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Hyperfine splittings in heavy-quark systems

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The hyperfine splittings in heavy-quark systems are calculated to fourth order in the strong coupling constant. The ψ-η_c mass difference gives $\Lambda_{\overline{\rm MS}}=0.16\pm0.02\pm0.07$ GeV ($\overline{\rm MS}$ refers to the modified minimal-subtraction scheme) where the first uncertainty is experimental and the second is a crude estimate of the uncertainty arising from scheme dependence. From the Υ-η_b mass difference we expect a more reliable and accurate determination of Λ.

Nonrelativistic bound-state spectroscopies (such as hydrogen atom, positronium, etc.) have provided invaluable tools to guide us in our understanding of quantum mechanics and quantum electrodynamics. They also allow us to determine accurately the fine-structure constant and the electron mass. Nonrelativistic bound systems are even more important in quantum chromodynamics (QCD) since there are no free quarks or free gluons upon which direct measurements can be performed. Although our present understanding of the large-distance behavior of QCD is still far from complete, quarkonia (i.e., heavy-quark—antiquark bound systems) can provide an accurate determination of the scale parameter Λ and heavy-quark masses in QCD.[1–3] This is possible only after care is taken to separate the short-distance effects from the large-distance effects. We argue that this can be achieved for the hyperfine splitting. Here we present the results of the complete one-loop calculation for the hyperfine splitting of heavy-quark systems. The result is then applied to the ψ and Υ spectroscopies.

To lowest order in perturbative QCD, the hyperfine splitting[4] is proportional to the square of the bound-state wave function at the origin, $|\phi(0)|^2$. Thus the spin force, which is responsible for the hyperfine splitting, is short ranged. As we shall see, this force remains short ranged even after the one-loop corrections have been incorporated. For the wave function at short distances we need the quarkonium potential which, however, can be accurately determined via a phenomenological approach.[2,5] Hence we believe that perturbative QCD is applicable to calculate the hyperfine splitting. This is in contrast to the fine structure (e.g., the splitting of the P states), where the responsible spin-dependent force extends to large distances. Consequently, the fine structure is more sensitive to the nature of the confining force and the result derived from perturbative QCD may be unreliable in this case.

Our calculation is carried out in two steps. We first calculate, in momentum space, the effective Hamiltonian ΔH which governs the spin-spin interaction of a quark-antiquark pair with on-shell quark (mass m). In the second step we obtain the hyperfine splittings ΔE by evaluating expectation values of the Fourier transform of ΔH with the bound-state wave functions. The Feynman diagrams which contribute to ΔH are shown in Fig. 1. This set of graphs is necessarily gauge invariant as can be easily checked. The results for the individual diagrams given below refer to the Feynman gauge. We have used dimensional regularization[6] for ultraviolet divergences, and a small gluon mass λ to regularize infrared divergences. The result reads

$$\Delta H=\frac{8\pi}{3}\frac{\alpha_s^{(0)}}{m^2}C_2(R)\left[1+\frac{\alpha_s^{(0)}}{\pi}K^{(0)}\right], \qquad (1)$$

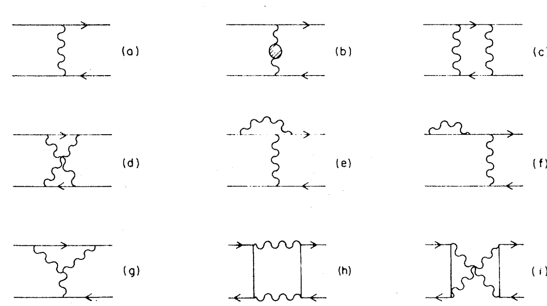

FIG. 1. Feynman diagrams contributing to the hyperfine splitting to fourth order in the strong coupling constant.

where $\alpha_s^{(0)}$ is the unrenormalized strong fine-structure constant, $\alpha_s = g^2/4\pi$, and $C_2(R) = \frac{4}{3}$. The first term arises from Fig. 1(a), and the various one-loop contributions are given by

$$K^{(0)}(1(\mathrm{b})) = \frac{1}{4}C_2(G)\left\{\frac{5}{3}\left[\frac{2}{\epsilon} + \ln(4\pi) - \gamma - \ln\left(\frac{Q^2}{\mu^2}\right)\right] + \frac{31}{9}\right\} - \frac{1}{4}T(R)N_f\left\{\frac{4}{3}\left[\frac{2}{\epsilon} + \ln(4\pi) - \gamma - \ln\left(\frac{Q^2}{\mu^2}\right)\right] + \frac{20}{9}\right\} , \tag{2}$$

$$K^{(0)}(1(\mathrm{c}) + 1(\mathrm{d})) = -\frac{3}{2}C_2(R) - \frac{3}{4}C_2(G)\left[\frac{1}{3} - \frac{1}{2}\ln\left(\frac{Q^2}{m^2}\right) + \frac{2}{3}\ln\left(\frac{Q^2}{\lambda^2}\right)\right] , \tag{3}$$

$$K^{(0)}(1(\mathrm{e}) + 1(\mathrm{f})) = C_2(R) - \frac{1}{4}C_2(G)\left[\frac{2}{\epsilon} + \ln(4\pi) - \gamma - \ln\left(\frac{m^2}{\mu^2}\right) + 6 + 2\ln\left(\frac{\lambda^2}{m^2}\right)\right] , \tag{4}$$

$$K^{(0)}(1(\mathrm{g})) = \frac{1}{4}C_2(G)\left\{3\left[\frac{2}{\epsilon} + \ln(4\pi) - \gamma - \ln\left(\frac{m^2}{\mu^2}\right) + \frac{4}{3}\right] + 2\ln\left(\frac{Q^2}{m^2}\right) + 2\right\} , \tag{5}$$

$$K^{(0)}(1(\mathrm{h}) + 1(\mathrm{i})) = \tfrac{9}{16}C_2(R)(1 - \ln 2) , \tag{6}$$

where we have systematically dropped terms of order Q^2/m^2. $\epsilon = 4 - D$ for D space-time dimensions, μ is the renormalization scale, Q the modulus of the spacelike momentum transfer, $\gamma = 0.5772\ldots$ the Euler constant, N_f the number of massless quark flavors, and in QCD the group factors read $T(R) = \frac{1}{2}$, $C_2(G) = 3$. In Eq. (3) the "$1/v$ singularity," which arises from Fig. 1(c), has been removed in the standard manner. Summing Eqs. (1)–(6) we obtain

$$\Delta H = \frac{8\pi}{3}\frac{\alpha_{\overline{\mathrm{MS}}}(\mu)}{m^2}C_2(R)\left[1 + \frac{\alpha_{\overline{\mathrm{MS}}}(\mu)}{\pi}K\right] , \tag{7a}$$

with

$$K = \tfrac{1}{16}(1 - 9\ln 2)C_2(R) + \tfrac{11}{18}C_2(G) - \tfrac{5}{9}T(R)N_f - \tfrac{1}{12}[11C_2(G) - 4T(R)N_f]\ln\left(\frac{Q^2}{\mu^2}\right) + \tfrac{7}{8}C_2(G)\ln\left(\frac{Q^2}{m^2}\right) , \tag{7b}$$

where we have used the modified minimal-subtraction ($\overline{\mathrm{MS}}$) scheme[7] to absorb the pole term as well as the constant $[\ln(4\pi) - \gamma]$ into the definition of the renormalized, scale-dependent coupling constant $\alpha_{\overline{\mathrm{MS}}}(\mu)$.[7] As expected, the infrared divergences which appear in Eqs. (3) and (4) have canceled in the sum Eq. (7b). The existence of the $\ln(Q^2/m^2)$ contribution in Eq. (7b) was first pointed out by Dine.[8] However, results quoted in the literature[8–10] for the coefficient of this term disagree with Eq. (7).

Nonperturbative effects[11] that can be absorbed into the potential would influence the wave function at short distances. We assume that any additional energy shift of the pseudoscalar state due to the U(1) anomaly is negligible. This assumption can eventually be tested in a quarkonium system where the quark and the antiquark carry different flavors [e.g., $(b\bar{c})$, $(t\bar{b})$, etc.]. In this case, the effective Hamiltonian is given by (m_1 and m_2 are the quark mass and the antiquark mass, respectively)

$$\Delta\tilde{H} = \frac{8\pi}{3}\frac{\alpha_{\overline{\mathrm{MS}}}(\mu)}{m_1 m_2}C_2(R)\,\vec{s}_1\cdot\vec{s}_2\left[1 + \frac{\alpha_{\overline{\mathrm{MS}}}(\mu)}{\pi}\tilde{K}\right] , \tag{8a}$$

with

$$\tilde{K} = \left[-3\left(\frac{m_1 m_2}{m_1^2 - m_2^2}\right)\ln\left(\frac{m_1}{m_2}\right) + 1\right]C_2(R) + \left[\frac{3}{8}\left(\frac{m_1 + m_2}{m_1 - m_2}\right)\ln\left(\frac{m_1}{m_2}\right) - \frac{5}{36}\right]C_2(G) - \frac{5}{9}T(R)N_f - \frac{1}{12}[11C_2(G) - 4T(R)N_f]\ln\left(\frac{Q^2}{\mu^2}\right) + \frac{7}{8}C_2(G)\ln\left(\frac{Q^2}{m_1 m_2}\right) . \tag{8b}$$

Note that the annihilation diagrams [Figs. 1(h), 1(i)] do not contribute in Eq. (8). Equations (7) and (8) are our final results.

From Eq. (7) one obtains the hyperfine splitting ΔE by evaluating the appropriate expectation values,[12]

$$\Delta E = \frac{32\pi}{9}\frac{\alpha_{\overline{MS}}(\mu)}{m^2}\langle 1\rangle\left[1+\frac{\alpha_{\overline{MS}}(\mu)}{\pi}\epsilon(\mu,\xi)\right] , \quad (9a)$$

with

$$\langle 1\rangle \equiv |\phi(0)|^2, \quad \xi \equiv \frac{\langle \ln(Q^2/m^2)\rangle}{\langle 1\rangle} , \quad (9b)$$

$$\epsilon(\mu,\xi) = 0.563 + 2.25\ln\left(\frac{\mu^2}{m^2}\right) + 0.375\xi , \quad (9c)$$

$$N_f = 3 .$$

The quantities $|\phi(0)|^2$ and ξ have to be evaluated in a specific potential model. From Ref. 2 we obtain[13] for J/ψ and Υ: $\xi(\psi) = 0.56$, $\xi(\Upsilon) = -0.26$. The smallness of ξ for ψ and Υ shows that momentum transfers of order m dominate the hyperfine splittings in both cases. Thus the appropriate number of flavors is $N_f = 3$ for ψ and $N_f = 4$ for Υ.

From the hyperfine splitting, the leptonic width[14] of the vector state (V), and the hadronic width[14] of the pseudoscalar state we can form the ratios r_1 and r_2, which are almost independent of the wave function at short distances, and hence any particular potential model that is used. Taking $\mu = m$, we have, for the ψ system with $N_f = 3$,

$$r_1 \equiv \frac{9}{8}e^2\alpha^2\frac{\Delta E}{\Gamma_{ee}^V} = \alpha_{\overline{MS}}(m)\left(1 + 6.1\frac{\alpha_{\overline{MS}}(m)}{\pi}\right) , \quad (10)$$

$$r_2 \equiv \frac{4}{3}\frac{\Gamma_{\rm had}^P}{\Delta E} = \alpha_{\overline{MS}}(m)\left(1 + 4.5\frac{\alpha_{\overline{MS}}(m)}{\pi}\right) , \quad (11)$$

where e and α are the quark charge and the electromagnetic fine-structure constant, respectively. Using[15] $\Gamma_{ee}^V(\psi) = 4.8 \pm 0.6$ keV and $\Delta E(\psi\text{-}\eta_c) = 119 \pm 9$ MeV, we obtain from the ratio r_1 ($N_f = 3$) and Eq. (10), $\Lambda_{\overline{MS}} = 0.40 \pm 0.07$ GeV (errors are experimental). However, the correction term in Eq. (10) is $\frac{3}{4}$ of the leading term, which clearly invalidates the perturbative approximation. Using the method suggested by Grunberg and others,[16] we obtain instead $\Lambda_{MS} = 0.170 \pm 0.02$ GeV. Using the wave functions of Ref. 2 and Eq. (9), we obtain $\Lambda_{\overline{MS}} = 0.35 \pm 0.16$ GeV, where the error includes the quark-mass uncertainty (see Ref. 2). Using the hadronic width of the η_c (Ref. 15), $\Gamma_{\eta_c} = 20^{+16}_{-11}$ MeV, we have, from r_2, $\Lambda_{\overline{MS}} = 55^{+110}_{-34}$ MeV. Taking the weighted average, we obtain

$$\Lambda_{\overline{MS}} = 0.16 \pm 0.02 \pm 0.07 \text{ GeV} ,$$

where the first uncertainty is experimental and the last is a crude estimate of the uncertainty due to scheme dependence. Also we predict $\Delta E(\psi'\text{-}\eta_c') = 80 \pm 10$ MeV.

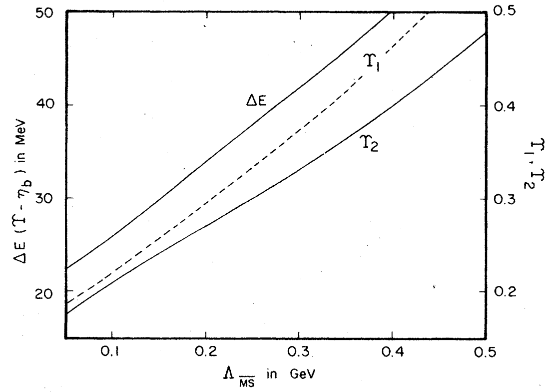

FIG. 2. Υ-η_b system. The hyperfine splitting $\Delta E(\Upsilon\text{-}\eta_b)$ and the ratios r_1 and r_2 (see text). For the hyperfine splitting, the wave functions of Ref. 2 have been used (Ref. 13). $N_f = 4$.

We conclude with the following remarks:

(i) The hypothesis that the hyperfine splittings of heavy-quark systems are dominated by short-distance effects is self-consistent.

(ii) As anticipated,[2,4] the radiative correction (i.e., the one-loop contribution) to the hyperfine splitting is small [$\epsilon(\psi) = 0.77$ in the $\overline{MS}$ scheme, see Eq. (9a)].

(iii) The inclusion of any additional U(1) anomaly effect would raise the η_c. This would result in an increase in the value of $\Lambda_{\overline{MS}}$.

(iv) The ψ-η_c mass difference gives a scale parameter compatible with the range of scale parameters ($\Lambda_{\overline{MS}} = 0.1-0.5$ GeV) considered in connection with the ψ and Υ spectroscopies.[2] However, theoretical uncertainties, such as relativistic corrections and higher-order radiative corrections, etc., are not known. All these corrections and ambiguities will be much less important in the Υ spectroscopy, for which we expect a more accurate determination of the scale parameter Λ. The predictions for the Υ-η_b mass difference and the ratios r_1 and r_2 are shown in Fig. 2 as functions of $\Lambda_{\overline{MS}}$.

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[12]The expectation value $\langle \ln(Q^2/m^2)\rangle$ reads in terms of coordinate-space wave functions

$$\left\langle \ln\left(\frac{Q^2}{m^2}\right)\right\rangle = \frac{1}{2\pi}\int d^3x \frac{1}{r}[\ln(mr)+\gamma]\,\Delta(\phi^*(\vec{x})\phi(\vec{x})) \ ,$$

where Δ is the Laplace operator, and $r=|\vec{x}|$.

[13]Here we have used the potential of Ref. 2 with the scale parameter of $\Lambda_{\overline{\rm MS}}=0.5$ GeV. For the potential with $\Lambda_{\overline{\rm MS}}=0.2$ GeV one obtains $\xi(\psi)=0.42$ and $\xi(\Upsilon)=-0.53$. The values for $|\phi(0)|^2$ read as follows: (a) potential 1 ($\Lambda_{\overline{\rm MS}}=0.5$ GeV): $|\phi(0)|^2{}_\psi=0.0628$ GeV^3, $|\phi(0)|^2{}_\Upsilon=0.488$ GeV^3; (b) potential 2 ($\Lambda_{\overline{\rm MS}}=0.2$ GeV): $|\phi(0)|^2{}_\psi=0.0584$ GeV^3, $|\phi(0)|^2{}_\Upsilon=0.390$ GeV^3. The quark masses are given by $m_c=1.48$ GeV, $m_b=4.88$ GeV, respectively. We have estimated the wave function at the origin for arbitrary scale parameters $\Lambda_{\overline{\rm MS}}$ by taking the linear interpolation determined by the two values of $|\phi(0)|^2$ for $\Lambda_{\overline{\rm MS}}=0.2$ GeV and $\Lambda_{\overline{\rm MS}}=0.5$ GeV.

[14]The leptonic width of the vector state reads

$$\Gamma^V_{ee}=\frac{4\pi e^2\alpha^2}{m^2}\langle 1\rangle\left(1-5.33\frac{\alpha_s}{\pi}\right)$$

(for a discussion, see Ref. 2, Appendix B). The hadronic width of the pseudoscalar state is given by (cf. Ref. 1), for $N_f=3$,

$$\Gamma^P_{\rm had}=\frac{8\pi}{3}\frac{\alpha_{\overline{\rm MS}}{}^2(m)}{m^2}\langle 1\rangle\times\left(1+5.27\frac{\alpha_{\overline{\rm MS}}(m)}{\pi}\right) \ .$$

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