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Pregeometry including fundamental gauge bosons

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Einstein's gravitation is derived as an induced effect due to quantum fluctuations of matter fields, including gauge bosons. It is found that the contribution from gauge bosons gives antigravity, while those from scalar and spinor particles give attractive gravity.

I. INTRODUCTION

Pregeometry[1] is the theoretical scheme to derive Einstein's gravitation (geometrodynamics) as an effective theory induced from more fundamental ingredients. The most remarkable idea is due to Sakharov,[2] who identified Einstein's gravitational action with that of the quantum fluctuation of matter. On the other hand, many composite models of gravitons were proposed,[2] none of which, however, could so clearly induce Einstein's action, because of lack of general covariance. In previous papers,[3-5] realizing Sakharov's idea, we presented the field-theoretical formulation of pregeometry, where the spacetime metric (hence, gravitons also) appears as a composite of fundamental matter, and the Einstein action is derived. Starting with the matter Lagrangian on a curved space, we extracted the divergent quantum fluctuations, which were interpreted as gravitational action. The divergence was cut off generally covariantly. The fundamental matter was taken as scalar and spinor particles, and they were called the scalar and spinor pregeometries. The purpose of this paper is to extend our previous work so as to include fundamental gauge fields and to investigate their implications. We call it, for short, gauge-boson pregeometry.

Recently, Adler[6] proposed the further refined program of "induced gravity," where a dynamical breakdown of the scale invariance is expected to induce the finite Einstein action without using a naive cutoff. In this case the fundamental matter should be spinor particles or gauge bosons, but not scalars. Here again the gauge-boson pregeometry is important. Unfortunately, no model has yet been found to realize this program without some rough approximations.

One of the motivations for the gauge-boson pregeometry is to find the sign in front of the induced Einstein action, i.e., whether the gravity is attractive or not. We showed in previous papers[3,4] that the scalar and spinor pregeometries give attractive gravity. If the photon, weak bosons, gluons, and/or many gauge bosons conjectured in the grand unification models are fundamental, it is important to determine their contributions to the gravitational action. Another motivation arises from subquark models[7] with subcolor.[8] In the ultimate subquark model, the subquarks should compose the basic constituents of everything including quarks, leptons, gauge bosons, and even gravitons, the last of which requires pregeometry. The subquarks are usually taken as spinor or scalar particles. Some people, however, consider subcolor gauge symmetry and subgluons,[8] mainly, expecting confinement of subquarks. The existence of such fundamental gauge bosons, i.e., subgluons, requires a reliable formulation of gauge-boson pregeometry.

There is another reason for introducing subcolor in the context of the subquark pregeometry. It is the saturation problem of the G-α relation,[9] the relation between the Newtonian gravitational constant G and the fine-structure constant α, which reads[3]

$$\frac{1}{\alpha}=\frac{1}{3\pi}\sum_i Q_i{}^2\ln\left[\frac{12\pi}{NGm_i{}^2}\right], \tag{1.1}$$

where Q_i and m_i are the charge and mass of the ith fundamental fermion and N is the number of fundamental fermions.[10] This is a straightforward consequence of pregeometry combined with pre-QED (i.e., the composite model of the photon which induces quantum electrodynamic[11]). If we take the photon and the graviton as composites of the fundamental quarks and leptons, instead of subquarks, and if we have six generations of quarks and leptons, relation (1) is approximately saturated.[12] In the subquark model, however, it is far from satisfactory, since the right-hand side of Eq.

(1.1) is too small. The realistic subquark model which reproduces the standard model of leptons and quarks with the Weinberg-Salam and color gauge symmetries is given in Ref. 13, where the leptons, quarks, and gauge bosons are all composed of three types of subquarks, the weak subquarks w_i ($i=1,2$), the color subquarks c_i ($i=0,1,2,3$), and the horizontal subquarks h_i ($i=1,2,\ldots, N_G$; N_G is the number of generations). Though the charge assignment for these subquarks is not unique, the relation[13]

$$\sin^2\theta_w=\frac{1}{4\sum_i Q_i{}^2}\times(\textit{ the number of weak isodoublets}) \tag{1.2}$$

in ths same model and the experimental value $\sin^2\theta_w = 0.23\pm0.02$ (Ref. 14) fix the value of $\sum_i Q_i{}^2$ to be 1.1 ± 0.1, and if we take $N_G\geq 3$ (i.e., $N\geq 9$), and $m_i{}^2 \gtrsim 10$ TeV,[5] we get the right-hand side of Eq. (1.1), to be $\lesssim 8$, while the left-hand side is 137. If the subquarks belong to an N_{sc}-plet of subcolor, the right-hand side is multiplied by N_{sc}. To be consistent, we should take $N_{sc}\gtrsim 17$.

In Sec. II, we introduce a new method of momentum cutoff, "the scale cutoff." The scalar, spinor, and gauge-boson pregeomentry is formulated in Sec. III. And finally Sec. IV is devoted to some discussions.

II. SCALE CUTOFF

The quantum fluctuation of matter involves severe ultraviolet divergence. It is this divergent term that is effectively identified with the gravitational action. Therefore, the momentum cutoff plays an important role, unlike in the renormalizable theory. The precise results depend on the mechanism determining how the physical cutoff takes place, which should ultimately be checked by experiment. The cutoff mechanism should be subject to the condition of general covariance in order to give the covariant form of the gravitational action. For example, for the scalar and spinor pregeometry, the Pauli-Villars regulator method was successful.[3,4] For the gauge-boson pregeometry, however, it is of no use, since the gauge bosons are massless. Dimensional regularization is also inadequate, because it gives no quadratically divergent term which is to give the Einstein action. (In general, the quartically, quadratically, and logarithmically divergent terms give the cosmological term, the Einstein action, and the term quadratic in the curvature tensor, respectively.) In order to circumvent these difficulties, we modify the dimensional regularization method to include an explicit cutoff parameter. First, for any Feynman amplitude $A(n)$ in n dimensions, we define the "scale parameter representation" $\widetilde{A}(\mu)$ as the inverse Mellin transform of $A(n)$,

$$A(n)=\int_0^\infty \widetilde{A}(\mu)\mu^{n-1}d\mu \ . \tag{2.1}$$

Then, the scale-cutoff amplitude $A_\Lambda(n)$ is defined by

$$A_\Lambda(n)=\int_0^\Lambda \widetilde{A}(\mu)\mu^{n-1}d\mu \ . \tag{2.2}$$

In the calculation below, the following formula is particularly useful. For the typical term in Feynman amplitudes

$$A(n)=\Gamma\left[-\frac{n}{2}\right]f(n) \ , \tag{2.3}$$

with some regular function $f(n)$, the scale-cutoff amplitude (in four-dimensional spacetime) becomes

$$A_\Lambda(4)=\tfrac{1}{2}f(0)\Lambda^4-f(2)\Lambda^2+\tfrac{1}{2}f(4)\ln\Lambda^2 +O(\Lambda^0) \ . \tag{2.4}$$

Thus we can extract not only the logarithmic divergence but also the quartic and quadratic ones. Now it is possible to formulate the gauge-boson pregeometry. Needless to say, it is also applicable to the scalar and the spinor pregeometry. The following section is devoted to them.

III. PREGEOMETRY

Though the scalar and spinor pregeometries were formulated in Refs. 3 and 4, here we reconsider them, since we use a new cutoff procedure. After that we will turn to gauge-boson pregeometry.

A. Scalar pregeometry

In the scalar pregeometry, the graviton is composed of the fundamental scalar particles. The basic Lagrangian is that for the scalar fields ϕ^i ($i=1,\ldots,N_0$) on a curved space[15]:

$$\mathscr{L}_0=\tfrac{1}{2}\sqrt{-g}\,[g^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^i-m_i{}^2(\phi^i)^2] \ , \tag{3.1}$$

where m_i is the mass of ϕ^i, the repeated index i implies summation over it, $g^{\mu\nu}$ is an auxiliary field, and $g=(\det g^{\mu\nu})^{-1}$. Note that the metric is merely auxiliary without its own kinetic term. It is to be interpreted as composite, when the kinetic term is induced through quantum fluctuations.

The effective Lagrangian $\mathscr{L}_0^{\text{eff}}$ due to the quantum fluctuation is given by

$$\exp\left[i\int \mathscr{L}_0^{\text{eff}} d^4x\right] = \int \int d\phi^i \exp\left[i\int \mathscr{L}_0 d^4x\right] . \tag{3.2}$$

Performing the path integration, we get

$$\int \mathscr{L}_0^{\text{eff}} d^4x = \frac{i}{2}\sum_i \text{Tr}\ \ln(\partial_\mu \sqrt{-g}\, g^{\mu\nu}\partial_\nu - m_i^2\sqrt{-g}) , \tag{3.3}$$

where Tr stands for trace operation with respect to spacetime index x. Then we expand the logarithm into the power series in the weak field $h^{\mu\nu} = g^{\mu\nu} - \eta^{\mu\nu}$ [$\eta^{\mu\nu} = \text{diag}(1,-1,-1,-1)$]. The term with the lth power of $h^{\mu\nu}$ corresponds to the scalar loop diagram with l external graviton lines attached. Then we perform the integration over the loop momentum in n-dimensional space (see Appendix). Finally, we extract the divergent parts by using the formula (2.4). The result is

$$\mathscr{L}_0^{\text{eff}} = \sum_i \sqrt{-g}\left[\left[\tfrac{1}{4}\Lambda^4 - \frac{1}{8\pi}m^2\Lambda^2 + \frac{1}{64\pi^2}m_i^4 \ln\Lambda^2\right] + \left[\frac{1}{48\pi}\Lambda^2 - \frac{1}{192\pi^2}m_i^2\ln\Lambda^2\right]R + \frac{1}{3840\pi^2}\ln\Lambda^2(R^2 + 2R_{\mu\nu}R^{\mu\nu}) + O(\Lambda^0)\right] , \tag{3.4}$$

where

$$R = g^{\mu\nu}R_{\mu\nu} , \tag{3.5}$$

$$R_{\mu\nu} = \partial_\nu\Gamma^\alpha_{\mu\alpha} - \partial_\alpha\Gamma^\alpha_{\mu\nu} + \Gamma^\alpha_{\beta\nu}\Gamma^\beta_{\mu\alpha} - \Gamma^\alpha_{\beta\alpha}\Gamma^\beta_{\mu\nu} , \tag{3.6}$$

and

$$\Gamma^\alpha_{\beta\gamma} = \tfrac{1}{2}g^{\alpha\sigma}(\partial_\beta g_{\gamma\sigma} + \partial_\gamma g_{\beta\sigma} - \partial_\sigma g_{\beta\gamma}) . \tag{3.7}$$

The first term in the square brackets in Eq. (3.4) corresponds to the cosmological term which can be renormalized (see Refs. 3 and 4). The second term is the induced Einstein Lagrangian. As long as we take $\Lambda >> m_i$, the sign of this term is positive and it gives attractive gravity. The third term gives a small correction to Einstein's gravity. The numerical results are slightly different from those with the Pauli-Villars cutoff.[4] It is not surprising, since we have no reason to identify the cutoff scale Λ with the Pauli-Villars regulator mass M.

B. Spinor pregeometry

Now we consider the case where the fundamental particles are spinors. The basic Lagrangian is that for the spinor fields ψ^j ($j = 1, \cdots, N_{1/2}$) on a curved space specified with vierbein $e^{k\mu}$,[15]

$$\mathscr{L}_{1/2} = (\det e^{l\nu})^{-1}\bar{\psi}^j(ie^{k\mu}\gamma_k D_\mu - m_j)\psi^j , \tag{3.8}$$

where D_μ is the covariant derivative (following the notations in Ref. 3). Note that $e^{k\mu}$ is auxiliary and to be interpreted as composite when the kinetic term is induced. The effective Lagrangian $L_{1/2}^{\text{eff}}$ is given by

$$\exp\left[i\int \mathscr{L}_{1/2}^{\text{eff}} d^4x\right] = \int\int d\bar{\psi}^j d\psi^j \exp\left[i\int \mathscr{L}_{1/2} d^4x\right] . \tag{3.9}$$

Performing the path integration, we get

$$\int \mathscr{L}_{1/2}^{\text{eff}} d^4x = -i\sum_j \text{Tr}\ \ln[(\det e^{l\nu})^{-1}(ie^{k\mu}\gamma_k D_\mu - m_j)] , \tag{3.10}$$

where Tr stands for trace operation with respect to the space-time index x and the γ matrix. A procedure similar to the scalar pregeometry leads to (see Appendix)

$$\mathscr{L}_{1/2}^{\rm eff}=\sum_j\sqrt{-g}\left[\left[-\tfrac{1}{4}\Lambda^2+\frac{1}{4\pi}m_j{}^2\Lambda^2-\frac{1}{16\pi^2}m_i{}^4\ln\Lambda^2\right]+\left[\frac{1}{48\pi}\Lambda^2-\frac{1}{96\pi^2}m_j{}^2\ln\Lambda^2\right]R\right.$$
$$\left.-\frac{1}{960\pi^2}\ln\Lambda^2(R^2-3R_{\mu\nu}R^{\mu\nu})+O(\Lambda^0)\right], \tag{3.11}$$

where R and $R_{\mu\nu}$ are given by Eqs. (3.5) and (3.6) with $g^{\mu\nu}=e^{k\mu}e_l^\nu$. Each term in Eq. (3.11) is interpreted as in Eq. (3.4). The positive sign of the second term indicates that the spinor pregeometry also gives attractive gravitation.

C. Gauge-boson pregeometry

Now we turn to the main theme of this paper. The basic Lagrangian for gauge-boson pregeometry is that for a gauge field A_μ^a $(a=1,\ldots,N_1)$ on a curved space,[15]

$$\mathscr{L}_1=-\tfrac{1}{4}\sqrt{-g}\,g^{\mu\nu}g^{\rho\sigma}F_{\mu\rho}^aF_{\nu\sigma}^a \tag{3.12}$$

with

$$F_{\mu\nu}^a=\partial_\mu A_\nu^a-\partial_\nu A_\mu^a-i\kappa f^{abc}A_\mu^bA_\nu^c\quad(a=1,\ldots,N_1)\ , \tag{3.13}$$

where κ is the gauge coupling constant, f^{abc} is the structure constant of the gauge group, and the metric $g^{\mu\nu}$ is again an auxiliary field to be interpreted as a composite.

The effective Lagrangian $\mathscr{L}_1^{\rm eff}$ due to the quantum fluctuations of the gauge field is given by

$$\exp\left[i\int\mathscr{L}_1^{\rm eff}d^4x\right]=\int\int dA_\mu dc\;dc^\dagger\exp\left[i\int(\mathscr{L}_1+\mathscr{L}_{\rm GF}+\mathscr{L}_{\rm FP})d^4x\right], \tag{3.14}$$

where $\mathscr{L}_{\rm GF}$ is the gauge-fixing term and $\mathscr{L}_{\rm FP}$ is the Faddeev-Popov ghost Lagrangian:

$$\mathscr{L}_{\rm GF}=-\tfrac{1}{2}\sqrt{-g}\,(g^{\mu\nu}\mathscr{D}_\mu A_\nu^a)^2\quad\text{(Feynman gauge)} \tag{3.15}$$

with

$$\mathscr{D}_\mu A_\nu^a=\partial_\mu a_\nu^a-\Gamma_{\mu\nu}^\lambda A_\lambda^a \tag{3.16}$$

and

$$\Gamma_{\mu\nu}^\gamma=\tfrac{1}{2}g^{\lambda\rho}(\partial_\mu g_{\rho\nu}+\partial_\nu g_{\rho\mu}-\partial_\rho g_{\mu\nu})\ , \tag{3.17}$$

$$\mathscr{L}_{\rm FP}=\sqrt{-g}\,g^{\mu\nu}\partial_\mu c^{a\dagger}(\partial_\nu c^a-i\kappa f^{abc}A_\nu^bc^c)\ . \tag{3.18}$$

Performing the path integration in Eq. (3.14), we get

$$\mathscr{L}_1^{\rm eff}=\mathscr{L}_G^{\rm eff}+\mathscr{L}_{\rm FP}^{\rm eff} \tag{3.19}$$

with

$$\int\mathscr{L}_G^{\rm eff}d^4x=\frac{i}{2}N_1{\rm Tr}\ln[\partial_\mu\sqrt{-g}\,(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\sigma}g^{\rho\nu}+g^{\mu\rho}g^{\nu\sigma})\partial_\sigma$$
$$+\sqrt{-g}\,g^{\alpha\beta}\Gamma_{\alpha\beta}^\mu g^{\nu\gamma}\partial_\gamma-\partial_\alpha g^{\mu\alpha}\sqrt{-g}\,g^{\gamma\delta}\Gamma_{\gamma\delta}^\nu$$
$$-\sqrt{-g}\,g^{\alpha\beta}\Gamma_{\alpha\beta}^\mu g^{\gamma\delta}\Gamma_{\gamma\delta}^\nu+O(\kappa)] \tag{3.20}$$

and

$$\int \mathscr{L}_{\mathrm{FP}}^{\mathrm{eff}} d^4x = -iN_1 \mathrm{Tr}\, \ln(\partial_\mu \sqrt{-g}\, g^{\mu\nu}\partial_\nu) + O(\kappa) \,, \tag{3.21}$$

where $\mathscr{L}_G^{\mathrm{eff}}$ corresponds to the series of the gauge-boson-loop diagrams with external graviton lines attached, while $\mathscr{L}_{\mathrm{FP}}^{\mathrm{eff}}$ corresponds to those of the ghost loops. A procedure similar to that in subsection A leads to the results (see Appendix)

$$\mathscr{L}_G^{\mathrm{eff}} = N_1\sqrt{-g}\left[-\frac{1}{12\pi}\Lambda^2 R - \frac{1}{1920\pi^2}\ln\Lambda^2 \times (3R^2 - 14R_{\mu\nu}R^{\mu\nu})\right], \tag{3.22}$$

$$\mathscr{L}_{\mathrm{FP}}^{\mathrm{eff}} = N_1\sqrt{-g}\left[-\tfrac{1}{2}\Lambda^4 - \frac{1}{24\pi}\Lambda^2 R - \frac{1}{1920\pi^2}\ln\Lambda^2 \times (R^2 + 2R_{\mu\nu}r^{\mu\nu})\right], \tag{3.23}$$

and, hence,

$$\mathscr{L}_1^{\mathrm{eff}} = N_1\sqrt{-g}\left[-\tfrac{1}{2}\Lambda^4 - \frac{1}{8\pi}\Lambda^2 R - \frac{1}{480\pi^2}\ln\Lambda^2 \times (R^2 - 3R_{\mu\nu}R^{\mu\nu})\right] \tag{3.24}$$

with R and $R_{\mu\nu}$ defined by Eqs. (3.5) and (3.6). Notice the negative sign of the $\Lambda^2 R$ term in Eq. (3.24), which implies that the gauge-boson pregeometry alone leads to antigravity.

In Table I, we compare the results from the various cutoff procedures, (i) dimensional regularization, (ii) Pauli-Villars regularization, and (iii) scale cutoff. The parameters a, b, c, and d in Table I are defined by

$$\mathscr{L}^{\mathrm{eff}} = \sqrt{-g}\,[a + bR + c(R^2 + dR_{\mu\nu}R^{\mu\nu})] \,. \tag{3.25}$$

The calculations with (i) for the scalar, spinor, and gauge-boson loops are given in Refs. 16, 17, and 18, respectively, though their purpose is not pregeometry. The calculations with (ii) are in our previous papers, Refs. 3 and 4. Those with (iii) are in this paper. Note that (i) does not give a and b, while (ii) is not applicable to gauge-boson pregeometry. Qualitatively, the results coincide with each other, but not precisely.

IV. DISCUSSIONS

The Lagrangian of the Einstein gravity is

$$\mathscr{L} = \frac{1}{16\pi G}\sqrt{-g}\,R \,, \tag{4.1}$$

TABLE I. The results from various cutoff procedures are compared. The coefficients a, b, c, and d are defined in Eq. (3.25). $\epsilon = 2/(4-n)$. M is the mass of the Pauli-Villars regulator.

Cutoff	Pregeometry	a	b	c	d	Ref.
Dimensional regularization	Scalar	···	···	$\frac{\epsilon}{3840\pi^2}$	2	16
	Spinor	···	···	$-\frac{\epsilon}{960\pi^2}$	-3	17
	Gauge boson	···	···	$-\frac{\epsilon}{480\pi^2}$	-3	18
Pauli-Villars regularization	Scalar	$\frac{M^4}{128\pi^2}$	$\frac{M^2}{384\pi^2}$	$\frac{\ln M^4}{3840\pi^2}$	2	4
	Spinor	$-\frac{M^4}{32\pi^2}$	$\frac{M^2}{192\pi^2}$	$\frac{-\ln M^2}{960\pi^2}$	-3	3
	Gauge boson	···	···	···	···	···
Scale cutoff	Scalar	$\frac{\Lambda^4}{4}$	$\frac{\Lambda^2}{48\pi}$	$\frac{\ln\Lambda^2}{3840\pi^2}$	2	This paper
	Spinor	$-\frac{\Lambda^4}{4}$	$\frac{\Lambda^2}{48\pi}$	$-\frac{\ln\Lambda^2}{960\pi^2}$	-3	This paper
	Gauge boson	$-\frac{\Lambda^4}{2}$	$-\frac{\Lambda^2}{8\pi}$	$-\frac{\ln\Lambda^2}{480\pi^2}$	-3	This paper

where G is the Newtonian gravitational constant. Comparing it with the combined results of the scalar, spinor, and gauge-boson pregeometry, we get

$$\frac{1}{G}=\left[\frac{N_0}{3}+\frac{N_{1/2}}{3}-2N_1\right]\Lambda^2\ , \tag{2}$$

where N_0, $N_{1/2}$, and N_1 are the numbers of the fundamental scalar, spinor, and gauge bosons, respectively. The gravitation is attractive or repulsive according to whether $N_0+N_{1/2}-6N_1$ is positive or not. If there exist fundamental gauge bosons, there must exist six times more scalar or spinor particles to compensate the antigravity due to gauge bosons. For example, for the combined theory of the quantum chromodynamics and the Weinberg-Salam model, with N_G generations of leptons and quarks and a complex isodoublet of Higgs scalar, $N_0=4$, $N_{1/2}=\frac{15}{2}N_G$, and $N_1=12$, and in order to make the induced gravity attractive, $N_G>10$; for the grand unified model with SU(5) with $\underline{24}$ and $\underline{5}$ (complex) Higgs scalar, $N_G>15$; for the case of SO(10) with $\underline{45}+\underline{10}+\underline{16}$ (complex) Higgs scalars, $N_G>23$. These seem to me somewhat tremendous. A way to avoid them is to take gauge bosons as composites[11–13] so as not to contribute to Eq. (4.2). In the subquark model[13] illustrated in Sec. I with subgluons of SU (N_{sc}),

$$N_G>\frac{6(N_{sc}{}^2-1)}{N_{sc}}-6\ . \tag{4.3}$$

If $N_G\gtrsim 17$, as is given in Sec. I, then $N_G\gtrsim 96$, which seems again too large. The subgluons may again be composite, or may be absent, with a global subcolor symmetry.

Then, combining the results in Eq. (4.2) with that from pre-QED,[11] we get the G-α relation in the presence of N_0 scalar particles, $N_{1/2}$ spinor particles, and N_1 gauge bosons:

$$\frac{1}{\alpha}=\frac{1}{3\pi}\sum_i Q_i{}^2\xi_i \ln[G^{-1}m_i{}^{-2}(\tfrac{1}{3}N_0+\tfrac{1}{3}N_{1/2}-2N_1)^{-1}]\ , \tag{4.4}$$

where $\xi_i=1$ for spinors and $\xi_i=\frac{1}{4}$ for scalars, and gauge bosons are taken as neutral. The main change in Eq. (4.4) from Eq. (1.1) is only in the logarithm, so that it does not cause any essential change in the numerical results given above.

The quartically divergent parts in $\mathscr{L}^{\text{eff}}$ is the cosmological term. It can be renormalized by subtraction of the counterterm like

$$\mathscr{L}^{\text{counter}}=c\sqrt{-g}\ , \tag{4.5}$$

where c is a constant.

As is illustrated in Ref. 4, in the scalar and spinor pregeometry, such terms naturally come out from the more basic Lagrangians written only with the fundamental fields.[15] Another possibility, however, is that the quartic divergences cancel each other without subtracting the counterterms. The condition is, from Eqs. (3.4), (3.11), and (3.24),

$$N_0-N_{1/2}-2N_1=0\ , \tag{4.6}$$

where we consider the case where $\Lambda>>m_i$. Similarly, if we assume intrinsic (and nonquantum) Einstein action, apart from the pregeometry viewpoints, the cancellation condition of the quadratic divergence is given by

$$N_0+N_{1/2}-6N_1=0\ . \tag{4.7}$$

The solution to Eqs. (4.6) and (4.7) is

$$N_0:N_{1/2}:N_1=4:2:1\ . \tag{4.8}$$

The cancellation conditions for the logarithmic divergence are

$$N_0-4N_{1/2}-8N_1=0\ , \tag{4.9}$$

$$N_0+6N_{1/2}+12N_1=0\ . \tag{4.10}$$

The only compatible solution for these is the absurd one

$$N_0=N_{1/2}=N_1=0\ .$$

In conclusion, we have formulated pregeometry including fundamental gauge boson, using the new method of scale cutoff, and found that the quantum fluctuations of gauge bosons give rise to antigravity. To keep the total induced gravity attractive, we need too many fundamental scalars and spinors. It seems to me rather favorable to take the gauge bosons as composite.[11–13]

After completion of this work, we received a report from Fradkin and Tseytlin,[19] who calculated the same quantities as we did, i.e., the contribution to gravitational action from the quantum fluctuations of matter. Their purpose is different from ours, to investigate asymptotic freedom of gravitation.[20] Their method of cutoff, the proper-time regularization, is also different from ours, giving different results.

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APPENDIX

A. Scalar pregeometry

The expansion of the logarithm in Eq. (3.3) is given in the Appendix of Ref. 4. The n-dimensional representation of $\mathscr{L}_0^{\rm eff}$ reads (in momentum space)

$$
\begin{aligned}
\mathscr{L}^0_{\rm eff}=\sum_i \frac{1}{4(4\pi)^{n/2}}\Gamma\left[-\frac{n}{2}\right]
&\times\int_0^1 dx\left\{m_i^{\,n}\left[h-\frac{h^2}{2}\right]-\tfrac{1}{4}L_i^{\,n/2}\left[2h_{(2)}+\frac{n^2-2n-4}{4}h^2\right]\right.\\
&\quad-\frac{n}{4}L_i^{\,n/2-1}\left[(1-2x)^2(p_r h^{\mu\alpha})^2-[1-(n+2)x(1-x)]p_\mu p_\nu h^{\mu\nu}h\right.\\
&\qquad\left.+\left[\tfrac{1}{4}-\frac{n}{4}x(1-x)\right]p^2h^2+m_i^{\,2}\left[1-\frac{n}{2}\right]h^2\right]\\
&\quad\left.+\frac{n}{2}\left[1-\frac{n}{2}\right]L_i^{\,n/2-2}\,[x(1-x)(p_\alpha p_\beta h^{\alpha\beta}-\tfrac{1}{2}p^2h)-\tfrac{1}{2}m_i^{\,2}h]^2\right\}+O(h^3)
\end{aligned}
\qquad (A1)
$$

with $h=h^\lambda{}_\lambda$, $h_{(2)}=h^{\mu\nu}h_{\mu\nu}$, and $L_i=m_i^{\,2}-x(1-x)p^2$.

B. Spinor pregeometry

The expansion of the logarithm in Eq. (3.10) is given in Ref. 3. The n-dimensional representation of $\mathscr{L}_{1/2}^{\rm eff}$ reads (in momentum space)

$$
\begin{aligned}
\mathscr{L}^{\rm eff}_{1/2}=\sum_i \frac{1}{2(2\pi)^{n/2}}\Gamma\left[-\frac{n}{2}\right]
&\times\int dx\left\{m_i^{\,n}(\varphi-\varphi^2)-\tfrac{1}{4}L_i^{\,n/2}[-(n-2)\varphi_{(2)}+(n^2-2)\varphi^2]\right.\\
&\quad-\frac{n}{4}L_i^{\,n/2-1}\left[[m_i^{\,2}+x(1-x)p^2]\varphi_{(2)}+\left[-\frac{n-2}{4}+(n-4)x(1-x)\right](p_\mu\varphi^{\mu\nu})^2\right.\\
&\qquad+\left[-\frac{n}{2}+(2n+4)x(1-x)\right]p_\mu p_\nu\varphi^{\mu\nu}\varphi\\
&\qquad\left.+\left[-2(n-2)+\frac{3n-2}{4}p^2-(2n+2)x(1-x)p^2\right]\varphi^2\right]\\
&\quad+\frac{n}{2}\left[1-\frac{n}{2}\right]L_i^{\,n/2-2}(\tfrac{1}{4}x(1-x)(1-2x)^2[p^2(p_\mu\varphi^{\mu\nu})^2+2p^2(p_\mu p_\nu\varphi^{\mu\nu})\varphi-p^4\varphi^2-2(p_\mu p_\nu\varphi^{\mu\nu})^2]\\
&\qquad+m_i^{\,2}\{-x(1-x)p^2\varphi^2+(\tfrac{1}{2}-x)^2[(p_\mu\varphi^{\mu\nu})^2+2(p_\mu p_\nu\varphi)\varphi\\
&\qquad\qquad\left.-3p^2\varphi^2]\})\right\}+O(\varphi^3)\,,
\end{aligned}
\qquad (A2)
$$

where $\varphi^{\mu\nu}=e^{\mu\nu}-\eta^{\mu\nu}=\frac{1}{2}h^{\mu\nu}+O(h^2)$, $\varphi=\varphi^{\lambda}{}_{\lambda}$, $\varphi_{(2)}=\varphi^{\mu\nu}\varphi_{\mu\nu}$, and $L_i=m_i{}^2-x(1-x)p^2$.

C. Vector pregeometry

We expand Eq. (3.20) as

$$\int \mathscr{L}_G^{\text{eff}}=\frac{i}{2}N_1\operatorname{Tr}\ln(\Box\eta^{\rho\sigma}+v^{\rho\sigma})+\text{const}=-\frac{i}{2}N_1\sum_{l=1}^{\infty}\frac{1}{l}\operatorname{Tr}\left[\left(-\frac{1}{\Box}V^{\beta}_{\alpha}\right)^l\right]+\text{const}\,, \tag{A3}$$

where

$$V^{\rho\sigma}=\partial_\mu(\eta^{\mu\nu}\rho^{\rho\sigma}+\eta^{\rho\sigma}\rho^{\mu\nu}+\eta^{\mu\rho}\rho^{\nu\sigma}+\eta^{\nu\sigma}\rho^{\mu\rho}-\eta^{\mu\sigma}\rho^{\rho\nu}-\eta^{\rho\nu}\rho^{\mu\sigma}+\rho^{\mu\nu}\rho^{\rho\sigma}+\rho^{\mu\rho}\rho^{\nu\sigma}-\rho^{\mu\sigma}\rho^{\rho\nu})\partial_\nu-B^{\rho}(\eta^{\nu\sigma}+\rho^{\nu\sigma})\partial_\nu$$
$$+\partial_\mu(\eta^{\mu\rho}+\rho^{\mu\rho})B^{\sigma}-B^{\rho}B^{\sigma} \tag{A4}$$

with

$$B^{\lambda}=\rho^{\alpha\lambda}{}_{,\alpha}-\frac{1}{n-4}(\rho_{,\lambda}+\rho^{\lambda\sigma}\rho_{,\sigma}-\rho_{\alpha\beta}\rho^{\alpha\beta,\lambda})\,,$$
$$\rho^{\mu\nu}=(-g)^{-1/4}g^{\mu\nu}-\eta^{\mu\nu}=h^{\mu\nu}-\tfrac{1}{4}\eta^{\mu\nu}h+O(h^2)\,, \tag{A5}$$
$$\rho=\rho^{\lambda}{}_{\lambda}\,.$$

Each term in Eq. (A3) corresponds to gauge-boson-loop diagrams. Performing the loop-momentum integration, we get (in n-dimensional momentum space)

$$\mathscr{L}_G^{\text{eff}}=N_1(4\pi)^{-n/2}\Gamma\left(-\frac{n}{2}\right)$$
$$\times\int_0^1 dx\left[\left(\frac{n(n-4)}{8}h-\frac{n}{16}h_{(2)}-\frac{n^2-16n+16}{32}h^2\right)\lambda^{n/2}\right.$$
$$+\tfrac{1}{16}L^{n/2}[n(n+4)h_{(2)}+\tfrac{1}{4}(n^3-6n^2-4n+16)h^2]$$
$$-\frac{n}{32}L^{n/2}x(1-x)[(n+2)p^2h_{(2)}+2n(p_\mu h^{\mu\nu})^2-\tfrac{1}{2}(n^2-8)p_\mu p_\nu h^{\mu\nu}h+\tfrac{1}{4}(n^24n-4)p^2h^2]$$
$$+\frac{n}{64}L^{n/2-1}[3p^2h_{(2)}+(n-2)(p_\mu h^{\mu\nu})^2-(n-4)p_\mu p_\nu h^{\mu\nu}h+\tfrac{1}{4}(n-8)p^2h^2]$$
$$+\tfrac{1}{16}n(n-2)L^{n/2-2}x^2(1-x)^2\left(p^4h_{(2)}-(n-2)p^2p_\mu p_\nu h^{\mu\nu}h+\frac{n-4}{4}p^4h^2+n(p_\mu p_\nu h^{\mu\nu})^2\right)$$
$$\left.-\tfrac{1}{8}n(n-2)L^{n/2-2}x(1-x)[p^2(p_\mu h^{\mu\nu})^2-\tfrac{1}{2}p^2p_\mu p_\nu h^{\mu\nu}h]+O(h^3)\right]\,, \tag{A6}$$

with $h=h^{\lambda}{}_{\lambda}$, $h_{(2)}=h^{\mu\nu}h_{\mu\nu}$, and $L=\lambda^2-x(1-x)p^2$, and λ is taken as zero after scale cutoff. The $\mathscr{L}_{\text{FP}}^{\text{eff}}$ is -2 times the result of the scalar pregeometry [up to $O(\kappa)$].

We apply the formula (2.4) to Eqs. (A1), (A2), and (A6), and get linear combinations of the following forms (up to a constant term):

$$1-\tfrac{1}{2}h+\tfrac{1}{4}h_{(2)}+\tfrac{1}{8}h^2,\quad (h^{\mu\nu}{}_{,\lambda})^2-2(h^{\mu\alpha}{}_{,\alpha})^2+2h^{\mu\alpha}{}_{,\alpha}h_{,\mu}-(h_{,\mu})^2\,,\quad (h^{\alpha\beta}{}_{,\alpha\beta})^2-2h^{\alpha\beta}{}_{,\alpha\beta}h^{,\gamma}{}_{\gamma}+(h^{,\alpha}{}_{\alpha})^2\,,$$

and

$$2(h^{\alpha\beta}{}_{,\alpha\beta})^2+(h_{\mu\nu}{}^{,\alpha}{}_{\alpha})^2-2(h^{\alpha\mu}{}_{,\alpha\nu})^2-2h^{\alpha\beta}{}_{,\alpha\beta}h^{,\gamma}{}_{\gamma}+(h^{,\alpha}{}_{\alpha})^2\,, \tag{A7}$$

which are the weak-field expansions of $\sqrt{-g}$, $\sqrt{-g}R$, $\sqrt{-g}R^2$, $\sqrt{-g}R_{\mu\nu}R^{\mu\nu}$, respectively, up to $O(h^2)$. Then, because of general covariance of the calculations, we uniquely get the final results (3.4), (3.11), (3.22), and (3.23).

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