PHYSICAL REVIEW D VOLUME 24, NUMBER 11 1 DECEMBER 1981

Inclusive J/ψ's in e^+e^- annihilations

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(Received 1 July 1981; revised manuscript received 14 September 1981)

Within the framework of perturbative quantum chromodynamics we calculate the cross section as well as energy and angular distributions for inclusive J/ψ production in e^+e^- annihilations. The correct color-singlet assignment of the J/ψ is taken into account. Our results are compared with the predictions of the "color-evaporation model."

I. INTRODUCTION

Inclusive J/ψ production has been investigated both theoretically and experimentally for a large number of hadron- and current-induced reactions. Since heavy charmed quarks are involved in these processes, it is generally believed that perturbative quantum chromodynamics (QCD) can be used to calculate the correct order of magnitude of the cross sections and the proper angular and energy distributions. The models which have been proposed so far fall into two different classes:

(a) In the "color-evaporation model"[1] one calculates the production of a pair of charmed quarks with an invariant mass between $2m_c$ and $2m_D$ and assumes roughly $\frac{1}{3}-\frac{1}{8}$ of these pairs convert into a J/ψ. The color content of the quark pair is ignored since radiating off an arbitrary number of soft gluons should ensure that the $(c\bar{c})$ pair is in a color-singlet state. No definite prediction is possible for the relative abundance of the various charmonium states.

(b) Another approach is to calculate the full charmonium color-singlet formation in the framework of perturbative QCD, as has been done for $gg \to (c\bar{c})_{\text{singlet}}$ by Carlson and Suaya.[2] This latter approach gives some predictions for the relative abundances of the different states.

Inclusive J/ψ production in e^+e^- annihilations is a particularly well suited reaction to distinguish between these two mechanisms. The predictions of the color-evaporation model [Fig. 1(a)] have been investigated by Fritzsch and Kühn.[3] Using a conversion factor of $\frac{1}{8}$ for $(c\bar{c}) \to J/\psi$, one obtains in this model

$$R_\psi \equiv \frac{\sigma(e^+e^- \to J/\psi + X)}{\sigma(e^+e^- \to \mu^+\mu^-)} \approx (1-5)\times 10^{-3}$$

for the energy region below 10 GeV and R_ψ decreases rapidly like $1/s$. Production rates of this order of magnitude seem to be accessible experimentally at DORIS[4] and eventually also at CESR. Apart from its own interest inclusive J/ψ production is also an important background to the indirect production from weak decays of B mesons which are expected to yield inclusive J/ψ particles with a branching ratio of $10^{-1}-10^{-3}$ (Ref. 5).

Thus we expect that experimental results on this subject will be available in the near future and we want to compare the predictions of the evaporation model with a calculation in which the $(c\bar{c})$ pair is produced as a $^3S_1$ color-singlet state together with two gluons [Fig. 1(b)]. We find a cross section that is smaller than before. R_ψ is roughly $(0.3-0.5)\times 10^{-3}$ in the energy region of interest but varies only moderately with energy. In addition, the characteristics of the final states differ drastically in the two models: Due to its two-body final state $(c\bar{c})+g$ the

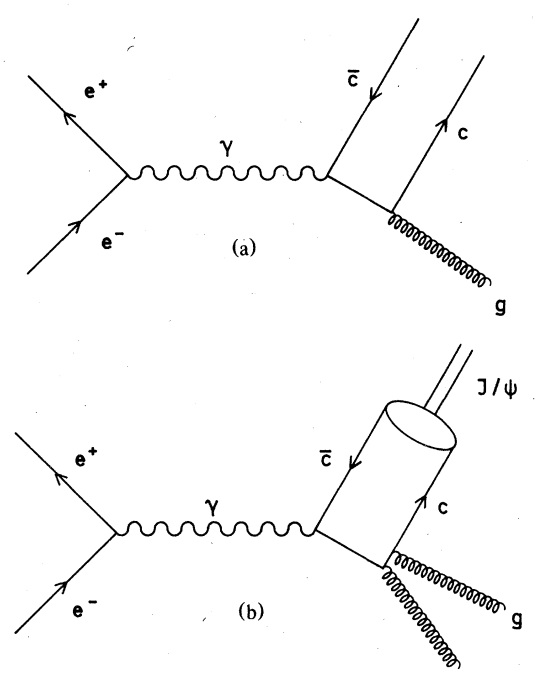

FIG. 1. Diagrams that contribute to inclusive J/ψ production for the evaporation model (a) and the model where the correct color assignment of J/ψ is ensured (b).

evaporation model leads to a rather hard spectrum[3] $dN/dE_\psi \propto \delta(E_\psi - s - m_\psi^2/2\sqrt{s})$ and one expects this feature to be true, at least qualitatively, also after the color-evaporation process. The angular distribution of the $(c\bar{c})$ system is $dN \propto (1+\cos^2\theta)d\cos\theta$. In contrast our present model predicts a rather soft energy distribution of the J/ψ and an angular distribution which is nearly flat.

II. EVALUATION OF THE RATE

To calculate the quarkonium production it is useful to apply the bound-state formalism of Ref. 6. It allows a straightforward evaluation of the covariant amplitude which describes the effective coupling of the virtual photon to a J/ψ and two gluons. The amplitude is given by

$$A^{\gamma^* gg} = \frac{1}{2}\frac{1}{\sqrt{4\pi m_\psi}} R_0 \mathrm{Tr}[\sigma_0(\not{P} - m_\psi)\not{E}] \ , \qquad (1)$$

where R_0 denotes the radial wave function of the bound state at the origin and P, m_ψ, and E denote its momentum, mass, and polarization. σ_0 is obtained from the relevant amplitude for the production of the free-quark pair at threshold

$$\sigma_0 = -\frac{i}{4}\left\{ \not{\epsilon}_2 \frac{[(-\not{K}_1+\not{K}_2+\not{K})+m_\psi]}{-(K_1-K)\cdot K_2} \not{\epsilon} \frac{[(-\not{K}_1+\not{K}_2-\not{K})+m_\psi]}{-(K_2-K)\cdot K_1} \not{\epsilon}_1 \right\}$$
$$+\text{five permutations of } K_1, K_2, -K \text{ and } \epsilon_1, \epsilon_2, \epsilon \ . \qquad (2)$$

Here K_1, K_2, K and $\epsilon_1, \epsilon_2, \epsilon$ stand for the momenta and polarization vectors of the two gluons and the virtual photon. Coupling constants and color matrices have been suppressed and contribute a factor

$$(\tfrac{2}{3}e)^2 \sum_{a,b} \left[\frac{\delta_{ik}}{\sqrt{3}} g_s (\tfrac{1}{2}\lambda^a)_{ij} g_s (\tfrac{1}{2}\lambda^b)_{jk} \right]^2 = (\tfrac{2}{3})^2 e^2 (\tfrac{2}{3}) g_s^4$$

to the rate. The differential cross section is then given by

$$d\sigma = \sigma_{\mu^+\mu^-}\left(\frac{1}{8\pi^3}\right)\left(\frac{\alpha_s}{\alpha}\right)^2 \frac{\Gamma(J/\psi\to\mu^+\mu^-)m_\psi^3}{s^2}\frac{|M|^2}{16} ds_{12} ds_{13} d\alpha\, d\beta\, d\cos\gamma \ , \qquad (3)$$

where s_{12} and s_{13} denote the invariant masses of the two different J/ψ-gluon systems and α, β, γ are three Euler angles. The squared and spin-averaged matrix element $|M|^2$ was obtained with the help of the algebraic-manipulation program SCHOONSHIP[7] and has the following form:

$$\begin{aligned}
\tfrac{1}{16}F_1^2F_2^2F_3^2|M|^2 = {} & 2[(F_1+F_2-F_3)^2 + (F_1+F_3-F_2)^2F_2^2 + (F_2+F_3-F_1)^2F_1^2] \\
& -2s\{3[(F_1+F_2-F_3)F_3^2 + (F_1+F_3-F_2)F_2^2 + (F_2+F_3-F_1)F_1^2] - 4F_1F_2F_3\} \\
& +(F_1^2+F_2^2-F_3^2)^2 + 4s^2(F_1+F_2-F_3)^2 + 4F_1F_2[s(F_1+F_2) - F_1F_2] \\
& +4\left(\frac{s}{m_\psi^2}\right)F_1F_2[s(F_1+F_2-F_3)+4F_1F_2] + 4\left(\frac{s-m_\psi^2}{s}\right)(F_1-F_2)F_3(K_+\cdot L_-)(K_-\cdot L_-) \\
& +\left\{(F_1+F_2-F_3)(3F_1+3F_2-F_3) - 2\left(\frac{s-m_\psi^2}{sm_\psi^2}\right)[(F_1-F_2)^2m_\psi^2 + 2sF_1F_2]\right\}(K_+\cdot L_-)^2 \\
& +\left[(F_1-F_2)^2 - F_3(4F_1+4F_2-F_3) + 2\left(\frac{m_\psi^2}{s}\right)F_3^2 + 4\left(\frac{s}{m_\psi^2}\right)F_1F_2\right](K_-\cdot L_-)^2 \ , \qquad (4)
\end{aligned}$$

where we used the following notations:

$$F_1 = K_1\cdot K_2 - K_1\cdot K \ , \quad F_2 = K_1\cdot K_2 - K_2\cdot K \ , \quad F_3 = -K_1\cdot K - K_2\cdot K \ ,$$
$$K_+ = K_1 + K_2 \ , \quad K_- = K_1 - K_2 \ , \quad L_- = L_1 - L_2 \ . \qquad (5)$$

s denotes the center-of-mass energy and L_1 and L_2 are the momenta of the e^+ and e^- particles. For the strong coupling constant we use

$$\alpha_s = \frac{12\pi}{27\ln[(m_\psi/0.5\text{ GeV})^2]} \quad . \tag{6}$$

The choice of the mass scale for α_s is to some extent ambiguous in this lowest-order calculation. As an alternative one might choose a scale of the order of the typical energy-momentum transfer to gluons, e.g., the invariant quantity M_{gg}.[8] Such a choice would enhance J/ψ production at small s by roughly a factor 5 and is suggested by the success of the corresponding model (with $\alpha_s \approx 1$) for the transition $\psi' \to \psi + X$. Of course beyond 10 GeV the rate would be depleted.

From these expressions we can calculate $R_\psi \equiv \sigma(e^+e^- \to J/\psi + X)/\sigma(e^+e^- \to \mu^+\mu^-)$ as well as the energy and the angular distribution of the production plane. In Fig. 2 we compare our result for R_ψ with the prediction of the evaporation model, assuming in the second model that $\frac{1}{8}$ of the produced $(c\bar{c})$ pairs convert into J/ψ. We find that the two models give quite different predictions for magnitude and energy dependence of R_ψ. In Fig. 3 we display the energy distribution of J/ψ, dN_ψ/dx with $x = 2E_\psi/\sqrt{s}$, for two energies $\sqrt{s} = 10$ and 20 GeV. Our calculation yields a spectrum far softer than the one expected from the evaporation model. The angular distribution of the J/ψ is characterized by a single constant A_ψ, since it is necessarily of the form

$$dN/d\cos\theta_\psi \propto 1 + A_\psi \cos^2\theta_\psi \quad , \tag{7}$$

where θ_ψ denotes the angle between the J/ψ momentum and the beam direction. The distribution of the vector $\vec{n}$ normal to the production plane can be represented in a similar form with a constant A_n. A_ψ and A_n vary of course with the total energy and are shown in Fig. 4.

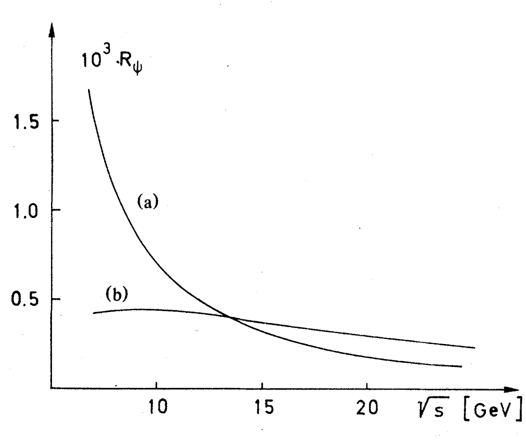

FIG. 2. R_ψ as a function of energy for models (a) and (b).

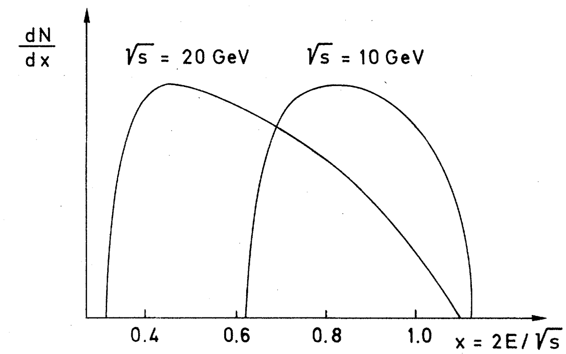

FIG. 3. Energy distribution dN/dx with $x = 2E_\psi/\sqrt{s}$ for $\sqrt{s} = 10$ and 20 GeV in model (b).

The color-evaporation model contains no information on the polarization of the J/ψ, and thus nothing can be said about the angular distribution of the lepton pair. Our amplitude (1) contains the J/ψ polarization vector, and thus we can predict the angular distributions of its decay products. The J/ψ particles would be detected through their decay into $\mu^+\mu^-$ (or e^+e^-). The distribution of the angle θ_μ between momenta of muon and J/ψ in the rest frame of the J/ψ has a form similar to Eq. (7) and is characterized by a constant A_μ. The evaluation of the relevant matrix elements is tedious and will be given elsewhere.[8] The variation of A_μ with energy is also shown in Fig. 4.

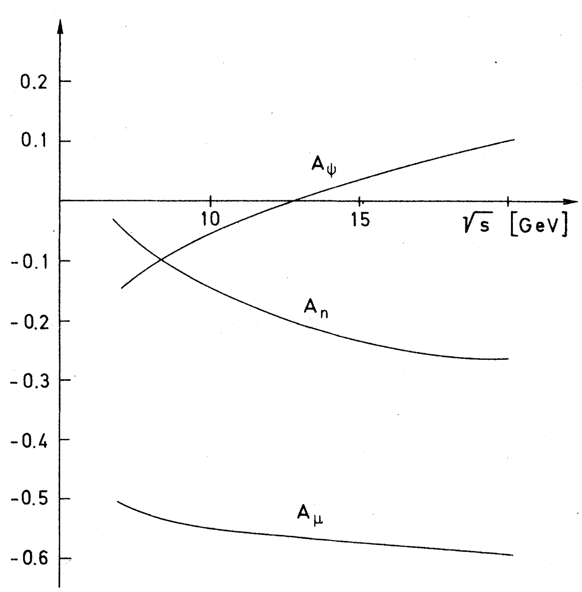

FIG. 4. Variation of A_ψ, A_n, and A_μ with energy in model (b).

III. SUMMARY

Inclusive J/ψ production in e^+e^- annihilations will be experimentally accessible down to a level of $R_\psi \approx 10^{-2}$–10^{-3}. Two competing models give predictions for R_ψ ranging between 0.4×10^{-3} and 2×10^{-3} in the energy region of interest. If inclusive J/ψ's were to be observed in e^+e^- collisions with a rate R_ψ of more than 10^{-3} at an energy around 5 GeV, then one would have to conclude either that the two-gluon-emission model does not apply or alternatively that the scale of α_s is determined by the soft gluons. The two models which we discussed differ also in their predictions for the energy and the angular distributions, and thus experiment might be able to decide between the two choices in the near future. Let us finally mention that the rates decrease in both models quite rapidly for large s. Thus for $\sqrt{s}$ above 20 GeV another process will dominate, the fragmentation of a quark into $\psi + X$, a reaction which has been discussed already in the literature.[9]

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