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Cosmology and broken scale invariance

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The cosmological consequences of a simple scalar field model for the generation of Newton's constant through the spontaneous breaking of scale invariance in a curved space are presented and discussed.

Considerable interest has been dedicated recently to the derivation of Einstein gravity as a symmetry-breaking effect.[1] In particular, so as to maintain renormalizability, the introduction of masses and dimensional parameters in the Lagrangian density is avoided and they are expected to arise through the spontaneous breaking of scale invariance introduced by scalar[1,2] or gauge fields.[1,3]

The purpose of this note is to examine the cosmological consequences of the following globally scale-invariant Lagrangian density for a scalar field σ in a curved space-time[4]:

$$L=-\tfrac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma-\frac{\lambda}{4}\sigma^4+\frac{\gamma}{2}\sigma^2R+L_m \quad , \tag{1}$$

where γ and λ are dimensionless positive constants, R is the curvature scalar, and L_m is the matter Lagrangian density which we assume does not contain σ. In flat space the above Lagrangian density has been shown to allow for spontaneous symmetry breaking,[5] therefore let us consider the vacuum a condensate of scalar particles and treat the presence of matter as a perturbation about a suitable ground-state (vacuum) solution to Eq. (1). Such an approach has been shown to lead,[6] in the weak-field limit, to essentially the same equations (up to a redefinition of the fields) as are obtained from the Brans-Dicke theory.[7] In contrast to that case, however, here one has a background solution (prior geometry) associated with a small, but nonzero, positive cosmological constant. This work further differs from other approaches for the generation of Einstein gravity through the use of scalar fields and spontaneous symmetry breaking,[2] in that in the latter approach the scalar field rather than being massless, as in our case, has a mass of the order of the Planck mass ($\sim 10^{19}$ GeV).

We therefore expect that the approach suggested by the Lagrangian density Eq. (1) will lead to cosmological predictions differing from Einstein gravity less than the Brans-Dicke theory (because of the stabilizing potential term) but more than other approaches involving massive scalar fields (or very deep potential wells) and the purpose of this note is to exhibit the results obtained.

Let us consider a Robertson-Walker universe with line element given by

$$ds^2=-dt^2+S^2(t)\left[\frac{dr^2}{1-kr^2}+r^2d\theta^2+r^2\sin^2\theta d\phi^2\right] \tag{2}$$

and matter behaving like an isentropic perfect fluid having energy-momentum tensor

$$T_m^{\alpha\beta}=pg^{\alpha\beta}+(\rho+p)u^\alpha u^\beta \quad , \tag{3}$$

where $\rho(t)$, $p(t)$, and u^α are the energy density, pressure, and velocity four-vector, respectively. The Einstein equations obtained from Eq. (1) will become

$$\dot\rho=-3\frac{\dot S}{S}(\rho+p) \quad , \tag{4}$$

$$\frac{\dot S^2}{S^2}+\frac{k}{S^2}=\frac{\rho}{3\gamma\sigma^2}+\frac{1}{6\gamma}\frac{\dot\sigma^2}{\sigma^2}-2\frac{\dot S}{S}\frac{\dot\sigma}{\sigma}+\frac{1}{12\gamma}\lambda\sigma^2 \quad , \tag{5}$$

$$\frac{d}{dt}(\sigma\dot\sigma S^3)=\frac{(\rho-3p)}{6\gamma+1}S^3 \quad , \tag{6}$$

where the dot denotes differentiation with respect to the time t.

In the absence of matter ($\rho=p=0$) we have the "vacuum" solution given by[6]

$$\sigma=\sigma_0=\left(\frac{\gamma R_0}{\lambda}\right)^{1/2}, \tag{7a}$$

where R_0 is a constant and corresponds to a space of constant curvature $R_0/12$,

$$k=0 , \tag{7b}$$

$$S=S_0(t)=S_0(0)\exp(H_0 t) , \tag{7c}$$

$$H_0=\left(\frac{\lambda}{12\gamma}\sigma_0^2\right)^{1/2} . \tag{7d}$$

Further, from the weak-field limit[6] Newton's coupling constant G is given by

$$\frac{1}{8\pi G_0}=\gamma\sigma_0^2\frac{6\gamma+1}{8\gamma+1} \tag{8}$$

leading to a vacuum expectation value σ_0 of the order of the Planck mass for σ and we note the above solution corresponds to a de Sitter universe.

On examining Eqs. (4)–(6) in the presence of matter, it is straightforward to see that because of the potential term ($\lambda\neq 0$) in contrast to the Brans-Dicke case a power in t solution for S and σ is not possible. We shall then treat the introduction of matter as a small perturbation and write

$$\sigma=\sigma_0[1+\chi(t)] , \tag{9a}$$

$$S=S_0(t)+s(t) , \tag{9b}$$

where χ and s are assumed small. We further take $k=0$ (zero-curvature three-space) and assume the following equation of state:

$$p=\alpha\rho(t) , \tag{10}$$

where α is a positive or zero constant.[8]

To lowest order Eqs. (4)–(6) will become

$$\frac{\dot\rho}{\rho}=-3(\alpha+1)\frac{\dot S}{S}=-3(\alpha+1)H_0 , \tag{11}$$

$$\frac{2H_0}{S_0}(\dot s-H_0 s)=\frac{\rho}{3\gamma\sigma_0^2}-2H_0\dot\chi+2H_0^2\chi , \tag{12}$$

$$\ddot\chi+3\dot\chi H_0=\frac{1}{6\gamma+1}\frac{(1-3\alpha)\rho}{\sigma_0^2} , \tag{13}$$

and the solutions are

$$\rho(t)=\rho(0)e^{-3(1+\alpha)H_0 t} , \tag{14a}$$

$$\chi=\frac{1}{6\gamma+1}\frac{(1-3\alpha)}{\sigma_0^2}\frac{\rho(0)}{9H_0^2\alpha}\left(\frac{e^{-3(1+\alpha)H_0 t}}{1+\alpha}-e^{-3H_0 t}+\frac{\alpha}{\alpha+1}\right) , \tag{14b}$$

$$s=\frac{(1-3\alpha)}{6\gamma+1}\frac{\rho(0)S_0(t)}{9\sigma_0^2H_0^2\alpha}\left((1-e^{-3H_0t(1+\alpha)})\frac{(4+3\alpha)}{3(1+\alpha)^2}-\tfrac{4}{3}(1-e^{-3H_0t})+\frac{\alpha H_0 t}{\alpha+1}\right)+\frac{\rho(0)S_0(t)}{18\gamma(1+\alpha)H_0^2\sigma_0^2}(1-e^{-3H_0t(1+\alpha)}) , \tag{14c}$$

where we have imposed the boundary conditions

$$\chi(0)=\dot\chi(0)=s(0)=0 . \tag{15}$$

We immediately observe that as a consequence of the above, the gravitational constant G acquires a time dependence given by

$$\frac{\dot G}{G}\approx-2\dot\chi=-\frac{(1-3\alpha)}{6\gamma+1}\frac{2\rho(t)}{3H_0\alpha\sigma_0^2}(-1+e^{+3\alpha H_0 t}) , \tag{16}$$

from which

$$G(t)=G_0\exp\left[-\frac{(1-3\alpha)}{6\gamma+1}\frac{2\rho(t)}{9H_0^2\alpha\sigma_0^2}\left(\frac{1}{1+\alpha}-e^{+3\alpha H_0 t}+\frac{\alpha e^{+3(1+\alpha)H_0 t}}{\alpha+1}\right)\right]\approx G_0\exp\left[-\frac{(1-3\alpha)\rho(0)t^2}{(6\gamma+1)\sigma_0^2}\right] \text{ for } H_0 t \text{ small}, \tag{17}$$

and we immediately see that for t large (with respect to H_0^{-1}) the gravitational constant is time independent whereas for t small it decreases or increases with time according to whether $\alpha<\frac{1}{3}$ or $>\frac{1}{3}$. The former case is associated with a matter-dominated universe ($\alpha=0$) and corresponds to the present situation. As one goes back in time one first expects $\alpha=\frac{1}{3}$ corresponding to a radiation-dominated universe ($\dot G=0$) and possibly $\alpha=1$ at very high densities arising from fermions (quarks) interacting via vector-meson (gauge) fields.[8,9] In particular, this last case could occur for energies in the vicinity of the grand unification scale (10^{15}–10^{17} GeV).[10]

In our approach we have considered matter as giving rise to a perturbation about the vacuum, or a ground-state solution, thus we expect all quantities not to be too different from their unperturbed values. Let us check the consistency of such an approach. If we take

$$\begin{aligned} &\rho(t_0) \approx 10^{-30}\ \mathrm{g/cm^3}\ , \\ &H_0^{-1} \approx \frac{S(t_0)}{\dot S(t_0)} \approx 0.7\times 10^{18}\ \mathrm{sec}\ , \\ &G(t_0) \approx 6.7\times 10^{-8}\ \mathrm{cm^3/g\,sec^2}\ , \\ &t_0 \approx 3\times 10^{17}\ \mathrm{sec}\ , \end{aligned} \tag{18}$$

we see that for $\alpha = 0$

$$\chi(t_0) \approx \frac{0.5\gamma}{8\gamma+1} \tag{19}$$

and

$$\left(\frac{\dot G}{G}\right)_{t=t_0} \approx -10^{-18}\frac{\gamma}{8\gamma+1}\ \mathrm{sec}^{-1}\ , \tag{20}$$

where in the above for G_0 we have taken $G(t_0)$ since we have

$$G(t_0) = G_0 \exp\left(-\frac{10^7\gamma}{8\gamma+1}G_0\right)\ , \tag{21}$$

the exponent being negligible for γ small and $G_0 \lesssim 10^{-7}$. It is worth observing that the smaller γ is, the closer the results are to those of Einstein gravity. Let us further observe that in the above approach one has a nonzero positive cosmological constant Λ given by

$$\Lambda = \frac{1}{4\gamma}\lambda\sigma_0{}^2 = 3H_0{}^2 \approx 10^{-56}\ \mathrm{cm}^{-2}\ , \tag{22}$$

and we have taken H_0 to be approximately the same as the present value since one can verify by using Eq. (14c) that $\dot S(t_0)/S(t_0)$ and H_0 agree very closely. Similar considerations also hold for the deceleration parameter which differs little from its initial value -1.

As one can then see, our approach, whereby the presence of matter introduces a small perturbation on an initial or vacuum universe of the de Sitter type, appears to be consistent. We further observe that our approach does not lead to difficulties such as the horizon problem since besides the fact that we are never far away from a de Sitter universe, the gravitational constant can actually decrease with time in an early universe when the equation of state can be expected to be $\rho = p$.

Let us observe that all the above considerations have been done for a zero-temperature field theory and the inclusion of finite-temperature effects will introduce a temperature dependence in σ_0 and corresponding changes in other parameters such as H_0 and G_0. Moreover, as we have mentioned, the relevant equation of state is also expected to be temperature dependent and α will vary from 1 to 0 as the Universe expands and cools. However, such effects have only been calculated for flat spaces[11] or in theories with local scale invariance and conformally flat metrics. Further, as one approaches extremely high temperatures and densities (of the order of the Planck mass), not only will finite-temperature effects introduce dramatic changes, but it may also be that other curved-space vacuum solutions become more significant.[6]

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