Momentum flow in weak decays of heavy mesons

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(Received 18 September 1981)

The momentum flow in a quarkonium weak decay is considered. The cancellation of contributions that could spoil the factorizability is found. The situation seems to be different in an analogous decay mode of a semiheavy meson. Possible consequences are discussed.

The factorization in exclusive channels is by now well understood and successfully applied in a variety of strong processes.[1] Following some earlier suggestions,[2] in this Communication an attempt to apply the same concept in the analyses of weak decays of (semi)heavy systems is presented. The first step in this direction is to understand the circulation of the internal momenta. Precisely, the momentum flow in the decay of quarkonium into a pair of light mesons is investigated and compared to the flow in the corresponding decay mode of "semiheavy" mesons. (The name "semiheavy" denotes here a meson built from one heavy and one light constituent.) The result is quite interesting. While the factorization can be done in the quarkonium case, some formal and physical arguments indicate that it is not likely to happen in other decays considered. Needless to say, the possible failure of the factorizability could radically change the entire existing interpretation of weak nonleptonic interactions.

It is convenient to start the analyses with quarkonium. The strong decays in this system are known to be factorizable,[3] and in the following it will be shown that the same is true for weak decays, too. Let me concentrate on the decay mode illustrated in Fig. 1 ("exchange contribution"). In the standard notation,[3,4] the dominant contribution to the decay in two pseudoscalars is given by the matrix element[5]

$$\mathfrak{M} = F\{[M\not{p}_1(1+x_1x_2-2x_1) - M\not{p}_2(1+x_1x_2+2x_2)](1-\gamma_5) - 4\not{p}_1\not{p}_2\} ,$$

where

$$F = \frac{16}{3}\frac{(g^2G/\sqrt{2})\times(\text{Cabibbo angles})}{M^5(1+x_1x_2)(1-x_1)(1+x_2)} .$$

The amplitude A for the process is determined by the trace, $A \sim \mathrm{Tr}\,\mathfrak{M}\Pi$, where Π is the projection operator depending on the spin structure of the initial state.[6]

Of course, one must proceed at least to the one-loop level in order to see whether the hard subprocess can be separated from the rest and analyzed by the renormalization group (RG). In this order, the additional gluon line connecting *light* quarks may be treated in the same way as used in the form-factor analysis,[1] and does not spoil the factorizability. However the situation becomes more cumbersome when the added gluon connects a light quark with a heavy one. Every such diagram (see, e.g., Fig. 2) has a regime in which the dominant, but nonhard,[7] contribution, survives. Due to the topology of the diagrams, such a contribution cannot be separated into the wave-function part, and the only hope is that in the total combination the unpleasant regimes are canceled. And, indeed, that happens in the quarkonium system. Matrix elements corresponding to Fig. 2 are,

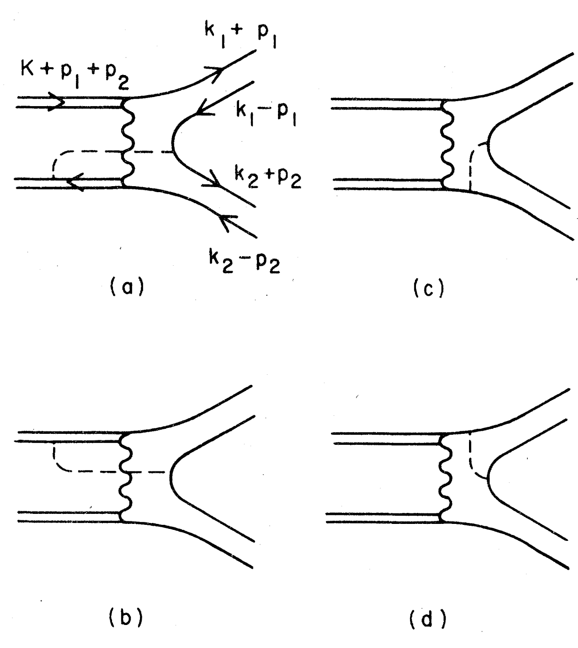

FIG. 1. Lowest-order contributions to the weak quarkonium decay. The double, wavy, and dashed lines denote heavy quark, weak boson, and gluon, respectively. $2p^{\alpha}_{1,2} \approx (M,0,0,\mp M)$.

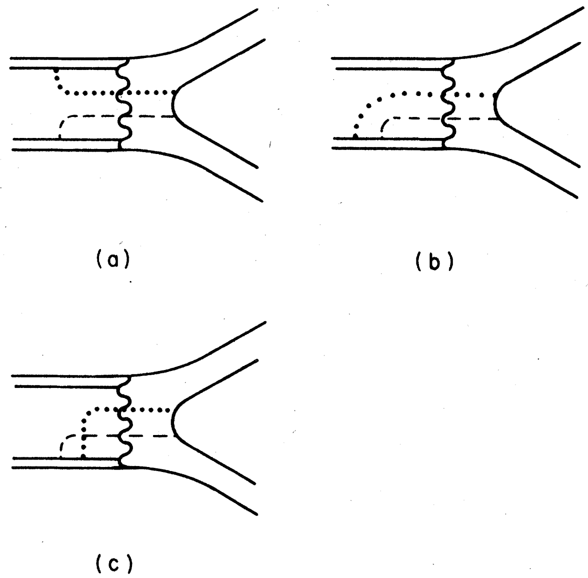

FIG. 2. Higher-order corrections to the diagram 1(a). The dotted line denotes the gluon with the momentum l, and collinear to p_1. $l \simeq x_- p_1$.

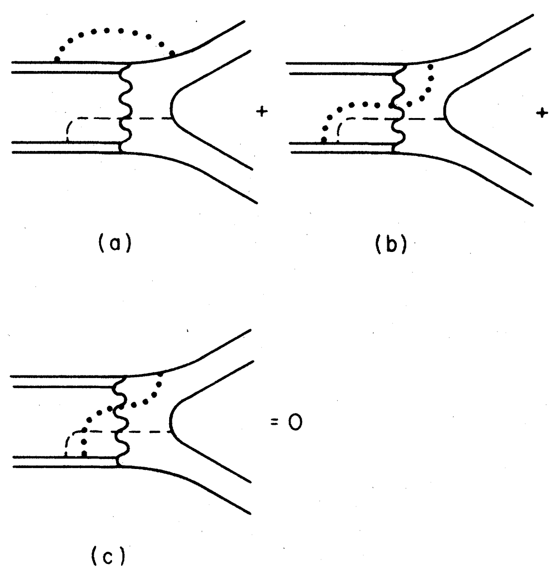

FIG. 3. Another set of diagrams for which the collinear contribution is canceled. The notation is the same as in Fig. 2.

for example,[5]

$$\mathfrak{M}_{(a)} \sim g^2 F \int d^4l \, \frac{1-x_1-x_-}{l^2(l+k_1-p_1)^2} \times \frac{(x_2 \not{p}_2 - M)\not{p}_1(1+\gamma_5)}{x_-(1+x_1x_2+x_-x_2)} ,$$

$$\mathfrak{M}_{(b)} \sim g^2 F \int d^4l \, \frac{(1-x_1-x_-)(1+x_2)}{l^2(l+k_1-p_1)^2} \frac{\not{p}_1\not{p}_2(1+\gamma_5)}{x_-(1+x_1x_2)} ,$$

$$\mathfrak{M}_{(c)} \sim g^2 F \int d^4l \, \frac{x_2(1-x_1-x_-)}{l^2(l+k_1-p_1)^2} \times \frac{(Mx_2-\not{p}_1)p_2(1-\gamma_5)}{(1+x_1x_2)(1+x_1x_2+x_-x_2)} .$$

The cancellation, still hidden in these expressions, becomes obvious when the proper amplitudes are calculated and summed. That is simply to check for the decay of the $^3S_1$ state. The projector Π is then proportional to[6] $(1+\gamma_0)\not{\epsilon}$, where ϵ^α is the polarization vector of the quarkonium. Some algebra leads to the result

$$A_{(a)} + A_{(b)} + A_{(c)} = 0 . \qquad (1)$$

The relation (1) is of course not restricted to $^3S_1$ decays, but is generally valid for any initial quarkonium state. Another example of the cancellation is represented in Fig. 3, and the reader may easily identify other groups of dominant diagrams with a similar property. The final result is that no nonhard correction survives. In the standard fashion[1] the one-loop result can be extended to any loop order.[8]

Although the cancellation illustrated above might look like a miracle, it has a clear physical explanation: The quarkonium, being a small and colorless object, is recognized only by short-distance gluons and the amplitude may depend only on short-distance contributions.

Let us turn our attention now to decays of semiheavy mesons. While the bound states of two equal-mass constituents are quite well understood, the proper framework for the analyses of the semiheavy mesons is still missing. However, some insight in the momentum flow can be reached even in a simple model based on the quarkonium picture. In this model the semiheavy meson is described as a weakly bound state of a heavy and a light quark, moving parallel in an S wave (Fig. 4). Investigating groups of

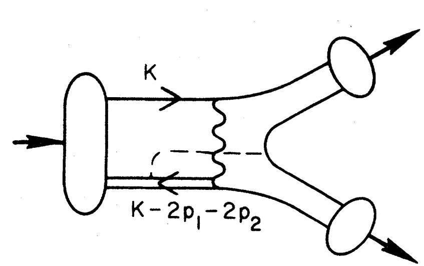

FIG. 4. The semiheavy meson decaying into the pair of light mesons. Only one of the four possible lowest-order diagrams is drawn. $4p^{\alpha}_{1,2} \approx (M,0,0,\mp M)$.

higher-order diagrams analogous, e.g., to those in Fig. 2, one easily concludes that the cancellation now does not occur:

$$A^{SH}_{(a)} + A^{SH}_{(b)} + A^{SH}_{(c)} \neq 0 \ . \qquad (2)$$

The inclusion of other one-loop corrections does not change this situation. Due to an asymmetry in the quark masses of the decaying particle, the dominant contribution of the form $\ln M/m$, which is not controlled by the RG, remains uncanceled.

Is it possible that (2) is just an artifact of the simplified model used in the description of the initial state? Convincing physical arguments support the opposite conclusion. The way in which the light and the heavy constituent react on the collinear gluons is not necessarily the same, and that might cause the appearance of collinear divergencies. Furthermore, it is the light quark that basically determines the size of a semiheavy particle, which thus gets the dimensions of *light* mesons. The mechanism that prevented the exchanges of nonhard gluons in the previous example is now lost, and therefore the "miraculous" cancellation of the type shown in (1) is not expected in decays of semiheavy mesons.[9] By analogy, the failure of the factorization strategy is likely to happen in the nonleptonic decays of light hadrons, too. Both initial and final states in these decays are large objects and there should not be any surprise if the physical amplitudes come out to be an unextricable mixture of hard and soft, dominant and nondominant contributions. Systems do spend a fraction of time close together; otherwise the exchange of the weak boson would not be possible. However, during the rest of the interaction time constituents are free to exchange long-distance gluons, nontreatable in the perturbative calculation.

How to confront this disastrous scenario with the well established effective-Hamiltonian scheme? The problem is that the effective Hamiltonian (EH) could hardly be fitted into the recent advance of perturbative quantum chromodynamics (QCD), and therefore is not a most suitable candidate for the comparative analyses. While the factorization approach is created to analyze interactions in the system of confined quarks, the EH describes just interactions of the selected pair of (unconfined) quarks. The dimensions of the system are introduced by hand (with the choice of the "renormalization point"), and the "spectator" hypothesis—although physically unjustified—is used in any calculation. There is no wonder that effects described in this note, and crucially related to the interactions with the spectator, could not be detected in the EH scheme. In fact, such a scheme might be of some relevance for the nonleptonic physics only if the factorization can be done, but even then, the relative importance of neglected hard contributions should be carefully reexamined.

In conclusion, instead of the standard EH scheme, the factorization strategy is used in the preliminary analyses of weak decays. Decays suitable for the QCD analyses and those in which the factorizability is not obvious are identified. In the latter decays, the failure of the factorizability, based on formal and physical arguments, is suggested. Such a failure would not be a serious challenge for QCD.[10] However, it *would be* a tremendous problem for the description of weak nonleptonic processes. An urgent, definite answer is of the greatest importance for this field of physics.

Author would like to thank S. Gupta for many helpful conversations. This paper is greatly influenced by his points of view. This work was supported by the Department of Energy under Contract No. DE-AC03-76SF00515.

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[1]For a review and references, see, for example, G. P. Lepage and S. J. Brodsky, Phys. Rev. D 22, 2157 (1980); A. H. Mueller, Phys. Rep. 73, 237 (1981); S. J. Brodsky and G. P. Lepage, Report No. SLAC-PUB-2762, 1981 (unpublished).

[2]A. Duncan, Phys. Scr. 23, 961 (1981); H. Galić, Phys. Rev. D 24, 2441 (1981).

[3]A. Duncan and A. Mueller, Phys. Lett. 93B, 119 (1980); Mueller, in Ref. 1; S. C. Chao, Columbia University Report No. CU-TP-199 (unpublished).

[4]M and M_W are masses of a heavy quark and a weak boson, respectively, and $M < M_W$; G is the Fermi coupling constant; all other masses and the binding energy are neglected; $k_i = x_i p_i$.

[5]For simplicity, only $K = 0$ terms (K^α is the relative momentum of quarks in the initial state) are written. If one is interested in a P-state decay, terms linear in K have to be taken into account, too.

[6]J. H. Kühn, J. Kaplan, and E. G. O. Safiani, Nucl. Phys. B157, 125 (1979).

[7]"Nonhard" denotes here the regime in which some of the internal lines in a diagram are close to the mass shell.

[8]In the same way, the proof of factorizability can be achieved for the "annihilation" decay mode too. Note, however, that the RG analysis becomes more complicated in the analyses of weak decays because of the presence of two mass scales, M and M_W.

[9]These physical arguments were brought to my attention long ago by S. Gupta (private communication).

[10]Nobody, for example, expects that QCD should describe strong decays of light hadrons. It is in the spirit of perturbative QCD to look just for the processes dominated by short distances. It could easily be that weak decays are not in this category.