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Invisible axion and neutrino oscillation in SU(11)

Jihn E. Kim
Department of Physics, Seoul National University, Seoul 151, Korea
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A global $U(1)_A$ symmetry is automatically embedded in the SU(11) model of Georgi which is the only known flavor unification with exactly 45 chiral fermions. The Higgs-field representations are 165's, 462's, and an adjoint. The $U(1)_A$ symmetry locates the ordinary fermions. Spontaneous breaking of $U(1)_A$ results in an invisible axion and Majorana masses of neutrinos of order $\lesssim 0.1-1$ eV.

Georgi's flavor-unification model[1] in SU(11) is very attractive in many respects. It is the minimal model which includes only three generations of nonexotic[2] quarks and leptons with nonrepetition[3] of fundamental representations. So far there have not appeared any interesting predictions from this model, mainly because it contains too many fermions (561 chiral) and Yukawa couplings are too complicated. At low energy there are exactly 45 chiral fields.

In this paper, we present, for the first time in this model,[4] reliable predictions: an invisible axion, mass bound for ν of order 0.1−1 eV, and location of ordinary fermions in the SU(11) representation. At first, this task seems to be hopeless due to the enormous spectrum of fermions. However, it has been suggested,[5] and recently emphasized,[6] that some fermion masses can be radiatively generated. This may hint that Yukawa couplings and the Higgs potential are in fact very simple.

To locate the low-energy fermions in the SU(11) representation, we need a symmetry; otherwise the location depends on the couplings, which is undesirable. There is a plausible candidate for such a symmetry: Peccei and Quinn's axial $U(1)_A$ symmetry.[7] This symmetry is needed for automatic strong CP invariance. At low energy there appears an axion[8] the mass of which depends on the symmetry-breaking scheme.[9]

The SU(11) can have an automatic $U(1)_A$, by introducing only the Higgs fields H^{abc}, H^{abcde}, and H^a_b, where a, b, c, etc., are SU(11) indices. Indices in H^{abc} and H^{abcde} are completely antisymmetrized. H^a_b is an adjoint Higgs field (real or complex). The number of introduced Higgs scalars is not restricted. The content of the surviving Higgs doublets is not known (hierarchy problem), but it does not affect our considerations in this paper. The fermion representation is $\psi^{abcd}(330)$, $\psi_{abc}(165^*)$, $\psi_{ab}(55^*)$, and $\psi_a(11^*)$. The $U(1)_A$ relevant Yukawa couplings and Higgs potential are

$$\begin{aligned}\mathcal{L} = {} & f_1\tilde{\psi}^{abcd}C\psi^{efgh}H^{ijk}\epsilon_{abcdefghijk} + f_2\tilde{\psi}^{abcd}C\psi_d H_{abc} + f_3\tilde{\psi}_{ab}C\psi_c H^{abc} + f_4\tilde{\psi}_{abc}C\psi_{de}H^{abcde} \\ & + MH^{abcde}H_1^{fgh}H_2^{ijk}\epsilon_{abcdefghijk} + (\text{irrelevant terms}) \ .\end{aligned} \tag{1}$$

Since we have completely general couplings, Higgs fields of the same type have the same global U(1) symmetry. Six of the seven U(1) phases (four fermion and two Higgs fields) are related by the couplings in (1), leading to a symmetry $SU(11)\times U(1)_A$. The resulting $U(1)_A$ symmetry is the automatic consequence of the Higgs representation. We denote the generator of this $U(1)_A$ symmetry as X; the X quantum numbers for the fields are

$$\begin{aligned} & X(\psi_a) = -3, \quad X(\psi_{ab}) = 5, \quad X(\psi_{abc}) = -9 \ , \\ & X(\psi^{abcd}) = 1, \quad X(H^{abc}) = -2 \ , \\ & X(H^{abcde}) = 4, \quad X(H^a_b) = 0 \ . \end{aligned} \tag{2}$$

Since there are 1, 9, 36, and 84 quark fields in ψ_a, ψ_{ab}, ψ_{abc}, and ψ^{abcd}, respectively, the X symmetry is the desired Peccei-Quinn symmetry,[9] viz., $1(-3)+9(5)+36(-9)+84(1)\neq 0$. The adjoint Higgs field is responsible only for separating $SU(3)_c$ and SU(2), keeping intact the X symmetry. Therefore, we classify fermions and Higgs scalars using SU(5) indices $\alpha, \beta, \gamma, \ldots = 1, 2, 3, 4$, and 5. Since the difference of the ranks of SU(11) and SU(5) is six, there are six more gauge U(1) symmetries whose generators are defined by

$$\begin{aligned} & Y_a = \mathrm{diag}(2,2,2,2,2,-5,-5,0,0,0,0) \ , \\ & Y_b = \mathrm{diag}(4,4,4,4,4,4,4,-7,-7,-7,-7) \ , \\ & Y_c = \mathrm{diag}(0,0,0,0,0,1,-1,0,0,0,0) \ , \\ & Y_d = \mathrm{diag}(0,0,0,0,0,0,0,1,-1,0,0) \ , \\ & Y_e = \mathrm{diag}(0,0,0,0,0,0,0,0,0,1,-1) \ , \\ & Y_f = \mathrm{diag}(0,0,0,0,0,0,0,1,1,-1,-1) \ . \end{aligned} \tag{3}$$

There are 27 SU(5)-singlet components in the Higgs representation: 20 in H^{abc}, and 7 in H^{abcde}. Giving vacuum expectation values (VEV's) to all the singlet components, we break the $SU(11)\times U(1)_A$ down to the SU(5) of Georgi and Glashow. But it is better to break the symmetry, at least as an illustration, first to $SU(5)\times U(1)_\Gamma$, so that we can classify SU(5) fermions. (The resulting physical implications do not depend on this specific pattern of symmetry breaking. In fact, we will assume in the end that all VEV's are comparable.) A natural choice for the six desired independent components are the six singlets (except H^{12345}) in H^{abcde}. They carry Y_a and Y_b hypercharges denoted as $H(Y_a,Y_b)$,

$$\begin{aligned}&H^{7890l}(-5,-24),\quad H^{6890l}(-5,-24)\ ,\\ &H^{6790l}(-10,-13),\quad H^{6780l}(-10,-13)\ ,\\ &H^{6789l}(-10,-13),\quad H^{67890}(-10,-13)\end{aligned} \tag{4}$$

The VEV's of the above fields lead to a global symmetry of the form $\Gamma=25X+aY_a+bY_b+cY_c+dY_d+eY_e+fY_f$. It turns out that $c=d=e=f=0$, and the remaining symmetry is

$$\Gamma=25X+\tfrac{1}{7}(44Y_a+20Y_b)\ . \tag{5}$$

This Γ is the one we employ for the study of fermion location. Since any SU(5)-singlet direction in SU(11) space is not preferred by (4), 5's of SU(5) descending from the same representation in SU(11) have the same Γ charge, etc. Therefore, we can easily find the Γ charges of 561 chiral fermions which we denote as $\psi(\Gamma)$,

$$\begin{aligned}&\psi^{\alpha\beta\gamma\delta}(121)[1],\quad \psi^{\alpha\beta\gamma6}\sim\psi^{\alpha\beta\gamma l}(77)[6]\ ,\\ &\psi^{\alpha\beta67}\sim\psi^{\alpha\beta0l}(33)[15]\ ,\\ &\psi^{\alpha678}\sim\psi^{\alpha90l}(-11)[20],\quad \psi^{6789}\sim\psi^{890l}(-55)[15]\ ,\\ &\psi_{\alpha\beta\gamma}(-297)[1],\quad \psi_{\alpha\beta6}\sim\psi_{\alpha\beta l}(-253)[6]\ ,\\ &\psi_{\alpha67}\sim\psi_{\alpha0l}(-209)[15],\quad \psi_{678}\sim\psi_{90l}(-165)[20]\ ,\\ &\psi_{\alpha\beta}(77)[1],\quad \psi_{\alpha6}\sim\psi_{\alpha l}(121)[6]\ ,\\ &\psi_{67}\sim\psi_{0l}(165)[15],\quad \psi_{\alpha}(-99)[1],\\ &\psi_6\sim\psi_l(-55)[6]\ ,\end{aligned} \tag{6}$$

where the number in square brackets is the number of SU(5) representations. It is clear from this classification that VEV's in (4) give masses only to SU(5) singlets: 15 of $\psi_{67}\sim\psi_{0l}$ and 15 out of $\psi_{678}\sim\psi_{90l}$. We should break the Γ symmetry to give masses to SU(5) real fermions. The other SU(5)-singlet components in antisymmetric Higgs fields carry nonvanishing Γ's,

$$\Gamma(H^{678}\sim H^{90l})=-110,\quad \Gamma(H^{12345})=220\ , \tag{7}$$

By giving VEV's ($\approx 10^{19}$ GeV) to these components, we break the global $U(1)_\Gamma$ symmetry and the resulting axion is invisible ($m_a\approx 10^{-14}$ eV).[10,11] Any SU(5) real fermion pair with total Γ charge equal to an integer multiple of 110 is removed: one (5^*) with $\Gamma=121$ and one (5) with $\Gamma=-11$, six (10^*) with $\Gamma=77$ and six (10) with $\Gamma=33$, one (10) with $\Gamma=-297$ and one (10^*) with $\Gamma=77$, and fifteen singlets with $\Gamma=-55$. These obtain masses at the tree level.

Unfortunately, there are too many fermions left light by tree graphs: $9(10)+6(10^*)+19(5)+22(5^*)$ in addition to 11 SU(5) singlets. The survival hypothesis[10] implies that they should get masses; otherwise we are left with additional global symmetries which, certainly, do not exist. These real fermions obtain masses.

One typical diagram is shown in Fig. 1 which represents removal of $(\psi^{\alpha\beta67}+\psi_{\alpha\beta6})$ at higher orders of Yukawa couplings and Higgs-potential parameters. The blob transforms like an effective H_7 which can couple to the fermion pair. The fermions which are removed at higher orders of Feynman diagrams are lighter than those removed at tree level. What is the possible lowest mass scale, say M_I, for these fermions? The mass scale for $\tilde{M}$ of SU(11) breaking to a low-energy gauge group [SU(5)] cannot be much smaller than $M_{\rm Pl}\equiv$ Planck mass; otherwise these relatively light fermions turn out to be too light and make coupling constants at 10^{15} GeV unacceptably large. We take therefore $\tilde{M}=M_{\rm Pl}$. This accords with Zee's original motivation for a spontaneously generated gravity.[12] Now we have at least three mass scales, M_W, $M_{\rm Pl}$, and $M_I=$ intermediate scale.[13] Do we need another mass scale for SU(5) breaking? Certainly we do, since the estimate of $\sin^2\theta_W(M_W)$ does not depend on the existence of extra fermions, viz., $\sin^2\theta_W=\frac{3}{8}-(55\alpha_{\rm EM}/24\pi)\ln(M_S/M_W)$ where M_S is the color-separation scale. (The Higgs-field

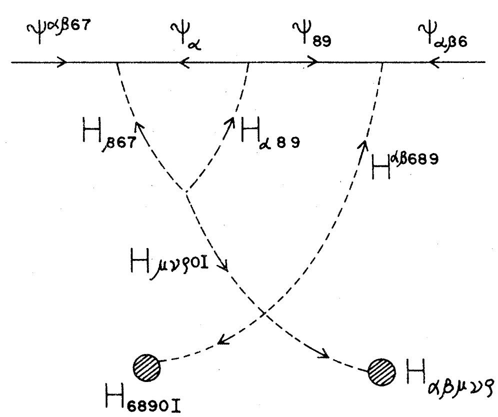

FIG. 1. A typical Feynman diagram which removes an SU(5) real fermion pair.

contribution is neglected. For one Higgs doublet, it is not important.) Above M_S, SU(5) is a good symmetry. Therefore, the estimate of $\sin^2\theta_W(M_W)$ and proton lifetime are identical to those of the SU(5) model.

For $\sin^2\theta_W=0.21$ [the SU(5) value] and $\Lambda=0.4$ GeV, $M_S=5.5\times10^{14}$ GeV.[14] For $M_I\approx10^{12}$ GeV, we obtain $\alpha_{11}=$ SU(11) unification constant at $M_{\rm Pl}=0.4$ which is little larger than $\alpha_c(M_W)$. Therefore, we set for simplicity $\alpha_c(M_W)=\alpha_{11}(M_{\rm Pl})$ and determine the intermediate mass scale M_I

$$M_I=M_W(M_{\rm Pl}/M_S)^{31/74}(M_S/M_W)^{53/74}$$
$$=0.84\times10^{13}\text{ GeV} \quad ,$$

for $\sin^2\theta_W=0.21$. This is the mass scale for heavy quarks and heavy leptons which get masses at higher orders. There are Majorana masses for SU(5)-singlet neutrinos surviving down to this scale. The flavor unification provides in general Dirac masses between these heavy neutrinos and the ordinary neutrinos. Taking this Dirac mass as the t-quark mass (up to mixing angles) which governs $Q=\frac{2}{3}$ quark masses, we obtain a rough upper bound for m_ν

$$m_\nu\lesssim m_t^2/M_I=0.05-1.2\text{ eV}$$

for $m_t=20-100$ GeV. This is an encouraging result.[15]

The low-energy fermions are located in ψ^{abcd} and ψ_{abc}. Three (10)'s belong to three combinations of $\psi^{\alpha\beta AB}$ ($A,B=6,7,\ldots,I$) and three (5*)'s belong to three combinations of $\psi_{\alpha AB}$. The exact forms of the six linear combinations for the finally surviving fermions depend on the VEV's (including signs) of SU(5)-singlet Higgs fields. Mixing angles and radiative fermion masses are deferred to a future communication.

We can introduce one or two (but not three or more) fundamental Higgs fields H^a and H'^a, preserving the automatic $U(1)_A$ symmetry. The surviving fermions are the same as discussed in this paper. However, the question, why there must not be three or more fundamentals, forbids us from consideration of this possibility.[16]

In conclusion, we have obtained reliable predictions out of the plethora of fermions in SU(11): the invisible axion, the neutrino-mass bound, and location of ordinary fermions. The automatic $U(1)_A$ symmetry[17] plays a pivotal role in classifying fermions of three different mass scales, 100, 10^{13}, and 10^{19} GeV.

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[3]This is a consequence of the principle postulated by Georgi.

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[13]M_I is a kind of average mass for fermions around $10^{12}-10^{14}$ GeV. In the following, M_I is defined as the effective fermion mass in the renormalization-group equation as if all these fermions are removed at M_I.

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[15]D. Silverman and A. Soni, Phys. Rev. Lett. 46, 467 (1981).

[16]In the scheme we presented in this paper, the t quark can get a mass at tree level, and all the other fermions must get masses at higher orders. With H^a included, this does not necessarily occur.

[17]Automatic embedding of the Peccei-Quinn symmetry has also been considered by H. M. Georgi, L. J. Hall, and M. B. Wise, Harvard University Report No. HUTP-81/A031, 1981 (unpublished), but their example is for one generation.