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Charged particles, magnetic monopoles, and topology in Einstein's unified field theory

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We report on recent progress in understanding the structure of charged particles in Einstein's theory of the nonsymmetric field. If one makes reasonable assumptions concerning the asymptotic behavior of the field far from a charged particle, assumptions which seem to be demanded by empirical evidence, one finds that the global topologies of certain spatial regions which contain the charged particle must be non-Euclidean if the field over those regions is to be nonsingular. One also finds under the same assumptions that the possibility that all charged particles possess a magnetic-monopole moment proportional to their charge cannot be ruled out. In an approximation which should be valid over macroscopic interaction distances it is found that such particles interact among themselves as if they had no magnetic-monopole moments. We do not yet know the effect of such magnetic moments on microscopic interactions.

I. INTRODUCTION

In this paper we shall investigate certain problems in Einstein's unified field theory—the theory of the nonsymmetric field—which have not been adequately investigated in previous papers.[1–3] They are (1) the question of the topology of a spatial region which contains a charged particle and (2) the question of whether in Einstein's theory a charged particle can possess a significant magnetic-monopole moment in addition to an electric charge and still not be in disagreement with experiment.

In the investigation we shall assume with Einstein that only regular (nonsingular) solutions to Einstein's field equations are realized in nature. We shall also make certain additional assumptions suggested by empirical evidence. We shall assume that in an appropriate coordinate system the field associated with an isolated charged particle can be approximated with negligible error sufficiently far from the particle by a time-independent spherically symmetric solution to Einstein's field equations and that the symmetric part of the field associated with such a solution is flat at infinity. We are restricting our study to stable particles. We shall also assume that charged particles interact to a good approximation with other charged particles over laboratory distances through the conventional classical electromagnetic interaction. This is necessary if Einstein's theory is to agree with observation.

Making the above assumptions, we show that the global topology of any spatial region which contains an isolated charged particle, and in which the above-mentioned asymptotic conditions far from the particle are satisfied, must be non-Euclidean. The idea that particles are associated with non-Euclidean spatial topologies is not new. Einstein and Rosen[4] suggested it in 1936, and the possibility has been extensively discussed by Wheeler[5] and his associates. What we show in this paper is that if the assumptions mentioned above are satisfied, charged particles in Einstein's theory must have non-Euclidean spatial topologies associated with them.

With respect to the question of whether a charged particle can possess a significant magnetic-monopole moment in addition to an electric charge and still not be in disagreement with experiment, we shall find that this possibility cannot be ruled out. If the magnetic-monopole moment associated with each particle is proportional to the particle's charge, and only in this case, then in an approximation which should be valid over macroscopic interaction distances (laboratory and astronomical distances) the interaction among particles is found to be independent of their magnetic-monopole moments.[6] In spite of the presence of magnetic-monopole moments on the charged particles, the particles interact over macroscopic distances as if they had no magnetic-monopole moments; they interact through what we have called Einstein electrodynamics.[7] We do not yet know the effect of such magnetic-monopole moments on microscopic interactions.[8]

The body of this paper begins in Sec. II with a brief description of Einstein's unified field theory. In Sec. III we discuss time-independent spherically symmetric solutions to Einstein's field equations and investigate the interaction over macroscopic distances of charged particles. In Sec. IV we describe the most general time-independent sphericially symmetric solution to Einstein's field equations which can represent the field of an isolated charged particle sufficiently far from the particle. In Sec. V we discuss the topology of spatial regions which contain a charged particle.

II. SPACE-TIME MANIFOLD

A. Field equations

In Einstein's theory of the nonsymmetric field, the structure of the space-time manifold is described through a second-rank tensor field $g_{\mu\nu}$. The fundamental field $g_{\mu\nu}$ satisfies the general-relativistic field equations[9]

$$\Gamma^{\nu}_{[\mu\nu]} = 0 \ , \tag{1a}$$

$$R_{[\mu\nu,\lambda]} = 0 \ , \tag{1b}$$

$$R_{(\mu\nu)} = 0 \ , \tag{1c}$$

where the affine connection $\Gamma^{\rho}_{\mu\nu}$ and the contracted curvature tensor $R_{\mu\nu}$ are defined through the equations

$$g_{\mu\nu;\rho}(= g_{\mu\nu,\rho} - g_{\sigma\nu}\Gamma^{\sigma}_{\mu\rho} - g_{\mu\sigma}\Gamma^{\sigma}_{\rho\nu}) = 0 \ , \tag{2}$$

$$R_{\mu\nu} = \Gamma^{\rho}_{\mu\nu,\rho} - \Gamma^{\rho}_{\mu\rho,\nu} - \Gamma^{\rho}_{\mu\sigma}\Gamma^{\sigma}_{\rho\nu} + \Gamma^{\rho}_{\mu\nu}\Gamma^{\sigma}_{\rho\sigma} \ . \tag{3}$$

B. Particles and physical fields

A region of the space-time manifold is called flat if a coordinate system can be found in the region so that the fundamental tensor field is equal to the Minkowski metric throughout the region, that is

$$g_{\mu\nu} = \eta_{\mu\nu} \ . \tag{4}$$

Particles are limited portions of the manifold—limited at least in the spatial directions—which have a very nonflat structure. Portions of the manifold between the particles and possessing a nearly flat structure are known as empty space or vacuum. The slight deviations from flatness in such portions of space-time indicate the presence of an electromagnetic field if $g_{[\mu\nu]} \neq 0$ and the presence of a gravitational field if $g_{(\mu\nu)} \neq \eta_{\mu\nu}$. Nearer the particles, where the deviations from flatness are larger, the field $g_{\mu\nu}$ may also be associated with weak and strong interactions.

In this paper we assume with Einstein that only regular (nonsingular) solutions to Einstein's field equations are realized in nature. Empirical evidence suggests that in an appropriate coordinate system deviations from time independence and spherical symmetry are negligible sufficiently far from an isolated charged stable elementary particle. Thus we assume that in an appropriate coordinate system the field associated with an isolated charged particle can be approximated with negligible error sufficiently far from the particle through a time-independent spherically symmetric solution to Einstein's field equations. We shall confine our study to stable particles. Further discussion of the above assumptions can be found in the literature.[3]

III. SOLUTIONS TO THE FIELD EQUATIONS

A. Time-independent spherically symmetric solutions

Assuming spherical symmetry about the origin of coordinates, it can be shown that in polar coordinates $x^1 = r$, $x^2 = \theta$, and $x^3 = \varphi$ the fundamental field $g_{\mu\nu}$ can be put into the form[10,11]

$$g_{\mu\nu} = \begin{pmatrix} -\alpha & 0 & 0 & w \\ 0 & -\beta & f\sin\theta & 0 \\ 0 & -f\sin\theta & -\beta\sin^2\theta & 0 \\ -w & 0 & 0 & \gamma \end{pmatrix} \ , \tag{5}$$

where α, β, γ, f, and w are functions of r and t.

The general time-independent spherically symmetric solution to Einstein's field equations was first found by Bandyopadhyay[12] and later, independently, by Vanstone.[13] Their results for α, β, γ, f, and w can be put in the form[14]

$$\begin{aligned} \alpha &= \frac{(f^2+\beta^2)}{4m_1^{\,2}} e^{-\delta}\left(\frac{d\delta}{dr}\right)^2 \ , \\ f + i\beta &= \frac{m_1^{\,2}(1+ih_1)e^{\delta}\,\mathrm{sech}^2[\tfrac{1}{2}(1+ih_1)^{1/2}\delta + a_1]}{c_1 + i} \ , \\ \gamma &= e^{-\delta} + \frac{w^2}{\alpha} \ , \\ w &= \left(\frac{\alpha e^{-\delta}}{f^2+\beta^2}\right)^{1/2} k_1 \ , \end{aligned} \tag{6}$$

where the integration constants m_1, c_1, h_1, and k_1 in (6) are real, and the constant a_1 is complex. The variable δ is an arbitrary real function of r.

Since we are only interested in solutions representing particles, we restrict our study to those Bandyopadhyay-Vanstone solutions for which the field $g_{(\mu\nu)}$ is flat at infinity, i.e., can take its Minkowski value $\eta_{\mu\nu}$ at infinity. For such solutions one can show

$$\sinh^2 a_1 = -1 \ , \tag{7}$$

so that f and β are given by

$$f + i\beta = -\frac{m_1^{\,2}(1+ih_1)e^{\delta}}{c_1 + i}\,\mathrm{csch}^2[\tfrac{1}{2}(1+ih_1)^{1/2}\delta] \ . \tag{8}$$

In investigating this solution we shall find it convenient to work in "standard" coordinates. Standard polar coordinates are defined as coordinates in which $g_{\mu\nu}$ takes the form (5) with $\beta = r^2$. Standard Cartesian coordinates are defined in terms of standard polar coordinates through the transformation

$$x^1 = r\sin\theta\cos\varphi\ ,$$
$$x^2 = r\sin\theta\sin\varphi\ , \tag{9}$$
$$x^3 = r\cos\theta\ .$$

In standard Cartesian coordinates the above solution takes the form

$$g_{st} = -\delta_{st} - (\alpha-1)\frac{x^s x^t}{r^2} - \epsilon_{stk} v\frac{x^k}{r}\ ,$$
$$g_{44} = \gamma,\ \ g_{4s} = -w\frac{x^s}{r},\ \ g_{s4} = w\frac{x^s}{r}\ , \tag{10}$$

where

$$\alpha = \frac{m_1{}^2(1+h_1{}^2)}{c_1{}^2+1}\frac{e^\delta}{(\cosh\xi-\cos\eta)^2}\left(\frac{d\delta}{dr}\right)^2\ ,$$
$$\gamma = e^{-\delta} + \frac{w^2}{\alpha}\ , \tag{11}$$
$$v = -\frac{f}{r^2} = \frac{2m_1{}^2}{c_1{}^2+1}\frac{e^\delta}{(\cosh\xi-\cos\eta)^2}\frac{1}{r^2}$$
$$\times[(c_1+h_1)(\cosh\xi\cos\eta-1) - (1-c_1h_1)\sinh\xi\sin\eta]\ ,$$
$$w = k_1\frac{e^{-\delta}}{2m_1}\left|\frac{d\delta}{dr}\right|\ ,$$

and

$$\xi = \mu\delta,\ \ \eta = \nu\delta\ ,$$
$$\mu = [\tfrac{1}{2}+\tfrac{1}{2}(1+h_1{}^2)^{1/2}]^{1/2}\ , \tag{12}$$
$$\nu = h_1[2+2(1+h_1{}^2)^{1/2}]^{-1/2}\ .$$

The variable δ in standard coordinates satisfies the equation

$$\frac{m_1{}^2}{c_1{}^2+1}\frac{2e^\delta}{(\cosh\xi-\cos\eta)^2}\times[(c_1+h_1)\sinh\xi\sin\eta + (1-c_1h_1)(\cosh\xi\cos\eta-1)] = r^2\ . \tag{13}$$

At distances sufficiently far from the origin of coordinates the functions α, γ, v, and w can be expanded in a power series in r^{-1}. We find in standard coordinates

$$\alpha = 1 + \frac{2m}{r} + \frac{4m^2}{r^2} + \frac{1}{2}\frac{q^2}{1+\frac{1}{4}(q^2/l^2)}\frac{1}{r^2} + O(r^{-3})\ ,$$
$$\gamma = 1 - \frac{2m}{r} + O(r^{-3}),$$
$$v = l\left(\frac{q}{r^2} - \frac{1}{2}\epsilon\frac{q}{l^2}\right) + O(r^{-3}), \tag{14}$$
$$w = l\frac{q_M}{r^2} + O(r^{-3}),$$

where the constants m, q, q_M, and l are defined in the following way:

$$m_1 = \left(1+\frac{1}{4}\frac{q^2}{l^2}\right)^{1/2} m\ ,$$
$$c_1 = -\frac{1}{2}\epsilon\frac{q}{l}\ ,$$
$$h_1 = -\frac{3lq}{m^2}\left(1+\frac{1}{4}\frac{q^2}{l^2}\right)^{-1}\ , \tag{15}$$
$$k_1 = \left(1+\frac{1}{4}\frac{q^2}{l^2}\right)^{1/2} l q_M\ ,$$

and ϵ can take the value 1 or -1. There is no loss in generality in assuming $l \geq 0$. The form of α, γ, v, and w at large distances from a particle suggests that we identify m with the mass of the particle, q with the charge, and q_M with the magnetic-monopole moment.[15] The form also suggests that the length l associated with each particle is universal, that is, the same for each particle. Finally, it suggests that the length l is an astronomical length, for we know that over laboratory distances the electric field produced by a charge falls off with distance as r^{-2}.

However, only by investigating the interaction among particles represented by the above solutions can we interpret the solutions physically and properly relate the arbitrary constants appearing in the solutions to the mass, charge, and magnetic-monopole moment of a particle. When this is done we will find that the above suggestions are correct.

B. Equations of motion

Approximation procedure. In order to investigate the interaction among particles in a nonlinear field theory one must in general use an approximation procedure to solve the field equations. We shall use a fast-motion approximation procedure developed by the author in a previous series of papers.[1] The approximation procedure is similar to the conventional slow-motion approximation procedure of Einstein, Infeld, and Hoffmann (the EIH procedure) in that one expands the field $g_{\mu\nu}$ in a power series in a parameter which parametrizes the quantities characterizing the particles (in our case the parameter will parametrize the mass m, charge q, and the magnetic-monopole moment q_M), but the procedure differs from the EIH procedure in that one does not consider time variation to be necessarily small and thus does not choose the parameter to also order time variation. The procedure leads to Lorentz-covariant equations of motion at each order of approximation. For further discussion see the papers of Ref. 1.

In the following discussion of the approximation procedure, unless otherwise stated, all indices will be raised and lowered with the Minkowski metric $\eta_{\mu\nu}=\eta^{\mu\nu}$. The subscript (k) to the left of a field will indicate order. We will be using the notation $\Box^2\psi=\eta^{\mu\nu}\psi_{,\mu\nu}$.

If we assume the expansion

$$g_{\mu\nu}=\eta_{\mu\nu}+\sum_{k=1}^{\infty}\kappa^k\,{}_{(k)}g_{\mu\nu} \tag{16}$$

for the fundamental field $g_{\mu\nu}$ (κ is the expansion parameter) the field equations (1) can be put into the form[1]

$$\begin{gathered}\Box^2\gamma^*_{[\mu\nu]}{}^{,\nu}=s_\mu\ ,\\ \gamma^*_{[\mu\nu,\lambda]}=0\ ,\\ \Box^2\gamma_{(\mu\nu)}-\gamma_{(\mu\rho)}{}^{,\rho}{}_{,\nu}-\gamma_{(\nu\rho)}{}^{,\rho}{}_{,\mu}+\eta_{\mu\nu}\gamma_{(\rho\sigma)}{}^{,\rho\sigma}=t_{\mu\nu}\ ,\end{gathered} \tag{17}$$

where

$$\begin{gathered}\gamma_{\mu\nu}=\eta_{\mu\rho}\eta_{\nu\sigma}\mathfrak{g}^{\rho\sigma}-\eta_{\mu\nu}\ ,\\ \gamma^*_{[\mu\nu]}=\tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}\mathfrak{g}^{[\rho\sigma]}\ ,\\ \gamma_{(\mu\nu)}=\eta_{\mu\rho}\eta_{\nu\sigma}\mathfrak{g}^{(\rho\sigma)}-\eta_{\mu\nu}\ ,\end{gathered} \tag{18}$$

and

$$s_\mu=-\eta_{\mu\rho}\epsilon^{\rho\sigma\kappa\lambda}R^N_{[\kappa\lambda,\sigma]}\ , \tag{19}$$

$$t_{\mu\nu}=-2(R^N_{(\mu\nu)}-\tfrac{1}{2}\eta_{\mu\nu}\eta^{\rho\sigma}R^N_{(\rho\sigma)})\ . \tag{20}$$

The field $\mathfrak{g}^{\mu\nu}$ is a contravariant tensor density associated with the fundamental field $g_{\mu\nu}$. It is defined through the equations

$$\mathfrak{g}^{\mu\nu}=(-g)^{1/2}g^{\mu\nu}\ , \tag{21}$$

where $g^{\mu\nu}$ is defined through

$$g_{\mu\rho}g^{\nu\rho}=g_{\rho\mu}g^{\rho\nu}=\delta^\nu_\mu \tag{22}$$

and g denotes the determinate of $g_{\mu\nu}$. The field $R^N_{\mu\nu}$ is that part of the tensor $R_{\mu\nu}$ which is nonlinear in $\gamma_{\mu\nu}$.

When investigating the physical consequences of Eqs. (17) it will be understood that we are investigating the fields only at points which are sufficiently far from the particles so that (16) can be considered valid.

For convenience we will impose the coordinate conditions

$$\gamma_{(\mu\nu)}{}^{,\nu}=0 \tag{23}$$

on the field at each order of approximation. Coordinates for which (23) are valid are known as harmonic coordinates. We do not impose (23) on the exact solution, only on the solution up to the order of approximation in which we choose to investigate the fields. Under these conditions the use of harmonic coordinates does not restrict the set of invariantly distinct solutions to the field equations.[16] In harmonic coordinates the field equations (17) take the form

$$\Box^2\gamma^*_{[\mu\nu]}{}^{,\nu}=s_\mu\ , \tag{24a}$$

$$\gamma^*_{[\mu\nu,\lambda]}=0\ , \tag{24b}$$

$$\Box^2\gamma_{(\mu\nu)}=t_{\mu\nu}\ , \tag{24c}$$

$$\gamma_{(\mu\nu)}{}^{,\nu}=0\ . \tag{24d}$$

Application of approximation procedure. To lowest order (first order) we have from (19) and (20)

$$s_\mu=0,\quad t_{\mu\nu}=0\ . \tag{25}$$

Because we do not want gravitational interaction to appear in the lowest-order interaction terms (second order), we will choose mass and therefore $\gamma_{(\mu\nu)}$ to be a second-order quantity. In this way we avoid having to investigate gravitational interaction in second order. Thus to lowest nontrivial order (second order) we find from (19) and (20)

$$s_\mu=0\ , \tag{26}$$

$$\begin{aligned}t_{\mu\nu}=&\tfrac{1}{2}\gamma_{[\rho\sigma],\mu}\gamma^{[\rho\sigma]}{}_{,\nu}+\gamma_{[\mu\rho]}{}^{,\sigma}\gamma_{[\nu\sigma]}{}^{,\rho}-\gamma_{[\mu\rho],\sigma}\gamma_{[\nu}{}^{\rho],\sigma}\\ &-\tfrac{1}{4}\eta_{\mu\nu}\gamma_{[\rho\sigma],\kappa}\gamma^{[\rho\sigma],\kappa}-\tfrac{1}{2}\eta_{\mu\nu}\gamma_{[\rho\sigma],\kappa}\gamma^{[\rho\kappa],\sigma}\\ &+\gamma^{[\rho\sigma]}\gamma_{[\mu\sigma],\nu\rho}+\gamma^{[\rho\sigma]}\gamma_{[\nu\sigma],\mu\rho}\\ &-\tfrac{1}{2}\eta_{\mu\nu}\gamma^{[\rho\sigma]}\Box^2\gamma_{[\rho\sigma]}\ .\end{aligned} \tag{27}$$

In a harmonic coordinate system, and keeping only terms linear in m, q, and q_M, one finds from solution (10)–(13), sufficiently far from an isolated particle,

$$\gamma_{[st]}=-l\,\epsilon_{stk}\left(\frac{q}{r^2}-\frac{1}{2}\epsilon\frac{q}{l^2}\right)\frac{x^k}{r},\quad \gamma_{[s4]}=l\,\frac{q_M}{r^2}\,\frac{x^s}{r}\ , \tag{28}$$

$$\gamma_{(st)}=0,\quad \gamma_{(s4)}=0,\quad \gamma_{(44)}=\frac{4m}{r}\ . \tag{29}$$

To investigate the electromagnetic interaction among these particles one must choose as the lowest-order solution to Eqs. (24)

$$\begin{aligned}&\gamma^*_{[\mu\nu]}=\gamma^{*E}_{[\mu\nu]}+\gamma^{*M}_{[\mu\nu]},\\ &\gamma^{*E}_{[\mu\nu]}=\tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}\gamma^{E[\rho\sigma]}=\gamma^E_{\mu,\nu}-\gamma^E_{\nu,\mu}\ ,\\ &\gamma^{*M}_{[\mu\nu]}=\tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}\gamma^{M[\rho\sigma]}=\epsilon_{\mu\nu\rho\sigma}\gamma^{M\rho,\sigma}\ ,\\ &\gamma_{(\mu\nu)}=0\ ,\end{aligned} \tag{30}$$

where

$$\gamma_{[\mu\nu]} = \gamma^E_{[\mu\nu]} + \gamma^M_{[\mu\nu]}\,,$$
$$\gamma^E_{[\mu\nu]} = \epsilon_{\mu\nu\rho\sigma}\gamma^{E\sigma,\rho},\quad \gamma^M_{[\mu\nu]} = \gamma^M_{\mu,\nu} - \gamma^M_{\nu,\mu}\,,$$
$$\gamma^E_\mu = \sum_p {}^{(p)}l^{(p)}\left[q u_\mu (r_\rho u^\rho)^{-1} + \frac{1}{2}\epsilon \frac{q}{l^2} r_\mu\right]_{\rm ret}\,,$$
$$\gamma^M_\mu = \sum_p {}^{(p)}l^{(p)}[q_M u_\mu (r_\rho u^\rho)^{-1}]_{\rm ret}\,. \tag{31}$$

We are using the notation

$$ {}^{(p)}r^\mu = x^\mu - {}^{(p)}\xi^\mu,\quad {}^{(p)}u^\mu = {}^{(p)}\dot{\xi}^\mu\,,$$
$$d^{(p)}\tau^2 = \eta_{\mu\nu} d^{(p)}\xi^\mu d^{(p)}\xi^\nu\,. \tag{32}$$

A superscript (p) to the left of an expression means that the quantities in the expression which are associated with a particle are associated with the pth particle. A dot over a quantity associated with the pth particle means the derivative of that quantity with respect to ${}^{(p)}\tau$. The subscript ret means that in the expression in brackets those quantities associated with the pth particle are to be evaluated at the "retarded point":

$$ {}^{(p)}r_\rho {}^{(p)}r^\rho = 0,\quad r^4 > 0\,.$$

The ${}^{(p)}\xi$ will be regarded as the coordinates of the pth particle. The quantities ${}^{(p)}q$, ${}^{(p)}q_M$, and ${}^{(p)}l$, are time independent. Let us investigate the interaction among such particles.

To find the equations of motion to second order satisfied by these particles one must study the solutions to Eqs. (24) to second order. The solutions to Eqs. (24a) and (24b) to second order are identical to the solutions to first order. No equations of motion are involved. In order to obtain equations of motion we must investigate Eqs. (24c) and (24d) to second order.

Placing the field (31) into (27) one can solve Eqs. (24c) to second order. From (29) we see that the solution will take the form

$$\gamma_{(\mu\nu)} = \sum_p {}^{(p)}[4m u_\mu u_\nu (r_\rho u^\rho)^{-1}]_{\rm ret} + \gamma^I_{(\mu\nu)}\,, \tag{33}$$

where

$$\Box^2 \gamma^I_{(\mu\nu)} = t_{\mu\nu}\,. \tag{34}$$

From (33), (34), (27), and (31) one finds for $\gamma_{(\mu\nu)}{}^{,\nu}$

$$\gamma_{(\mu\nu)}{}^{,\nu} = \sum_p {}^{(p)}[C_\mu (r_\rho u^\rho)^{-1}]_{\rm ret}\,, \tag{35}$$

where

$$ {}^{(p)}C_\mu = {}^{(p)}\left[\frac{d}{d\tau}(4m u_\mu) - lq\Box^2\gamma^{*\rm ext}_{[\mu\nu]}u^\nu + \epsilon\frac{q}{l}\gamma^{*\rm ext}_{[\mu\nu]}u^\nu - \tfrac{4}{3}\epsilon q^2(\ddot{u}_\mu + \dot{u}_\rho \dot{u}^\rho u_\mu) - l\, q_M \Box^2 \gamma^{\rm ext}_{[\mu\nu]}u^\nu\right]\,, \tag{36}$$

with

$$ {}^{(p)}\gamma^{*\rm ext}_{[\mu\nu]} = {}^{(p)}\gamma^{*M\rm ext}_{[\mu\nu]} + {}^{(p)}\gamma^{*E\rm ext}_{[\mu\nu]}\,,$$
$$ {}^{(p)}\gamma^{*E\rm ext}_{[\mu\nu]} = {}^{(p)}\gamma^{E\rm ext}_{\mu,\nu} - {}^{(p)}\gamma^{E\rm ext}_{\nu,\mu}\,,$$
$$ {}^{(p)}\gamma^{*M\rm ext}_{[\mu\nu]} = \epsilon_{\mu\nu\rho\sigma}{}^{(p)}\gamma^{M\rm ext\ \rho,\sigma}\,, \tag{37}$$

where

$$ {}^{(p)}\gamma^{E\rm ext}_\mu = \sum_{p'\neq p} {}^{(p')}l^{(p')}\left[q u_\mu (r_\rho u^\rho)^{-1} + \frac{1}{2}\epsilon\frac{q}{l^2}r_\mu\right]_{\rm ret}\,,$$
$$ {}^{(p)}\gamma^{M\rm ext}_\mu = \sum_{p'\neq p} {}^{(p')}l^{(p')}[q_M u_\mu (r_\rho u^\rho)^{-1}]_{\rm ret}\,, \tag{38}$$

and

$$ {}^{(p)}\gamma^{\rm ext}_{[\mu\nu]} = \tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}{}^{(p)}\gamma^{*\rm ext\,[\sigma\rho]}\,. \tag{39}$$

The procedure used to find $\gamma_{(\mu\nu)}{}^{,\nu}$ is discussed in previous papers.[1] Since Eqs. (24d) must also be satisfied we must have

$$ {}^{(p)}C_\mu = 0\,. \tag{40}$$

Particle motion is restricted by (40). The equations of motion satisfied by the particles to second order are

$$m\dot{u}_\mu = \tfrac{1}{4}lq\Box^2\gamma^{*\rm ext}_{[\mu\nu]}u^\nu - \tfrac{1}{4}\epsilon(q/l)\gamma^{*\rm ext}_{[\mu\nu]}u^\nu + \tfrac{1}{3}\epsilon q^2(\ddot{u}_\mu + \dot{u}_\rho\dot{u}^\rho u_\mu) + \tfrac{1}{4}l q_M\Box^2\gamma^{\rm ext}_{[\mu\nu]}u^\nu. \tag{41}$$

These equations follow from (36) and (40). We find from (41) that the particles can interact to a good approximation over laboratory distances through the laws of classical electrodynamics if and only if ${}^{(p)}l$ is a universal astronomical length, ${}^{(p)}q_M$ is proportional to ${}^{(p)}q$, and ${}^{(p)}\epsilon = 1$.[17] Under these conditions one finds from (41) (Ref. 17)

$$m\dot{u}_\mu = \tfrac{1}{2}(q/l)\tilde{\gamma}^{\rm ext}_{[\nu\mu]}u^\nu + \tfrac{1}{3}q^2(\ddot{u}_\mu + \dot{u}_\rho\dot{u}^\rho u_\mu)\,, \tag{42}$$

where the effective external electromagnetic field $\tilde{\gamma}^{\rm ext}_{[\mu\nu]}$ appearing in (42) is given by

$$ {}^{(p)}\tilde{\gamma}^{\rm ext}_{[\mu\nu]} = {}^{(p)}\tilde{\gamma}^{\rm ext}_{\mu,\nu} - {}^{(p)}\tilde{\gamma}^{\rm ext}_{\nu,\mu}\,,$$
$$ {}^{(p)}\tilde{\gamma}^{\rm ext}_\mu = l\sum_{p'\neq p}{}^{(p')}\left[q u_\mu (r_\rho u^\rho)^{-1} + \frac{1}{4}\frac{q}{l^2}r_\mu\right]_{\rm ret}\,. \tag{43}$$

We see that under these conditions, to second order, the interaction among particles does not depend on the particles' magnetic-monopole moments ${}^{(p)}q_M$.[18]

If we introduce the mass M, charge e, and effective electromagnetic field $F_{\mu\nu}$ in practical units,

$$m = \frac{GM}{c^2}\,,$$
$$q = \left(\frac{G}{2\pi\epsilon_0 c^4}\right)^{1/2} e\,,$$
$$\tilde{\gamma}_{[\mu\nu]} = \frac{(8\pi\epsilon_0 G)^{1/2}}{c}\, l\, F_{\mu\nu}\,, \tag{44}$$

Eqs. (42) take the form

$$M\dot{u}_\mu = \frac{e}{c}\,F^{\rm ext}_{\nu\mu}\,u^\nu + \frac{2}{3}\left(\frac{e^2}{4\pi\epsilon_0 c^2}\right)(\ddot{u}_\mu + \dot{u}_\rho\dot{u}^\rho u_\mu)\,, \tag{45}$$

where

$$\begin{aligned} {}^{(p)}F^{\rm ext}_{\mu\nu} &= {}^{(p)}A^{\rm ext}_{\mu,\nu} - {}^{(p)}A^{\rm ext}_{\nu,\mu}\,, \\ {}^{(p)}A^{\rm ext}_{\mu} &= \frac{1}{4\pi\epsilon_0}\sum_{p'\neq p}{}^{(p')}\left[\frac{e}{c}\,u_\mu(r_\rho u^\rho)^{-1}\right]_{\rm ret} \\ &\quad + \frac{1}{4\pi\epsilon_0}\sum_{p'\neq p}{}^{(p')}\left[\frac{e}{c}\left(\frac{1}{4l^2}\right)r_\mu\right]_{\rm ret}\,. \end{aligned} \tag{46}$$

The particles interact to second order through what we have called Einstein electrodynamics. A discussion of this electrodynamics can be found in the literature.[3]

If the ${}^{(p)}q_M$ are not excessive, and by this we mean ${}^{(p)}q_M \lesssim {}^{(p)}q$, the approximations used in arriving at (45) and (46) should be valid over macroscopic interaction distances. The arguments for why this is so are identical to those previously discussed in the literature for the case of particles possessing no magnetic-monopole moments,[19] and lead to the conclusion that as long as l is only a moderate astronomical length Eqs. (45) and (46) should be valid over both laboratory and astronomical distances. Reasons for believing that l satisfies this criterion can be found in the literature.[3]

From the above results we see that our tentative identifications of m, q, and q_M in Sec. III were correct. The quantity m represents the mass of a particle, q represents the charge, and thus q_M represents the magnetic-monopole moment.[20] We also find that the length l associated with a particle must be a universal (moderate) astronomical length, and ϵ must be equal to 1.

With respect to the question of whether a charged particle can possess a significant magnetic-monopole moment in addition to an electric charge and still not be in disagreement with experiment, we see that this possibility cannot be ruled out. If the magnetic-monopole moment associated with each particle is proportional to the particle's charge, and not excessive, then over macroscopic interaction distances the interaction among such particles is independent of their magnetic moments. In spite of the presence of magnetic-monopole moments on the particles, the particles interact over macroscopic distances as though they possessed no magnetic-monopole moments. We do not yet know the effect of such magnetic moments on microscopic interactions.

IV. ISOLATED CHARGED PARTICLE

We have found that the most general time-independent spherically symmetric solution to Einstein's field equations which can, under the assumptions mentioned in the Introduction, represent the field of an isolated charged particle sufficiently far from the particle is characterized by four parameters: a mass m, a charge q, a magnetic-monopole moment q_M, and a universal length l. The ratio of q_M to q is a universal constant.

The solution is given in standard coordinates by

$$\begin{aligned} g_{st} &= -\delta_{st} - (\alpha-1)\frac{x^s x^t}{r^2} - \epsilon_{stk}\,v\,\frac{x^k}{r}\,, \\ g_{44} &= -\gamma\,,\quad g_{4s} = -w\frac{x^s}{r}\,,\quad g_{s4} = w\frac{x^s}{r}\,, \end{aligned} \tag{47}$$

where, introducing the definition

$$r_0 = (|q|\,l)^{1/2} \tag{48}$$

and the universal constant k_M,

$$k_M = q_M/q\,, \tag{49}$$

one can write

$$\begin{aligned} \alpha &= \frac{6e^\delta}{(\cosh\xi - \cos\eta)^2}\left[\left(1+\frac{1}{4}\frac{q^4}{r_0^4}\right)^{-1} + \frac{1}{9}\left(1+\frac{1}{4}\frac{q^4}{r_0^4}\right)\left(\frac{m}{r_0}\right)^4\right]\left(r_0\frac{d\lambda}{dr}\right)^2, \\ \gamma &= e^{-\delta} + \frac{w^2}{\alpha}\,, \\ v &= \frac{q}{|q|}\,\frac{6e^\delta}{(\cosh\xi-\cos\eta)^2}\left\{\left[\left(1+\frac{1}{4}\frac{q^4}{r_0^4}\right)^{-1} + \frac{1}{6}\,\frac{q^2}{r_0^2}\left(\frac{m}{r_0}\right)^2\right](1-\cosh\xi\cos\eta)\right. \\ &\qquad \left. - \left[\frac{1}{2}\,\frac{q^2}{r_0^2}\left(1+\frac{1}{4}\,\frac{q^4}{r_0^4}\right)^{-1} - \frac{1}{3}\left(\frac{m}{r_0}\right)^2\right]\sinh\xi\,\sin\eta\right\}\left(\frac{r_0}{r}\right)^2, \\ w &= \frac{q}{|q|}\,k_M\,e^{-\delta}\left[\frac{1}{6}\left(1+\frac{1}{4}\,\frac{q^4}{r_0^4}\right)\right]^{1/2}\left|r_0\frac{d\lambda}{dr}\right|, \end{aligned} \tag{50}$$

with

$$\delta = (\tfrac{2}{3})^{1/2}\left(1+\frac{1}{4}\,\frac{q^4}{r_0^{\,4}}\right)^{1/2}\left(\frac{m}{r_0}\right)\lambda\,,$$

$$\xi = \left\{\left[1+\tfrac{1}{9}\left(1+\frac{1}{4}\,\frac{q^4}{r_0^{\,4}}\right)^2\left(\frac{m}{r_0}\right)^4\right]^{1/2}+\tfrac{1}{3}\left(1+\frac{1}{4}\,\frac{q^4}{r_0^{\,4}}\right)\left(\frac{m}{r_0}\right)^2\right\}^{1/2}\lambda\,, \tag{51}$$

$$\eta = \left\{\left[1+\tfrac{1}{9}\left(1+\frac{1}{4}\,\frac{q^4}{r_0^{\,4}}\right)^2\left(\frac{m}{r_0}\right)^4\right]^{1/2}+\tfrac{1}{3}\left(1+\frac{1}{4}\,\frac{q^4}{r_0^{\,4}}\right)\left(\frac{m}{r_0}\right)^2\right\}^{-1/2}\lambda\,.$$

The parameter λ in (51) is a function of r defined through the equation

$$\frac{6e^{\delta}}{(\cosh\xi-\cos\eta)^2}\left\{\left[\left(1+\frac{1}{4}\,\frac{q^4}{r_0^{\,4}}\right)^{-1}+\frac{1}{6}\,\frac{q^2}{r_0^{\,2}}\left(\frac{m}{r_0}\right)^2\right]\sinh\xi\,\sin\eta - \left[\frac{1}{2}\,\frac{q^2}{r_0^2}\left(1+\frac{1}{4}\,\frac{q^4}{r_0^{\,4}}\right)^{-1}-\frac{1}{3}\left(\frac{m}{r_0}\right)^2\right](\cosh\xi\,\cos\eta-1)\right\}=\left(\frac{r}{r_0}\right)^2. \tag{52}$$

If we understand the values of m and q appearing in this solution to be those appropriate to an elementary particle, it is not difficult to show that for $q\neq 0$ the solution is singular only at the origin of coordinates. For $q=0$, $q_M=0$, the solution reduces to the well-known Schwarzschild solution to Einstein's gravitational equations.

In the special case where $k_M=0$, the properties of the above solution have been studied in a previous paper.[3] Because α, v, and $e^{-\delta}$ are independent of k_M, the properties of these functions are discussed in that paper. We shall briefly discuss some of the properties of γ and w.

In the vicinity of the origin it is convenient to expand γ and w in a power series in r/r_0. One finds from (50)–(52), in the vicinity of the origin,

$$\gamma = \left[1+k_M^{\,2}\tfrac{1}{36}(1+\cosh\pi)^2+\cdots\right]+O\!\left(\left(\frac{r}{r_0}\right)^2\right),$$

$$w = k_M\,\frac{q}{|q|}\left[\frac{1}{6}\left(\frac{2}{3}\right)^{1/2}\frac{(1+\cosh\pi)^2}{\sinh\pi}+\cdots\right]\left(\frac{r}{r_0}\right)+O\!\left(\left(\frac{r}{r_0}\right)^3\right). \tag{53}$$

In (53) we have written out only the lowest-order terms in the power-series expansion in m/r_0 and $q^2/r_0^{\,2}$ of the coefficients of $(r/r_0)^n$. Note w vanishes at the origin of coordinates while γ remains finite. The behavior of γ and w at large distances from the origin has been discussed in Sec. III. In (20) of Sec. III one should set $\epsilon=1$.

V. CHARGED PARTICLES AND TOPOLOGY

In this section we shall show that the global topology of any spatial region which contains an isolated charged particle and in addition satisfies sufficiently far from the particle certain asymptotic conditions suggested by empirical evidence and discussed in the Introduction must be non-Euclidean. We are assuming only regular (nonsingular) solutions to Einstein's field equations are realized in nature.

We first note that from the generalized Stokes's theorem[21] it follows for any antisymmetric tensor field $A_{[\mu\nu]}$ satisfying the condition

$$A_{[\mu\nu,\lambda]}=0\,, \tag{54}$$

that

$$\int_{\Omega_2} A_{[\mu\nu]}\,d\tau^{\mu\nu}=0 \tag{55}$$

if the integral extends over a closed two-dimensional surface Ω_2 enclosing a three-dimensional spatial region having a Euclidean global topology. We are assuming the field $A_{[\mu\nu]}$ is nonsingular over the region. The $d\tau^{\mu\nu}$ in (55) is called the extension of an infinitesimal two-cell. It is a contravariant antisymmetric second-rank tensor. The concept of extension is discussed further in Ref. 21.

That (55) follows from (54) is easy to see. From the generalized Stokes's theorem we have[21]

$$\int_{\Omega_3} A_{[\mu\nu,\rho]}\,d\tau^{\mu\nu\rho}=\int_{\Omega_2} A_{[\mu\nu]}\,d\tau^{\mu\nu}\,, \tag{56}$$

where the integral on the left-hand side of (56) extends over a three-dimensional spatial region Ω_3 enclosed by a closed two-dimensional surface Ω_2. The integral on the right-hand side extends over Ω_2. The quantities $d\tau^{\mu\nu\rho}$ and $d\tau^{\mu\nu}$ are the extensions of an infinitesimal three-cell and two-cell, respectively. They are completely antisymmetric tensors (antisymmetric under the interchange of any two indices). The tensor $A_{[\mu\nu]}$ is nonsingular over Ω_3 and the global topology of Ω_3 is Euclidean. It is clear from (56) that (55) follows from (54).

We note that field equations (1b) are of the form (54). From the field equations (1b) it then follows that

$$\int_{\Omega_2} R_{[\mu\nu]}\, d\tau^{\mu\nu} = 0 \tag{57}$$

if the integral extends over a closed two-dimensional surface enclosing a spatial region having a Euclidean global topology. We are assuming that only regular (nonsingular) solutions to Einstein's field equations are realized in nature.

We next make use of the assumption that in an appropriate coordinate system and sufficiently far from an isolated charged particle the field $g_{\mu\nu}$ can be approximated with negligible error by the solution (47)–(52). This is equivalent to the assumption that (1) in an appropriate coordinate system the field associated with an isolated charged particle can be represented with negligible error sufficiently far from the particle by a time-independent spherically symmetric solution to Einstein's field equations, (2) the symmetric part of the field associated with such a charged particle is flat at spatial infinity, and (3) charged particles interact to a good approximation with other charged particles over laboratory distances through the conventional classical electromagnetic interaction.

From (47)–(52) we find

$$R_{[st]} = \frac{1}{2}\,\frac{q}{l}\,\epsilon_{stk}\,\frac{x^k}{r^3}\,, \tag{58}$$

thus

$$\int_{\Omega_2} R_{[\mu\nu]}\, d\tau^{\mu\nu} = 4\pi\frac{q}{l}\,, \tag{59}$$

where the integral in (59) extends over a closed two-dimensional spherical surface surrounding the isolated charged particle but sufficiently far from the particle so that with negligible error the field over the surface can be approximated by (47)–(52). Comparing (59) with (57) we conclude that such surfaces do not enclose a region of space having a Euclidean global topology. This means that in Einstein's theory, if assumptions (1)–(3) are valid, assumptions suggested by empirical evidence, charged particles must have non-Euclidean spatial topologies associated with them.

APPENDIX A: f AND w

Restricting ourselves to fields of physical interest (fields for which the determinate of $g_{\mu\nu}$ is negative except possibly at coordinate singularities), we shall show that a spherically symmetric field $g_{\mu\nu}$ for which $f \neq 0$ is invariantly distinct from a spherically symmetric field for which $f=0$, and that a spherically symmetric field for which $w \neq 0$ is invariantly distinct from a spherically symmetric field for which $w=0$.

We first define the invariant I_1 and the oriented invariant I_2,

$$I_1 = \tfrac{1}{2} g^{[\mu\nu]} g_{[\mu\nu]}\,, \tag{A1}$$

$$I_2 = \tfrac{1}{8}(-g)^{-1/2}\epsilon^{\mu\nu\rho\sigma} g_{[\mu\nu]} g_{[\rho\sigma]}\,, \tag{A2}$$

where $g^{\mu\nu}$ is defined through

$$g_{\mu\rho} g^{\nu\rho} = g_{\rho\mu} g^{\rho\nu} = \delta^\nu_\mu \tag{A3}$$

and g denotes the determinate of $g_{\mu\nu}$. Since we are only interested in fields of physical interest we shall restrict ourselves except at coordinate singularities to fields for which

$$g < 0\,. \tag{A4}$$

From (5) one finds

$$g = -(\alpha\gamma - w^2)(\beta^2 + f^2)\sin^2\theta\,, \tag{A5}$$

so that the above-mentioned restriction is equivalent to

$$\alpha\gamma - w^2 > 0, \quad \beta^2 + f^2 > 0\,. \tag{A6}$$

We also find from (5)

$$I_1 = \frac{f^2}{(\beta^2+f^2)} - \frac{w^2}{(\alpha\gamma - w^2)}\,, \tag{A7}$$

$$I_2 = \frac{fw}{(\beta^2+f^2)^{1/2}(\alpha\gamma - w^2)^{1/2}}\,. \tag{A8}$$

From (A6)–(A8) we see that if $f=0$ and $w=0$, then $I_1=0$, $I_2=0$; if $f=0$ and $w\neq 0$, then $I_1<0$, $I_2=0$; if $f\neq 0$ and $w=0$, then $I_1>0$, $I_2=0$; and if $f\neq 0$ and $w\neq 0$, then $I_2\neq 0$. This means that if we restrict ourselves to fields of physical interest (fields for which the determinate of $g_{\mu\nu}$ is negative except possibly at coordinate singularities), we find that a spherically symmetric field $g_{\mu\nu}$ for which $f \neq 0$ is invariantly distinct from one for which $f=0$, and that a spherically symmetric field for which $w\neq 0$ is invariantly distinct from one for which $w=0$.

APPENDIX B: PHYSICAL MEANING OF m, q, q_M, AND l

In this appendix we shall investigate the physical meaning of the parameters m, q, q_M, and l associated with particles in Einstein's unified field theory.

The equations of motion satisfied by the particles we have been studying are given to second order by (41). If we introduce the effective external electromagnetic fields ${}^{(p)}\tilde{\gamma}^{\,\mathrm{ext}}_{[\mu\nu]}$ and ${}^{(p)}\tilde{\gamma}^{\,C\,\mathrm{ext}}_{[\mu\nu]}$,

$$^{(p)}\tilde{\gamma}^{\text{ext}}_{[\mu\nu]} = \tfrac{1}{2}\,{}^{(p)}(\gamma^{*\text{ext}}_{[\mu\nu]} - \epsilon l^2 \Box^2 \gamma^{*\text{ext}}_{[\mu\nu]}) , \tag{B1}$$

$$^{(p)}\tilde{\gamma}^{C\,\text{ext}}_{[\mu\nu]} = {}^{(p)}(-\epsilon l^2 \Box^2 \gamma^{*\text{ext}}_{[\mu\nu]}) , \tag{B2}$$

and define the fields $^{(p)}\tilde{\gamma}^{*\text{ext}}_{[\mu\nu]}$ and $^{(p)}\tilde{\gamma}^{*C\,\text{ext}}_{[\mu\nu]}$ through

$$^{(p)}\tilde{\gamma}^{*\text{ext}}_{[\mu\nu]} = \tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}\,{}^{(p)}\tilde{\gamma}^{\text{ext}\,[\sigma\rho]} , \tag{B3}$$

$$^{(p)}\tilde{\gamma}^{*C\,\text{ext}}_{[\mu\nu]} = \tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}\,{}^{(p)}\tilde{\gamma}^{C\,\text{ext}\,[\sigma\rho]} , \tag{B4}$$

and make use of (39), equations of motion (41) reduce to

$$m\dot{u}_\mu = \tfrac{1}{2}\epsilon \frac{q}{l}\,\tilde{\gamma}^{\text{ext}}_{[\nu\mu]} u^\nu + \tfrac{1}{2}\epsilon \frac{q_M}{2l}\,\tilde{\gamma}^{*C\,\text{ext}}_{[\nu\mu]} u^\nu + \tfrac{1}{3}\epsilon q^2(\ddot{u}_\mu + \dot{u}_\rho \dot{u}^\rho u_\mu) . \tag{B5}$$

From the definitions (B1) and (B2), and from (37) and (38), we have for the fields $^{(p)}\tilde{\gamma}^{\text{ext}}_{[\mu\nu]}$ and $^{(p)}\tilde{\gamma}^{C\,\text{ext}}_{[\mu\nu]}$,

$$^{(p)}\tilde{\gamma}^{\text{ext}}_{[\mu\nu]} = {}^{(p)}\tilde{\gamma}^{E\,\text{ext}}_{\mu,\nu} - {}^{(p)}\tilde{\gamma}^{E\,\text{ext}}_{\nu,\mu} + \epsilon_{\mu\nu\rho\sigma}\,{}^{(p)}\tilde{\gamma}^{M\,\text{ext}\,\rho,\sigma} , \tag{B6}$$

$$^{(p)}\tilde{\gamma}^{C\,\text{ext}}_{[\mu\nu]} = {}^{(p)}\tilde{\gamma}^{C\,\text{ext}}_{\mu,\nu} - {}^{(p)}\tilde{\gamma}^{C\,\text{ext}}_{\nu,\mu} , \tag{B7}$$

where

$$^{(p)}\tilde{\gamma}^{E\,\text{ext}}_{\mu} = {}^{(p)}l \sum_{p'\neq p} \frac{1}{2}\, \frac{^{(p')}l}{^{(p)}l}\left[1 + \frac{^{(p)}\epsilon\,{}^{(p)}l^2}{^{(p')}\epsilon\,{}^{(p')}l^2}\right] \times {}^{(p')}[q u_\mu (r_\rho u^\rho)^{-1}]_{\text{ret}} + {}^{(p)}l \sum_{p'\neq p} \frac{^{(p')}l}{^{(p)}l}\,{}^{(p')}\left[\frac{1}{4}\epsilon \frac{q}{l^2} r_\mu\right]_{\text{ret}} , \tag{B8}$$

$$^{(p)}\tilde{\gamma}^{M\,\text{ext}}_{\mu} = {}^{(p)}l \sum_{p'\neq p} \frac{^{(p')}l}{^{(p)}l}\,{}^{(p')}[\tfrac{1}{2} q_M u_\mu (r_\rho u^\rho)^{-1}]_{\text{ret}} , \tag{B9}$$

$$^{(p)}\tilde{\gamma}^{C\,\text{ext}}_{\mu} = {}^{(p)}l \sum_{p'\neq p} \left\{\frac{^{(p)}\epsilon\,{}^{(p)}l}{^{(p')}\epsilon\,{}^{(p')}l}\right\} {}^{(p')}[q u_\mu (r_\rho u^\rho)^{-1}]_{\text{ret}} . \tag{B10}$$

If we introduce the mass M, charge e, magnetic-monopole moment e_M, and effective external electromagnetic fields $F^{\text{ext}}_{\mu\nu}$ and $F^{C\text{ext}}_{\mu\nu}$ in practical units,

$$m = \frac{GM}{c^2} , \tag{B11}$$

$$q = \left(\frac{G}{2\pi\epsilon_0 c^4}\right)^{1/2} e , \tag{B12}$$

$$q_M = \left(\frac{G}{2\pi\epsilon_0 c^4}\right)^{1/2} \frac{2e_M}{c} , \tag{B13}$$

$$\tilde{\gamma}^{E\,\text{ext}}_{[\mu\nu]} = (8\pi\epsilon_0 G)^{1/2} \frac{l}{c} F^{\text{ext}}_{\mu\nu} , \tag{B14}$$

$$\tilde{\gamma}^{C\,\text{ext}}_{[\mu\nu]} = (8\pi\epsilon_0 G)^{1/2} \frac{l}{c} F^{C\,\text{ext}}_{\mu\nu} , \tag{B15}$$

equations of motion (B5) take the form

$$M\dot{u}_\mu = \epsilon \frac{e}{c} F^{\text{ext}}_{\nu\mu} u^\nu + \epsilon \frac{e}{c^2} F^{*C\,\text{ext}}_{\nu\mu} u^\nu + \frac{2}{3}\,\frac{e^2}{4\pi\epsilon_0 c^2}(\ddot{u}_\mu + \dot{u}_\rho u^\rho \dot{u}_\mu) . \tag{B16}$$

where

$$^{(p)}F^{\text{ext}}_{\mu\nu} = {}^{(p)}A^{\text{ext}}_{\mu,\nu} - {}^{(p)}A^{\text{ext}}_{\nu,\mu} + \epsilon_{\mu\nu\rho\sigma}\,{}^{(p)}M^{\text{ext}\,\rho,\sigma} , \tag{B17}$$

$$^{(p)}F^{*C\,\text{ext}}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}\,{}^{(p)}A^{C\,\text{ext}\,\sigma,\rho} , \tag{B18}$$

and

$$^{(p)}A^{\text{ext}}_{\mu} = \frac{1}{4\pi\epsilon_0} \sum_{p'\neq p} \frac{1}{2}\,\frac{^{(p')}l}{^{(p)}l}\left(1 + \frac{^{(p)}\epsilon\,{}^{(p)}l^2}{^{(p')}\epsilon\,{}^{(p')}l^2}\right) \times {}^{(p')}\left[\frac{e}{c} u_\mu (r_\rho u^\rho)^{-1}\right]_{\text{ret}} + \frac{1}{4\pi\epsilon_0}\sum_{p'\neq p} \frac{^{(p')}l}{^{(p)}l}\,{}^{(p')}\left[\epsilon \frac{e}{c}\left(\frac{1}{4l^2}\right) r_\mu\right]_{\text{ret}} , \tag{B19}$$

$$^{(p)}M^{\text{ext}}_{\mu} = \frac{\mu_0}{4\pi} \sum_{p'\neq p} \frac{^{(p')}l}{^{(p)}l}\,{}^{(p')}[e_M u_\mu (r_\rho u^\rho)^{-1}]_{\text{ret}} , \tag{B20}$$

$$^{(p)}A^{C\,\text{ext}}_{\mu} = \frac{1}{4\pi\epsilon_0} \sum_{p'\neq p} \left\{\frac{^{(p)}\epsilon\,{}^{(p)}l}{^{(p')}\epsilon\,{}^{(p')}l}\right\} {}^{(p')}\left[\frac{e}{c} u_\mu (r_\rho u^\rho)^{-1}\right]_{\text{ret}} . \tag{B21}$$

We see from (B16)–(B21) that the interaction among the particles can approximate that of conventional classical electrodynamics over laboratory distances only if $^{(p)}l$ is a universal astronomical length and if $^{(p)}\epsilon = 1$. We shall therefore restrict our study to such particles. Under these conditions we find as the equations of motion of the particles to second order

$$M\dot{u}_\mu = \frac{e}{c} F^{\text{ext}}_{\nu\mu} u^\nu + \frac{e_M}{c^2} F^{*C\,\text{ext}}_{\nu\mu} u^\nu + \frac{2}{3}\,\frac{e^2}{4\pi\epsilon_0 c^2}(\ddot{u}_\mu + \dot{u}_\rho \dot{u}^\rho u_\mu) , \tag{B22}$$

where

$$^{(p)}F^{\text{ext}}_{\mu\nu} = {}^{(p)}A^{\text{ext}}_{\mu,\nu} - {}^{(p)}A^{\text{ext}}_{\nu,\mu} + \epsilon_{\mu\nu\rho\sigma}\,{}^{(p)}M^{\text{ext}\,\rho,\sigma} , \tag{B23}$$

$$^{(p)}F^{*C\,\text{ext}}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}\,{}^{(p)}A^{C\,\text{ext}\,\sigma,\rho} , \tag{B24}$$

and

$$^{(p)}A^{\text{ext}}_{\mu} = \frac{1}{4\pi\epsilon_0} \sum_{p'\neq p} {}^{(p')}\left[\frac{e}{c} u_\mu (r_\rho u^\rho)^{-1}\right]_{\text{ret}} + \frac{1}{4\pi\epsilon_0} \sum_{p'\neq p} {}^{(p')}\left[\frac{e}{c}\left(\frac{1}{4l^2}\right) r_\mu\right]_{\text{ret}} , \tag{B25}$$

$$^{(p)}M^{\text{ext}}_{\mu} = \frac{\mu_0}{4\pi} \sum_{p'\neq p} {}^{(p')}[e_M u_\mu (r_\rho u^\rho)^{-1}]_{\text{ret}} , \tag{B26}$$

$$ {}^{(p)}A_\mu^{C\,\text{ext}} = \frac{1}{4\pi\epsilon_0} \sum_{p'\neq p}{}^{(p')} \left[\frac{e}{c}\, u_\mu (r_\rho u^\rho)^{-1}\right]_{\text{ret}} . \quad \text{(B27)} $$

The particles interact to second order through what we shall call generalized Einstein electrodynamics.[22]

If the ${}^{(p)}q_M$ are not excessive, and by this we mean ${}^{(p)}q_M \lesssim {}^{(p)}q$, the approximations used in arriving at (B22)–(B27) should be valid over macroscopic interaction distances. The arguments for why this is so are identical to those previously discussed in the literature for the case of charged particles possessing no magnetic-monopole moments ($q_M=0$),[19] and lead to the conclusion that as long as l is a moderate astronomical length Eqs. (B22)–(B27) should be valid over both laboratory and astronomical distances. Reasons for believing that l satisfies this criterion can be found in the literature.[3]

From the above results we see that our tentative identifications of m, q, and q_M in Sec. III were correct. The quantity m represents the mass of a particle, q represents the charge, and q_M represents the magnetic-monopole moment. We also find that the length l associated with a particle must be a universal (moderate) astronomical length, and ϵ must be equal to 1.

We see from the second order equations of motion (B22)–(B27) that over macroscopic interaction distances electric charge in Einstein's theory interacts with electric charge through a weak long-range nonconventional electromagnetic interaction in addition to the conventional classical electromagnetic interaction. In the same order of approximation there is also a self interaction associated with electric charge which gives rise to the conventional classical radiation reaction force acting on a charged particle. The above properties of electric charge in Einstein's theory have been discussed in previous papers and will not be discussed further here. We also see from (B22)–(B27) that in Einstein's theory in an approximation which should be valid over macroscopic interaction distances, magnetic-monopole moments do not interact with magnetic-monopole moments. In this way Einstein's theory differs from what is usually assumed in electrodynamics so that if magnetic monopoles are ever discovered in nature a study of their interaction over macroscopic distances should provide a test of Einstein's theory. Finally we note from (B22)–(B27) that in Einstein's theory the interaction over macroscopic distances between magnetic-monopole moments and and electric charge is the same as that usually assumed in electrodynamics.

Next we ask the question, can charged particles in Einstein's theory possess sizable magnetic-monopole moments in addition to electric charge and still interact over laboratory distances through the laws of conventional classical electrodynamics, i.e., Maxwell electrodynamics? We shall find that they can if the magnetic-monopole moment associated with each particle is proportional to the particle's charge. Under these conditions the particles we have been studying are found to interact over macroscopic distances as if they had no magnetic-monopole moments.

If the magnetic-monopole moments associated with the particles are not to effect the motion of the particles when interacting over macroscopic distances, we see from (B22)–(B27) that along the world line of each particle one must have

$$ \frac{e}{c}\,\epsilon_{\mu\nu\rho\sigma} M^{\text{ext}\,\rho,\sigma} u^\nu + \frac{e_M}{c^2} F^{*C\,\text{ext}}_{\mu\nu} u^\nu = 0 . \quad \text{(B28)} $$

Making use of (B24), we see (B28) is equivalent to

$$ e\epsilon_{\mu\nu\rho\sigma} M^{\text{ext}\,\rho,\sigma} u^\nu = \frac{e_M}{c}\,\epsilon_{\mu\nu\rho\sigma} A^{C\,\text{ext}\,\rho,\sigma} u^\nu , \quad \text{(B29)} $$

and from (B26) and (B27) we find (B29) is equivalent to

$$ {}^{(p)}e\epsilon_{\mu\nu\kappa\lambda} \sum_{p'\neq p}{}^{(p')}[e_M u^\kappa (r_\rho u^\rho)^{-1}]_{\text{ret}}{}^{,\lambda}\,{}^{(p)}u^\nu $$

$$ = {}^{(p)}e_M\,\epsilon_{\mu\nu\kappa\lambda} \sum_{p'\neq p}{}^{(p')}[e u^\kappa (r_\rho u^\rho)^{-1}]_{\text{ret}}{}^{,\lambda}\,{}^{(p)}u^\nu . \quad \text{(B30)} $$

Equations (B30) will in general only be satisfied if the magnetic-monopole moment associated with each particle is proportional to the particle's charge, that is if

$$ {}^{(p)}q_M = k_M\,{}^{(p)}q \quad \text{(B31)} $$

or equivalently,

$$ {}^{(p)}e^M = \tfrac{1}{2}(k_M c)\,{}^{(p)}e , \quad \text{(B32)} $$

where k_M is a universal constant. If (B31) is satisfied, equations of motion (B22)–(B27) take the form

$$ M\dot{u}_\mu = \frac{e}{c} F^{\text{ext}}_{\nu\mu} u^\nu + \frac{2}{3}\,\frac{e^2}{4\pi\epsilon_0 c^2}(\ddot{u}_\mu + \dot{u}_\rho \dot{u}^\rho u_\mu), \quad \text{(B33)} $$

where

$$ {}^{(p)}F^{\text{ext}}_{\mu\nu} = {}^{(p)}A^{\text{ext}}_{\mu,\nu} - {}^{(p)}A^{\text{ext}}_{\nu,\mu} \quad \text{(B34)} $$

and

$$ {}^{(p)}A^{\text{ext}}_\mu = \frac{1}{4\pi\epsilon_0} \sum_{p'\neq p}{}^{(p')} \left[\frac{e}{c}\, u_\mu (r_\rho u^\rho)^{-1}\right]_{\text{ret}} $$

$$ + \frac{1}{4\pi\epsilon_0} \sum_{p'\neq p}{}^{(p')} \left[\frac{e}{c}\left(\frac{1}{4l^2}\right) r_\mu\right]_{\text{ret}} . \quad \text{(B35)} $$

The particles interact to second order through what we have called Einstein electrodynamics.[3] If (B31) is valid, to second order the interaction among the particles does not depend on the particles' magnetic-monopole moments ${}^{(p)}e_M$. Equations of motion (B33)–(B35) are equivalent to (42) and (43).

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[1]C. R. Johnson, Phys. Rev. D 4, 295 (1971); 4, 318 (1971); 4, 3555 (1971); 5, 282 (1972); 5, 1916 (1972); 7, 2825 (1973); 7, 2838 (1973); 8, 1645 (1973). In subsequent references we shall refer to these papers as papers I–VIII, respectively.

[2]C. R. Johnson, Nuovo Cimento 8B, 391 (1972).

[3]C. R. Johnson and J. R. Nance, Phys. Rev. D 15, 377 (1977); 16, 533(E) (1977).

[4]A. Einstein and N. Rosen, Phys. Rev. 48, 73 (1935).

[5]J. A. Wheeler, *Geometrodynamics* (Academic, New York, 1962).

[6]We are assuming the magnitudes of the magnetic monopole moments are not excessive. This is discussed in Sec. III.

[7]Over laboratory distances Einstein electrodynamics differs insignificantly from conventional Maxwell electrodynamics, but over astronomical distances, where Maxwell electrodynamics has not been adequately tested, the differences between the two can become significant. This has been discussed in the literature and provides a possible way of testing Einstein's theory. See Ref. 3.

[8]The interaction of charged particles over microscopic distances in Einstein's theory is at present unknown. Although techniques exist for investigating such interactions, the techniques involve a great deal of labor and have not yet been fully applied to the problem. For further discussion see papers I, II, and VIII of Ref. 1.

[9]The notation $A_{(\mu\nu)}=\frac{1}{2}(A_{\mu\nu}+A_{\nu\mu})$, $A_{[\mu\nu]}=\frac{1}{2}(A_{\mu\nu}-A_{\nu\mu})$, $A_{[\mu\nu,\lambda]}=\frac{1}{3}(A_{[\mu\nu],\lambda}+A_{[\nu\lambda],\mu}+A_{[\lambda\mu],\nu})$ will be used in this paper. Lower case Greek indices take the values 1–4; lower case Latin indices take the values 1–3. The Levi-Civita symbols $\epsilon^{\mu\nu\rho\sigma}$ and ϵ_{stk} will be chosen so that $\epsilon^{1234}=1$ and $\epsilon_{123}=1$. The Minkowski metric $\eta_{\mu\nu}=\eta^{\mu\nu}$ will be defined through the equations $\eta_{st}=\delta_{st}$, $\eta_{s4}=\eta_{4s}=0$, $\eta_{44}=1$.

[10]A. Papapetrou, Proc. R. Ir. Acad. A52, 69 (1948). Papapetrou's analysis can be generalized to include time-dependent systems.

[11]Restricting ourselves to fields $g_{\mu\nu}$ of physical interest (fields for which the determinate of $g_{\mu\nu}$ is negative except possibly at coordinate singularities), we show in Appendix A that a spherically symmetric field for which $f\neq 0$ is invariantly distinct from a spherically symmetric field for which $f=0$, and that a spherically symmetric field for which $w\neq 0$ is invariantly distinct from a spherically symmetric field for which $w=0$.

[12]G. Bandyopadhyay, Sci. and Cult. (Calcutta) 25, 427 (1960). By a time-independent spherically symmetric solution we mean a solution which is time independent in a coordinate system in which the field takes the form (5).

[13]J. R. Vanstone, Can. J. Math. 14, 568 (1962).

[14]The additional solutions found by Vanstone, corresponding to $m_1=0$, cannot represent charged particles and will not be discussed here.

[15]That q not q_M is to identified with the charge of the particle is suggested by the work of Ref. 3.

[16]See Ref. 1 (paper III).

[17]See Appendix B.

[18]That ${}^{(p)}q_M$ should be regarded as the magnetic-monopole moment of the pth particle is discussed in Appendix B.

[19]See Ref. 1 (paper II, Appendix C, and paper VIII). See also Ref. 2.

[20]For a more thorough discussion of the physical meaning of q and q_M, see Appendix B.

[21]A discussion and proof of the generalized Stokes's theorem can be found in J. L. Synge and A. Schild, *Tensor Calculus* (University of Toronto Press, Toronto, 1949), Chap. VII, pp. 240–281.

[22]Generalized Einstein electrodynamics is a generalization of what we have called Einstein electrodynamics. See Ref. 3 for a discussion of Einstein electrodynamics.