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# On baryon semileptonic decays

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We make comments on baryon semileptonic decays motivated by recent work on quark models. We find the decay rate of $\Lambda_c \to \Lambda \bar{e} \nu$ to be reduced by a factor of 2 from its naive value due to wave-function distortion and form-factor effects. Configuration mixing implies the value of $\pm 0.04$ for the vector-to-axial-vector ratio in $\Sigma^{\pm} \to \Lambda \beta$ decay. Small changes in $SU_6$ predictions due to configuration mixing are also discussed. The Appendix describes a very simple method of evaluation of the axial-vector-to-vector ratio in the symmetric quark model with static quarks.

A small but positive progress has occurred in our understanding of the spectrum and strong decays of baryons following ideas conjectured from quantum chromodynamics.[1] Two ideas received special emphasis: (a) the effects of different quark masses in a universal, flavor-independent, confining potential,[2] and (b) the effects of configuration mixing induced by quark hyperfine interactions.[3] The purpose of this paper is to discuss some simple consequences of these ideas on baryon semileptonic decays. Thus, we discuss corrections to Cabibbo theory.[4] Since Cabibbo theory is known to work well, many of the corrections we discuss are small and will require very accurate measurements for their confirmation. We emphasize that this paper does not give a complete description of any of the topics it deals with, but only comments on the few features suggested by the quark-model work alluded to above.

We start by discussing a mass effect. Several authors[2] have noted that the assumption of flavor-independent confining forces leads to a distortion of the orbital wave function of a hadron containing a heavy quark $Q$ relative to the orbital wave function of a hadron composed of light quarks only. This can change, for example, the expectation value of the hyperfine interaction in one hadron to a different value in the other one. In the present context, we recall that the transition amplitude for the semileptonic decay of a heavy baryon into a light baryon contains, at low momentum transfer, the overlap integral of the orbital wave functions of the two hadrons:

$$I = \int d\tau\, \Psi^*_{\text{final}}\, \Psi_{\text{initial}} \quad , \tag{1}$$

and this overlap will be smaller than unity if the two wave functions are not identical.[5] The overlap integral can be estimated in a harmonic-oscillator model where it is

$$I^2 = \left(\frac{2\alpha_1\alpha_2}{\alpha_1^2+\alpha_2^2}\right)^3 . \tag{2}$$

Here $\alpha_1$ and $\alpha_2$ are parameters describing the relevant oscillators in the initial and final states:

$$\psi_{\text{final}} = N_f e^{-\alpha_1^2(\rho^2+\lambda^2)/2} ,$$

$$\psi_{\text{initial}} = N_i e^{-(\alpha_1^2\rho^2+\alpha_2^2\lambda^2)/2} .$$

The quantity $I^2$ is one of the factors in the decay rate of the initial hadron into the final hadron. The parameters $\alpha_1$ and $\alpha_2$ depend on the quark masses and the only case in which $I$ differs perceptibly from unity is that of a baryon containing a charmed or a heavier quark $(b, t)$. In the approximation of neglecting the light-quark mass relative to the heavy-quark mass we find

$$I^2 = 2^3 3^{3/4}(1+\sqrt{3})^{-3} \simeq 0.894 \tag{3}$$

and therefore a diminution of the semileptonic decay rate of a charmed baryon by about 10% from its value predicted on the assumption of undistorted wave functions.

It should be remembered that the integral (1) is itself an approximation which holds only in the limit of low momentum transfer $\vec{q}$ to the hadron from the electron neutrino system. More generally the integrand (1) contains an additional factor $e^{-i\vec{q}\cdot\vec{r}}$ where $\vec{r}$ is the coordinate of the quark undergoing weak decay. The momentum transfer $\vec{q}$ can be quite appreciable in charmed-hadron decays and so we have to retain this momentum dependence. If we work again in the harmonic-oscillator approximation the result (2) is modified by a multiplicative factor:

$$I^2 \to \left(\frac{2\alpha_1\alpha_2}{\alpha_1^2+\alpha_2^2}\right)^3 e^{-2\vec{q}^2/3(\alpha_1^2+\alpha_2^2)} . \tag{4}$$

The numerical value of this factor can be quite small—for example, for very energetic or very slow electrons—as small as about 0.33. We have taken here $M_i \simeq 2.2$ GeV, $M_f \simeq 1.1$ GeV, $m_e = m_\nu = 0$. This loss of probability of producing a final baryon in its ground state is compensated by producing orbitally excited baryons; these will give rise to additional pions in the final state. On the other hand, if the hadronic system is produced at low momentum transfer, the form factor $e^{-q^2/3\alpha^2}$ will be of order unity. The next generation of experiments on charmed baryons should be capable of observing this effect.

To estimate the effect of the form factor (4) on the overall semileptonic decay rate one will have to integrate this expression with the three-body phase space appropriate for the decay. We have approximated this integration by taking incorrectly the phase-space volume element appropriate to a static final hadron but a correct maximal momentum transfer $q_{\max}=E_{\max}=W\cong 0.82$ GeV as follows from the values $M(\Lambda_c)\simeq 2.2$ GeV, $M(\Lambda)\simeq 1.1$ GeV. Then the average value of the rate is given by

$$R=\left(\frac{2\alpha_1\alpha_2}{\alpha_1^2+\alpha_2^2}\right)^3\frac{15}{W^5}\int_0^W dE\int_{-1}^{1} d\cos\theta E^2(W-E)^2e^{-A^2\tilde{q}^2}$$

$$=0.894\times\frac{15}{2(AW)^5}\left[\sqrt{\pi}\,\phi(AW)\left(\frac{A^2W^2}{8}-16^{-1}\right)-e^{-A^2W^2}\left(\frac{AW}{8}-\frac{A^3W^3}{6}\right)\right]$$

$$=0.894\times\frac{15}{2}\left(\sqrt{\pi}\frac{\phi(1)}{16}\right)-\left(\frac{e^{-1}}{24}\right)\simeq 0.528\,. \qquad (5)$$

Here we have taken $AW\simeq 1$ as follows from $\alpha_1^2=0.17$ GeV$^2$ and the value $\tilde{q}_{\max}^2=0.68$ GeV$^2$ noted above. It is implicitly assumed that $(G_A/G_V)=1$ as appropriate for $\Lambda_c\to\Lambda e+\nu$ (this is derived in the Appendix); otherwise there would be an additional correlation between the electron and neutrino angle $\theta$, neglected in the integral (5). We have checked numerically that the approximation (5) is reasonably good.

Therefore, we find that the semileptonic decay rate of $\Lambda_c$ into $\Lambda$ and $N$ is diminished by about a factor of 2 from its value computed without taking form factors into account. On the other hand, if one has used form factors normalized to unity at low-momentum transfer one has to apply only the overlap integral (2). This is the case with the very thorough estimate of Buras[6] from which we deduce

$$\Gamma(\Lambda_c\to\Lambda,\, N\bar{e}\nu)=I^2\Gamma_{\text{Buras}}=0.894\times 1.18\times 10^{12}\ \text{sec}^{-1}=1.05\times 10^{12}\ \text{sec}^{-1}\,.$$

We now discuss some effects which are due to configuration mixing through hyperfine interactions. The most interesting case is that of the decays of $\Sigma\to\Lambda e\nu$. Cabibbo and Gatto[7] pointed out in the context of conserved vector theory that $\Sigma^\pm$ decays into $\Lambda$ only through the axial-vector current. The strangeness-conserving vector current is an isospin generator and cannot change the isospin of the initial state. The same result continues to hold in Cabibbo theory.[4] The experimental data[8] available so far agrees, with large errors, with this prediction. Larger samples of decaying $\Sigma$'s are available with hyperon beams. We discuss the leading nonvanishing contribution to the matrix element of the vector current in $\Sigma\to\Lambda\ \beta$ decay.

This contribution arises when we recall that the physical state $\Lambda^P$ is a linear superposition of the $I=0$ ($\Lambda$) and the $I=1$ ($\Sigma$) states:

$$|\Lambda^P\rangle=\cos\theta\,|\Lambda\rangle+\sin\theta\,|\Sigma^0\rangle\,. \qquad (6)$$

The mixing (6) has been noted very early[9] in the context of $SU_3$, and the mixing (angle) $\sin\theta$ has been predicted from the observed electromagnetic mass differences of baryons and simple assumptions about the transformation properties of the mass operator. More recently, Isgur[10] has shown that this mixing is mainly a consequence of the color hyperfine interaction between quarks and the mass difference between $u$ and $d$ quarks. All the estimates in the literature give $\sin\theta\simeq+0.014$ to $+0.02$; we shall take the value $+0.02$. The sign implies a choice of relative phase between the $\Lambda$ and $\Sigma^0$.

With a small $\Sigma^0$ component in the physical $\Lambda$ state the vector current can now connect $\Sigma^\pm$ to this physical state. If we compute the matrix elements using static quarks, as shown in detail in the Appendix, we find for $\Sigma^-$ decay

$$\left(\frac{G_V}{G_A}\right)_{\Sigma^-\to\Lambda}=-\sqrt{3}\tan\theta=-0.035\,. \qquad (7)$$

The convention we use (as seen in the Appendix) has $G_A/G_V$ positive ($+\frac{5}{3}$) in neutron $\beta$ decay. As in neutron $\beta$ decay we expect that the static estimate of $G_A$ is too large, probably because of relativistic effects. This might increase the ratio (7) to $-0.04$. In the decay $\Sigma^+\to\Lambda\bar{e}\nu$ the ratio should have opposite sign $+0.04$. Though small, these ratios should be accessible to experimental measurement with sufficiently large samples of decaying hyperons. The best value available in the Particle Data Group tables at the moment is $0.1\pm 0.22$ for $\Sigma^-\to\Lambda$. This has the sign we expect, since the tables have the opposit convention to the one we use. However, the errors are far too large for a meaningful comparison. The precise measurement of these ratios will constitute direct experimental evidence for the isospin mixing (6).

The mixing of $\Lambda$ and $\Sigma^0$ also affects the axial-vector-to-vector ratio of other semileptonic decays which start from or end in the $\Lambda$ particle, for example, $\Lambda\to pe\nu$, $\Xi^-\to\Lambda e\nu$. The same ratios will also change when we allow in addition isospin-conserving configuration mixing in all ground-state baryons. The situation is analogous to that of the magnetic moments of ground-state baryons.[11] As in that case we shall only consider the effect of mixing the configuration $^2S_M$ into the ground state $^2S_S$.

For example, the proton and neutron have a small admixture (of −0.27 in amplitude[12]) of a $^2S_M$ state. This small mixing changes the ratio $G_A/G_V$: $(G_A/G_V)_{N\to p} = \frac{5}{3}(1-\frac{4}{5}\sin^2\theta_N) \simeq 1.57$ from the value of $\frac{5}{3}$ corresponding to $^2S_S$ (see the Appendix). We assume that the remaining discrepancy is due to relativistic effects.[13]

We have computed the corrections due to configuration mixing using the mixing angles of Ref. 11, and listed the results in Table I for some cases of experimental interest. We have included relativistic effects on all static results by simply multiplying with the ratio (1.25/1.57) appropriate to neutron $\beta$ decay. A more sophisticated calculation would distinguish between the case of strange-quark decay and nonstrange-quark decay, which have different masses. In the bag model the strange-quark decay would be less affected by relativistic corrections. Our results are compared to the experimental data available as given in the Particle Data Group tables. It is clear that more experimental data would be rather useful.

There is one other interesting case not discussed here, the vector coupling in $\Omega^-$ semileptonic decay. This is sensitive to $D$-wave admixtures in $\Omega^-$ and $\Xi^0$. We hope to discuss this case in a future report.

Clearly the axial-vector-to-vector ratios are very sensitive to configuration mixing and their accurate determination will help in understanding the properties of ground-state baryons.

*Note added in proof.* The discussion of overlap integrals was anticipated in A. Amer *et al.*, Phys. Lett. 81B, 48 (1979), for charmed-hadron production and M. B. Gavela, Phys. Lett. 83B, 367 (1979), for charmed-hadron decay. The formulas (2) and (4) differ from these references which include Lorentz-contraction factors.

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## APPENDIX: THE COMPUTATION OF $G_A/G_V$ WITH STATIC, SYMMETRIC QUARKS

In the body of this paper we used several values of $G_A/G_V$ for transitions between ground-state baryons: 56-plet in $SU_6$. Although these values are well known we discuss their computation with a method which is rather simple. We do not use $SU_6$ but rather construct states which satisfy the requirements of the symmetric quark model.[14]

We start with the decay $n\to pe\nu$. The neutron has the composition $duu$, and we choose the up quark to be the third one. From the requirement of the symmetric quark model for the two down quarks 1 and 2 (Greenberg statistics), the spin wave function of the neutron (with spin up) will be[14]

$$\chi_{12}^{\lambda} = \frac{1}{\sqrt{6}}(\alpha\beta\,\alpha + \beta\alpha\alpha - 2\alpha\alpha\beta)\,. \qquad \text{(A1)}$$

Let us assume that the second down quark is $\beta$ decaying. Then the proton state so obtained has the form $duu$ and its spin wave function has to be symmetric between quarks 2 and 3:

$$\chi_{23}^{\lambda} = \frac{1}{\sqrt{6}}(\alpha\beta\,\alpha + \alpha\alpha\beta - 2\beta\alpha\alpha)\,. \qquad \text{(A2)}$$

In the static limit the value of $G_A/G_V$ is the ratio of expectation values:

$$(G_A/G_V)_{n\to p} = \frac{\langle\chi_{23}^{\lambda}\,|\,\sigma_z^{(2)}\,|\,\chi_{12}^{\lambda}\rangle}{\langle\chi_{23}^{\lambda}\,|\,1\,|\,\chi_{12}^{\lambda}\rangle} = \frac{-\frac{5}{6}}{-\frac{1}{2}} = \frac{5}{3}\,. \qquad \text{(A3)}$$

It should be emphasized we only want here the ratio $G_A/G_V$. If we were to add the contribution of both down quarks 1 and 2 then both the denominator and numerator would double. The sign of the denominator in (A3) is also unusual; it results from a phase convention in (A1) and (A2).

A somewhat simpler example is the decay $\Sigma^- \to \Sigma^0 e\nu$. If we choose $dds$ for $\Sigma^-$ the spin wave

TABLE I. Ratio of axial-vector to vector amplitudes $(G_A/G_V)$ for semileptonic decays.

| Process | $SU_6$ value | $+\Delta\Sigma$ mixing + configuration mixing | + relativistic effects | Experiment |
|---|---|---|---|---|
| $N\to pe^-\nu$ | 1.67 | 1.57 | 1.25 | 1.254 ± 0.007 |
| $\Lambda\to pe^-\nu$ | 1.00 | 0.99 | 0.79 | 0.62 ± 0.05 |
| $\Sigma^-\to ne^-\nu$ | −0.33 | −0.30 | −0.24 | ±(0.38 ± 0.07) |
| $\Xi^0\to\Sigma^+e^-\nu$ | 1.67 | 1.63 | 1.30 | |
| $\Xi^-\to\Lambda e^-\nu$ | 0.33 | 0.31 | 0.25 | |
| $\Xi^-\to\Xi^0 e^-\nu$ | −0.33 | −0.32 | −0.25 | |

function is $\chi_{12}^{\lambda}$ and after $\beta$ decay to $\Sigma^0=2^{-1/2}(ud+du)s$ the same spin state results, so that

$$(G_A/G_V)_{\Sigma^-\to\Sigma^0}=\frac{\langle\chi_{12}^{\lambda}|\sigma_z^{(2)}|\chi_{12}^{\lambda}\rangle}{\langle\chi_{12}^{\lambda}|\chi_{12}^{\lambda}\rangle}=\tfrac{1}{6}(-1+1+4)=\tfrac{2}{3}\ . \quad (A4)$$

Considering now $\Sigma^-\to\Lambda$ and recalling $\Lambda=2^{-1/2}(ud-du)s$ the spin wave function $\chi_{12}^{\rho}$ of the $\Lambda$ has to be antisymmetrized in quarks 1 and 2:

$$\chi_{12}^{\rho}=\frac{1}{\sqrt{2}}(\alpha\beta\alpha-\beta\alpha\alpha) \quad (A5)$$

and therefore

$$(G_A/G_V)_{\Sigma^-\to\Lambda}=\frac{\langle\chi_{12}^{\rho}|\sigma_z^{(2)}|\chi_{12}^{\lambda}\rangle}{\langle\chi_{12}^{\rho}|\chi_{12}^{\lambda}\rangle}=\frac{-1\sqrt{3}}{0}\ , \quad (A6)$$

a purely axial-vector transition, as noted in the text. With $\Lambda$, $\Sigma^0$ mixing we have for the inverse ratio

$$(G_V/G_A)_{\Sigma^-\to\Lambda}\simeq\frac{\sin\theta\langle\chi_{12}^{\lambda}|\chi_{12}^{\lambda}\rangle}{\cos\theta\langle\chi_{12}^{\rho}|\sigma_z^{(2)}|\chi_{12}^{\lambda}\rangle}=-\sqrt{3}\tan\theta\ , \quad (A7)$$

which is Eq. (7) of the main text. The phase change between $\Sigma^+$ and $\Sigma^-$ comes from the denominator which changes sign between the two components of the $\Lambda$ : *uds* and *dus*.

Finally we list briefly other possible values of the ratio $G_A/G_V$:

$$(G_A/G_V)_{\Sigma^-\to N}=\frac{\langle\chi_{12}^{\lambda}|\sigma_z^{(3)}|\chi_{12}^{\lambda}\rangle}{\langle\chi_{12}^{\lambda}|\chi_{12}^{\lambda}\rangle}=-\frac{1}{3}\ , \quad (A8)$$

$$(G_A/G_V)_{\Lambda\to p}=\frac{\langle\chi_{12}^{\rho}|\sigma_z^{(3)}|\chi_{23}^{\lambda}\rangle}{\langle\chi_{12}^{\rho}|\chi_{23}^{\lambda}\rangle}=1\ , \quad (A9)$$

$$(G_A/G_V)_{\Xi^-\to\Lambda}=\frac{\langle\chi_{12}^{\rho}|\sigma_z^{(2)}|\chi_{23}^{\lambda}\rangle}{\langle\chi_{12}^{\rho}|\chi_{23}^{\lambda}\rangle}=\frac{1}{3}\ . \quad (A10)$$

The value $-3^{-1}$ holds more generally for any decay of the type $ddQ\to ddQ'$ (with $Q,Q'\neq d$); the value 1 is also relevant for the decay $\Lambda_c\to\Lambda$ as mentioned in the main text after Eq. (5); and the value $\frac{5}{3}$ also holds more generally for decays like $QQq\to Qqq$ (with $Q\neq q$), as, for example, in $\Xi^0\to\Sigma^+$ or the yet to be discovered $ccs\to css$.

As noted in the main text all these values are computed in the static limit and should be corrected for relativistic effects[13] which are believed to be responsible for shifts like $1.57\to1.25$. The method we used here is very simple for states belonging to $SU_6$ 56-plets (or 20-plets); this is important in practice since ground-state baryons belong here. In the case of 70-plets of $SU_6$ the ratio $G_A/G_V$ is more complicated to deduce with the present procedure.

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[1]For reviews see Nathan Isgur, in *New Aspects of Subnuclear Physics*, edited by A. Zichichi (Plenum, New York, 1980), pp. 107–203; G. Karl, in *Proceedings of the 19th International Conference on High Energy Physics, Tokyo, 1978*, edited by S. Homma, M. Kawaguchi, and H. Miyazawa (Phys. Soc. of Japan, Tokyo, 1979), p. 135.

[2]Nathan Isgur and Gabriel Karl, Phys. Lett. 74B, 353 (1978); U. Ellwanger, Nucl. Phys. B139, 422 (1978); A. Le Yaouanc *et al.*, Phys. Rev. D 18, 1591 (1978).

[3]Nathan Isgur and Gabriel Karl, Phys. Lett. 72B, 109 (1978); D. Gromes and I. O. Stamatescu, Nucl. Phys. B112, 213 (1976); R. Carlitz, S. D. Ellis, and R. Savit, Phys. Lett. 68B, 443 (1977).

[4]N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963).

[5]The possibility of overlap integrals which can be less than unity was noted by Nathan Isgur [Phys. Rev. D 12, 3666 (1975)] in the context of kaon semileptonic decay.

[6]A. F. Buras, Nucl.Phys. B109, 373 (1976).

[7]N. Cabibbo and R. Gatto, Nuovo Cimento 15, 159 (1960); V. P. Belov, B. S. Mingalev, and V. M. Shekhter, Zh. Eksp. Teor. Fiz. 38, 541 (1960) [Sov. Phys. JETP 11, 392 (1960)].

[8]J. A. Thompson *et al.*, Phys. Rev. D 21, 25 (1980); W. Tanenbaum *et al.*, *ibid.* 12, 1871 (1975). These are the most recent measurements which contain references to earlier work.

[9]R. H. Dalitz and F. von Hippel, Phys. Lett. 10, 153 (1964); A. J. MacFarlane and E. C. G. Sudarshan, Nuovo Cimento 31, 1176 (1964).

[10]Nathan Isgur, Phys. Rev. D 21, 779 (1980).

[11]Nathan Isgur and Gabriel Karl, Phys. Rev. D 21, 3175 (1980).

[12]Nathan Isgur, Gabriel Karl, and Roman Koniuk, Phys. Rev. Lett. 44, 1269 (1978).

[13]See, for example, Cameron Hayne and Nathan Isgur, Toronto report, 1981 (unpublished).

[14]For ground-state baryons these wave functions were first proposed by J. Franklin, Phys. Rev. 172, 1807 (1968); for the excited baryons they were introduced in N. Isgur and G. Karl, Phys. Rev. D 18, 4187 (1978).