PHYSICAL REVIEW D VOLUME 24, NUMBER 12 15 DECEMBER 1981

Interacting supergravity in ten dimensions: The role of the six-index gauge field

Ali H. Chamseddine
Physics Department, Northeastern University, Boston, Massachusetts 02115
(Received 3 September 1981)

We construct the interaction of supergravity with Yang-Mills supersymmetry in 10 dimensions. The formulation is only possible with the use of a six-index antisymmetric gauge field in place of the familiar two-index field in the supergravity sector. We are interested in breaking supersymmetry and thus apply the generalized dimensional reduction. The nonpropagating field strength $F_{\mu\nu\rho\sigma\alpha\beta\gamma}$ plays a vital role in making the scalar potential nontrivial. It would be possible to test the predictions of this model once the internal-symmetry group is specified and solutions of the potential minima are obtained.

I. INTRODUCTION

Since the foundation of supersymmetry[1,2] and extended supergravity,[3,4] it was hoped that they could accommodate the present particle phenomenology. Two models offered some hope. The $N=4$ supersymmetric Yang-Mills theory[5] with its remarkable finiteness properties unifies a vector, a quartet of two-component Dirac spinors, and a sextet of scalars, all in the adjoint representation of the internal-symmetry group. The physical use of this theory was problematic essentially due to the difficulty of spontaneously breaking supersymmetry.[6] It was later realized that this difficulty could be resolved by coupling supersymmetry theory to supergravity in 10 dimensions, and then employing the generalized dimensional reduction.[7] However, this last model has not been worked out. On the other hand, $N=8$ supergravity, the most general realization of local supersymmetry (with only one graviton), is too restrictive, with the vectors lying in the adjoint representation of SO(8).[8] Interesting predictions are obtained only after postulating the dynamical generation of an SU(8) current and assuming the decoupling of the higher-spin fields.[9] Even then, it is difficult to proceed in determining the effective field theory.[10]

Thus the only possibility left for investigation is supergravity ($N=1$) coupled to matter in 10 dimensions. This model is intimately related to the spinning-string theory.[11] The matter multiplet belongs to the adjoint representation of a Yang-Mills group which, to start with, is arbitrary.

In four dimensions, before symmetry breaking, this corresponds to $N=4$ supergravity coupled to six vector matter multiplets (the dual spinor model of closed strings) all interacting with $N=4$ supersymmetry Yang-Mills matter (the dual spinor model of open strings).[12] The fact that the spin-$\frac{1}{2}$ fermions also lie in the adjoint representation of the internal-symmetry group would limit the possible groups. It was demonstrated[13] that only SO($N\geq 11$) and the exceptional groups E_6, E_7, and E_8, can accommodate in their adjoint representations conventional quark-lepton generations classified by the SU(5) scheme.[14] There will be four copies of the above multiplets together with the $V+A$ conjugates and many other particles. The main test is to obtain, with the few allowed parameters in the theory, an adequate breaking mechanism that keeps only the desired particle spectrum.

Having outlined our motive, we shall address ourselves in this paper to the construction of the coupled 10-dimensional supergravity theory, and to study the general properties of this model. This will pave the way for any future study along the above lines.

Free supergravity theory in 10-dimensions has recently been constructed.[15] It contains, in particular, a second-rank antisymmetric tensor. To couple it to Yang-Mills supersymmetry in 10 dimensions we proceed by using the Noether-current method[16] order by order in the gravitational coupling k. This program, surprisingly, runs into difficulty. We find a term in the supersymmetry variation of the action that cannot be canceled by the available field variations in the supergravity sector. To be more specific, the spoiling term implies the existence of a sixth-rank antisymmetric tensor among the supergravity fields.

But we know that a totally antisymmetric six-index gauge field, $A_{M_1\cdots M_6}$, has the same number of physical degrees of freedom as $A_{M_1M_2}$ in 10 dimensions, given by $C_6^{10-2}=C_2^{10-2}=28$.

This suggests that an alternative form of supergravity exists with $A_{M_1\cdots M_6}$ replacing $A_{M_1M_2}$. Only the new form with $A_{M_1\cdots M_6}$ will couple to supersymmetry. This is in complete contrast with 11-dimensional supergravity where the construction is possible with the three-index field but not with the six-index field.[17]

The generalized dimensional reduction of this theory is rather involved and the fields are completely entangled. The particle spectrum can only be deduced correctly after the symmetry breaking. The field strengths $F_{\mu\nu\rho\sigma\alpha\beta\gamma}$ play a special role in this model in contributing to the scalar potential,[18] making it nontrivial. We did not commit ourselves to any particular model because the results are sensitive to the choice of the internal-symmetry group and the group of internal transformations. This program needs a separate study.

The plan of this paper is as follows. In Sec. II we construct pure supergravity in 10 dimensions using the six-index antisymmetric gauge field. In Sec. III we obtain the coupling to Yang-Mills supersymmetry (up to quartic fermionic terms). In Sec. IV we perform the generalized dimensional reduction on the bosonic fields (the reduction of the fermionic-bosonic interaction terms is extremely lengthy and it would simplify a great deal if we first specify the symmetry breaking). We also determine the scalar potential after substituting the contributions of $F_{\mu\nu\rho\sigma\alpha\beta\gamma}$. In Sec. V we give the conclusion.

II. SUPERGRAVITY IN 10 DIMENSIONS WITH THE SIX-INDEX POTENTIAL

Supergravity in 10 dimensions was constructed with the following fields[15]: the 10 bien, the Majorana-Weyl gravitino, a Majorana-Weyl spinor, a scalar, and an antisymmetric tensor.

As we have discussed in the Introduction, it is essential to formulate supergravity theory again with the six-index gauge field $A_{M_1\cdots M_6}$ replacing $A_{M_1M_2}$, in order to be able to couple it to Yang-Mills supersymmetry.

We first write down the most general expressions for the variation of the fields e_M^A, ϕ, $A_{M_1\cdots M_6}$, ψ_M, and χ. We then vary the linearized Lagrangian

$$-\frac{1}{4k^2}[R(\omega)]^{\text{lin}}-\frac{i}{2}\bar{\psi}_M\Gamma^{MNP}\partial_N\psi_P+\frac{i}{2}\chi\Gamma^M\partial_M\chi+\frac{1}{2\times 7!}F_{M_1\cdots M_7}F^{M_1\cdots M_7}+\tfrac{1}{2}(\partial_M\phi)(\partial^M\phi)\,.$$

Supersymmetry invariance then rules out a possible term, for $\delta A_{M_1\cdots M_6}$, of the form $\bar{\epsilon}\Gamma^M_{M_1\cdots M_6}\psi_M$ and fixes the coefficients of all other terms in a function of a single constant. This, in turn, is determined by requiring the field transformations to form a representation of the supersymmetry algebra.

The order-by-order construction determines the Lagrangian to be

$$\begin{aligned}\mathcal{L}=&-\frac{1}{4k^2}VR(\omega)+\frac{V}{2\times 7!}\exp(2k\phi)F_{M_1\cdots M_7}F^{M_1\cdots M_7}+\frac{V}{2}(\partial_M\phi)(\partial^M\phi)-\frac{iV}{2}\bar{\Psi}_M\Gamma^{MNP}D_N\left(\frac{\omega+\hat{\omega}}{2}\right)\Psi_P+\frac{iV}{2}\bar{\chi}\Gamma^M D_M\left(\frac{\omega+\hat{\omega}}{2}\right)\chi\\&+\frac{ikV}{2\times 8!}\exp(2k\phi)[\bar{\Psi}_M(\Gamma^{MNM_1\cdots M_7}-42g^{MM_1}\Gamma^{M_2\cdots M_6}g^{M_7N})\Psi_N+i\sqrt{2}\,\bar{\Psi}_M(\Gamma^{MM_1\cdots M_7}-7g^{MM_1}\Gamma^{M_2\cdots M_7})\chi]\\&\times(\hat{F}_{M_1\cdots M_7}+F_{M_1\cdots M_7})+\frac{kV}{2\sqrt{2}}(\partial_M\phi+\hat{D}_M\phi)(\bar{\Psi}^M\chi)\,,\end{aligned}\tag{2.1}$$

where

$$V=\det e_M^A\tag{2.2}$$

and all quartic terms are absorbed in the supercovariant quantities. We note that we made use of the previous formulation of this theory[15] in grouping the terms in the form appearing in (2.1).

The gravitational connection obtained from the equations of motion is

$$\begin{aligned}\omega_{MAB}=\omega_{OMAB}(e,\partial e)+\frac{ik^2}{4}\Big[&-\bar{\Psi}_N\Gamma^{NP}{}_{MAB}\Psi_P+i\sqrt{2}\,\bar{\chi}\Gamma^P{}_{MAB}\Psi_P+\tfrac{5}{9}\bar{\chi}\Gamma_{MAB}\chi+2(\bar{\Psi}_M\Gamma_B\Psi_A-\bar{\Psi}_M\Gamma_A\Psi_B+\bar{\Psi}_B\Gamma_M\Psi_A)\\&+\frac{i\sqrt{2}}{3}(2e_{M[A}\bar{\Psi}_{B]}\chi-\bar{\Psi}_M\Gamma_{AB}\chi)\Big]\,,\end{aligned}\tag{2.3}$$

while the supercovariant form of it is

$$\hat{\omega}_{MAB}=\omega_{OMAB}(e,\partial e)+\frac{ik^2}{4}\left[2(\bar{\Psi}_M\Gamma_B\Psi_A-\bar{\Psi}_M\Gamma_A\Psi_B+\bar{\Psi}_B\Gamma_M\Psi_A)-\tfrac{1}{36}\bar{\chi}\Gamma_{MAB}\chi+\frac{i\sqrt{2}}{3}(2e_{M[A}\bar{\Psi}_{B]}\chi-\bar{\Psi}_M\Gamma_{AB}\chi)\right].\tag{2.4}$$

The gauge field $A_{M_1\cdots M_6}$ is subject to the Abelian gauge transformations

$$\delta A_{M_1\cdots M_6}=6\partial_{[M_1}\Lambda_{M_2\cdots M_6]}\tag{2.5}$$

and the field strength invariant under (2.5) is

$$F_{M_1\cdots M_7}=7\partial_{[M_1}A_{M_2\cdots M_7]}\,.\tag{2.6}$$

The Lagrangian (2.1) is invariant under the supersymmetry transformations

$$\delta e_M^A=-ik\bar{\epsilon}\Gamma^A\Psi_M\,,$$

$$\delta\phi=-\frac{1}{\sqrt{2}}\bar{\epsilon}\chi,$$

$$\delta A_{M_1\cdots M_6}=-3i\exp(-k\phi)\left(\bar{\epsilon}\Gamma_{[M_1\cdots M_5}\Psi_{M_6]}+\frac{i}{6\sqrt{2}}\bar{\epsilon}\Gamma_{M_1\cdots M_6}\chi\right),$$

$$\delta\Psi_M=\frac{1}{k}D_M(\hat{\omega})\epsilon+\frac{1}{6!}\exp(k\phi)(3\Gamma_M^{M_1\cdots M_7}-7\delta_M^{M_1}\Gamma^{M_2\cdots M_7})\epsilon\hat{F}_{M_1\cdots M_7}, \tag{2.7}$$

$$\delta\chi=\frac{1}{k\sqrt{2}}\hat{D}_M\phi\epsilon+\frac{i}{2\sqrt{2}\times 7!}\exp(k\phi)(\Gamma^{M_1\cdots M_7}\epsilon)\hat{F}_{M_1\cdots M_7}.$$

The supercovariant field strength $\hat{F}_{M_1\cdots M_7}$ appearing in (2.1) and (2.7) is given by

$$\hat{F}_{M_1\cdots M_7}=F_{M_1\cdots M_7}-\frac{21i}{\sqrt{2}}\exp(-k\phi)\left(\bar{\Psi}_{[M_1}\Gamma_{M_2\cdots M_6}\Psi_{M_7]}+\frac{i}{3\sqrt{2}}\bar{\Psi}_{[M_1}\Gamma_{M_2\cdots M_7]}\chi+\tfrac{1}{72}\bar{\chi}\Gamma_{M_1\cdots M_7}\chi\right). \tag{2.8}$$

Also,

$$\hat{D}_M\phi=\partial_M\Psi+\frac{1}{\sqrt{2}}\bar{\Psi}_M\chi. \tag{2.9}$$

The Majorana spinors Ψ_M, χ, and ϵ are also subject to the Weyl condition,

$$\gamma^{11}\Psi_M=\Psi_M,\quad \gamma^{11}\chi=-\chi,\quad \gamma^{11}\epsilon=\epsilon, \tag{2.10}$$

$$\gamma^{11}=i\Gamma^0\Gamma^1\cdots\Gamma^9,\quad (\gamma^{11})^2=1. \tag{2.11}$$

The two conditions can be imposed simultaneously only in space-times of dimensions 10 modulo 8.

To conclude this section, we note that the Lagrangian (2.1) is equivalent, on the mass shell and after duality transformations are performed, to the Lagrangian involving the gauge field $A_{M_1M_2}$.

III. COUPLING TO YANG-MILLS SUPERSYMMETRY IN 10 DIMENSIONS

Yang-Mills supersymmetry in 10 dimensions has a very simple structure. It consists of a vector and a Majorana-Weyl spinor, both in the adjoint representation of the group. In four dimensions this corresponds to $N=4$ supersymmetry, a theory whose β function vanishes at the three-loop level[19] (and may vanish to all orders). In this theory it is very difficult to break supersymmetry,[6] and it appears that the most natural way to generate the breaking is by coupling the theory to supergravity (in 10 dimensions). The flat-space limit of the spontaneously broken theory ($k\to 0$) would not be invariant under supersymmetry transformations because the variation of the Goldstone spinor fields is proportional to k^{-1}.[7] Thus the coupling to supergravity is crucial to generate the breaking mechanism.

The generators of the internal-symmetry gauge group H obey the algebra

$$[X_i,X_j]=-iC_{ijk}X_k, \tag{3.1}$$

where the structure constants C_{ijk} are real and completely antisymmetric, and X_i are normalized such that

$$\mathrm{Tr}(X_iX_j)=\delta_{ij}. \tag{3.2}$$

The vector and spinor matrices are defined by

$$A_M=A_M^iX_i,\quad \lambda=\lambda^iX_i, \tag{3.3}$$

and the Majorana spinor λ is also subject to the Weyl condition

$$\gamma^{11}\lambda=\lambda. \tag{3.4}$$

We then have for the field strength and spinor covariant derivative

$$G_{MN}=\partial_MA_N-\partial_NA_M+ig[A_M,A_N],\qquad D_M\lambda=\partial_M\lambda+ig[A_M,\lambda]. \tag{3.5}$$

The supersymmetric Lagrangian reads[5]

$$\mathcal{L}_{\mathrm{YM}}=V\,\mathrm{Tr}\left(-\tfrac{1}{4}G_{MN}G^{MN}+\frac{i}{2}\bar{\lambda}\Gamma^MD_M\lambda\right) \tag{3.6}$$

and is invariant under the supersymmetry transformations

$$\delta A_M=\frac{i}{\sqrt{2}}\bar{\epsilon}\Gamma_M\lambda,\qquad \delta\lambda=\frac{1}{2\sqrt{2}}(\Gamma^{MN}\epsilon)G_{MN}. \tag{3.7}$$

Promoting the transformations (3.7) from global to local, by making the spinor ϵ space-time dependent $\epsilon(X)$, a new term $N^M\partial_M\epsilon$ appears in the variation of the action. This term can be eliminated by adding to the action the interaction term[20]

$$-\frac{iVk}{2\sqrt{2}}\mathrm{Tr}(\bar{\lambda}\Gamma^M\Gamma^{NP}\Psi_MG_{NP}). \tag{3.8}$$

This would make the action supersymmetric at order k^0. At order k, all terms of the form $G^2\Psi\epsilon$ in the variation of the action cancel, except the term

$$-\frac{kV}{8\times 5!}\epsilon^{NPQRM_1\cdots M_6}\bar{\epsilon}\Gamma_{M_1\cdots M_5}\Psi_{M_6}\mathrm{Tr}(G_{NP}G_{QR}). \tag{3.9}$$

This term would be impossible to cancel if we did not have the six-index gauge potential at hand. By using it we can cancel (3.9) by adding to the Lagrangian the interaction term

$$\frac{iVk}{4\times 6!}\exp(k\phi)A_{M_1\cdots M_6}\mathrm{Tr}(G_{NP}G_{QR})\epsilon^{NPQRM_1\cdots M_6}. \tag{3.10}$$

The variation of $A_{M_1\cdots M_6}$ given in (2.7) involves also the spinor χ and would thus generate a new term from (3.10) that can be canceled by adding to the Lagrangian the interaction term

$$\frac{Vk}{4}\mathrm{Tr}(\bar\lambda\Gamma^{QR}\chi G_{QR})\,. \tag{3.11}$$

Finally, the variation of (3.11) will leave one uncanceled term:

$$\frac{k}{4\sqrt{2}}\bar\epsilon\chi\,\mathrm{Tr}(G_{NP}G^{NP})\,. \tag{3.12}$$

Evidently, this would cancel if the scalar field ϕ couples nonpolynomially to the Yang-Mills kinetic term, in the form

$$-\tfrac{1}{4}V\exp(-k\phi)\,\mathrm{Tr}(G_{MN}G^{MN})\,. \tag{3.13}$$

This, in turn, will fix the coupling of the scalar field ϕ with matter and in the transformation rules. Variations of the form $\lambda\epsilon FG$ would cancel out completely if we add the following term to the Lagrangian:

$$\frac{ikV}{2\times 7!}\exp(k\phi)\,\mathrm{Tr}(\bar\lambda\Gamma^{M_1\cdots M_7}\lambda)F_{M_1\cdots M_7}\,. \tag{3.14}$$

In principle, we can continue our procedure to determine all quartic terms (of order k^2) in the action, and cubic terms in the transformation rules, and the modification of $\delta\Psi_M$ and $\delta\chi$ by λ. This presumably can be done by finding the supercovariant field strength $\hat G_{MN}$ and covariant derivative $\hat D_M\lambda$. However, we shall not dwell on this laborious calculation here because quartic terms in the action will hardly affect any physical results that might be obtained from this model (except when quantum loop calculations are concerned). Our results, up to cubic terms in the transformations and quartic terms in the action, are

$$\delta A_M=\frac{i}{\sqrt{2}}\exp(\tfrac{1}{2}k\phi)\bar\epsilon\Gamma_M\lambda\,,$$
$$\delta\lambda=\frac{1}{2\sqrt{2}}\exp(-\tfrac{1}{2}k\phi)(\Gamma^{MN}\epsilon)G_{MN}\,, \tag{3.15}$$

$$\begin{aligned}\mathcal{L}_{\mathrm{matter}}=V\,\mathrm{Tr}\Big[&-\tfrac{1}{4}\exp(-k\phi)G_{MN}G^{MN}+\frac{i}{2}\bar\lambda\Gamma^M(\partial_M\lambda+ig[A_M,\lambda]+\tfrac{1}{4}\omega_{MAB}\Gamma^{AB}\lambda)\\&-\frac{ik}{2\sqrt{2}}\exp(-\tfrac{1}{2}k\phi)\Big(\bar\lambda\Gamma^M\Gamma^{NP}\Psi_M+\frac{i}{\sqrt{2}}\bar\lambda\Gamma^{NP}\chi\Big)G_{NP}+\frac{ik}{4\times 6!}V^{-1}\epsilon^{NPQRM_1\cdots M_6}A_{M_1\cdots M_6}G_{NP}G_{QR}\\&+\frac{ik}{4\times 7!}\bar\lambda\Gamma^{M_1\cdots M_7}\lambda F_{M_1\cdots M_7}\Big]\,.\end{aligned} \tag{3.16}$$

Later we shall need an equivalent form of the term

$$\frac{ik}{4\times 6!}\epsilon^{NPQRM_1\cdots M_6}A_{M_1\cdots M_6}\,\mathrm{Tr}(G_{NP}G_{QR})\,, \tag{3.17}$$

involving only the field strength $F_{M_1\cdots M_7}$. This can be achieved by noting that

$$\epsilon^{NPQRM_1\cdots M_6}\,\mathrm{Tr}(G_{NP}G_{QR})=2\epsilon^{NPQRM_1\cdots M_6}\mathrm{Tr}\,\partial_N\Big[A_P\Big(G_{QR}-\frac{2ig}{3}A_QA_R\Big)\Big]\,. \tag{3.18}$$

Thus, up to a total divergence, (3.17) is equivalent to

$$\frac{i}{2\times 7!}\epsilon^{PQRM_1\cdots M_7}F_{M_1\cdots M_7}\,\mathrm{Tr}\Big[A_P\Big(G_{QR}-\frac{2ig}{3}A_QA_R\Big)\Big]\,. \tag{3.19}$$

IV. REDUCTION TO FOUR DIMENSIONS

We are now ready to reduce the theory down to four dimensions. We shall use the generalized dimensional reduction as prescribed by Scherk and Schwarz.[7] All fields depend on the internal coordinates in such a way that the final action is y independent. The details of reducing the different fields are given in Ref. 7, except for the six-index gauge field. Here we shall be as brief as possible.

The results for ordinary dimensional reduction (or the absence of symmetry breaking) are easy to predict. One must get $N=4$ supergravity interacting with six vector multiplets from the supergravity sector all interacting with $N=4$ Yang-Mills supersymmetry from the matter sector. However, in the case of spontaneous breakdown of symmetry, the spectrum depends on the particular breaking under consideration. Therefore, our main aim is to determine the scalar potential of the theory. Breaking the symmetry and obtaining the particle spectrum for a particular internal group H deserves a separate study.

Our notation differs slightly from that of Ref. 7. We use early capital Latin letters for tangent space indices (A, B, $C,\ldots$) and late Latin letters for the curved indices ($M,N,P,\ldots$) in the 10-dimensional space. For space-time indices taking four-values, late Greek letters are used in the curved case ($\mu,\nu,\rho,\ldots$) and late Latin letters in the flat case

$(m,n,p,\ldots)$. For internal indices taking six-values, early Greek letters are used in the curved case $(\alpha,\beta,\gamma,\ldots)$ and early Latin letters in the flat case $(a,b,c,\ldots)$. The general coordinates x^M consist of space-time coordinates x^M and internal coordinates y^α. For convenience, we shall use caretted indices for the components of the 10-dimensional fields, and noncaretted indices for the four-dimensional rescaled fields.

We can extract a Lie group G of six generators out of the general coordinate transformations whose infinitesimal parameters $\xi^M(x,y)$ are given by

$$\begin{aligned}&\xi^{\hat\mu}(x,y)=\xi^\mu(x)\,,\\&\xi^{\hat\alpha}(x,y)=[U^{-1}(y)]^\alpha_\beta\xi^\beta(x)\,.\end{aligned}\tag{4.1}$$

The structure constants of the group, determined from the commutator of two internal transformations, are

$$f^\gamma_{\alpha\beta}=(U^{-1})^{\alpha'}_\alpha(U^{-1})^{\beta'}_\beta(\partial_{\beta'}U^\gamma_{\alpha'}-\partial_{\alpha'}U^\gamma_{\beta'})\,.\tag{4.2}$$

A general rule for the dependence of the fields on the internal coordinates is that every field with a lower internal Greek index must be accompanied by a $U(y)$ factor and every upper internal Greek index by a $U^{-1}(y)$. For example, in a special gauge, the 10-bien is written as

$$\begin{aligned}&e^{\hat r}_{\hat\mu}(x,y)=(\delta)^{-1/8}e^r_\mu(x)\,,\\&e^{\hat a}_{\hat\mu}(x,y)=(\delta)^{-1/8}2k\,(A^\alpha_\mu(x)e^a_\alpha(x))\,,\\&e^{\hat r}_{\hat\alpha}(x,y)=0\,,\\&e^{\hat a}_{\hat\alpha}(x,y)=(\delta)^{-1/8}[U(y)]^\beta_\alpha e^a_\beta(x)\,,\end{aligned}\tag{4.3}$$

where

$$\delta=\det(e^a_\alpha)\,,\quad e=\det(e^r_\mu)\,.\tag{4.4}$$

Then

$$V=\det(e^A_M)=(\delta)^{-1/4}e\,.\tag{4.5}$$

Now we shall give the resulting contributions of the bosonic terms in the Lagrangians (2.1) and (3.16), in terms of the four-dimensional components.

A. The gravity sector

The curvature reduces to

$$\begin{aligned}-\frac{1}{4k^2}VR(\omega)=&-\frac{e}{4k^2}R_4(\omega)-\tfrac14 g^{\mu\rho}g^{\nu\sigma}h_{\alpha\beta}F^\alpha_{\mu\nu}F^\beta_{\rho\sigma}\\&-\frac{1}{16k^2}g^{\mu\nu}(D_\mu h_{\alpha\beta})(D_\nu h^{\alpha\beta})\\&-\frac{1}{16k^2}f^\gamma_{\alpha\beta}(2f^\beta_{\alpha\gamma'}+f^{\alpha'}_{\beta'\gamma'}h_{\alpha\alpha'}h^{\beta\beta'})h^{\gamma\gamma'}\,,\end{aligned}\tag{4.6}$$

where

$$\begin{aligned}&g_{\mu\nu}=e^r_\mu\eta_{rs}e^s_\nu\,,\\&h_{\alpha\beta}=-e^a_\alpha\eta^{ab}e^b_\beta=e^a_\alpha\delta_{ab}e^b_\beta\,,\end{aligned}\tag{4.7}$$

and $F^\alpha_{\mu\nu}$ is the gauge field strength of the group G:

$$F^\alpha_{\mu\nu}=\partial_\mu A^\alpha_\nu-\partial_\nu A^\alpha_\mu-2kf^\alpha_{\beta\gamma}A^\beta_\mu A^\gamma_\nu\,.\tag{4.8}$$

The covariant derivatives $D_\mu h_{\alpha\beta}$ and $D_\mu h^{\alpha\beta}$ can be deduced from

$$\begin{aligned}&D_\mu e^a_\alpha=\partial_\mu e^a_\alpha-2kf^\gamma_{\alpha\beta}A^\beta_\mu e^a_\gamma\,,\\&D_\mu e^\alpha_a=\partial_\mu e^\alpha_a-2kf^\alpha_{\beta\gamma}A^\beta_\mu e^\gamma_a\,.\end{aligned}\tag{4.9}$$

The structure constants $f^\gamma_{\alpha\beta}$ provide some degrees of freedom to the model. Maximally, they can describe the group SO(4) ~SU(2) × SU(2).

We note that a 10-dimensional space-time is a special case in which the Weyl scale factor δ is absorbed completely and do not appear in (4.6). The last term in (4.6) is the first contribution to the total scalar potential.

B. The scalar field

We simply set

$$\phi(x,y)=\phi(x)\,,\tag{4.10}$$

and the kinetic term would reduce to

$$\frac{V}{2}g^{MN}\partial_M\phi\partial_N\phi=\frac{e}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\,.\tag{4.11}$$

C. The six-index antisymmetric gauge field

The gauge field $A_{M_1\cdots M_6}$ transforms under Abelian and general coordinate transformations:

$$\begin{aligned}\delta A_{M_1\cdots M_6}=&6\partial_{[M_1}\Lambda_{M_2\cdots M_6]}+6\partial_{[M_1}\xi^N A_{N|M_2\cdots M_6]}\\&+\xi^N\partial_N A_{M_1\cdots M_6}\,.\end{aligned}\tag{4.12}$$

The y dependence of the A fields is specified according to the general rule we stated:

$$\begin{aligned}&A_{\hat\mu_1\cdots\hat\mu_6}=A_{\hat\mu_1\cdots\hat\mu_5\hat\alpha}=0\,,\\&A_{\hat\mu_1\cdots\hat\mu_4\hat\alpha_1\hat\alpha_2}=U^{\alpha_1'}_{\alpha_1}(y)U^{\alpha_2'}_{\alpha_2}(y)A_{\mu_1\cdots\mu_4\alpha_1'\alpha_2'}(x)\,,\\&A_{\hat\mu_1\cdots\hat\mu_3\hat\alpha_1\cdots\hat\alpha_3}(x,y)=U^{\alpha_1'}_{\alpha_1}(y)\cdots U^{\alpha_3'}_{\alpha_3}(y)A_{\mu_1\cdots\mu_3\alpha_1'\cdots\alpha_3'}(x)\,,\end{aligned}\tag{4.13}$$

and so on.

The x-dependent A fields transforms noncovariantly under ξ^α transformations. To find the fields that transform covariantly, we define the tangent-space form

$$A'_{A_1\cdots A_6}(x)\equiv\delta^{-3/4}e^{M_1}_{A_1}\cdots e^{M_6}_{A_6}A_{M_1\cdots M_6}(x,y)\,.\tag{4.14}$$

Then the desired fields are

$$\begin{aligned}&A'_{\mu_1\mu_2\mu_3\alpha_1\alpha_2\alpha_3}(x)\equiv e^{r_1}_{\mu_1}\cdots e^{r_3}_{\mu_3}A_{r_1\cdots r_3a_1\cdots a_3}(x)\,,\\&A'_{\mu_1\mu_2\alpha_1\cdots\alpha_4}(x)\equiv e^{r_1}_{\mu_1}e^{r_2}_{\mu_2}e^{a_1}_{\alpha_1}\cdots e^{a_4}_{\alpha_4}A_{r_1r_2a_1\cdots a_4}\,,\end{aligned}\tag{4.15}$$

and so on.

The expressions (4.15) imply

$$\begin{aligned} A'_{\alpha_1\cdots\alpha_6} &= A_{\alpha_1\cdots\alpha_6}\,,\\ A'_{\mu\alpha_1\cdots\alpha_5} &= A_{\mu\alpha_1\cdots\alpha_5} + 2kA^{\alpha_6}_{\mu}A_{\alpha_1\cdots\alpha_6}\,,\\ A'_{\mu_1\mu_2\alpha_1\cdots\alpha_4} &= A_{\mu_1\mu_2\alpha_1\cdots\alpha_4} + 4kA^{\alpha_5}_{[\mu_1}A_{\mu_2]\alpha_1\cdots\alpha_5}\\ &\quad + A^{\alpha_5}_{\mu_1}A^{\alpha_6}_{\mu_2}A_{\alpha_1\cdots\alpha_6}\,,\\ A'_{\mu_1\mu_2\mu_3\alpha_1\alpha_2\alpha_3} &= A_{\mu_1\mu_2\mu_3\alpha_1\alpha_2\alpha_3} + 6kA^{\alpha_4}_{[\mu_1}A_{\mu_2\mu_3]\alpha_1\cdots\alpha_4}\\ &\quad + 12k^2A^{\alpha_4}_{[\mu_1}A^{\alpha_5}_{\mu_2}A_{\mu_3]\alpha_1\cdots\alpha_5}\\ &\quad + 8k^3A^{\alpha_4}_{\mu_1}A^{\alpha_5}_{\mu_2}A^{\alpha_6}_{\mu_3}A_{\alpha_1\cdots\alpha_6}\,. \end{aligned} \tag{4.16}$$

Similarly, we define the tangent-space field strength

$$F'_{A_1\cdots A_7} \equiv \delta^{-7/8}e^{M_1}_{A_1}\cdots e^{M_7}_{A_7}F_{M_1\cdots M_7}\,, \tag{4.17}$$

and

$$\begin{aligned} F_{\mu_1\cdots\mu_4\alpha_1\cdots\alpha_3} &= e^{r_1}_{\mu_1}\cdots e^{r_4}_{\mu_4}e^{a_1}_{\alpha_1}\cdots e^{a_3}_{\alpha_3}F_{r_1\cdots r_4a_1\cdots a_3}\,,\\ F'_{\mu_1\cdots\mu_3\alpha_1\cdots\alpha_4} &= e^{r_1}_{\mu_1}\cdots e^{r_3}_{\mu_3}e^{a_1}_{\alpha_1}\cdots e^{a_4}_{\alpha_4}F'_{r_1\cdots r_3a_1\cdots a_4}\,. \end{aligned} \tag{4.18}$$

We deduce then that

$$\begin{aligned} F'_{\mu\alpha_1\cdots\alpha_6} &= D_\mu A'_{\alpha_1\cdots\alpha_6} + 15f^{\gamma}{}_{[\alpha_1\alpha_2}A'_{\mu|\alpha_3\cdots\alpha_6]\gamma}\,,\\ F'_{\mu_1\mu_2\alpha_1\cdots\alpha_5} &= 2D_{[\mu_1}A'_{\mu_2]\alpha_1\cdots\alpha_5} - 2kF^{\alpha_6}_{\mu_1\mu_2}A'_{\alpha_1\cdots\alpha_6}\,,\\ F'_{\mu_1\cdots\mu_3\alpha_1\cdots\alpha_4} &= 3D_{[\mu_1}A'_{\mu_2\mu_3]\alpha_1\cdots\alpha_4}\\ &\quad - 6kF^{\alpha_5}_{[\mu_1\mu_2}A'_{\mu_3]\alpha_1\cdots\alpha_5}\,,\\ F'_{\mu_1\cdots\mu_4\alpha_1\cdots\alpha_3} &= 4D_{[\mu_1}A'_{\mu_2\mu_3\mu_4]\alpha_1\cdots\alpha_3}\\ &\quad - 12kF^{\alpha_4}_{[\mu_1\mu_2}A'_{\mu_3\mu_4]\alpha_1\cdots\alpha_4}\,. \end{aligned} \tag{4.19}$$

The covariant derivatives in (4.19) are defined with respect to the group G:

$$\begin{aligned} D_\mu A'_{\alpha_1\cdots\alpha_6} &= \partial_\mu A'_{\alpha_1\cdots\alpha_6} + 6(2k)f^{\gamma}{}_{[\alpha_1|\beta}A^{\beta}_{\mu}A_{\alpha_2\cdots\alpha_6]\gamma}\,,\\ D_{\mu_1}A'_{\mu_2\alpha_1\cdots\alpha_5} &= \partial_{\mu_1}A'_{\mu_2\alpha_1\cdots\alpha_5} - 5(2k)f^{\gamma}{}_{[\alpha_1|\beta}A^{\beta}_{\mu_1}A'_{\mu_2\alpha_2\cdots\alpha_5]\gamma}\,,\\ D_{\mu_1}A'_{\mu_2\mu_3\alpha_1\cdots\alpha_4} &= \partial_{\mu_1}A'_{\mu_2\mu_3\alpha_1\cdots\alpha_4}\\ &\quad + 4(12k)f^{\gamma}{}_{[\alpha_1|\beta}A^{\beta}_{\mu_1}A'_{\mu_2\mu_3\alpha_2\cdots\alpha_4]\gamma}\,, \end{aligned} \tag{4.20}$$

and so on. Therefore, the reduced kinetic term is

$$\begin{aligned} \frac{V}{2\times 7!}\exp(2k\phi)F_{M_1\cdots M_7}F^{M_1\cdots M_7} = \frac{e}{2\times 6!}\delta^{3/2}\exp(2k\phi)(&-5F'_{\mu_1\cdots\mu_4\alpha_1\cdots\alpha_3}F'_{\mu'_1\cdots\mu'_4\alpha'_1\cdots\alpha'_3}g^{\mu_1\mu'_1}\cdots g^{\mu_4\mu'_4}h^{\alpha_1\alpha'_1}\cdots h^{\alpha_3\alpha'_3}\\ &+5F'_{\mu_1\cdots\mu_3\alpha_1\cdots\alpha_4}F'_{\mu'_1\cdots\mu'_3\alpha'_1\cdots\alpha'_4}g^{\mu_1\mu'_1}\cdots g^{\mu_3\mu'_3}h^{\alpha_1\alpha'_1}\cdots h^{\alpha_4\alpha'_4}\\ &-3F'_{\mu_1\mu_2\alpha_1\cdots\alpha_5}F'_{\mu'_1\mu'_2\alpha'_1\cdots\alpha'_5}g^{\mu_1\mu'_1}g^{\mu_2\mu'_2}h^{\alpha_1\alpha'_1}\cdots h^{\alpha_5\alpha'_5}\\ &+F'_{\mu\alpha_1\cdots\alpha_6}F'_{\mu'\alpha'_1\cdots\alpha'_6}g^{\mu\mu'}h^{\alpha_1\alpha'_1}\cdots h^{\alpha_6\alpha'_6})\,. \end{aligned} \tag{4.21}$$

D. The Yang-Mills sector

This part has the invariance under the gauge group G gauged by A^α_μ and the Yang-Mills group H gauged by A^i_μ.

The y dependence is given by

$$\begin{aligned} A^i_{\hat\mu}(x,y) &= A^i_\mu(x)\,,\\ A^i_{\hat\alpha}(x,y) &= U^{\beta}_{\alpha}(y)A^i_{\beta}(x)\,, \end{aligned} \tag{4.22}$$

just as we did in the previous subsection C, we define the vector A'^i_μ which is covariant under the ξ^α transformations:

$$\begin{aligned} A'^i_\mu &= A^i_\mu - 2kA^\alpha_\mu A^i_\alpha\,,\\ A'^i_\alpha &= A^i_\alpha\,. \end{aligned} \tag{4.23}$$

We also define the tangent-space field strength

$$G'_{A_1A_2} \equiv \delta^{-1/4}e^{M_1}_{A_1}e^{M_2}_{A_2}G_{M_1M_2} \tag{4.24}$$

and the components

$$\begin{aligned} G'^i_{\mu\nu} &\equiv e^{r_1}_{\mu}e^{r_2}_{\nu}G'^i_{r_1r_2}\,,\\ G'^i_{\mu\alpha} &\equiv e^{r}_{\mu}e^{a}_{\alpha}G'^i_{ra}\,,\\ G'^i_{\alpha\beta} &\equiv e^{a}_{\alpha}e^{b}_{\beta}G'^i_{ab}\,. \end{aligned} \tag{4.25}$$

The results of these definitions are

$$\begin{aligned} G'^i_{\mu\nu} &= (\partial_\mu A'^i_\nu - \partial_\nu A'^i_\mu + gC^i_{jk}A'^j_\mu A'^k_\nu) + 2kF^{\alpha}_{\mu\nu}A^i_\alpha\,,\\ G'^i_{\mu\alpha} &= (\partial_\mu A^i_\alpha + gC^i_{jk}A^j_\alpha A^k_\nu - 2kf^{\gamma}_{\alpha\beta}A^i_\gamma A^\beta_\nu)\,,\\ G'^i_{\alpha\beta} &= -(f^{\gamma}_{\alpha\beta} - gC^i_{jk}A^i_\alpha A^k_\beta)\,. \end{aligned} \tag{4.26}$$

The reduced Yang-Mills kinetic term gives

$$\begin{aligned} -\frac{V}{4}\exp(-k\phi)\,\mathrm{Tr}(G_{MN}G^{MN}) = e\delta\tfrac{1}{4}\exp(-k\phi)[&-\tfrac14 G'^i_{\mu\nu}G^{\mu\nu i} + \tfrac12 h^{\alpha\beta}g^{\mu\nu}G'^i_{\mu\alpha}G'^i_{\nu\beta}\\ &-\tfrac14 h^{\alpha\alpha'}h^{\beta\beta'}(f^{\gamma}_{\alpha\beta}A^i_\gamma - gC^i_{jk}A^j_\alpha A^k_\beta)(f^{\gamma'}_{\alpha'\beta'}A^i_{\gamma'} - gC^i_{j'k'}A^{j'}_{\alpha'}A^{k'}_{\beta'})]\,. \end{aligned} \tag{4.27}$$

The last term in (4.27) is the second contribution to the total scalar potential.

E. The bosonic interaction term

This term is of particular importance because it would make it possible for the nonpropagating field strength $F_{\mu\nu\rho\sigma\alpha\beta\gamma}$ to give a contribution to the scalar potential and to still further mix the fermionic mass terms.

We were able to write this interaction term in the form (3.19). Taking into account the previous definitions, we obtain

$$\frac{ik}{2\times 6!}\epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha_1\cdots\alpha_6}\Big[F'_{\sigma\alpha_1\cdots\alpha_6}A'^{i}_{\mu}\big(G'^{i}_{\nu\rho}+\tfrac{2}{3}C_{ijk}A'^{j}_{\nu}A'^{k}_{\rho}\big)-3F'_{\rho\sigma\alpha_1\cdots\alpha_5}\big(2A'^{i}_{\mu}G'^{i}_{\nu\alpha_6}+A^{i}_{\alpha_6}G'^{i}_{\mu\nu}+gC_{ijk}A'^{i}_{\mu}A'^{j}_{\nu}A^{k}_{\alpha_6}\big)$$
$$+5F'_{\nu\rho\sigma\alpha_1\cdots\alpha_4}\big(A'^{i}_{\mu}G'^{i}_{\alpha_5\alpha_6}-2A^{i}_{\alpha_5}G'^{i}_{\mu\alpha_6}+gC_{ijk}A'^{k}_{\mu}A^{i}_{\alpha_5}A^{j}_{\alpha_6}\big)$$
$$+5F'_{\mu\nu\rho\sigma\alpha_1\alpha_2\alpha_3}A^{i}_{\alpha_4}\big(\tfrac{2}{3}gC^{i}_{jk}A^{j}_{\alpha_5}A^{k}_{\alpha_6}-f^{\gamma}_{\alpha_5\alpha_6}A^{i}_{\gamma}\big)\Big]. \tag{4.28}$$

Now we can make use of the fact that $A'_{\mu\nu\rho\alpha_1\alpha_2\alpha_3}$ appears only through its field strength $F'_{\mu\nu\rho\sigma\alpha_1\alpha_2\alpha_3}$, and because in this case the Bianchi identity $\partial_{[k}F'_{\mu\nu\rho\sigma]\alpha_1\alpha_2\alpha_3}$ imposes no restriction on $F'_{\mu\nu\rho\sigma\alpha_1\alpha_2\alpha_3}$, we can consider $F'_{\mu\nu\rho\sigma\alpha_1\alpha_2\alpha_3}$ as the independent field and solve for its equations of motion.[18] It appears quadratically in (4.21) and linearly in (4.28). The equations of motion imply

$$F^{\mu\nu\rho\sigma}=\frac{ik}{2}\exp(-2k\phi)\,\delta^{-3/2}(e^{-1}\epsilon^{\mu\nu\rho\sigma})\,\epsilon^{\alpha_1\cdots\alpha_6}A^{i}_{\alpha_4}\big(\tfrac{2}{3}gC_{ijk}A^{j}_{\alpha_5}A^{k}_{\alpha_6}-f^{\gamma}_{\alpha_5\alpha_6}A^{i}_{\gamma}\big)+\text{terms involving the fermions}. \tag{4.29}$$

Substituting (4.29) back into the Lagrangian gives the following contribution to the scalar potential:

$$3\frac{k^2e}{4}(\delta)^{\frac{1}{2}}\exp(-2k\phi)h^{\alpha\alpha'}h^{\beta\beta'}h^{\gamma\gamma'}A^{i}_{\alpha}\big(\tfrac{2}{3}gC^{i}_{jk}A^{j}_{\beta}A^{k}_{\gamma}-f^{\delta}{}_{\beta\gamma}A^{i}_{\delta}\big)\big(\tfrac{2}{3}gC^{i'}_{i'k'}A^{i'}_{[\alpha'}A^{j'}_{\beta'}A^{k'}_{\gamma']}-A^{i'}_{[\alpha'}f^{\delta'}{}_{\beta'\gamma']}A^{i'}_{\delta'}\big). \tag{4.30}$$

Therefore, the full scalar potential of the theory is

$$V(\phi,h_{\alpha\beta},A^{i}_{\alpha})=\frac{1}{16k^2}f^{\alpha}_{\beta\gamma}\big[2f^{\beta}_{\alpha\gamma'}+f^{\alpha'}_{\beta'\gamma'}h_{\alpha\alpha'}h^{\beta\beta'}\big]h^{\gamma\gamma'}$$
$$+\tfrac{1}{4}(\delta)^{\frac{1}{4}}\exp(-k\phi)h^{\alpha\alpha'}h^{\beta\beta'}\big(f^{\gamma}_{\alpha\beta}A^{i}_{\gamma}-gC_{ijk}A^{j}_{\alpha}A^{k}_{\beta}\big)\big(f^{\gamma'}_{\alpha'\beta'}A^{i}_{\gamma'}-gC_{ij'k'}A^{j'}_{\alpha'}A^{k'}_{\beta'}\big)$$
$$-\frac{3k^2}{4}\delta^{\frac{1}{2}}\exp(-2k\phi)h^{\alpha\alpha'}h^{\beta\beta'}h^{\gamma\gamma'}\Big(A^{i}_{\alpha}\big(\tfrac{2}{3}gC_{ijk}A^{j}_{\beta}A^{k}_{\gamma}-f^{\delta}_{\beta\gamma}A^{i}_{\delta}\big)\Big)\big(\tfrac{2}{3}gC_{i'j'k'}A^{i'}_{[\alpha'}A^{j'}_{\beta'}A^{k'}_{\gamma']}-A^{i'}_{[\alpha'}f^{\delta'}_{\beta'\gamma']}A^{i'}_{\delta'}\big) \tag{4.31}$$

minimizing the potential (4.31) in its general form, with arbitrary $f^{\alpha}{}_{\beta\gamma}$ and C_{ijk} is not a trivial matter. We are only interested in solutions such that the potential is zero at its minimum (implying the vanishing of the cosmological constant) and is positive otherwise. This last requirement will restrict our freedom in choosing the group G. To see this, we first write the positive-definite internal metric $h_{\alpha\beta}$ in the form $h_{\alpha\beta}=\exp(2kS)_{\alpha\beta}$ and expand around $S_{\alpha\beta}=0$:

$$h_{\alpha\beta}=\delta_{\alpha\beta}+2kS_{\alpha\beta}+2k^2S_{\alpha\gamma}S_{\gamma\beta}+\cdots. \tag{4.32}$$

Then the condition

$$V(\phi=0,S_{\alpha\beta}=0,A^{i}_{\alpha}=0)=\frac{1}{16k^2}f^{\alpha}_{\beta\gamma}(2f^{\beta}_{\alpha\gamma}+f^{\alpha}_{\beta\gamma})>0 \tag{4.33}$$

would impose severe restrictions of $f^{\alpha}_{\beta\gamma}$ and can only be satisfied by almost-trivial groups. For semisimple groups (4.33) is not satisfied.

Therefore, we have to find the class of groups satisfying (4.33) and which yields the desired minima for a specific choice of H.

F. The fermionic contributions

Apart from the resulting fermionic kinetic terms, the spin-$\frac{1}{2}$ fields are completely entangled, and in order to obtain meaningful results we have to obtain the breaking mechanism. This is the program of our future study.

V. CONCLUSION

We have constructed a supersymmetric model which is a candidate for superunification. The theory is supergravity coupled to Yang-Mills supersymmetry in 10 dimensions. Before symmetry breaking the spectrum described $N=4$ supergravity coupled to $N=4$ Yang-Mills matter.

We determine that the theory is only possible to construct with a six-index antisymmetric gauge field. The resulting spectrum in general depends on the breaking mechanism. Therefore, it is important to study the scalar potential, which we find to be nontrivial. A special role is played by the nonpropagating field strength $F_{\mu\nu\rho\sigma\alpha\beta\gamma}$.

Although the internal-symmetry group H is arbitrary, the property that the spin-$\frac{1}{2}$ fermions lie in the adjoint representation H would limit our choice to one of the groups[13] SO($N\geq 11$) and the exceptional groups E_6, E_7, E_8. Moreover, we are constrained by the requirements of a non-negative potential and the vanishing of the cosmological constant.

We hope to return to this project in a future publication.

ACKNOWLEDGMENTS

I would like to thank Professor P. Nath and Dr. N. S. Baaklini for many useful discussions. I am also grateful to Professor R. Arnowitt for a critical reading of the manuscript. This research was supported in part by the National Science Foundation under Grant No. PHY-80-0833.

---

[1]J. Wess and B. Zumino, Nucl. Phys. B78, 1 (1974).

[2]A. Salam and J. Strathdee, Phys. Rev. D 11, 1521 (1975).

[3]D. Z. Freedman, D. van Nieuwenhuizen, and S. Ferrara, Phys. Rev. D 13, 3214 (1976); S. Deser and B. Zumino, Phys. Lett. 62B, 335 (1976).

[4]S. Ferrara and P. van Nieuwenhuizen, Phys. Rev. Lett. 37, 1664 (1976); D. Z. Freedman, Phys. Lett. 38, 105 (1977); S. Ferrara, J. Scherk, and B. Zumino, *ibid.* 66B, 35 (1977); E. Cremmer, J. Scherk, and S. Ferrara, *ibid.* 68B, 234 (1977); E. Cremmer and J. Scherk, Nucl. Phys. B127, 259 (1977).

[5]L. Brink, J. H. Schwarz, and J. Scherk, Nucl. Phys. B121, 171 (1977).

[6]P. Fayet, Nucl. Phys. B149, 137 (1977).

[7]J. Scherk and J. H. Schwarz, Nucl. Phys. B153, 61 (1979).

[8]B. de Wit and D. Z. Freedman, Nucl. Phys. B130, 105 (1977); E. Cremmer, B. Julia, and J. Scherk, Phys. Lett. 76B, 409 (1978); E. Cremmer and B. Julia, Nucl. Phys. B159, 141 (1979).

[9]J. Ellis, M. K. Gaillard, and B. Zumino, Phys. Lett. 94B, 343 (1980).

[10]B. Zumino, in *Proceedings of the Nuffield Supergravity Workshop*, edited by S. Hawking and M Roĉek (Cambridge University Press, Cambridge, 1981), p. 423.

[11]J. H. Schwarz, in *New Frontiers in High Energy Physics*, proceedings of the Orbis Scientiae, Coral Gables, 1978, edited by Arnold Perlmutter and Linda F. Scott (Plenum, New York, 1978).

[12]F. Gliozzi, J. Scherk, and D. Olive, Nucl. Phys. B122, 253 (1977).

[13]N. S. Baaklini, Phys. Rev. D 22, 3118 (1980).

[14]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 483 (1974).

[15]Ali H. Chamseddine, Nucl. Phys. B185, 403 (1981).

[16]S. Ferrara, F. Gliozzi, J. Scherk, and P. van Nieuwenhuizen, Nucl. Phys. B117, 333 (1976).

[17]H. Nicolai, P. K. Townsend, and P. van Nieuwenhuizen, Lett. Nuovo Cimento 30, 315 (1981).

[18]A. Aurilia, H. Nicolai, and P. K. Townsend, Nucl. Phys. B176, 509 (1981); M. Duff and P. van Nieuwenhuizen, Phys. Lett. 94B, 179 (1980).

[19]L. F. Abbott and M. T. Grisaru, Nucl. Phys. B154, 429 (1979); M. T. Grisaru, M. Roĉek, and W. Siegel, Phys. Rev. Lett. 45, 1063 (1980); Nucl. Phys. B183, 141 (1981); W. Caswell and D. Zanon *ibid.* B182, 152 (1981).

[20]P. van Nieuwenhuizen, Phys. Rep. 68C, 189 (1981).