the light cone will transform into itself under Lorentz transformations), and indeed Hellwig and Kraus (Ref. 6) have shown that, despite appearances, it will yield the correct probabilities for all measurements of local observables.

The probabilities for nonlocal observables, however, are another matter. If, for example, a momentum measurement is carried out at $t = 0 - \epsilon$, then the measurement will with certainty confirm that the state at $0 - \epsilon$ is not the one depicted in Fig. 4, but rather the same momentum eigenstate in which the particle was prepared at $t = -\infty$: the one depicted in Fig. 2. Hellwig and Kraus will now respond that no such momentum measurement is possible (at $t = 0 - \epsilon$), since Landau and Peierls have shown that such a measurement would violate causality; however, the conclusion of Landau and Peierls is, as we have already remarked, incorrect. Attempts to design a satisfactory covariant reduction postulate will, therefore, fail. The second paradox, like the first, demands not to be tampered with superficially but rather to be carefully reconsidered from the ground up, and this we shall also undertake to do in the present work.

We turn now to a detailed study of these problems. The free one-particle system with which we have been concerned above, as we shall pres-

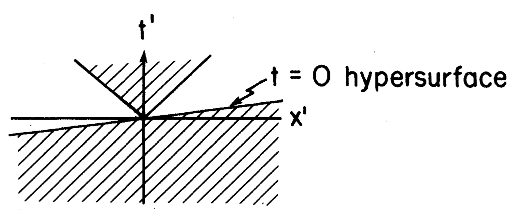

FIG. 3. A Lorentz-transformed verion of Fig. 2.

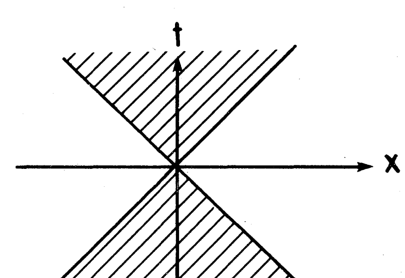

FIG. 4. The "covariant" reduction postulate of Ref. 6.

ently see, is not well suited to such a study, and we shall consider instead a system of two spin-$\frac{1}{2}$ particles, one of which we take to be fixed (by some potential, perhaps) at the point x_1, and the other at x_2. We shall be concerned only with measurements of the spin of these particles, and thus the particular simplicity of this system is that it has a four-dimensional relevant Hilbert space of states, whereas the dimension of the Hilbert space for the free particle is infinite.

The organization of our paper is as follows. Section II will review and expand upon the work of Ref. 1. It will begin with a description of the two paradoxes as they arise for the system of two fixed spin-$\frac{1}{2}$ particles, and then proceed to a more detailed analysis of the first paradox. We will describe explicitly how certain nonlocal properties of the system can in fact be measured at a well-defined time without any violation of causality, and in particular we will design experimental procedures for verifying that the system is in either of the nonlocal states:

$$|\alpha\rangle_\pm \equiv \frac{1}{\sqrt{2}}\,(|\uparrow\rangle_{x_1}|\downarrow\rangle_{x_2} \pm |\downarrow\rangle_{x_1}|\uparrow\rangle_{x_2}) \qquad (1)$$

(which we have written in a hopefully obvious notation) at a well-defined time, without violating causality. In the latter part of Sec. II we will consider the importance of these nonlocal procedures for the discussion of the second paradox. As we have already remarked, the possibility of carrying out such procedures will invalidate the "covariant" reduction postulate of Hellwig and Kraus, and indeed it will turn out that *no* covariant succession of states at a given time can consistently be associated with the system, although the notion of a state will continue to make sense within a given Lorentz frame. In Sec. III we will begin to consider the question of nonlocal measurement in a more general way.

The experiments for verifying the states $|\alpha\rangle_\pm$, which are described in Sec. II, are repeatable: the experiment for $|\alpha\rangle_+$, say, will leave a system in the state $|\alpha\rangle_+$ entirely undisturbed (just as a momentum measurement, for example, will not disturb a system in a momentum eigenstate), and can therefore be carried out again and will with certainty find the same result (hereafter such experiments will be referred to as nondemolition experiments[7] for $|\alpha\rangle_\pm$). $|\alpha\rangle_\pm$, then, can be verified at a well-defined time by means of nondemolition experiments, but in Sec. III we shall present a proof that states of the form

$$|\beta\rangle_\varphi \equiv \sin\varphi\,|\uparrow\rangle_{x_1}|\downarrow\rangle_{x_2} + \cos\varphi\,|\downarrow\rangle_{x_1}|\uparrow\rangle_{x_2},$$
$$\text{where } \varphi \neq \frac{n\pi}{4}, \quad n = 0, 1, 2, \ldots \qquad (2)$$

cannot (i.e., that any nondemolition experiment for $|\beta\rangle_\phi$ will necessarily give rise to violations of causality).

A procedure for *preparing* some particular state need not be repeatable (on the same system), however, and indeed in Sec. IV it will be shown that although $|\beta\rangle_\phi$ cannot be verified by means of a nondemolition experiment at a well-defined time, it can nevertheless be prepared at a well-defined time, without invoking the equations of motion of the system, by means of a definite sequence of observations. Whether this preparation may be considered a measurement of $|\beta\rangle_\phi$ is a matter of taste; indeed what will emerge here is that in the relativistic case the subject of measurement embraces a variety of very different processes, which have different causal properties, and which perhaps are oversimplified in being referred to collectively as "measurements". Such differences arise again and again; it turns out, for example, that what can be measured at a well-defined time for a single system differs from what can be measured at a well-defined time for an ensemble of systems.

In Sec. V we will describe how differences arise between what can be measured at a well-defined time for fermions and for bosons, and also for ensembles of these.

II. NONLOCAL OBSERVABLES

We will begin by describing our system of two fixed spin-$\frac{1}{2}$ particles in somewhat pedantic detail. The Hilbert space of spin states for this system is as we have already remarked four-dimensional, and it may be spanned, for example, by the four state vectors

$$\begin{aligned} |\uparrow\rangle_{x_1}|\uparrow\rangle_{x_2} &\equiv |A\rangle, \\ |\uparrow\rangle_{x_1}|\downarrow\rangle_{x_2} &\equiv |C\rangle, \\ |\downarrow\rangle_{x_1}|\uparrow\rangle_{x_2} &\equiv |B\rangle, \\ |\downarrow\rangle_{x_1}|\downarrow\rangle_{x_2} &\equiv |D\rangle, \end{aligned} \qquad (3)$$

where

$$\sigma_z^{(x_1)}|\uparrow\rangle_{x_1} = +\tfrac{1}{2}|\uparrow\rangle_{x_1}, \quad \sigma_z^{(x_2)}|\downarrow\rangle_{x_2} = -\tfrac{1}{2}|\downarrow\rangle_{x_2}, \text{ etc.,}$$
$$[\sigma_i^{(x_l)}, \sigma_j^{(x_m)}] = \delta_{lm}\epsilon_{ijk}\sigma_k^{(x_m)}. \qquad (4)$$

In this basis the states $|\alpha\rangle_\pm$ defined in (1) will take the form

$$|\alpha\rangle_\pm = (|C\rangle \pm |B\rangle)\frac{1}{\sqrt{2}}. \qquad (5)$$

The operators $\sigma_i^{(x_l)}$, which refer to the spins of the two particles individually, can in the usual way be combined to form total spin operators for the two-particle system as follows:

$$J_i \equiv \sigma_i^{(x_1)} + \sigma_i^{(x_2)}, \qquad J^2 \equiv \sum_{i=1}^{3} J_i^2 \tag{6}$$

and we may alternately form a basis out of eigenstates of, say, J^2 and J_z, viz.

$$|J_z=0, J^2=0\rangle = |\alpha\rangle_-, \quad |J_z=0, J^2=2\rangle = |\alpha\rangle_+, \quad |J_z=1, J^2=2\rangle = |A\rangle, \quad |J_z=-1, J^2=2\rangle = |D\rangle \tag{7}$$

as a reader can easily confirm.

Now it is possible to conveniently restate the two paradoxes as they arise for this system.

(1′) If the system is initially prepared in the state $|A\rangle$, say, and at some well-defined time t_1 a measurement of J^2 is carried out, the measurement will leave the system entirely undisturbed (since $|A\rangle$ is already an eigenstate of J^2); and if, for example, a measurement of $\sigma_z^{(x_2)}$ is carried out at time $t_1+\epsilon$, the result will with certainty be $\sigma_z^{(x_2)}=+\frac{1}{2}$. Suppose, on the other hand, that an experimenter flips the spin of the particle at x_1 at time $t_1-\epsilon$, just before the J^2 measurement, and thereby produces the state $|B\rangle$ at $t_1-\epsilon$. In this case the J^2 measurement *will* disturb the system (since $|B\rangle$ is not an eigenstate of J^2); in particular it will change the state instantaneously into either $|\alpha\rangle_-$ if the result is $J^2=0$, or $|\alpha\rangle_+$ if $J^2=2$ (since $J_z|B\rangle = 0|B\rangle$). So in the event that the spin at x_1 is flipped as we have described, then, whatever the result of the measurement of J^2 is, the probability of finding $\sigma_z^{(x_2)}=+\frac{1}{2}$ at time $t_1+\epsilon$ will be $\frac{1}{2}$, whereas if the spin at x_1 is *not* flipped, then as we have seen the probability of finding $\sigma_z^{(x_2)}=+\frac{1}{2}$ at $t_1+\epsilon$ will be 1. *Thus if a measurement of J^2 is carried out at time t_1, the probabilities at x_2 at $t_1+\epsilon$ will depend upon the spin at x_1 at $t_1-\epsilon$* (where ϵ may be arbitrarily small and $|x_1-x_2|$ arbitrarily large). The J^2 measurement is thus apparently capable of transferring discernible information between x_1 and x_2 at superluminal velocities; that is, the possibility of measuring J^2 at a well-defined time apparently gives rise to violations of causality.

(2′) Suppose that at time $t=-\infty$ the system has been prepared in the state $|\alpha\rangle_-$, and that at time $t=0$ a measurement of $\sigma_z^{(x_1)}$ is carried out by means of an apparatus positioned at x_1 and which interacts locally with the particle at x_1, with the result that $\sigma_z^{(x_1)}=+\frac{1}{2}$. The state of the system will change instantaneously at $t=0$, according to the reduction postulate, from $|\alpha\rangle_-$ to $|C\rangle$. What is paradoxical in this, as in the case of the one-particle system, is that the $t=0$ hypersurface will *not* be an equal-time hypersurface in any other frame, and so although the local action of the apparatus is entirely covariant and cannot be attached to any particular frame, nevertheless the collapse cannot possibly be defined so as to be covariantly instantaneous.

That the argument of the first paradox is flawed may best be made clear by means of a counterexample, wherein we will design a procedure which combines several local (and so necessarily causal) interactions between the system and a measuring device in such a way as to end up measuring a *nonlocal* property of the system.

To begin with, consider a device which is designed to measure $\sigma_z^{(x_1)}$, say, by means of a local interaction with the particle at x_1 as follows: The device interacts with the particle through a term in the Hamiltonian of the form

$$H_{\text{int}} = g(t) q \sigma_z^{(x_1)}, \tag{8}$$

where q is some internal variable associated with the measuring device, and $g(t)$ is a coupling which is nonzero only during a short interval $t_1<t<t_2$, when the device is switched on. Then in the Heisenberg picture

$$\frac{\partial \Pi}{\partial t} = -g(t)\sigma_z^{(x_1)}, \tag{9}$$

where Π is the momentum canonically conjugate to q, and so if we consider a very short interval $(t_1 \approx t_2)$ wherein we may approximate q by a constant then we have

$$\sigma_z^{(x_1)} = \frac{\Pi(t<t_1) - \Pi(t>t_2)}{\int_{t_1}^{t_2} dt\, g(t)} \tag{10}$$

and this is how the device is used to measure $\sigma_z^{(x_1)}$.

Now consider two such devices, which interact with the system through the Hamiltonian

$$H_{\text{int}} = h_1 + h_2, \tag{11}$$

where

$$h_i = g_i(t) q_i \sigma_z^{(x_i)} \tag{12}$$

and where for the moment we will set $g_1(t)=g_2(t)$. Our intent here is to design some combination of these two devices which will collectively measure $J_z \equiv \sigma_z^{(x_1)} + \sigma_z^{(x_2)}$ *without* measuring either $\sigma_z^{(x_1)}$ or $\sigma_z^{(x_2)}$ individually. This can be accomplished as follows: Imagine that at some time $t_0<t_1$ [t_1 is the time at which $g_1(t)$ begins to be nonzero] we bring the two devices together and prepare them in an initial state which has the properties

$$\Pi_1(t_0) + \Pi_2(t_0) = 0, \tag{13a}$$

$$q_1(t_0) - q_2(t_0) = 0. \tag{13b}$$

Then we separate the devices again and allow them to interact with the particles. When the interaction is over, the devices will have measured $\sigma_z^{(x_1)} + \sigma_z^{(x_2)}$, that is,

$$J_z = \sigma_z^{(\alpha 1)} + \sigma_z^{(\alpha 2)} = \frac{-[\Pi_1(t>t_2) + \Pi_2(t>t_2)]}{\int_{t_1}^{t_2} dt\, g(t)} \tag{14}$$

where we have used (13a) to eliminate $\Pi_1(t<t_1) + \Pi_2(t<t_1)]$. On the other hand they will *not* have measured $\sigma_z^{(\alpha 1)}$ or $\sigma_z^{(\alpha 1)}$ or $\sigma_z^{(\alpha 1)} - \sigma_z^{(\alpha 2)}$. In order to measure, say, $\sigma_z^{(\alpha 1)}$, we need to know $\Pi_1(t<t_1) - \Pi_1(t>t_2)$; however Π_1 does not commute with $q_1 - q_2$, that is, Π_1 is not well defined for the state (13) in which the devices were initially prepared. So no measurement of $\sigma_z^{(\alpha 1)}$ has occurred; i.e., no information about $\sigma_z^{(\alpha 1)}$ can be discerned from the devices. Similarly,

$$[\Pi_2, q_1 - q_2] \neq 0, \qquad [\Pi_1 - \Pi_2, q_1 - q_2] \neq 0 \tag{15}$$

so no measurement of $\sigma_z^{(\alpha 2)}$ or $\sigma_z^{(\alpha 1)} - \sigma_z^{(\alpha 2)}$ has taken place either. We have thus succeeded in designing a system of purely local experiments which measures a nonlocal property of the system at a well-defined time (since $t_2 - t_1$ may be arbitrarily short).

Let us now design an experiment which will verify at a well-defined time that the system is in the state $|\alpha\rangle_- \equiv |J^2 = 0\rangle$. This is simple enough: $|\alpha\rangle_-$ may be uniquely defined by the requirements

$$J_z \equiv \sigma_z^{(\alpha 1)} + \sigma_z^{(\alpha 2)} = 0, \quad J_x \equiv \sigma_x^{(\alpha 1)} + \sigma_x^{(\alpha 2)} = 0, \quad J_y \equiv \sigma_y^{(\alpha 1)} + \sigma_y^{(\alpha 2)} = 0 \tag{16}$$

and therefore we shall construct our measuring devices so as to interact with the particles through Hamiltonian

$$H_{\text{int}} = g(t)(\sigma_x^{(\alpha 1)} q_1 + \sigma_y^{(\alpha 1)} q_2 + \sigma_z^{(\alpha 1)} q_3 + \sigma_x^{(\alpha 2)} q_4 + \sigma_y^{(\alpha 2)} q_5 + \sigma_z^{(\alpha 2)} q_6), \tag{17}$$

and the devices shall be prepared initially so that

$$q_1 = q_4, \quad q_2 = q_5, \quad q_3 = q_6, \qquad \Pi_1 = -\Pi_4, \quad \Pi_2 = -\Pi_5, \quad \Pi_3 = -\Pi_6 \tag{18}$$

with the aid of (18), (17) may be rewritten as

$$H_{\text{int}} = g(t)[(\sigma_x^{(\alpha 1)} + \sigma_x^{(\alpha 2)})(q_1 + q_4) + (\sigma_y^{(\alpha 1)} + \sigma_y^{(\alpha 2)})(q_2 + q_5) + (\sigma_z^{(\alpha 1)} + \sigma_z^{(\alpha 2)})(q_3 + q_6)]\tfrac{1}{2} \tag{19}$$

and the resulting equations of motion for the σ's will be

$$\begin{aligned}
\frac{\partial}{\partial t}(\sigma_x^{(\alpha 1)} + \sigma_x^{(\alpha 2)}) &= g(t)[(q_2 + q_5)(\sigma_z^{(\alpha 1)} + \sigma_z^{(\alpha 2)}) - (q_3 + q_6)(\sigma_y^{(\alpha 1)} + \sigma_y^{(\alpha 2)})]\tfrac{1}{2}, \\
\frac{\partial}{\partial t}(\sigma_y^{(\alpha 1)} + \sigma_y^{(\alpha 2)}) &= g(t)[-(q_1 + q_4)(\sigma_z^{(\alpha 1)} + \sigma_z^{(\alpha 2)}) + (q_3 + q_6)(\sigma_y^{(\alpha 1)} + \sigma_y^{(\alpha 2)})]\tfrac{1}{2}, \\
\frac{\partial}{\partial t}(\sigma_z^{(\alpha 1)} + \sigma_z^{(\alpha 2)}) &= g(t)[(q_1 + q_4)(\sigma_y^{(\alpha 1)} + \sigma_y^{(\alpha 2)}) - (q_2 + q_5)(\sigma_x^{(\alpha 1)} + \sigma_x^{(\alpha 2)})]\tfrac{1}{2}.
\end{aligned} \tag{20}$$

The constant functions (16) are, by inspection, a solution of (20); and thus the procedure we have designed is indeed a nondemolition experiment for $|\alpha\rangle_-$, whereby $|\alpha\rangle_-$ can be verified at a well-defined time, using only local interactions, and, therefore, without any violation of causality.[8]

The argument of the first paradox will not apply to this procedure, since it is not a measurement of J^2 in the usual sense (which hereafter will be called an *operator-specific* experiment), but rather a measurement of whether or not $J^2 = 0$ (which hereafter will be called a *state-specific* experiment). It is composed of three measurements of three variables, the J_i, which commute *only* for the state $|\alpha\rangle_-$. Thus, whereas this procedure will not disturb the state $|\alpha\rangle_- \equiv |J^2 = 0, J_z = 0\rangle$, it *will*, for example, disturb the state $|A\rangle \equiv |J^2 = 2, J_z = 1\rangle$; this contradicts the assumption of the paradox. Indeed it is not difficult to show explicitly that if *this* procedure is carried out at time t_1, then the probabilities of x_2 at $t_2 + \epsilon$ will be independent of the conditions at x_1 at $t_1 - \epsilon$.

We have stipulated that the local interactions (12) at x_1 and x_2 are simultaneous [i.e., that $g_1(t) = g_2(t)$]; but this is certainly not a covariant statement, and in general, in any other frame, it will not be the case. Consider, therefore, the following experiment: The system is initially prepared in the state $|\alpha\rangle_- \equiv |J_x = J_y = J_z = 0\rangle$ and subsequently a measurement of J_z is carried out according to the prescription of Eqs. (11)–(14), but where $g_1(t)$ is nonzero for $t_1 < t < t_1 + \delta$, and $g_2(t) = g_1(t - \gamma)$, with $\gamma \gg \delta$. We have studied a case analogous to this one in some detail in Sec. III of Ref. 1, and it will be sufficient for our purposes here simply to review the main conclusions:

First, insofar as the end result of the measurement is concerned, it will make no difference whether the two interactions (12) are simultaneous or not. That is, the measurement will with certainty record that $J_z = 0$,[9] and it will leave the system undisturbed and in its original state ($|\alpha\rangle_-$) when the process is completed, irrespective of the timing of the two interactions. During the interval following the first interaction and preceding the second, on the other hand, the system will not be in *any* eigenstate of J_x or J_y, and indeed the full state will not be separable into a state of the two-particle system and a state of the measuring apparatus (i.e., the full state will not be any direct product of a state of the system and a state of the apparatus). The interaction at x_1 *will*, therefore, disturb the system, and subsequently the interaction at x_2 will undo this disturbance and restore the original state ($|\alpha\rangle_-$); in the event that these two interactions are simultaneous, as we have seen, no disturbance will occur at all.

Thus, if within the interval between these two interactions a measurement of J_x is carried out by means of interactions at the space-time points c and d in Fig. 5, the result will *not* necessarily be $J_x=0$, and the original J_z measurement, having thus been interrupted in midcourse, will *not* necessarily find $J_z=0$, and, finally, the state $|\alpha\rangle_-$ will *not* necessarily be restored at the end of the measurement. If on the other hand a measurement of J_x is carried out at the points e and f in Fig. 6, this will with certainty find $J_x=0$, the original measurement, now uninterrupted, will with certainty find $J_z=0$, and the state $|\alpha\rangle_-$ will with certainty be restored at the end of the process.

All of this leads to a curious observation about the covariance of the state-vector. Consider a measuring process which verifies the state $|\alpha\rangle_-$ without disturbing the system, which hereafter will be called a nondemolition experiment for $|\alpha\rangle_-$. Such a measurement can be repeated an arbitrary number of times and at arbitrarily short intervals, and in principle a limit can be approached in which the state is checked at every instant by a nondemolition experiment, without any disturbance of the system; which shall be called a monitoring of the state history of this system. Now suppose that A and B are two observers in different Lorentz frames K and K', respectively. Since the condition $g_1(t)=g_2(t)$ in (12) cannot be satisfied in *both* frames by any single process, a measurement which verifies $|\alpha\rangle_-$ without disturbing the system as observed by A will necessarily disturb the system, during some finite interval, as observed by B. Furthermore if both A and B attempt to monitor the state history of the two-particle system in their own frames and in overlapping regions of space-time, then these two monitoring procedures will disrupt one another as in Fig. 5. The formal covariance of the state history will therefore be destroyed by any attempt to monitor this history experimentally, since such a procedure can only leave the system undisturbed in one particular frame [the frame for which $g_1(t)=g_2(t)$], and must necessarily disturb it in all others.

Let us apply this observation to the second paradox. The two observers witness the following events: at $t=-\infty$ the two-particle system is prepared in the state $|\alpha\rangle_-$, and at $t=0$ a measurement of $\sigma_z^{(x_1)}$ is carried out, with the result $\sigma_z^{(x_1)}=+\frac{1}{2}$. The paradox is that if each observer applies the postulate of instantaneous reduction in his own frame, then according to A the reduction will occur along the $t=0$ hypersurface, whereas according to B the reduction will occur along the $t'=0$ hypersurface (see Fig. 7), which certainly is not covariant, and, apparently, is a contradiction.

The proposal that the reduction be taken to occur covariantly along the backward light cone of ($x=x_1$, $t=0$), or, indeed, that it be taken to occur along *any* hypersurface other than $t=0$, will fail, as we have remarked already, since it cannot account

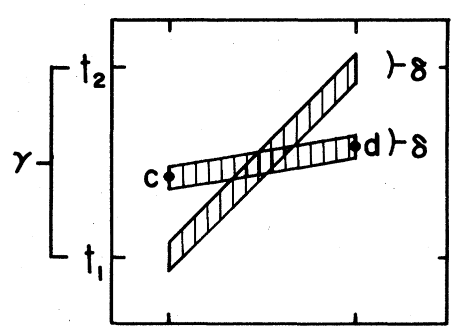

FIG. 5. A measurement of J_z is carried out by means of interactions in the vicinities of (x_1,t_1) and (x_2,t_2) and a measurement of J_x is carried out by means of interactions in the vicinities of c and d. The two measurements will disrupt one another, as explained in the text.

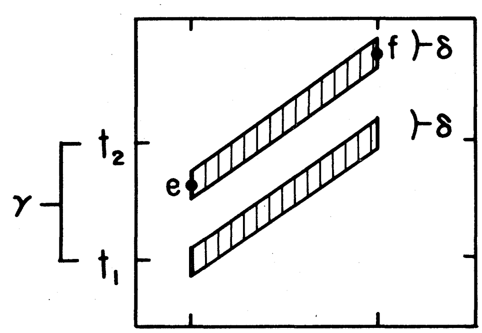

FIG. 6. A measurement of J_z is carried out at (x_1,t_1) and (x_2,t_2) and a measurement of J_y is carried out at e and f. The two measurements will not disrupt one another and the original state $|\alpha\rangle_-$ will be restored at the end of the process.

for the results of nonlocal measurements of the kind we have described here. Thus a measurement carried out at time $t = 0 - \epsilon$ will with certainty confirm that $J^2 = 0$, whereas according to the covariant prescription of Hellwig and Kraus the system will *not* be in an eigenstate of J^2 at $t = 0 - \epsilon$, nor indeed in any eigenstate of any observable whatever.

So it seems that the reduction process *must* be instantaneous, and this, puts us back where we started. If each observer imposes this condition in his own frame, then A and B will give conflicting accounts of the reduction process which cannot possibly be incorporated into any single covariant state history of this system. Apparently we must either accept both of these accounts and so relinquish the covariance of the state, or decide, somehow, which of them is correct.

Suppose that the entire state history of the system is monitored experimentally so as to determine where the reduction "really" occurs. The trouble with this is that the state history cannot be monitored *covariantly*, since any procedure which monitors this history as observed by A will disturb the history as observed by B; and if on the other hand each observer were to monitor the history in his own frame, these two procedures would disrupt one another as in Fig. 5.

If A monitors the succession of states at a given time in his own frame, this will with certainty confirm that the reduction process occurs along $t = 0$, and it will alter the state history as observed by B; and, conversely, if B monitors the state history in k', then *this* will with certainty confirm that the reduction occurs along $t' = 0$, and will alter the history as observed by A. Either of these two conflicting accounts, therefore, can be confirmed by experiment, and in this sense each of them is correct; and this involves no contradiction, since the two different measuring procedures whereby these two accounts can, respectively, be confirmed cannot *both* be carried out on the same system.

No single covariant state history of our system may be defined, therefore, which properly accounts for the experimental results. The covariance of relativistic quantum theories (about which we shall have more to say in Appendix A) resides *exclusively* in the experimental probabilities, and not in the underlying quantum states. The states themselves make sense only within a given frame, or, more abstractly, along some given family of parallel spacelike hypersurfaces; and this is markedly in contrast to the nonrelativistic case. Hereafter we shall continue to use the terminology of states, since in subsequent sections we shall be concerned mainly with questions of causality and not of covariance, but, in light of what we have found, this terminology must be understood to refer only to noncovariant states within a given frame.

III. A CAUSAL RESTRICTION

We return now to the study of what can and what cannot be measured at a well-defined time (in some particular frame) without violating causality; and in the present section we shall be concerned particularly with what can be measured, causally, by means of nondemolition experiments (various other kinds of measuring procedures, whose causal properties are markedly different, will be considered later).

In Sec. II we have described state-specific nondemolition experiments for the nonlocal states $|\alpha\rangle_\pm$ [as they are defined in (1)], which can be carried out, causually, at an arbitrarily well-defined time. This abolishes the suspicion that only local properties of physical systems can be measured in relativistic quantum theories, and therby it raises a question: Is it, then, possible to verify *every* linear combination of the states (3), by means of nondemolition experiments, without violating causality? The answer, curiously, is no. In the present section it will be shown, in particular, that any nondemolition experiment for $|\beta\rangle_\varphi$ [as defined in (2)] will necessarily give rise to violations of causality. We shall assume nothing about this experiment other than that it is capable of verifying the state $|\beta\rangle_\varphi$ without disturbing a system in that state, and that it may be carried out at a well-defined time; and thereby we shall take care to avoid the error of the first paradox.

We will begin by describing the experiment in somewhat greater detail. First of all, it is to be capable of verifying the state $|\beta\rangle_\varphi$; that is, it is to be capable of *distinguishing* between $|\beta\rangle_\varphi$ and any state orthogonal to $|\beta\rangle_\varphi$. The experiment must with certainty yield one particular result if it is carried out on a system in the state $|\beta\rangle_\varphi$, and it must with certainty yield some *other* result if it is carried out on a system in any state orthogonal to $|\beta\rangle_\varphi$.

Let us make this notion more precise. Our experiment must, for example, distinguish

$$|\beta\rangle_\varphi \equiv \sin\varphi\,|C\rangle + \cos\varphi\,|B\rangle$$

from

$$|\beta\rangle_{\varphi\perp} \equiv \sin\varphi\,|\beta\rangle - \cos\varphi\,|C\rangle \tag{21}$$

that is, it must necessarily involve the measurement of some *observable* M_φ which distinguishes between $|\beta\rangle_\varphi$ and $|\beta\rangle_{\varphi\perp}$ as follows: If a measurement of M_φ is carried out on a system in the state $|\beta\rangle_\varphi$, then the result will with certainty be $M_\varphi = \gamma$, where $M_\varphi|\beta\rangle_\varphi = \gamma|\beta\rangle_\varphi$ ($|\beta\rangle_\varphi$ must, clearly, be an eigenstate of any observable measured in the course of a nondemolition experiment for $|\beta\rangle_\varphi$); and if a measurement of M_φ is carried out on a system in the state $|\beta\rangle_{\varphi_\perp}$, then the result will with certainty *not* be $M_\varphi = \gamma$. The experiment, to put it another way, must necessarily involve the measurement of some observable M_φ (where $M_\varphi|\beta\rangle_\varphi = \gamma|\beta\rangle_\varphi$) of which any eigenstate degenerate with $|\beta\rangle_\varphi$ is orthogonal to $|\beta\rangle_{\varphi_\perp}$.[10] In what follows, we will refer to such observables as observables of the type M_φ.

At this point it will be convenient to introduce some further notation. We will associate the four states (3) with the orthonormal basis vectors of a four-dimensional space as follows:

$$|A\rangle \equiv \begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \quad |B\rangle \equiv \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \quad |C\rangle \equiv \begin{pmatrix}0\\0\\1\\0\end{pmatrix}, \quad |D\rangle \equiv \begin{pmatrix}0\\0\\0\\1\end{pmatrix}, \tag{22}$$

wherein $|\alpha\rangle_\pm$, $|\beta\rangle_\varphi$, and $|\beta\rangle_{\varphi_\perp}$ will take the form

$$|\alpha\rangle_\pm = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\\pm 1\\0\end{pmatrix}, \quad |\beta\rangle_\varphi = \begin{pmatrix}0\\\cos\varphi\\\sin\varphi\\0\end{pmatrix}, \quad |\beta\rangle_{\varphi_\perp} = \begin{pmatrix}0\\-\sin\varphi\\\cos\varphi\\0\end{pmatrix}. \tag{23}$$

The three-dimensional subspace of states orthogonal to $|\beta\rangle_\varphi$ is spanned by $|\beta\rangle_{\varphi_\perp}$, $|A\rangle$, and $|D\rangle$, and we will associate these three states with the basis vectors of a three-dimensional space $\varphi\perp$, wherein

$$|\beta\rangle_{\varphi_\perp} \equiv \begin{pmatrix}1\\0\\0\end{pmatrix}_{\varphi_\perp}, \quad |A\rangle \equiv \begin{pmatrix}0\\1\\0\end{pmatrix}_{\varphi_\perp}, \quad |D\rangle \equiv \begin{pmatrix}0\\0\\1\end{pmatrix}_{\varphi_\perp}. \tag{24}$$

Every observable of this system will have four (perhaps degenerate) eigenstates; and any observable 0_φ which is to be measured in the course of a nondemolition experiment for $|\beta\rangle_\varphi$ must necessarily have for its four eigenstates $|\beta\rangle_\varphi$ itself and some three mutually orthogonal vectors in $\varphi\perp$

$$R(\lambda,\mu,\nu)\begin{pmatrix}1\\0\\0\end{pmatrix}_{\varphi_\perp}, \quad R(\lambda,\mu,\nu)\begin{pmatrix}0\\1\\0\end{pmatrix}_{\varphi_\perp}, \quad R(\lambda,\mu,\nu)\begin{pmatrix}0\\0\\1\end{pmatrix}_{\varphi_\perp}, \tag{25}$$

where $R(\lambda,\mu,\nu)$ represents the three-dimensional matrix of rotations through the Eulerian angles λ, μ, and ν (these, hereafter, will be called the Eulerian angles of 0_φ).

Now we are in a position to consider the central issue of the present section. Suppose that a measurement of some observable of the type M_φ is carried out on the two-particle system at some well-defined time t_1. If the measurement is not to give rise to violations of causality, then

(A) the local probabilities at x_2 at $t_1+\epsilon$ must be independent of the conditions at x_1 at $t_1-\epsilon$ *and*

(B) the local probabilities at x_1 at $t_1+\epsilon$ must be independent of the conditions at x_2 at $t_1-\epsilon$.

We will presently show that (A) and (B) can both be satisfied (for such a measurement) *only* in the event that $\varphi = n\pi/4$ (where $n = 0, 1, 2, \ldots$); thereby (since, as we have seen, any nondemolition experiment for $|\beta\rangle_\varphi$ will involve the measurement of an observable of the type M_φ) we shall have proven that any nondemolition experiment for $|\beta\rangle_\varphi$ (where $\varphi \neq n\pi/4$) will necessarily give rise to violations of causality.

Consider the following two scenarios:

(1) The two-particle system is initially prepared in the state $|\alpha\rangle_-$, and at the well-defined time t_1 a measurement of some particular observable 0_φ, of the type M_φ, is carried out. The probability that a measurement of $\sigma_z^{(x_2)}$, carried out at time $t_1+\epsilon$, will find that $\sigma_z^{(x_2)} = +\frac{1}{2}$, is, then:

$$P_1^{(x_2)}(+\tfrac{1}{2}) = [(1-2\sin\varphi\cos\varphi)\cos^2\varphi + (1+2\sin\varphi\cos\varphi)\sin^2\varphi + \eta_1)]\tfrac{1}{2}, \tag{26}$$

where η_1 will depend on the Eulerian angles of 0_φ (the calculation of the local probabilities at $t_1+\epsilon$, for both scenarios, shall be considered in detail in Appendix B; here, so as not to obscure the main line of the argument, we will simply quote the results).

(2) The system is initially prepared in the state $|\alpha\rangle_-$, as above. At time $t_1-\epsilon$ an experimenter at x_1 rotates the x and y components of the spin of the particle at x_1 about $\hat{z}$ by 180°, and thereby changes the state at $t_1-\epsilon$ from $|\alpha\rangle_-$ to $|\alpha\rangle_+$ [this can be done, for example, by means of a magnetic field in the z direction in the vicinity of x_1 (if the particles are charged); that is, it can be done entirely by means of local interactions with the particle at x_1]. Finally at t_1, as in the first scenario, a measurement of 0_φ is carried out. In this case the probability that $\sigma_z^{(x_2)} = \frac{1}{2}$ at $t_1+\epsilon$ will be

$$P_2^{(x_2)}(+\tfrac{1}{2}) = [(1+2\sin\varphi\cos\varphi)\cos^2\varphi + (1-\sin\varphi\cos\varphi)(\sin^2\varphi + \eta_2)]\tfrac{1}{2}, \tag{27}$$

where (in accordance with the results of Appendix B)

$$\eta_2 = \eta_1 = \eta \,. \tag{28}$$

In the event that $P_1^{(x_2)}(+\frac{1}{2}) \neq P_2^{(x_2)}(+\frac{1}{2})$, the probabilities at x_2 at $t_1+\epsilon$ will depend on whether or not the spin at x_1 is rotated at $t_1-\epsilon$; and thereby the measurement of 0_φ will violate (*A*), and give rise to violations of causality. If the measurement is to *satisfy* (*A*) then $P_1^{(x_2)}(+\frac{1}{2})$ must equal $P_2^{(x_2)}(+\frac{1}{2})$; that is, either φ must be chosen so that

$$\sin\varphi \cos\varphi = 0 \tag{29}$$

or 0_φ (more particularly: the Eulerian angles of 0_φ) must be chosen so that

$$\eta = \cos^2\varphi - \sin^2\varphi \,. \tag{30}$$

Now let us attend to the question of satisfying (*B*). Consider another scenario, (2′), wherein $|\alpha\rangle_-$ is changed to $|\alpha\rangle_+$ at $t_1-\epsilon$ by rotating the particle of x_2 (instead of the particle at x_1), and which in all other respects is identical to scenario (2). The probability, in scenario (2′), that a measurement of $\sigma_z^{(x_2)}$, carried out at $t_1+\epsilon$, will find that $\sigma_z^{(x_1)} = +\frac{1}{2}$ is

$$P_{2'}^{(x_2)}(+\tfrac{1}{2}) = \tfrac{1}{2}[(1+2\sin\varphi\cos\varphi)\sin^2\varphi + (1-2\sin\varphi\cos\varphi)(\cos^2\varphi+\eta)] \,. \tag{31}$$

The probability that $\sigma_z^{(x_1)} = +\frac{1}{2}$ at $t_1+\epsilon$ in scenario (1), on the other hand, is

$$P_1^{(x_1)}(+\tfrac{1}{2}) = [(1-2\sin\varphi\cos\varphi)\sin^2\varphi + (1+2\sin\varphi\cos\varphi)(\cos^2\varphi+\eta)]\,\tfrac{1}{2} \,. \tag{32}$$

If the measurement of 0_φ is to satisfy (B), then $P_1^{(x_1)}(+\frac{1}{2})$ must equal $P_{2'}^{(x_1)}(+\frac{1}{2})$; that is, either $\sin\varphi\cos\varphi = 0$ or

$$\eta = \sin^2\varphi - \cos^2\varphi \,. \tag{33}$$

Finally, if the measurement of 0_φ is to be *causal* [that is, if the measurement is to satisfy both (A) *and* (B)], then either $\sin\varphi\cos\varphi = 0$ or [combining (30) and (33)]

$$\eta = \sin^2\varphi - \cos^2\varphi = \cos^2\varphi - \sin^2\varphi = 0 \,. \tag{34}$$

The possibility of carrying out any nondemolition experiment for $|\beta\rangle_\varphi$ at a well-defined time, therefore, will necessarily give rise to violations of causality unless

$$\varphi = \frac{n\pi}{4}\,, \quad n = 0, 1, 2, \ldots$$

and thereby our proof is complete.

It is not difficult to extend this analysis to arbitrary complex linear combinations of $|B\rangle$ and $|C\rangle$; and it turns out that states of the form

$$\cos\varphi\,|C\rangle + e^{i\theta}\sin\varphi\,|B\rangle \tag{35}$$

will be verifiable by means of nondemolition experiments if and only if $\varphi = n\pi/4$, for all values of θ.

IV. OTHER VARIETIES OF MEASUREMENT

As we have remarked already, there are procedures other than nondemolition experiments, with very different casual properties, whereby a system can be measured to be in some particular state. We shall presently show, for example, how the two-particle system can be *prepared* in the state $|\beta\rangle_\varphi$, in an arbitrarily short and well-defined time, by means of a definite sequence of measurements without any violation of causality [although, as we have seen, any *nondemolition* experiment for $|\beta\rangle_\varphi$ (where $\varphi \neq n\pi/4$) will necessarily give rise to violations of causality].

Consider the following scenario.

(1) The two-particle system is initially prepared in the state $|\alpha\rangle_+$ (by means, say, of the state-specific experiments described in Sec. II); an experimental apparatus of the kind described in Eqs. (8)–(10) for measuring $\sigma_z^{(x_1)}$ (where $t_1 \approx t_2$ and $\int_{t_1}^{t_2} dt\, g(t) = 1$) is initially prepared in the state

$$\sin\varphi\,|\pi = -\tfrac{1}{2}\rangle + \cos\varphi\,|\pi = +\tfrac{1}{2}\rangle \,. \tag{36}$$

(2) At time $t_1-\epsilon$, the state of the two-particle system ($|\alpha\rangle_+$) is verified by means of a nondemolition experiment, and at time $t_1 \approx t_2$ the experimental apparatus for measuring $\sigma_z^{(x_1)}$ is allowed to interact with the particle at x_1 [as described in (8) and (9)]; whereby, as the reader can easily confirm, the full state of the system plus the measuring apparatus becomes

$$\frac{1}{\sqrt{2}}\sin\varphi(|\pi=0\rangle|B\rangle + |\pi=-1\rangle|C\rangle) + \frac{1}{\sqrt{2}}\cos\varphi(|\pi=0\rangle|C\rangle + |\pi=+1\rangle|B\rangle) \,. \tag{37}$$

(3) Finally, at time $t_2+\epsilon$, a measurement of π is carried out on the apparatus.

In the event that π is found to be zero at $t = t_2+\epsilon$, we shall have *measured*[11] that the state of the two-particle system at $t_2+\epsilon$ is with certainty $|\beta\rangle_\varphi$ (without giving rise to any violation of causality, since all of the interactions involved are purely local ones). Consider, however, what sort of measurement this is; the experiment will "succeed" (that is, π will be found to be zero at $t_2+\epsilon$) only half the time [as is obvious from (37)] and the procedure requires that the initial state of the system be $|\alpha\rangle_+$, rather than $|\beta\rangle_\varphi$. This experiment, therefore, will not suffice to *verify* the state $|\beta\rangle_\varphi$ (we have already seen indeed that such a verification is impossible); rather, it is an experiment whereby the state $|\beta\rangle_\varphi$ can be *prepared*, on those occasions when the procedure is successful, at a well-defined time. Thus the causal properties of a preparation experiment for $|\beta\rangle_\varphi$ are very different from those of a verification experiment for the same state.

Such discrepancies also arise between the causal properties of verification experiments for single systems and those of verification experiments for *ensembles*. The reason is this: Any state of the two-particle system will be uniquely determined by the associated local probabilities at x_1 and x_2 and the probabilities of the results of state-specific experiments for $|\alpha\rangle_+$ and $|\alpha\rangle_-$ [the latter are required in order to determine the phase relations between the various local states (3)]. The state of any ensemble of identical systems (all of which have been prepared in identical states) *can*, therefore, be determined at a well-defined time by means of an ensemble of causal experiments (whereby the required probabilities can be measured statistically). The causal observables of this system, then, are precisely enough to verify an arbitrary state for an ensemble; but too few to verify an arbitrary state for an individual pair of particles. (It is tempting to suppose that, in this sense, the relativistic theory is fundamentally a theory about ensembles, and not about individual systems; but this seems to us at best premature.)

V. BOSONS AND FERMIONS

Let us now apply some of what we have learned to the case of free particles. Consider, for example, a single-particle state $|\delta\rangle_-$ in which the particle is in a superposition of two localized states: one at x_1 and the other at x_2, i.e.,

$$|\delta\rangle_- \equiv \frac{1}{\sqrt{2}}(|x_1\rangle - |x_2\rangle). \tag{38}$$

Now we define an operator $\sigma_z^{(x_1)}$ by

$$\begin{aligned}\sigma_z^{(x_1)}|x=x_1\rangle &= +\tfrac{1}{2}|x=x_1\rangle,\\ \sigma_z^{(x_1)}|x\neq x_1\rangle &= -\tfrac{1}{2}|x\neq x_1\rangle\end{aligned} \tag{39}$$

and we define $\sigma_x^{(x_1)}$ and $\sigma_y^{(x_1)}$ so as to satisfy the appropriate spin commutation relations with $\sigma_z^{(x_1)}$. If the particle is a boson, then the various $\sigma_j^{(x_1)}$ will all be local observables, and in particular

$$[\sigma_i^{(x_1)}, \sigma_j^{(x_2)}] = 0 \quad \text{if} \quad x_1 \neq x_2. \tag{40}$$

The conditions (16), then, will uniquely define $|\delta\rangle_-$, and everything will now proceed exactly as before; that is, the process described by Eqs. (17)–(20) will now constitute a causal nondemolition experiment for the state $|\delta\rangle_-$.

By the same token, *whatever* we have learned about the two-spin system will apply to the one-boson states as well: the localized states $|x_1\rangle$ and $|x_2\rangle$ and the maximally nonlocal states $|\delta\rangle_\pm$ are verifiable by means of nondemolition experiments at a well-defined time without any violation of causality; the "intermediate" states of the form

$$|z\rangle_\varphi \equiv \sin\varphi\,|x_1\rangle + \cos\varphi\,|x_2\rangle, \quad \text{where } \varphi \neq \frac{n\pi}{4}, \; n=0,1,2,\ldots \tag{41}$$

can, causally, be prepared, but *not* verified at a given time; arbitrary linear combinations of $|x_1\rangle$ and $|x_2\rangle$ can be verified at a well-defined time, without violating causality, for *ensembles* of bosons.

The measurement of superpositions of n localized boson states (and, as $n\to\infty$, of momentum eigenstates) is more complicated. Apparently, such states are all preparable; whether any of them are verifiable, on the other hand, remains to be seen and shall be the subject of a future work. [It has on occasion been suggested[12] that the momentum could be measured by means of interactions with the "local" momentum density $(\rho(x))$ at every point in space, where

$$P \equiv \int_{-\infty}^{\infty} d^3x\,\rho(x), \tag{42}$$

but this would constitute an *operator-specific* measurement of P, and thereby, as we learn from the first paradox, would necessarily give rise to violations of causality.]

Now let us consider the case of fermions. The fermion field *anti*commutes with itself at spacelike separations; and therefore the operators $\sigma_x^{(x)}$ and $\sigma_y^{(x)}$, which are both linear in the field at x, will *not* be local observables for systems of $\frac{1}{2}$-integral spin. A measurement, say of $\sigma_y^{(x_1)}$ or $\sigma_y^{(x_1)} + \sigma_y^{(x_2)}$ will observably disrupt the system at points spacelike separated from x_1 and x_2, and will thereby give rise to violations of causality. This changes everything: nondemolition experiments for $|\delta\rangle_\pm$ cannot causally be carried out for fermion systems, nor can the preparation experiments for $|z\rangle_\phi$, nor can the verification experiments for ensembles.

There are, however, yet other procedures; albeit of limited usefulness. A *second* fermion, for example, can be introduced into the system, which, together with the first, will (not with certainty but with some finite probability, as in the preparation experiments) constitute a single boson whose state can then be measured by the methods described above. We can in this way verify that the state of the original fermion was, say $|\delta\rangle_-$; but this requires that the second fermion (which is to be introduced in the course of the measurement) be initially prepared in the state $|\delta\rangle_-$ (which cannot, so far as we know, be accomplished at a well-defined time), and once the measurement is complete (*if* it succeeds) the original one-fermion state will have been irrepairably destroyed.

Yet, whatever can or cannot be achieved along these lines, it is certain that no causal nondemolition experiment can be constructed for any nonlocal fermion state; thus it is certain that what is measurable for fermions is, oddly, very different from what is measurable for bosons.

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APPENDIX A: THE COVARIANCE OF RELATIVISTIC QUANTUM FIELD THEORIES

Let us consider in somewhat more detail how the covariance of the probabilities (which is necessary if the theory is to make any sense) and the noncovariance of the state history (which we have described in Sec. II) manage to peacefully coexist.

The local observables of any relativistic quantum field theory are required to commute at spacelike separations, and therefore the results of two spacelike-separated local experiments will not depend upon the order in which these experiments are carried out. Suppose, for example, that our two-particle system is prepared at time $t=-\infty$ in the state $|\alpha\rangle_-$, and that at $t=0$ a measurement of $\sigma_z^{(x_1)}$ is carried out, with the result $\sigma_z^{(x_1)}=+\frac{1}{2}$. Any spacelike separated measurement of $\sigma_z^{(x_2)}$, *whether before or after* $t=0$, will then with certainty find that $\sigma_z^{(x_2)}=-\frac{1}{2}$ [see Fig. 7(a)]. Alternately, a measurement of $\sigma_z^{(x_2)}$ carried out at the point A in Fig. 7(b) will with certainty find that $\sigma_z^{(x_2)}=-\frac{1}{2}$ whether the state reduction associated with the measurement of x_1 is taken to occur along $t=0$, or along another spacelike hypersurface $t'=0$; indeed the reduction may be taken to occur along *any* spacelike hypersurface which passes through $(x,0)$ without affecting the results of local experiments (see Ref. 6). If, therefore, each observer applies the postulate of instantaneous reduction in his *own* frame, all will nonetheless derive identical (i.e., covariant) experimental predictions for *local* observables.

It remains, then, only to account for the covariance of the probabilities of nonlocal observables. Let us recall precisely how these observables are measured: The physical system is allowed to interact with the measuring apparatus in such a way that the various nonlocal properties of the system become correlated to purely *local*

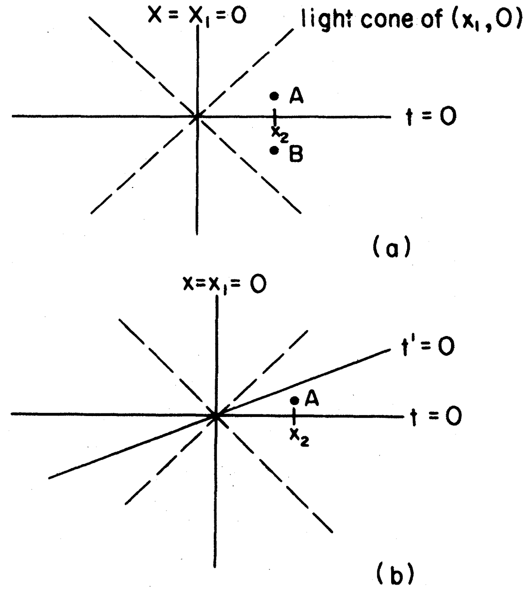

FIG. 7. (a) The system is prepared at $t=-\infty$ in the state $|\alpha\rangle_-$, and at $t=0$ $\sigma_z^{(x_1)}$ is measured to be $+\frac{1}{2}$. Any measurement of $\sigma_z^{(x_2)}$ at A or B (i.e., whether before or after $t=0$) will with certainty find that $\sigma_z^{(x_2)}=-\frac{1}{2}$. (b) The system is prepared at $t=-\infty$ in the state $|\alpha\rangle_-$, and at $t=0$ $\sigma_z^{(x_1)}$ is measured to be $+\frac{1}{2}$. A measurement of $\sigma_z^{(x_2)}$ at A will with certainty find that $\sigma_z^{(x_2)}=-\frac{1}{2}$, whether the collapse is taken to occur along $t=0$ or $t'=0$.

properties of the apparatus (in the experiment described by Eqs. (11)–(14), for example, the nonlocal observable J_z of the two-particle system becomes correlated to Π_1 and Π_2 [or, more particularly, $\Pi_1-\Pi_2$), which are purely local variables of the apparatus]. The results of these experiments, then, may be characterized in terms of purely local observables of the apparatus; that is, our nonlocal measurements on, say, the two-particle system, consist of purely *local* measurements on the larger system of the measuring apparatus *plus* the two particles, and so for this larger system we may invoke the above conclusions about local observables.

It is a requirement of relativistic causality, then, that although we may measure nonlocal properties of various systems, we must always carry out such measurements by means of *local* observations on the measuring apparatus; all measurements must *ultimately* (in the sense we have just described) be local ones.

APPENDIX B: ON THE CALCULATION OF THE PROBABILITY FUNCTIONS OF SEC. III

In this appendix we shall derive the various $P_j^{(x_1)}$ of Eqs. (20), (21), (25), and (26) of Sec. III. We will begin with a particular illustrative example, and afterwards proceed to the more general case.

Suppose, then that in scenario (1) (of Sec. III) the observable 0_φ has four eigenstates of the form[13]

$$\begin{pmatrix} 0 \\ \cos\varphi \\ \sin\varphi \\ 0 \end{pmatrix} \equiv |\beta\rangle_\varphi, \quad \frac{1}{\sqrt{2}}\begin{pmatrix} \lambda \\ -\sin\varphi \\ \cos\varphi \\ \tau \end{pmatrix} \equiv |l\rangle,$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix} -\lambda \\ -\sin\varphi \\ \cos\varphi \\ -\tau \end{pmatrix} \equiv |m\rangle, \quad \begin{pmatrix} -\tau \\ 0 \\ 0 \\ \lambda \end{pmatrix} \equiv |n\rangle,$$

where $0_\varphi|\beta\rangle_\varphi = \gamma|\beta\rangle_\varphi$, $0_\varphi|n\rangle = \gamma|n\rangle$, $0_\varphi|l\rangle = l|l\rangle$, and $0_\varphi|m\rangle = m|m\rangle$. Since 0_φ is to be an observable of the type M_φ, we require that $\gamma \neq l$ and $\gamma \neq n$.

Now, if, as in scenario 1, the system is initially prepared in the state $|\alpha\rangle_-$, the probability that a measurement of 0_φ will find that $0_\varphi = \gamma$ will be $|{}_-\langle\alpha|\beta\rangle_\varphi|^2 + |{}_-\langle\alpha|n\rangle|^2 = (1 - 2\sin\varphi\cos\varphi) + 0$. Similarly, the probability, in this scenario, that $0_\varphi = l$ plus the probability that $0_\varphi = m$ will be $1 + 2\sin\varphi\cos\varphi$. If the result is $0_\varphi = \gamma$ (in which case the state of the system after the measurement will be $|\beta\rangle_\varphi$, since $\langle n|\alpha\rangle_- = 0$) then the probability of finding $\sigma_z^{(x_2)} = +\frac{1}{2}$ at $t_1 + \epsilon$ will be $\cos^2\varphi$; if the result is either $0_\varphi = l$ or $0_\varphi = m$, then the probability that $\sigma_z^{(x_2)} = +\frac{1}{2}$ at $t_1 + \epsilon$ will be $\sin^2\varphi + \lambda^2$. The *total* probability, therefore, that $\sigma_z^{(x_2)} = +\frac{1}{2}$ at $t_1 + \epsilon$ will be

$$\tfrac{1}{2}[(1 - 2\sin\varphi\cos\varphi)\cos^2\varphi + (1 + 2\sin\varphi\cos\varphi)(\sin^2\varphi + \lambda^2)]$$

and the calculation of the various other $P_j^{(x_1)}(+\frac{1}{2})$ will proceed in a similar manner.

Now consider the more general case. Whatever the Eulerian angles of 0_φ (i.e., whatever its eigenstates), so long as it is an observable of the type M_φ, the probability (in scenario 1) that $0_\varphi = \gamma$, where $0_\varphi|\beta\rangle_\varphi = \gamma|\beta\rangle_\varphi$, will be $(1 - 2\sin\varphi\cos\varphi)$, and the probability of finding any other result will be $(1 + 2\sin\varphi\cos\varphi)$. Furthermore, in the event that $0_\varphi = \gamma$, then the probability that $\sigma_z^{(x_2)} = +\frac{1}{2}$ at $t_1 + \epsilon$ will be $\cos^2\varphi$, and the probability that $\sigma_z^{(x_1)} = +\frac{1}{2}$ will be $\sin^2\varphi$; in the event that $0_\varphi \neq \gamma$, the probability that $\sigma_z^{(x_2)} = +\frac{1}{2}$ will be $\sin^2\varphi + \eta$ (where η depends upon the Eulerian angles of 0_φ), and the probability that $\sigma_z^{(x_1)} = +\frac{1}{2}$ will be $\cos^2\varphi + \eta$ (where η will be the same as above, and the same for *all* scenarios).

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[1] Yakir Aharonov and David Z. Albert, Phys. Rev. D 21, 3316 (1980).

[2] Here, and throughout the present paper, we shall be concerned with localizing particles only to within regions whose dimensions are of the order of (or larger than) their Compton wavelengths; that is, we shall consider only those localization experiments wherein the effects of pair creation are negligible.

[3] L. D. Landau and R. Peierls, Z. Phys. 69, 56 (1931).

[4] Indeed, they went so far as to write down new "uncertainty relations" of the form $\Delta p \Delta t \geq \hbar/c$.

[5] Which apparently was first described in I. Bloch, Phys. Rev. 156, 1377 (1967).

[6] K. E. Hellwig and K. Kraus, Phys. Rev. D 1, 566 (1970).

[7] See, for example, Carlton M. Caves, Kip S. Thorne, Ronald W. P. Drever, Vernon D. Sandberg, and Mark Zimmerman, Rev. Mod. Phys. 52, 341 (1980), and references therein.

[8] $|\alpha\rangle_+$, conversely, may be uniquely defined by the requirements

$$\sigma_z^{(x_1)} + \sigma_z^{(x_2)} = 0,$$
$$\sigma_y^{(x_1)} - \sigma_y^{(x_2)} = 0,$$
$$\sigma_x^{(x_1)} - \sigma_x^{(x_2)} = 0.$$

Proceeding by analogy with (16)–(20), then, it is simple to design a nondemolition experiment for $|\alpha\rangle_+$.

[9] That is, for $t > t_1 + \gamma$, $\pi_1 + \pi_2$ will with certainty be equal to zero [where π_1 and π_2 are the internal variables of the apparatus described in (13a)] .

[10] Note that $\langle\beta\rangle_{\varphi\perp}$ need not be an eigenstate of M_φ.

[11] "Measured" in the sense that we shall be able to say with certainty that the system is in the state $|\beta\rangle_\phi$, *without* invoking the equations of motion (i.e., we shall be able to read what the state is in our measuring device, rather than computing what it ought to be by means of the equations of motion).

[12] In various private communications with the authors.

[13] Where $|\lambda|^2 + |\tau|^2 = 1$.