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Grand unification and proton stability based on a chiral SU(8) theory

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We present a grand unified model of the strong, electromagnetic, and weak interactions based on a local $SU(8)_L \times SU(8)_R$ gauge theory which possesses a global $U(8)_L \times U(8)_R$ invariance. The model is spontaneously broken by the recently introduced neutrino-pairing mechanism, in which a Higgs field which transforms like a pair of right-handed neutrinos acquires a vacuum expectation value. This neutrino pairing breaks the model down to the standard Weinberg-Salam phenomenology. Further, the neutrino pairing causes the two initial global currents of the model, fermion number and axial fermion number, to mix with the non-Abelian local currents to leave unbroken two new global currents, namely, baryon number and a particular lepton number which counts charged leptons and left-handed neutrinos only. The exact conservations of these two resulting currents ensure the absolute stability of the proton, the masslessness of the observed left-handed neutrinos, and the standard lepton-number conservation of the usual weak interactions. A further feature of our model is the simultaneous absence of both strong *CP* violations and of observable axions. The model has a testable prediction, namely, the existence of an absolutely stable, relatively light, massive neutral lepton generated entirely from the right-handed-neutrino sector of the theory.

I. INTRODUCTION

During the past few years a considerable amount of evidence has accumulated to support the idea that each of the fundamental interactions is derivable from a gauge principle. The observed low-energy structure of the weak and electromagnetic interactions is completely in accord with the spontanously broken $SU(2)_L \times U(1)$ gauge theory developed by Weinberg,[1] by Salam,[2] and by Glashow,[3] while at higher energies the structure of strong interactions conforms well to the expectations of quantum chromodynamics (QCD). Moreover, the success of $SU(2)_L \times U(1)$ established not only the relevance of the gauge principle, but also the fact that the weak and electromagnetic interactions are unified into one comprehensive theory. The subsequent identification of a gauge structure for the strong interactions as well then led to the speculation[4,5] that all of the three interactions are combined into one and the same grand unified gauge theory.

In the grand unified theories the weak, electromagnetic, and strong-interaction currents are embedded into the adjoint representation of some large grand unifying group which is then spontaneously broken down to produce the standard low-energy structure of $SU(2)_L \times U(1) \times SU(3)_C$. The additional new (hitherto unobserved) currents which are needed to fill out the adjoint representation have associated with them new gauge bosons which acquire superheavy masses, perhaps as high as 10^{15} GeV. Further, the matter fields of the theory, the leptons and the quarks, are also combined into irreducible representations of the grand unifying group, so that the above new currents lead to possible transitions between leptons and quarks. Such transitions constitute one of the very few ways to study effects due to a 10^{15}-GeV scale at presently accessible laboratory energies, and there is currently much excitement because the theoretical upper bound[6] on the lifetime for the proton decay which these transitions cause in the most popular grand unified theory, the SU(5) theory of Georgi and Glashow,[5] is within an order of magnitude of the present experimental lower bound,[7] so that the theory can actually be (unambiguously) tested in the near future. While the observation of proton decay would provide dramatic confirmation of the grand unification idea, its nonobservation would only serve to eliminate theories such as the SU(5) theory itself, and leave the situation regarding grand unification unclear as access to the 10^{15}-GeV region would unfortunately be lost. Thus, while pursuing improvements in the experimental situation, we should also examine further the theoretical issues involved, particularly since the origin and nature of baryon number and the degree to which we might expect it to be conserved is of such prime theoretical concern.

It has long been the view of the authors (as exemplified for instance in their spontaneously broken theory of flavor[8]) that there is no content to a broken or an approximate symmetry unless there exists some well-defined limit in which the symmetry is exact. Consequently, we would expect any theory, including a grand unified theory, to be baryon-number conserving in the symmetry limit. In this respect we find the SU(5) theory to be somewhat unappealing since baryon number is not a generator of that theory with the proton already being unstable in the symmetry limit. (In fact we regard it as perhaps fortuitous that the superheavy gauge bosons which mediate proton decay just managed to become heavy enough after the spontaneous breakdown so as to keep the theory compatible with the present experimental lack of proton decay.) In this paper we shall therefore reopen the question of baryon conservation by studying an alternate grand unified theory in which baryon number is an explicit generator which is conserved in the symmetry limit.

In selecting an explicit grand unified gauge theory we recall that the leptons and quarks appear to occur together in families, with the first such family containing the familiar u_i and d_i quarks in three colors ($i=R,G,B$) together with e and ν_e. In the absence of right-handed neutrinos there would be altogether 15 particles in the first family, and given these particles the smallest grand unified group in which they can all be accommodated is the SU(5) of Georgi and Glashow, as they just nicely fill out a $\underline{5}$ and a $\underline{10}^*$-dimensional representation of the group. This is then the simplest possible grand unification, and it is so very popular precisely because of this simplicity. However, with such a classification of the particles we note that the $\underline{10}$, unlike the fundamental of the group, contains both quarks and antiquarks. Consequently, some of the 24 gauge bosons of the model can mediate a $\underline{5}\times\underline{5}^*\to\underline{10}\times\underline{10}^*$ transition in the symmetry limit to allow the baryon-number-violating leptoquark process $l+\bar{q}\to q+q$ to occur in the unbroken theory. It is the existence of this process which gives rise to an unstable proton in the SU(5) theory, and not just the fact of grand unification; i.e., it is not so much a consequence of putting leptons and quarks in common multiplets, but rather of the particular choice of multiplets made. It is this specific aspect of the theory which we find undesirable, and in seeking an alternate grand unified theory we shall therefore only consider situations in which no particles and antiparticles appear in common multiplets.

The simplest way to meet our above requirement is to put the basic fermions only into the fundamental representation of some alternative group, a situation which is anyway quite natural since the obvious way to classify the ultimate building blocks of matter is in the fundamental of a group. (Moreover this view accords with the historical development of particle theory since our confidence for instance in the quark concept at all stems from the insistence that the fundamental of the flavor group be of physical significance, while its utility in QCD stems from the fact that the quarks transform only as triplets under the color group.) Thus, for the purposes of grand unification we are led to considering groups based on SU(8) as we can then classify the eight left-handed fermions in a common fundamental. However, under a single SU(8) we note that the seven right-handed fermions in the first family could only be classified as singlets, which is undesirable phenomenologically since, for instance, the right-handed quarks would then have to be color singlets. Thus we must introduce one further right-handed fermion, a right-handed neutrino, to fill out another $\underline{8}$ and so we are led to consider the local gauge group $SU(8)_L\times SU(8)_R$ based on families of 16 fermions. This group is then the opposite of SU(5) in the sense that it is (one of) the largest rather than the smallest group we could use as it contains all possible fermion currents within each chiral sector. The inclusion of right-handed neutrinos constitutes an unavoidable departure from present phenomenology, but as we shall see below the right-handed neutrinos will play a crucial role in fixing the symmetry-breaking pattern and will ultimately provide the model with a new and novel low-energy test of our ideas. We note that since the sum of the electric charges of the fermions within each $\underline{8}$ is zero the theory contains electromagnetism. Further, the model contains the popular left-right $SU(2)_L\times SU(2)_R\times U(1)$ extension of the Weinberg-Salam model. The final additional imposition of a discrete parity invariance then reduces the theory to that of a single coupling constant, and thus our model is indeed a grand unified theory.

While we have now constructed a model without leptoquark transitions, we note that baryon number is not yet an explicit generator of our theory. The local SU(8) chiral theory only contains the generators $Y_i=\frac{1}{2}(B_i-L_i)$, where B_i and L_i are baryon and lepton numbers ($i=L,R$), and not the individual B_i and L_i generators separately. However, we note that in the Lagrangian of our model the fermion–gauge-boson sector actually possesses the slightly larger global invariance $U(8)_L\times U(8)_R$ with two additional conserved global fermion-number generators $F_i=L_i+3B_i$ ($i=L,R$). (We take the baryon number of the quarks to be $\frac{1}{3}$.) We shall therefore extend this symmetry to the rest of the Lagrangian as well[9] so that the complete theory is now $U(8)_L\times U(8)_R$ invariant, with baryon number (and also lepton number) being exactly conserved in the symmetry limit. Since the additional F_L and F_R currents are not gauged, we note that baryon number is for the moment neither local nor global as it is a linear combination of the Y_i and F_i generators.

In breaking the symmetry spontaneously we note that the local $SU(8)_L\times SU(8)_R$ gauge group contains 126 gauge bosons of which 117 are to acquire masses by the Higgs mechanism so as to leave unbroken a local $SU(3)_C \times Q$ symmetry. Now the full $U(8)_L \times U)8)_R$ group contains 128 currents, and since its breaking is only allowed to produce 117 Goldstone bosons, 2 global currents must therefore remain unbroken in addition to the 9 local currents of $SU(3)_C \times Q$. Thus necessarily after the breaking there must still be some residual global symmetry in the model, and we shall pick the breaking direction so that one of the resulting global symmetries corresponds to baryon number (in a certain sense this is just a definition of the names of the quantum numbers associated with the unbroken directions—the nontrivial aspect of our work is that there will be residual unbroken global symmetries). Hence our model has a rationale not only for why baryon number is conserved in the symmetry limit, but also for why it does not get spontaneously broken either.

In order to implement this conservation explicitly while ensuring that we produce no extraneous Goldstone bosons we shall rely upon a mechanism apparently first due to 't Hooft.[10] He pointed out that when a set of Higgs fields simultaneously breaks both a local and a global group down to a residual global group generated by a linear combination of the currents of the two groups it is possible for the local gauge bosons to absorb all the Goldstone bosons leaving none of them in the spectrum. Thus we can trade an initial global symmetry for a final mixed residual global symmetry and finish up with no ungauged Goldstone bosons and no superfluous massless gauge bosons. Since baryon number is a linear combination of the local Y_i and the global F_i generators we can explicitly use the 't Hooft mechanism in our model so that baryon number emerges after the breaking as a residual global symmetry. In this way baryon number emerges as an exact symmetry to which, unlike electric charge, no massless gauge boson is coupled. Thus we explain not only the fact of baryon-number conservation, but also why it is not a local symmetry like electromagnetism.

To take advantage of the 't Hooft mechanism the Higgs fields employed must also break both the local and the global symmetries. Now recently a new mechanism for obtaining parity violation in chiral weak-interaction theories has been introduced,[11,12] namely neutrino pairing, in which a Higgs field which transforms as a pair of right-handed neutrinos acquires a vacuum expectation value. This dilepton pairing gives maximal parity violation in the charged-current sector and also breaks the $B-L$ symmetry in just the right way to give the familiar Weinberg mixing pattern of parity violation in the neutral-current sector.[11,12] Thus neutrino pairing breaks chiral theories down to the usual Weinberg-Salam phenomenology. We will therefore incorporate these features of neutrino pairing into our grand unified chiral model. For our purposes here we now note further that the neutrino pairing term also breaks the global $L+3B$ symmetry, and thus it exactly produces the 't Hooft mechanism to leave baryon number B_V $(=B_L+B_R)$ as an unbroken global symmetry, as required. Additionally, we note that an alternative way to break $B-L$ and $L+3B$ would be quark pairing. However, electric-charge conservation prevents fractionally charged quarks from pairing in the vacuum, a pairing which would violate baryon number. The electrical neutrality of the neutrinos is thus crucial to the pairing mechanism, and we thus see that with the conventional assignment of quantum numbers there is an intricate connection between baryon-number conservation and electric-charge conservation in our model.

Noting that there are not one but two global generators in our model, some other current in the model must also undergo the 't Hooft mechanism and emerge as a residual global symmetry which is conserved like baryon number. [This second current could not now be the lepton number $L_V = L_L + L_R$, since $B_V - L_V$ is contained entirely within the local $SU(8)_L \times SU(8)_R$ sector of the theory.] Because the neutrino pairing only acts on right-handed neutrinos the second current turns out to also involve the Y_i and F_i generators, being a particular lepton number (to be denoted by $L_V - N_R$) which counts charged leptons and left-handed neutrinos only. (The pairing broke N_R, the right-handed neutrino counting operator.) The resulting conserved current (given by $\bar{e}\gamma_\lambda e + \bar{\nu}_L^e \gamma_\lambda \nu_L^e$ for the first family) we recognize as the conventionally defined lepton number of the usual Weinberg-Salam weak interactions. While this current has always been assumed to be conserved in weak interactions, as far as we know this is the first time that an explanation has been given for the fact, and since its conservation is given simultaneously with that of baryon number it makes the structure of our grand unified model very appealing.

As well as explaining baryon- and lepton-number conservation our model also possesses some other noteworthy features. Firstly, the conservation of our particular lepton number $L_V - N_R$ prevents the left-handed neutrinos from acquiring a mass, and hence they remain massless to all orders in the interaction to give a group-theoretical explanation for the masslessness of the observed neutrinos. Secondly, since no such constraint exists in the right-handed sector, the neutrino pairing acts to give the right-handed neutrinos a Majorana mass.[12] However, because of the conservation of $L_V - N_R$ this now massive right-handed neutrino is decoupled from the usual leptons and is hence absolutely stable. Consequently the model predicts the existence of a light (i.e., not superheavy) absolutely stable neutral lepton whose detection and study would provide a way to explore grand unified theories without needing to go to superheavy energies.

With regard to the quarks we note that since [unlike SU(5)] the theory is a chiral theory it possesses a conserved axial-vector baryon-number current (which is yet another linear combination of the Y_i and F_i generators) which then enables us to remove all strong θ-vacua CP-violation effects arising from the instanton structure of QCD.[13] However, because of the mixing of the local and the global currents, the would-be axion Goldstone boson[14] (which appears when axial baryon number is spontaneously broken) is removed by the same 't Hooft mechanism and hence does not appear in the observable spectrum. Thus the axion is eliminated by the Higgs mechanism without needing to gauge the full chiral $U(8)_L \times U(8)_R$ group, and so in our model there are both no observable strong CP violations and no observable axion Goldstone bosons.

Our grand unified chiral model is thus seen to be a very rich theory which explains some of the outstanding issues of the fundamental interactions. The neutrino-pairing mechanism is central to achieving these features in such a compact way, and thus analogously to the work of Ref. 12 our model not only has a rationale for why there should be right-handed neutrinos at all, but also takes advantage of their existence in a crucial way to produce the appropriate breaking mechanism. Some of the results of our work have already been presented in a Letter[15] and in this paper we give the details.

The present paper is organized as follows. In Sec. II we introduce the model and the appropriate pattern of spontaneous breakdown. In Sec. III we discuss the renormalization of the coupling constants of the model from the superheavy unification scale down to normal energies. In Sec. IV we show how the axion problem is solved in our model, and in Sec. V we study the structure of the Higgs potential needed to produce the basic features of our theory. In Sec. VI we present our conclusions. Finally, in the Appendix, we present a self-contained section on a method for calculating the anomaly factor for an arbitrary irreducible representation of an SU(N) group, with the results being used in the body of the text to make our model anomaly-free.

II. THE MODEL

In our model the basic fermions transform according to the $(8,1)\oplus(1,8)$ representation of $SU(8)_L \times SU(8)_R$. Within each chiral sector we label the basis for each family as $(\nu_e, u_R, u_G, u_B, e, d_R, d_G, d_B)$. Each U(8) possesses eight diagonal λ matrices for which we use the basis

$$\lambda_A = \tfrac{1}{2}\begin{pmatrix} 1 &&&&&&& \\ & 1 &&&&&& \\ && 1 &&&&& \\ &&& 1 &&&& \\ &&&& 1 &&& \\ &&&&& 1 && \\ &&&&&& 1 & \\ &&&&&&& 1 \end{pmatrix}, \quad \lambda_B = \tfrac{1}{2}\begin{pmatrix} 1 &&&&&&& \\ & 1 &&&&&& \\ && 1 &&&&& \\ &&& 1 &&&& \\ &&&& -1 &&& \\ &&&&& -1 && \\ &&&&&& -1 & \\ &&&&&&& -1 \end{pmatrix},$$

$$\lambda_C = \frac{1}{2\sqrt{3}}\begin{pmatrix} 3 &&&&&&& \\ & -1 &&&&&& \\ && -1 &&&&& \\ &&& -1 &&&& \\ &&&& 3 &&& \\ &&&&& -1 && \\ &&&&&& -1 & \\ &&&&&&& -1 \end{pmatrix}, \quad \lambda_D = \frac{1}{2\sqrt{3}}\begin{pmatrix} 3 &&&&&&& \\ & -1 &&&&&& \\ && -1 &&&&& \\ &&& -1 &&&& \\ &&&& -3 &&& \\ &&&&& 1 && \\ &&&&&& 1 & \\ &&&&&&& 1 \end{pmatrix},$$

$$\lambda_E=\begin{bmatrix}0&&&&&&&\\&1&&&&0&&\\&&-1&&&&&\\&&&0&&&&\\&&&&0&&&\\&0&&&&0&&\\&&&&&&0&\\&&&&&&&0\end{bmatrix},\quad \lambda_F=\frac{1}{\sqrt{3}}\begin{bmatrix}0&&&&&&&\\&1&&&&0&&\\&&1&&&&&\\&&&-2&&&&\\&&&&0&&&\\&0&&&&0&&\\&&&&&&0&\\&&&&&&&0\end{bmatrix},$$

$$\lambda_G=\begin{bmatrix}0&&&&&&&\\&0&&&&0&&\\&&0&&&&&\\&&&0&&&&\\&&&&0&&&\\&0&&&&1&&\\&&&&&&-1&\\&&&&&&&0\end{bmatrix},\quad \lambda_H=\frac{1}{\sqrt{3}}\begin{bmatrix}0&&&&&&&\\&0&&&&0&&\\&&0&&&&&\\&&&0&&&&\\&&&&0&&&\\&0&&&&1&&\\&&&&&&1&\\&&&&&&&-2\end{bmatrix}. \tag{2.1}$$

We denote by T_L^3 and T_R^3 the third components of the weak isospins $SU(2)_L$ and $SU(2)_R$ contained within $SU(8)_L$ and $SU(8)_R$, respectively. The baryon and lepton numbers are related to the operators introduced in Sec. I, viz.,

$$Y_L=\tfrac{1}{2}(B_L-L_L)\,,\quad Y_R=\tfrac{1}{2}(B_R-L_R)\,,\quad F_L=L_L+3B_L\,,\quad F_R=L_R+3B_R\,, \tag{2.2}$$

so that the electric charge operator is given as

$$Q=T_L^3+T_R^3+Y_L+Y_R\,. \tag{2.3}$$

When these operators act on the fundamental of each SU(8) we have the correspondence

$$T^3\approx\lambda_B,\quad Y\sim-\frac{1}{\sqrt{3}}\lambda_C,\quad F\sim 2\lambda_A,\quad Q\sim\lambda_B-\frac{1}{\sqrt{3}}\lambda_C\,, \tag{2.4}$$

while the diagonal $SU(3)_C$ generators transform as $\lambda_E+\lambda_G$, $\lambda_F+\lambda_H$. Within each SU(8) we can represent the 63 gauge bosons as a rank two tensor

$$W_{ij}=\sum_{a=1}^{63}\lambda_{ij}^a W_a \tag{2.5}$$

for L and R indices. In terms of the basis of Eq. (2.1) we obtain (in an obvious notation)

$$\begin{aligned}
&W_{11}=\tfrac{1}{2}W_B+\frac{\sqrt{3}}{2}W_C+\frac{\sqrt{3}}{2}W_D,\quad W_{22}=\tfrac{1}{2}W_B-\frac{1}{2\sqrt{3}}W_C-\frac{1}{2\sqrt{3}}W_D+W_E+\frac{1}{\sqrt{3}}W_F,\\
&W_{33}=\tfrac{1}{2}W_B-\frac{1}{2\sqrt{3}}W_C-\frac{1}{2\sqrt{3}}W_D-W_E+\frac{1}{\sqrt{3}}W_F,\quad W_{44}=\tfrac{1}{2}W_B-\frac{1}{2\sqrt{3}}W_C-\frac{1}{2\sqrt{3}}W_D-\frac{2}{\sqrt{3}}W_F,\\
&W_{55}=-\tfrac{1}{2}W_B+\frac{\sqrt{3}}{2}W_C-\frac{\sqrt{3}}{2}W_D,\quad W_{66}=-\tfrac{1}{2}W_B-\frac{1}{2\sqrt{3}}W_C+\frac{1}{2\sqrt{3}}W_D+W_G+\frac{1}{\sqrt{3}}W_H,\\
&W_{77}=-\tfrac{1}{2}W_B-\frac{1}{2\sqrt{3}}W_C+\frac{1}{2\sqrt{3}}W_D-W_G+\frac{1}{\sqrt{3}}W_H,\quad W_{88}=-\tfrac{1}{2}W_B-\frac{1}{2\sqrt{3}}W_C+\frac{1}{2\sqrt{3}}W_D-\frac{2}{\sqrt{3}}W_H
\end{aligned} \tag{2.6}$$

for the diagonal elements of W_{ij}. The coupling of these diagonal gauge bosons to the fermion currents of the first family is then given in terms of a single coupling constant g as

$$\mathcal{L}_{\text{int}} = g\sum \bar{L}\tfrac{1}{2}\lambda^a\gamma_\lambda L(W_L^a)_\lambda + g\sum \bar{R}\tfrac{1}{2}\lambda^a\gamma_\lambda R(W_R^a)_\lambda$$

$$= \frac{g}{4} W_\lambda^B\left[\bar{\nu}_e\gamma_\lambda\nu_e - \bar{e}\gamma_\lambda e + \sum(\bar{u}_i\gamma_\lambda u_i - \bar{d}_i\gamma_\lambda d_i)\right] + \frac{g}{4\sqrt{3}} W_\lambda^C\left[3\bar{\nu}_e\gamma_\lambda\nu_e + 3\bar{e}\gamma_\lambda e - \sum(\bar{u}_i\gamma_\lambda u_i + \bar{d}_i\gamma_\lambda d_i)\right]$$

$$+ \frac{g}{4\sqrt{3}} W_\lambda^D\left[3\bar{\nu}_e\gamma_\lambda\nu_e - 3\bar{e}\gamma_\lambda e - \sum(\bar{u}_i\gamma_\lambda u_i - \bar{d}_i\gamma_\lambda d_i)\right] + \frac{g}{2} W_\lambda^E(\bar{u}_R\gamma_\lambda u_R - \bar{u}_G\gamma_\lambda u_G)$$

$$+ \frac{g}{2\sqrt{3}} W_\lambda^F(\bar{u}_R\gamma_\lambda u_R + \bar{u}_G\gamma_\lambda u_G - 2\bar{u}_B\gamma_\lambda u_B)$$

$$+ \frac{g}{2} W_\lambda^G(\bar{d}_R\gamma_\lambda d_R - \bar{d}_G\gamma_\lambda d_G) + \frac{g}{2\sqrt{3}} W_\lambda^H(\bar{d}_R\gamma_\lambda d_R + \bar{d}_G\gamma_\lambda d_G - 2\bar{d}_B\gamma_\lambda d_B) \tag{2.7}$$

summed over both chiral sectors. The off-diagonal gauge bosons and currents can be constructed analogously, but we shall not refer to them explicitly in the subsequent development of this paper, as we shall essentially be able to monitor the full breaking pattern by studying the diagonal sector only.

In our work $SU(3)_C \times SU(2)_L \times U(1)_V \times T_R^3$ will emerge as a residual light local subgroup. Here $SU(3)_C$ is the usual QCD color group, $SU(2)_L$ is the weak isospin of Weinberg and Salam, $U(1)_V$ has $(B_V - L_V)/2$ as its generator, and T_R^3 is the third component of a right-handed counterpart of the Weinberg-Salam weak isospin. The familiar U(1) of the Weinberg-Salam theory has its generator $T_R^3 + (B_V - L_V)/2$ according to Eq. (2.3), so this U(1) is a linear combination of our $U(1)_V$ and T_R^3. Associated with our light subgroup are diagonal colored gauge bosons $W_V^{EG} = (W_L^E + W_L^G + W_R^E + W_R^G)/2$ and $W_V^{FH} = (W_L^F + W_L^H + W_R^F + W_R^H)/2$, while W_L^B and W_R^B are coupled to T_L^3 and T_R^3, respectively, and $W_V^C = (W_L^C + W_R^C)/\sqrt{2}$ is coupled to $(B_V - L_V)/2$. In the symmetry limit our model is a chiral, parity-conserving, SU(8)-flavor-conserving theory with processes such as the usual β decay $\bar{u}_L + d_L \to \bar{\nu}_L^e + e_L$, for instance, being forbidden prior to the breaking, as we are able to make all possible phase transformations on the eight left-handed fermions. Additionally, we also note that as well as possessing eight gauge bosons which are coupled to the $SU(3)_C$ vector currents the model possesses another set of gauge bosons which are coupled to an $SU(3)_C$ octet of axial-vector currents.

We note that our model is not yet free of Adler-Bell-Jackiw triangle anomalies. To eliminate all possible $SU(8)_L \times SU(8)_R$ anomalies we must introduce additional fermions. The simplest possibility is to have the anomalies due to each separate $(8,1)\oplus(1,8)$ family canceled by an associated presumably heavy companion $(8^*,1)\oplus(1,8^*)$ "mirror" family. Alternatively (as we show in the Appendix) the anomalies of an entire set of four families of fermions can be canceled by adding one additional $(28^*,1)\oplus(1,28^*)$ representation of fermions. [Since the $\underline{28}^*$ is the antisymmetric representation contained in the direct product of two $\underline{8}^*$ representations of SU(8), we see that our cancellation of anomalies mimics that of the SU(5) theory where the $\underline{10}^*$, an antisymmetric $\underline{5}^* \times \underline{5}^*$, cancels the $\underline{5}$.] Also this latter possibility is actually quite economical as the $(28^*,1)\oplus(1,28^*)$ representation only contains one state which is not a confined color nonsinglet, with there then being only one extra observable fermion in our model, specifically a charged lepton which presumably would have a large mass. While we can thus formally keep our model anomaly-free we are not sure whether to attach too much significance to these additional fermions at this time, as the whole anomaly question is really an expression of our ignorance of the short-distance behavior of field theory.

We now construct the breaking pattern for our model. For simplicity we shall only use Higgs fields which transform as fermion bilinears and quadrilinears.[9] In the first stage of breaking, the superheavy stage, we shall use the quadrilinear representations, reserving the lower dimension bilinears for the more familiar breaking of the light subgroup. At the superheavy stage we note that we want to break the local axial color and the local $B_A - L_A$ symmetries. Consequently the superheavy Higgs fields must transform nontrivially according to both $SU(8)_L$ and $SU(8)_R$. Specifically we therefore introduce $\rho \sim (36, 36^*) \oplus (36^*, 36)$ and $\omega \sim (28, 28^*) \oplus (28^*, 28)$ as the superheavy Higgs fields. [In SU(8), $\underline{8} \times \underline{8} = \underline{36} \oplus \underline{28}$.] We represent each of these Higgs fields by a four-index tensor [with Latin indices for $SU(8)_L$ and Greek indices for $SU(8)_R$] so that the gauge-invariant minimal coupling leads for ρ to a gauge-boson mass term

$$g^2 \left| W_{ac}^L \rho_{cb\alpha\beta} + W_{bc}^L \rho_{ac\alpha\beta} - W_{\alpha\gamma}^R \rho_{ab\gamma\beta} - W_{\beta\gamma}^R \rho_{ab\alpha\gamma} \right|^2 \tag{2.8}$$

with an analogous expression for ω. In order to find the superheavy breaking pattern which will leave $SU(3)_C \times SU(2)_L \times U(1)_V \times T_R^3$ unbroken we decompose SU(8) according to $SU(3) \times SU(2)$, viz.,

$$\begin{aligned} 8 &= (3,2) + (1,2)\,, \\ 36 &= (6,3) + (3^*,1) + (1,3) + (3,3) + (3,1)\,, \\ 28 &= (3^*,3) + (6,1) + (1,1) + (3,3) + (3,1)\,. \end{aligned} \tag{2.9}$$

Since $SU(2)_L$ is to be preserved the left-handed piece of ρ or its conjugate must transform as $(3,1)$. To preserve the vector $SU(3)_C$ we then need the right-handed piece to transform as a color 3^*. We will find below that we need to break $SU(2)_R$ already at the superheavy stage, and so we break the right-handed sector of ρ as $(3^*,3^*)$. Thus we take the nonvanishing vacuum expectation values of ρ to transform as (the fermion fields are used only to indicate the quantum numbers)

$$\begin{aligned} \rho &\equiv (36,36^*) \oplus (36^*,36) \\ &\sim \sum_{i=R,G,B} \langle (u_i e + e u_i - d_i \nu_e - \nu_e d_i)_L (\bar{u}_i \bar{e} + \bar{e}\bar{u}_i)_R \rangle \\ &\quad + \text{H.c.} \end{aligned} \tag{2.10}$$

With this choice of breaking we see that the usual left-handed β decay has now been permitted by spontaneous breakdown. For the second superheavy Higgs field ω we note that if its left-hand sector transforms the same way as that of ρ it will not be able to complete the breaking of the left-hand sector down all the way to $SU(3)_C \times SU(2)_L \times U(1)_V \times T_R^3$. Thus for the left-hand sector of ω we have to take the $(6,1)$ term in the decomposition of the $\underline{28}$. Then in order to maintain $SU(3)_C$ the right-hand sector of ω must transform as $(6^*,1)$. Hence, finally for ω we take

$$\begin{aligned} \omega &\equiv (28,28^*) \oplus (28^*,28) \\ &\sim \sum_{i=R,G,B} \langle (u_i d_i - d_i u_i)_L (\bar{u}_i \bar{d}_i - \bar{d}_i \bar{u}_i)_R \rangle \\ &\quad + [\langle (u_R d_G - d_G u_R + u_G d_R - d_R u_G)_L (\bar{u}_R \bar{d}_G - \bar{d}_G \bar{u}_R + \bar{u}_G \bar{d}_R - \bar{d}_R \bar{u}_G)_R \rangle + \text{cyclic permutations}] + \text{H.c.} \end{aligned} \tag{2.11}$$

From Eq. (2.8) we find that the symmetry is broken down to the local $SU(3)_C \times SU(2)_L \times U(1)_V \times T_R^3$ for both the diagonal and nondiagonal elements of W_{ij} as required. In particular, for the diagonal sector of W_{ij} Eqs. (2.6) and (2.8) give a mass term (ρ_0 and ω_0 denote mean expectation values for the respective representations)

$$\begin{aligned} \mathcal{L}_{\text{mass}} &= g^2 \rho_0^2 [(W_L^C - W_R^C + 2W_R^D)^2 + 4W_L^{D^2} + (W_L^E - W_R^E)^2 + (W_L^G - W_R^E)^2 + (W_L^F - W_R^F)^2 + (W_L^H - W_R^F)^2] \\ &\quad + g^2 \omega_0^2 [5(W_L^C - W_R^C)^2 + 4(W_L^E + W_L^G - W_R^E - W_R^G)^2 + 3(W_L^E - W_L^G - W_R^E + W_R^G)^2 \\ &\qquad + 3(W_L^E - W_L^G + W_R^E - W_R^G)^2 + 4(W_L^F + W_L^H - W_R^F - W_R^H)^2 \\ &\qquad + 3(W_L^F - W_L^H - W_R^F + W_R^H)^2 + 3(W_L^F - W_L^H + W_R^F - W_R^H)^2] \end{aligned} \tag{2.12}$$

to leave W_V^{EG}, W_V^{FH}, W_L^B, W_V^C, and W_R^B so far massless within the diagonal sector of the local $SU(3)_C \times SU(2)_L \times U(1)_V \times T_R^3$. The couplings of these latter diagonal bosons to the currents is obtained from Eq. (2.7) as

$$\begin{aligned} \mathcal{L}_{\text{int}} &= \frac{g}{4} (W_L^B)^\lambda \Big[\bar{\nu}_e \gamma_\lambda \nu_e - \bar{e}\gamma_\lambda e + \sum (\bar{u}_i \gamma_\lambda u_i - \bar{d}_i \gamma_\lambda d_i)\Big]_L + \frac{g}{4} (W_R^B)^\lambda \Big[\bar{\nu}_e \gamma_\lambda \nu_e - \bar{e}\gamma_\lambda e + \sum (\bar{u}_i \gamma_\lambda u_i - \bar{d}_i \gamma_\lambda d_i)\Big]_R \\ &\quad + \frac{g}{4\sqrt{6}} (W_V^C)^\lambda \Big[3\bar{\nu}_e \gamma_\lambda \nu_e + 3\bar{e}\gamma_\lambda e - \sum (\bar{u}_i \gamma_\lambda u_i + \bar{d}_i \gamma_\lambda d_i)\Big] + \frac{g}{4} (W_V^{EG})^\lambda \bar{q}\gamma_\lambda (\lambda_E + \lambda_G) q + \frac{g}{4} (W_V^{FH})^\lambda \bar{q}\gamma_\lambda (\lambda_F + \lambda_H) q\,. \end{aligned} \tag{2.13}$$

To complete the breaking we follow Refs. 11 and 12 and choose the low-energy Higgs fields to transform as the fermion bilinears $\sigma_L \sim (36,1)$, $\sigma_R \sim (1,36)$, and $\chi \sim (8,8^*) \oplus (8^*,8)$. Here σ_L and σ_R are difermions whose only electrically neutral elements are the dineutrinos, and χ is the fermion mass term. For these Higgs fields the gauge-invariant minimal coupling leads to a gauge-boson mass term

$$\begin{aligned} \mathcal{L}_{\text{mass}} &= \tfrac{1}{4} g^2 \,|W_{\alpha\gamma}^R \sigma_{\gamma\beta}^R + W_{\beta\gamma}^R \sigma_{\alpha\gamma}^R|^2 \\ &\quad + \tfrac{1}{4} g^2 \,|W_{ab}^L \chi_{b\alpha} - W_{\beta\alpha}^R \chi_{a\beta}|^2\,. \end{aligned} \tag{2.14}$$

For the vacuum expectation values we follow Refs. 11 and 12 and take

$$\sigma_L \equiv (36,1) \sim 0\,, \tag{2.15}$$

$$\sigma_R \equiv (1,36) \sim \langle \nu_R^e \nu_R^e \rangle\,, \tag{2.16}$$

$$\chi \equiv (8, 8^*) \oplus (8^*, 8)$$
$$\sim m_e\langle \bar{e}e\rangle + \sum_i (m_u \langle \bar{u}_i u_i\rangle + m_d\langle \bar{d}_i d_i\rangle) . \qquad (2.17)$$

The neutrino-pairing term σ_R breaks $(B_V - L_V)/2$ and T^3_R but not Q so it leaves invariant an $SU(2)_L \times U(1)$ group whose U(1) generator is $T^3_R + (B_V - L_V)/2$. The fermion mass term χ then gives the fermions their masses[16] and breaks this latter $SU(2)_L \times U(1)$ group down to the U(1) of electric charge. Thus the above pattern completes the breaking down to the local $SU(3)_C \times Q$ as required. Moreover, as was noted in Refs. 11 and 12, when the expectation value of σ_R is much greater than that of χ, our breaking pattern also produces the standard Weinberg-Salam phenomenology in both the charged- and neutral-current sectors of the theory. Thus neutrino pairing breaks the theory in exactly the right way. Before studying the ensuing phenomenology however, a little caution is required in applying the analysis of Refs. 11 and 12 to our grand unified model since now the various coupling constants undergo renormalization as we go down from the superheavy unification scale to usual energies; and although our results in fact remain true we shall defer discussion of this point until Sec. III where we shall study the renormalization explicitly.

From our breaking pattern it is clear that as well as having broken the local $SU(8)_L \times SU(8)_R$ down to a local $SU(3)_C \times Q$ we have additionally also broken the global fermion and axial fermion numbers of $U(8)_L \times U(8)_R$, and our breaking pattern has instead left two other global symmetries unbroken, namely B_V and $L_V - N_R$. Thus the final overall resulting symmetry of our model is the direct product of a local $SU(3)_C \times Q$ and a global $B_V \times (L_V - N_R)$. Since the model has to produce two residual global symmetries it is necessary that the fermion mass term χ also respect our choice of symmetry with the conservation of $L_V - N_R$ thereby preventing χ from generating a neutrino mass term in Eq. (2.17). With a Majorana mass for the left-handed neutrinos also forbidden by the conservation of $L_V - N_R$ we thus see that the left-handed neutrinos are unable to acquire either a Dirac ($\sim\chi$) or a Majorana ($\sim\sigma_L$) mass, and must thus remain massless to all orders in the interaction. We believe this to be the first time that the masslessness of the observed neutrinos has been explained by the conservation of a continuous symmetry. (Theories which try to use γ_5 invariance to keep neutrinos massless are usually unable to explain why that symmetry is not spontaneously broken at the same time as it is broken by the other fermion masses.) With regard to the right-handed sector we note that the right-hand β decay $\bar{u}_R + d_R \to \bar{\nu}^e_R + e_R$ is forbidden in the unbroken theory by the local $SU(8)_R$ invariance of phase transformations on the right-handed fermions, and is also forbidden after the symmetry breaking by the global $L_V - N_R$. Consequently $SU(2)_R$ can never be an approximate symmetry in our theory, but rather only T^3_R, with the right-handed neutrino currents remaining diagonal in the canonical $SU(8)_L \times SU(8)_R$ basis. Thus in order to leave B_V and $L_V - N_R$ unbroken the superheavy ρ and ω also must not leave an $SU(2)_R$ symmetry unbroken but rather only T^3_R. Thus the parity violation and fermion-number violation due to right-handed neutrino pairing must propagate to the superheavy sector as well, thus explaining the choice of breaking given in Eqs. (2.10) and (2.11).

Since our residual $L_V - N_R$ symmetry does not act on the right-handed neutrinos, the right-handed neutrinos are able to acquire mass, and indeed the σ_R pairing term generates a Majorana mass for them.[12] With the left-handed neutrinos remaining massless there is thus a clear and unambiguous disparity between the left- and right-handed sectors of our model which will eventually make our ideas amenable to experimental testing. As we noted above the right-handed neutrino currents remain diagonal in the canonical basis with the conservation of $L_V - N_R$ thus preventing the decay of the massive right-handed neutrinos into the usual leptons, and hence we expect at least one absolutely stable neutral fermion in our model. This is then the lepton analog of the absolute stability of the proton in our model, and provides a clear experimental test of our model. (The neutral lepton would have to be pair produced by the neutral currents [see explicitly Eq. (3.14) below], be found to be stable, and be found not to be produced together with e_R at a comparable rate in the charged-current sector.) Finally, since the neutrinos get their masses from the Higgs field σ_R which can have a relatively small vacuum expectation value, the neutrinos are relatively light. Thus the model can be tested at normal energies way below the superheavy unification scale.

In the above way neutrino pairing leads us to a breaking pattern which breaks our grand unified model down to the standard phenomenology, while retaining B_V and $L_V - N_R$ as conserved global currents. We note that our counting of broken currents due to the pattern chosen for ρ, ω, σ_L, σ_R, and χ agrees exactly with the counting (117) and with the explicit quantum numbers of the gauge bosons that acquire mass from the minimal coupling in the gauge-boson mass matrix. The 't Hooft mechanism ensures us that if we are at a minimum of the Higgs potential there will then be no superfluous Goldstone bosons or extraneous massless gauge bosons in the theory. However, it is unfortunately beyond our computational ability to check this point explicitly since our model possesses 4432 Higgs fields. We have, however, verified that all of the phenomena we require do obtain in a somewhat simplified but nontheless sufficiently complicated model which we present in Sec. V. Based on this later analysis we are confident that the application of the 't Hooft mechanism (whose content is primarily group theoretical anyway) to our breaking pattern is correct in essence.

As a mechanism for obtaining proton stability our model is one of a broad class of grand unified models discussed in the review of Gell-Mann, Ramond, and Slansky.[17] In particular the authors of Ref. 17 noted that breaking according to a dilepton pair in SU(N)-based models can lead to a stable proton by means of the 't Hooft mechanism in much the manner discussed here. What is new in our work is the identification of right-handed neutrinos as the explicit dilepton pair, and the use of the chiral group which leads to several novel features. In particular, since our model is chiral it possesses not one but two global symmetries so that while the application of the 't Hooft mechanism to fermion number leads to the conservation of B_V exactly as in Ref. 17, its application to axial fermion number leads, at the same time and through the same neutrino pairing term, to the conservation of $L_V - N_R$ and also, as we shall discuss in detail in Sec. IV, to the elimination of the axion by the Higgs mechanism. None of these additional features were anticipated in the review of Ref. 17, and so they now provide us with a specific physical reason (other than proton stability) to pick just one of the general analyses and cases of Ref. 17 as special.

Before discussing some of the above features of our model in more detail, we turn first instead in Sec. III to a study of the renormalization of the coupling constants of our model so that the low-energy structure of the neutral currents of our model can be put in a form suitable for phenomenological testing of our ideas, and in particular for studying the physics associated with the massive right-handed neutrinos.

III. RENORMALIZATION OF THE NEUTRAL-CURRENT SECTOR

In this section we study the mass dependence of our model as the theory is renormalized downwards from the superheavy unification mass scale M to a normal mass scale m. Before we discuss this effect explicitly though, it is convenient to first discuss $SU(3)_C \times SU(2)_L \times U(1)_V \times T_R^3$ as an independent theory and only then embed it into $SU(8)_L \times SU(8)_R$. We shall define the diagonal sector of this theory in terms of four momentarily independent coupling constants with an interaction for the diagonal sector of the form

$$\mathcal{L}_{\rm int} = \tfrac{1}{2} g_C G_3^\lambda \bar{q}\gamma_\lambda(\lambda_E + \lambda_G)q + \tfrac{1}{2} g_C G_8^\lambda \bar{q}\gamma_\lambda(\lambda_F + \lambda_H)q + g_L L_\lambda J_L^\lambda + g_B B_\lambda J_B^\lambda + g_R R_\lambda J_R^\lambda . \tag{3.1}$$

Here G_3 and G_8 are the diagonal vector gluons, L, B, and R are the neutral gauge bosons of $SU(2)_L \times U(1)_V \times T_R^3$, and the currents are

$$\begin{aligned} J_{L,R}^\lambda &= \tfrac{1}{4}\sum_i [\bar{u}_i\gamma_\lambda(1\mp\gamma_5)u_i - \bar{d}_i\gamma_\lambda(1\mp\gamma_5)d_i] + \tfrac{1}{4}[\bar{\nu}_e\gamma_\lambda(1\mp\gamma_5)\nu_e - \bar{e}\gamma_\lambda(1\mp\gamma_5)e] , \\ J_B^\lambda &= \tfrac{1}{6}\sum_i (\bar{u}_i\gamma_\lambda u_i + \bar{d}_i\gamma_\lambda d_i) - \tfrac{1}{2}(\bar{\nu}_e\gamma_\lambda\nu_e + \bar{e}\gamma_\lambda e) \end{aligned} \tag{3.2}$$

for the first family. The $SU(2)_L \times U(1)_V \times T_R^3$ sector of $\mathcal{L}_{\rm int}$ can be reexpressed as[18]

$$\mathcal{L}_{\rm int} = eA_\lambda Q^\lambda + g_L \sec\theta_W Z_\lambda J_z^\lambda + (g_B{}^2 + g_R{}^2)^{1/2} C_\lambda \left[J_R^\lambda + \frac{g_L{}^2}{g_R{}^2}\tan^2\theta_W (J_z^\lambda - \cos^2\theta_W Q^\lambda)\right], \tag{3.3}$$

where we have introduced

$$\begin{aligned} A &= (\tilde{g}^2 + g_L{}^2)^{-1/2}\left[\tilde{g}L + \frac{g_L}{(g_R{}^2+g_B{}^2)^{1/2}}(g_B R + g_R B)\right] , \\ Z &= (\tilde{g}^2 + g_L{}^2)^{-1/2}\left[g_L L - \frac{\tilde{g}}{(g_R{}^2+g_B{}^2)^{1/2}}(g_B R + g_R B)\right] , \\ C &= (g_B{}^2 + g_R{}^2)^{-1/2}(g_R R - g_B B) , \\ Q^\lambda &= J_L^\lambda + J_B^\lambda + J_R^\lambda , \\ J_z^\lambda &= J_L^\lambda - \sin^2\theta_W Q^\lambda , \\ \tilde{g} &= g_R g_B/(g_B{}^2 + g_R{}^2)^{1/2} , \\ \sin^2\theta_W &= \tilde{g}^2/(g_L{}^2 + \tilde{g}^2) , \\ e &= g_L \sin\theta_W . \end{aligned} \tag{3.4}$$

While Eq. (3.3) is merely an identical rewriting of Eq. (3.1) we note that $\mathcal{L}_{\rm int}$ has been expressed in terms of the physical electromagnetic current Q_λ and the familiar Weinberg-Salam neutral current J_Z^λ. The form of Eq. (3.3) is completely general and is the most convenient one to use in theories which contain both left-and right-handed neutral currents.

In order to break the theory we shall imitate Sec. II by introducing a right-handed neutrino-pairing Higgs field $\hat{\sigma}_R$ which transforms as (0, -1, 3) under $SU(2)_L \times U(1)_V \times T_R^3$ and a fermion-mass Higgs field $\hat{\chi}$ which transforms as $(2,0,2^*)$. Only $\hat{\chi}$ gives a mass (M_W) to the charged left-handed gauge bosons of $SU(2)_L$. Further, only $\hat{\sigma}_R$ gives a mass to the neutral C, while $\hat{\chi}$ gives masses to both Z and C and also mixes them. The resulting Z-C mass matrix is given as

$$M^2=\begin{pmatrix} M_Z^2 & M_{Z\text{-}C}^2 \\ M_{Z\text{-}C}^2 & M_C^2 \end{pmatrix}, \tag{3.5}$$

where

$$\begin{aligned} M_Z^2 &= \tfrac{1}{4}(g_L^2+\tilde{g}^2)(\langle\hat{\chi}_{11}\rangle^2+\langle\hat{\chi}_{22}\rangle^2) \\ &= M_W^2\sec^2\theta_W, \\ M_{Z\text{-}C}^2 &= -\tfrac{1}{4}g_R^2\left(\frac{g_L^2+\tilde{g}^2}{g_R^2+g_B^2}\right)^{1/2}(\langle\hat{\chi}_{11}\rangle^2+\langle\hat{\chi}_{22}\rangle^2), \\ M_C^2 &= \frac{g_R^4}{4(g_R^2+g_B^2)}(\langle\hat{\chi}_{11}\rangle^2+\langle\hat{\chi}_{22}\rangle^2) \\ &\quad +(g_B^2+g_R^2)\langle\hat{\sigma}^R_{11}\rangle^2. \end{aligned} \tag{3.6}$$

After some manipulation we find[18] that the effective weak interaction at small momentum transfer due to Eq. (3.3) can be written identically as

$$\mathcal{L}_{\text{eff}}=\mathcal{L}_{\text{int}}^\dagger\mathcal{L}_{\text{int}}=-\frac{g_L^2}{M_Z^2\cos^2\theta_W}\left\{J_Z^\lambda J_Z^\lambda+\alpha^2\left[J_Z^\lambda+\beta\left(J_R^\lambda+\frac{g_L^2}{g_R^2}\tan^2\theta_W(J_Z^\lambda-\cos^2\theta_W Q^\lambda)\right)\right]^2\right\}, \tag{3.7}$$

where

$$\alpha^2=M_{Z\text{-}C}^4/\det M^2, \tag{3.8}$$

$$\beta=-\left(\frac{g_R^2+g_B^2}{g_L^2+\tilde{g}^2}\right)^{1/2}\frac{M_Z^2}{M_{Z\text{-}C}^2}=\frac{(g_R^2+g_B^2)}{g_R^2},$$

so that the α^2 term generates all possible deviations from the Weinberg-Salam theory. From the above relations we can now recover the results of Refs. 11 and 12, namely that the theory breaks down to the standard Weinberg-Salam model in the limit in which $\langle\hat{\sigma}_R\rangle$ is much greater than $\langle\hat{\chi}\rangle$, a limit in which α tends to zero.

All of our analysis so far would be true for any $SU(2)_L\times U(1)_V\times T_R^3$ theory. However, our theory here is a low-energy limit of $SU(8)_L\times SU(8)_R$. Consequently, we can relate the various coupling constants appearing in Eq. (3.1) by treating them as running coupling constants which are a function of the mass scale m. At the unification mass scale M the symmetry is restored, and thus at that scale we can equate the Lagrangian of Eq. (3.1) with that of Eq. (2.13) to obtain

$$\begin{aligned} &g_C(M)=g/2,\quad g_L(M)=g/2, \\ &g_B(M)=-g\sqrt{3}/2\sqrt{2},\quad g_R(M)=g/2, \end{aligned} \tag{3.9}$$

with the gauge bosons G_3, G_8, L, B, R being identified as W_V^{EG}, W_V^{FH}, W_L^B, W_V^C, and W_R^B, respectively. With the reduction of the theory to one coupling constant the Weinberg angle at the unification scale M is then determined entirely by the group theory, so that from Eq. (3.4) we obtain

$$\sin^2\theta_W(M)=\tfrac{3}{8}. \tag{3.10}$$

Thus our model gives an acceptable value for the unrenormalized Weinberg angle, and a familiar one in fact, as it is the same one as obtained in the SU(5) theory.[19]

In breaking the theory down from the unification scale M we note that the superheavy breaking acts before we break $SU(3)_C\times SU(2)_L\times U(1)_V\times T_R^3$. Thus by the time the fermion bilinears get to act the various couplings of Eq. (3.1) have already assumed low-energy values which differ from the symmetry-limit values of Eq. (3.9). (To simplify the discussion we have taken the scales of all the fermion-bilinear Higgs fields to be much smaller than the scale M.) Consequently, the spontaneous breakdown associated with these bilinears should be discussed in terms of $\hat{\sigma}_R$ and $\hat{\chi}$ rather than in terms of σ_R and χ of Eq. (2.14). In this way we can incorporate a running coupling-constant dependence into Eq. (2.14). Thus it is not only the Weinberg angle which undergoes renormalization but also the Higgs-field couplings which appear in the gauge-boson mass matrix. This latter renormalization effect can most easily be described simply as a renormalization of the running coupling constants of Eq. (3.7).

To evaluate the explicit renormalization of the coupling constants we follow the one-loop renormalization-group analysis of Ref. 19. The most significant contributions are due to the gauge bosons and the fermions, and of these only the light particles (i.e., masses of order m, not M) affect the renormalization. With our choice of only the gauge bosons of $SU(3)_C\times SU(2)_L\times U(1)_V\times T_R^3$ being light, and with all the fermions being light, we obtain

$$\begin{aligned} \frac{1}{g_C^2(m)} &= \frac{4}{g^2}-\frac{11}{8\pi^2}\ln\frac{M}{m}+\frac{N}{6\pi^2}\ln\frac{M}{m}, \\ \frac{1}{g_L^2(m)} &= \frac{4}{g^2}-\frac{11}{12\pi^2}\ln\frac{M}{m}+\frac{N}{6\pi^2}\ln\frac{M}{m}, \\ \frac{1}{g_B^2(m)} &= \frac{2}{3}\left(\frac{4}{g^2}+\frac{N}{6\pi^2}\ln\frac{M}{m}\right), \\ \frac{1}{g_R^2(m)} &= \frac{4}{g^2}+\frac{N}{6\pi^2}\ln\frac{M}{m}, \end{aligned} \tag{3.11}$$

where N is the number of families of fermions. From these relations we see that $\tilde{g}^2$ is not renormalized and after a little algebra we obtain

$$\sin^2\theta_W(m)=\frac{3}{8}-\frac{55e^2(m)}{96\pi^2}\ln\frac{M}{m}$$
$$=\frac{1}{6}+\frac{5}{9}\frac{e^2(m)}{g_C{}^2(m)}. \qquad (3.12)$$

Thus the renormalization of the Weinberg angle is exactly the same as in the SU(5) theory.[19] Thus, as there, we have unification at 10^{15} GeV. However, since we do not encounter the problem of proton decay in our model we still have the freedom to make order of magnitude changes in the value of M as the experimental uncertainty in the Weinberg angle decreases. From Eqs. (3.11) we note further that β and $(g_L/g_R)\tan\theta_W$ are renormalization invariants with values

$$\beta(m)=\tfrac{5}{2},\quad \frac{g_L{}^2(m)}{g_R{}^2(m)}\tan^2\theta_W(m)=\frac{\tilde g^2(m)}{g_R{}^2(m)}=\tfrac{3}{5}. \qquad (3.13)$$

Thus $\mathcal{L}_{\rm eff}$ of Eq. (3.7) takes the final form at mass m,

$$\mathcal{L}_{\rm eff}=-\frac{8G_F}{\sqrt{2}}\big(J_Z^\lambda J_Z^\lambda + \alpha^2\{\tfrac{5}{2}(J_L^\lambda+J_R^\lambda)-[\tfrac{3}{2}+\sin^2\theta_W(m)]Q^\lambda\}^2\big). \qquad (3.14)$$

We see that the second term in $\mathcal{L}_{\rm eff}$ is completely parity conserving. Since the full theory was parity invariant in the unbroken limit we thus find that in our theory the entire departure from parity conservation in the neutral-current sector of the model is that already given by Weinberg-Salam, and as before the theory reduces to exact Weinberg-Salam for small α. The phenomenological limits on the parameter α in Eq. (3.14) have been studied in Ref. 18, where it was found that there is little mixing in Eq. (3.5) and that M_C must be greater than 2.74 M_Z.

As noted, the relations of Eqs. (3.12) are obtained both in our model where $SU(2)_L\times U(1)_V\times T_R^3$ is the light flavor subgroup and also in the popular SU(5) theory where only $SU(2)_L\times U(1)$ is light. The reason for this is that there is only one extra light gauge boson in our model, and with it being coupled to an Abelian group it makes no contribution in Eqs. (3.11). Consequently, the dominant gauge-boson contributions are the same in the two cases. Moreover, there is even no difference between the two cases in the fermion sector either. We obtained Eqs. (3.11) by taking all the fermions to be light. If instead we were to make σ_R superheavy so that the right-handed neutrino would acquire a superheavy Majorana mass, we note from Eq. (3.7) that, at the same time, only $SU(2)_L\times U(1)$ would be left as a residual light flavor subgroup and only the remaining fermions (the usual light ones) would renormalize the Weinberg angle. This of course is exactly the situation met in SU(5) where Eqs. (3.12) were first obtained, and hence they would still be obtained in our model even when σ_R is superheavy. Consequently, the renormalization of the Weinberg angle in our model is not affected at all by whether or not the right-handed neutrino is light or superheavy if the low-energy flavor subgroup of the theory is no larger than $SU(2)_L\times U(1)_V\times T_R^3$. Hence, the known success of Eqs. (3.12) does not constrain our theory to possess only $SU(2)_L\times U(1)$ as its light flavor subgroup, and in no way prevents the right-handed neutrinos from acquiring light as opposed to superheavy masses.

Having now completed the analysis for $SU(2)_L\times U(1)_V\times T_R^3$ as the low-energy flavor subgroup, it is of interest to ask what would have happened had the low-energy flavor subgroup been the popular $SU(2)_L\times U(1)_V\times SU(2)_R$ instead. Since both the groups have the same neutral-current structure all of the analysis of this section would go through the same (and neutrino pairing would also make the right-handed charged gauge bosons much heavier than the left-handed ones[11,12]), except that the renormalization of g_R would be modified as it is now non-Abelian to give

$$\frac{1}{g_R{}^2(m)}=\frac{4}{g^2}-\frac{11}{12\pi^2}\ln\frac{M}{m}+\frac{N}{6\pi^2}\ln\frac{M}{m}. \qquad (3.15)$$

Consequently, the renormalization of the Weinberg angle is changed, and we would obtain instead

$$\sin^2\theta_W(m)=\frac{3}{8}-\frac{11}{48\pi^2}e^2(m)\ln\frac{M}{m}$$
$$=\frac{1}{4}+\frac{e^2(m)}{3g_C{}^2(m)}. \qquad (3.16)$$

Since $\sin^2\theta_W(m)$ is apparently less than $\frac{1}{4}$ experimentally we thus see that Eq. (3.16) would appear to be ruled out. Thus necessarily $SU(2)_R$ must not be a good low-energy symmetry but rather only T_R^3. This is completely in accord with our development of the breaking pattern in Sec. II where we maintained the conservation of L_V-N_R by breaking $SU(2)_R$ but not T_R^3 in the superheavy sector. Thus we can interpret the observed value of the Weinberg angle as an indirect justification for our choice of breaking pattern and as an indirect manifestion of the relevance of the conservation of L_V-N_R.

IV. SOLUTION TO THE AXION PROBLEM

In this section we show how the axion problem is resolved in our model. We begin by reviewing the nature of the problem. Due to the global properties of instantons[20] the usual local QCD Lagrangian alone does not fully specify the dynamics but needs to be augmented by an extra term of the form $\theta F\tilde{F}$. Each different value of θ specifies a different theory, and all of them with $\theta \neq 0$ are CP violating at the level of the Lagrangian, i.e., even before the symmetry is spontaneously broken. The origin of this CP violation is due to the fact that the n = odd index instantons tunnel between states of opposite CP, making the vacuum of QCD after tunneling a CP-violating superposition of states. Such a CP violation is diagonal in the flavor indices and would lead to an unacceptable experimental value for the neutron electric dipole moment unless for some reason θ is as small as 10^{-8}. Apart from this possible experimental problem the situation is even more disturbing theoretically as there does not appear to be any physical principle which would fix the value of θ at all.

A possible resolution of this dilemma has been provided by Peccei and Quinn,[13] who noted that in a chiral theory the effects due to θ can be rotated away completely, making the dynamics then independent of this arbitrary parameter θ. Specifically, they noted that due to the Adler-Bell-Jackiw anomaly the divergence of the axial baryon-number current B_A is proportional to a term of the $F\tilde{F}$ just like the instanton contribution to the Lagrangian above, and hence the dependence on θ can be removed by a chiral rotation. However, in order to effect this rotation it is necessary to augment the Weinberg-Salam weak interaction with an additional $U(1)_A$ symmetry. Consequently, when the fermions acquire their masses by spontaneous breakdown this continuous $U(1)_A$ symmetry will be broken also, to produce an extra observable massless Goldstone boson, the axion of Ref. 14, for which there is no apparent experimental support. Consequently, we have either an uncalculable parameter θ in the theory or an unwanted axion.

What would have been the obvious solution to the axion problem would have been to remove it by the Higgs mechanism. However this is not possible since anomalies in the $U(1)_A$ current prevent this current from being given a local extension in the first place. In order to gauge the $U(1)_A$ current we would thus need the current to be anomaly free, while in order to induce a θ dependence into the divergence of the current we would need an explicit $F\tilde{F}$ anomaly term. Thus the anomaly resolves one problem only to create another.

We now note that in our model a new possibility is now open to us as it is now possible to remove the axion by the Higgs mechanism without needing to gauge the extra $U(1)_A$ symmetry. Specifically, in our model B_A is not purely a global symmetry but is a linear combination of the local $Y_A = \frac{1}{2}(B_A - L_A)$ and the global axial fermion number $F_A = L_A + 3B_A$. Consequently, when the fermion masses spontaneously break B_A in Eq. (2.17) they simultaneously break both a global and a local symmetry, so that by the 't Hooft mechanism the local gauge bosons now absorb the axion to remove it from the physical spectrum, with a new residual global symmetry, $L_V - N_R$, emerging in place of F_A, exactly as discussed in the previous sections. Moreover, while Y_A is anomaly free since it is part of the gauged $SU(8)_L \times SU(8)_R$, we note that we still have the freedom to give anomalies to F_L and F_R [which belong to $U(8)_L \times U(8)_R$] as we do not gauge their currents. (In the Appendix we show how this is done explicitly.) Thus in our model B_A is not anomaly free, so that through its divergence the dependence on θ can be rotated away; while at the same time B_A is not coupled only to a purely global symmetry, so that through the 't Hooft mechanism its associated axion is removed by the Higgs mechanism. Thus we eliminate the axion by the Higgs mechanism without needing to gauge the full $U(8)_L \times U(8)_R$ symmetry.

We note the explicit role played by neutrino pairing. It breaks both the global and the local symmetries in just the right way to produce along with the fermion mass term the 't Hooft mechanism in the axial sector. Axial fermion number is spontaneously broken everywhere by neutrino pairing and by the fermion masses except in the left-handed neutrino sector. The masslessness of these left-handed neutrinos then provides the additional global symmetry necessary to get the Goldstone counting correct, and thus in our model the masslessness of the observed neutrinos has as a consequence the lack of observable CP violations in the strong interactions.

It is also of interest to see how the Goldstone counting is explicitly maintained in the Higgs potential. While we shall discuss the full potential itself in more detail in Sec. V we can illustrate the main part of the Goldstone counting mechanism by considering just $V(\chi)$, the piece of the potential which involves only the fermion-mass Higgs field χ. Every term in $V(\chi)$ possesses the full $U(8)_L \times U(8)_R$ invariance except for a possible term of the form $\kappa \det \chi$. When $\kappa \neq 0$ the symmetry is reduced to $SU(8)_L \times SU(8)_R \times U(1)_V$. In the event that $\kappa \neq 0$ the symmetry-breaking pattern of Eq. (2.17) produces a certain number of Goldstone bosons (107 to be precise, since the pattern breaks 107 chiral currents) in $V(\chi)$, with all of them then being absorbed by the local $SU(8)_L \times SU(8)_R$ gauge bosons in the standard way. However, since $m_\nu = 0$ we note that the det χ term vanishes in our solution with the breaking pattern possessing an extra $U(1)_A$ invariance. Consequently, the potential $V(\chi)$ will still only produce 107 Goldstone bosons even if the det χ term is absent from $V(\chi)$ altogether, i.e., even if $\kappa = 0$, and they will again be absorbed by the same set of gauge bosons as previously. Thus when $m_\nu = 0$ a $U(8)_L \times U(8)_R$ invariant potential will not produce any superfluous Goldstone bosons which cannot be gauged by the $SU(8)_L \times SU(8)_R$ gauge bosons. (This effect was first noted in Ref. 12.) Thus the conservation of $L_V - N_R$ maintains the Goldstone counting in our model.

It is further of interest to compare our solution with the "non-axion" possibility discussed in Ref. 14, the $m_u = 0$ solution. It was noted in Ref. 14 that if $m_u = 0$ there would still be sufficient residual axial symmetry in the theory to be able to rotate θ away; and with the $U(1)_A$ current of the up-quark simply not being spontaneously broken in the first place there would be no axion in the theory either. Since m_u is known to be nonzero experimentally this solution had of course to be rejected. However, it has a reflection in our proposed solution. Specifically in our case, because we combine leptons and quarks together we have to introduce axial fermion number and not just axial baryon number, and now a piece of F_A (other than B_A) is left unbroken by retaining $m_\nu = 0$, so that there is one fewer Goldstone boson than would be the case if the neutrinos were also to get a Dirac mass. (In that respect it is the analog of the effect that $m_u = 0$ would have had on B_A in Ref. 14.) However, the axial baryon number in our model is still completely broken by all the quark masses, so that there is a B_A axion in the theory which is then gauged out of the spectrum. What there is not is an analogous F_A axion since $m_\nu = 0$. Thus with grand unification we can move a conserved axial-vector current completely into the neutrino sector while still being able to rotate θ in the quark sector and still being able to give a mass to each of the quarks. Thus through $m_\nu = 0$ the leptons have an explicit influence on the QCD structure of the strongly interacting quarks.

In concluding this section we note that despite possessing QCD with all its instanton structure, most grand unified theories have not addressed the axion problem at all, with SU(5) for instance not even being able to rotate θ out of the spectrum. Further, other grand unified theories which do have a chiral structure, such as SO(10), (Ref. 21), are then not able to eliminate the axion from the spectrum once the quarks acquire their masses. Consequently our $SU(8)_L \times SU(8)_R$ theory is the first grand unified theory which can completely accommodate all the implications of the instanton structure of its QCD subgroup, in a manner compatible with experiment.

V. STRUCTURE OF THE HIGGS POTENTIAL IN A SIMPLIFIED MODEL

One of the major difficulties with large grand unified gauge groups is that the analysis of the Higgs sector of the theory poses enormous computational difficulties. However, the Higgs sector does not raise any really conceptual difficulties as the group theory alone already dictates the structure of the breaking pattern and of the associated surviving residual symmetry. Nonetheless, it would be nice to minimize the $SU(8)_L \times SU(8)_R$ potential fully to check the consistency of our model. Unfortunately, this is beyond our computational ability. Noting, however, that color is playing no explicit role in the Goldstone counting needed for the 't Hooft mechanism in our model, we have, alternatively, constructed an abbreviated colorless version of our model. We are able to carry out the complete minimization of the Higgs potential of this simpler model to the bitter end, to thus confirm that the basic ideas of this paper are mutually consistent. This strongly suggests that they will also be consistent in the full SU(8) model as well.

For our simplified model we introduce quartets $(\nu_e, e, \mathfrak{N}, \mathcal{P})_{L,R}$, where $\mathfrak{N}, \mathcal{P}$ are "quarks" with charges zero and one, respectively, so chosen so that the sum of the electric charges within each quartet is zero. Based on these quartets we introduce a local $SU(4)_L \times SU(4)_R$ gauge theory which has a global $U(4)_L \times U(4)_R$ invariance. The model contains the appropriate Weinberg-Salam model and is essentially a colorless version of our $SU(8)_L \times SU(8)_R$ theory. We take the breaking pattern to closely resemble our previous pattern by introducing 344 Higgs fields with expectation values which transform as

$$\begin{aligned}
&\rho \equiv (10, 10^*) \oplus (10^*, 10) \\
&\quad \sim \langle (\mathcal{P}e + e\mathcal{P} - \mathfrak{N}\nu_e - \nu_e\mathfrak{N})_L (\bar{\mathcal{P}}\bar{e} + \bar{e}\bar{\mathcal{P}})_R \rangle + \text{H.c.}\,, \\
&\omega \equiv (6, 6^*) \oplus (6^*, 6) \\
&\quad \sim \langle (\mathcal{P}\mathfrak{N} - \mathfrak{N}\mathcal{P})_L (\bar{\mathcal{P}}\bar{\mathfrak{N}} - \bar{\mathfrak{N}}\bar{\mathcal{P}})_R \rangle + \text{H.c.}\,, \\
&\sigma_L \equiv (10, 1) \sim 0\,, \qquad\qquad (5.1) \\
&\sigma_R \equiv (1, 10) \sim \langle (\nu_e \nu_e)_R \rangle\,, \\
&\chi \equiv (4, 4^*) \oplus (4^*, 4) \\
&\quad \sim m_e \langle \bar{e}e \rangle + m_{\mathfrak{N}} \langle \bar{\mathfrak{N}}\mathfrak{N} \rangle + m_{\mathcal{P}} \langle \bar{\mathcal{P}}\mathcal{P} \rangle\,.
\end{aligned}$$

As previously, this pattern breaks the theory down completely to the Weinberg-Salam phenomenology (giving incidentally a value of $\frac{1}{4}$ for $\sin^2\theta_W$) with the local Q and the global B_V and $L_V - N_R$ remaining unbroken. For consistency we must now show that our pattern is a minimum of the Higgs potential.

In discussing the Higgs potential we consider first only the self-coupling terms,

$$V(\rho) = -b_1 \operatorname{Tr}\rho^\dagger\rho + b_2(\operatorname{Tr}\rho^\dagger\rho)^2 + b_3\rho^\dagger_{ab\alpha\beta}\rho_{cd\alpha\gamma}\rho^\dagger_{cd\delta\gamma}\rho_{ab\delta\beta} + b_3\rho^\dagger_{ab\alpha\beta}\rho_{ac\gamma\delta}\rho^\dagger_{dc\gamma\delta}\rho_{db\alpha\beta}$$
$$+ b_5\rho^\dagger_{ab\alpha\beta}\rho_{ab\gamma\delta}\rho^\dagger_{cd\gamma\delta}\rho_{cd\alpha\beta} + b_6\rho^\dagger_{ab\alpha\beta}\rho_{ac\alpha\gamma}\rho^\dagger_{cd\gamma\delta}\rho_{bd\beta\delta}\,, \tag{5.2}$$

$$V(\omega) = -a_1 \operatorname{Tr}\omega^\dagger\omega + a_2(\operatorname{Tr}\omega^\dagger\omega)^2 + (a_3, a_3, a_5, a_6)\operatorname{Tr}\omega^\dagger\omega\omega^\dagger\omega(\text{analogous to } \rho)\,, \tag{5.3}$$

$$V(\sigma_L, \sigma_R) = -d(\operatorname{Tr}\sigma_L^\dagger\sigma_L + \operatorname{Tr}\sigma_R^\dagger\sigma_R) + e[(\operatorname{Tr}\sigma_L^\dagger\sigma_L)^2 + (\operatorname{Tr}\sigma_R^\dagger\sigma_R)^2] + f[\operatorname{Tr}(\sigma_L^\dagger\sigma_L)^2 + \operatorname{Tr}(\sigma_R^\dagger\sigma_R)^2]\,, \tag{5.4}$$

$$V(\chi) = -a\operatorname{Tr}\chi^\dagger\chi + b(\operatorname{Tr}\chi^\dagger\chi)^2 + c\operatorname{Tr}(\chi^\dagger\chi)^2\,, \tag{5.5}$$

where as usual b_1, a_1, d, and a are all positive. The pattern of Eq. (5.1) is a minimum of the separate $V(\omega)$ and $V(\sigma_R)$ pieces of the potential if

$$4a_2 + 4a_3 + 4a_5 + a_6 > 0, \quad e + f > 0, \quad f < 0\,, \tag{5.6}$$

but the proposed pattern produces tachyons in each of the ρ, σ_L, and χ self-coupled sectors. There are also cross terms which couple the various representations in the Higgs potential,

$$V_{\text{cross}} = c_1\operatorname{Tr}\omega^\dagger\omega\operatorname{Tr}\rho^\dagger\rho + c_2\omega^\dagger_{ab\alpha\beta}\omega_{ac\alpha\gamma}\rho^\dagger_{cd\gamma\delta}\rho_{bd\beta\delta} + c_3\omega^\dagger_{ab\alpha\beta}\omega_{ab\alpha\gamma}\rho^\dagger_{cd\gamma\delta}\rho_{cd\beta\delta} + c_3\omega^\dagger_{ab\alpha\beta}\omega_{ac\alpha\beta}\rho^\dagger_{cd\gamma\delta}\rho_{bd\gamma\delta}$$
$$+ c_5\operatorname{Tr}\omega^\dagger\omega(\operatorname{Tr}\sigma_L^\dagger\sigma_L + \operatorname{Tr}\sigma_R^\dagger\sigma_R) + c_6(\sigma^{L\dagger}_{ac}\sigma^L_{cb}\omega^\dagger_{bd\alpha\beta}\omega_{ad\alpha\beta} + \sigma^R_{\alpha\gamma}\sigma^{R\dagger}_{\gamma\beta}\omega^\dagger_{ab\beta\delta}\omega_{ab\alpha\delta}) + c_7\operatorname{Tr}\rho^\dagger\rho(\operatorname{Tr}\sigma_L^\dagger\sigma_L + \operatorname{Tr}\sigma_R^\dagger\sigma_R)$$
$$+ c_8(\sigma^{L\dagger}_{ac}\sigma^L_{cb}\rho^\dagger_{bd\alpha\beta}\rho_{ad\alpha\beta} + \sigma^R_{\alpha\gamma}\sigma^{R\dagger}_{\gamma\beta}\rho^\dagger_{ab\beta\delta}\rho_{ab\alpha\delta})$$
$$+ c_9(\sigma^{L\dagger}_{ab}\sigma^L_{cd}\rho^\dagger_{cd\alpha\beta}\rho_{ab\alpha\beta} + \sigma^R_{\alpha\beta}\sigma^{R\dagger}_{\gamma\delta}\rho^\dagger_{ab\gamma\delta}\rho_{ab\alpha\beta})$$
$$+ d_1\operatorname{Tr}\omega^\dagger\omega\operatorname{Tr}\chi^\dagger\chi + d_2(\chi_{a\alpha}\chi^\dagger_{a\beta}\omega^\dagger_{bc\alpha\gamma}\omega_{bc\beta\gamma} + \chi_{a\alpha}\chi^\dagger_{b\alpha}\omega^\dagger_{ac\beta\gamma}\omega_{bc\beta\gamma}) + d_3\chi_{a\alpha}\chi^\dagger_{b\beta}\omega^\dagger_{ac\alpha\gamma}\omega_{bc\beta\gamma} + d_4\operatorname{Tr}\rho^\dagger\rho\operatorname{Tr}\chi^\dagger\chi$$
$$+ d_5(\chi_{a\alpha}\chi^\dagger_{a\beta}\rho^\dagger_{bc\alpha\gamma}\rho_{bc\beta\gamma} + \chi_{a\alpha}\chi^\dagger_{b\alpha}\rho^\dagger_{ac\beta\gamma}\rho_{bc\beta\gamma}) + d_6\chi_{a\alpha}\chi^\dagger_{b\beta}\rho^\dagger_{ac\alpha\gamma}\rho_{bc\beta\gamma} + h\operatorname{Tr}\sigma_L^\dagger\sigma_L\operatorname{Tr}\sigma_R^\dagger\sigma_R$$
$$+ k(\operatorname{Tr}\chi^\dagger\chi\sigma_L^\dagger\sigma_L + \operatorname{Tr}\chi\chi^\dagger\sigma_R^\dagger\sigma_R) + l\operatorname{Tr}\chi^\dagger\chi(\operatorname{Tr}\sigma_L^\dagger\sigma_L + \operatorname{Tr}\sigma_R^\dagger\sigma_R)$$
$$+ [\rho_{ab\alpha\beta}\omega^\dagger_{ac\alpha\gamma}(\kappa_1\chi^\dagger_{b\beta}\chi_{c\gamma} + \kappa_2\rho^\dagger_{bd\beta\delta}\rho_{cd\gamma\delta} + \kappa_3\omega^\dagger_{bd\beta\delta}\omega_{cd\gamma\delta}) + \kappa_4\chi_{a\alpha}\chi_{b\beta}\rho^\dagger_{ab\alpha\beta} + \kappa_5\chi_{a\alpha}\chi_{b\beta}\omega^\dagger_{ab\alpha\beta} + \text{H.c.}]\,. \tag{5.7}$$

Explicit exhausting calculation then shows that there will be a complete mutual cancellation of all of these tachyons if

$$\begin{aligned} &b, c \gg 0\,,\\ &b_2 \gg -b_5 \gg b_3, b_6 > 0,\\ &h - 2(e+f) > 0\,,\\ &k(c+3b)d - 2c(e+f)a > 0\,,\\ &8b_2ca - |d_6|(c+3b)b_1 \gg 0\,,\\ &-2d_6b_2a - (b_6 + 2b_3)(c+3b)b_1 > 0\,, \end{aligned} \tag{5.8}$$

all other coefficients very small. Under the set of conditions of Eqs. (5.6) and (5.8) our proposed pattern of Eq. (5.1) is then a minimum of the full Higgs potential for a continuous range of its parameters.

To complete the analysis we need to check that there are no superfluous Goldstone bosons generated by the potential in the diagonal sector of the theory, the only place where they could in principle appear. Our model contains eight diagonal $U(4)_L \times U(4)_R$ currents. If the three currents Q, B_V, and $L_V - N_R$ are to remain unbroken there should then only be five Goldstone bosons in this sector, all of which would then necessarily be gauged by the diagonal gauge bosons of the local $SU(4)_L \times SU(4)_R$ to leave only the photon massless. As can be seen from Eq. (5.1) ρ possesses two independent components which acquire vacuum expectation values $[\sim\langle(\mathcal{P}e)(\overline{\mathcal{P}e})\rangle$ and $\langle(\mathfrak{N}\nu_e)(\overline{\mathcal{P}e})\rangle]$, ω possesses one, σ_R one, and χ three. Consequently, in its self-potential $V(\rho)$ ρ produces two Goldstone bosons in the diagonal sector due to the imaginary parts of the above two components, while ω produces one, σ_R produces one, and χ produces three Goldstone bosons in their respective self-potentials. The self-potentials thus generate an apparent seven Goldstone bosons altogether in the diagonal sector. Explicit diagonalization, however, of the 14×14 matrix containing these seven bosons and their real parts is then found in the presence of the cross terms of Eq. (5.7) to give a positive mass squared to two linear combinations of the seven would-be Goldstone bosons to finally leave only five Goldstone bosons as required. (The cross terms which achieve this explicitly are essentially the κ_i terms with the parameters κ_i all being negative.)

The simple model is thus an explicitly solvable model which contains all the phenomena of interest in this paper, i.e., a completely stable proton, no unwanted axions, with the conservation of $L_V - N_R$ being maintained by the masslessness of the left-handed neutrinos.

VI. CONCLUSIONS

In this paper we have attempted to understand baryon number from a somewhat more basic viewpoint than is usually taken in grand unified models. We have found a rationale for why baryon number has to be an explicitly conserved symmetry at the level of the Lagrangian, for why it is then not subsequently spontaneously broken, and for why it ultimately emerges as an absolutely conserved global symmetry with no associated massless gauge boson. We cannot conclude that baryon number is conserved in the universe as a whole however, but only that grand unification does not provide a microscopic origin for phenomena such as the observed baryon excess. To the extent that such an excess does exist it must be due to macroscopic cosmological effects and not due to the fundamental weak, electromagnetic, and strong interactions. While we have yet to make an explicit study, we note in passing that because of the very fact that baryon number is short range in our theory while electric charge is long range, these two symmetries could have quite different cosmological behaviors. Apart from these macroscopic issues, the model does of course make a strong microscopic prediction that the proton be stable, and this issue is currently under experimental investigation. On the face of it would appear to be quite difficult to perform an experiment to show that the proton is completely stable; thus the upcoming proton-decay experiments will only be able to eliminate our theory but not confirm it. However, subsequent generations of experiments could in principle test proton lifetimes up to times associated with the Planck mass, and if by then proton decay is still not seen we would be least be able to conclude that grand unification is not causing proton decay.

Apart from the key feature of proton stability our model has several other noteworthy features. Since the model is a chiral theory, parity violation arises from spontaneous breakdown, unlike say SU(5). We have a reason for the masslessness of the left-handed neutrinos unlike the SU(4)[4] model of Pati and Salam[4] or the SO(10) theory of Ref. 21. Unlike all other grand unified theories, as far as we know, there is lepton-number conservation in our model. The axion problem is solved in our model. Finally, the emergence of a new stable massive neutral lepton is a novel aspect of our model which will eventually provide a clear experimental way of testing our ideas. With this neutral lepton having a relatively light mass our grand unified model can be tested at normal energies way below the unification mass scale, a fact that is useful in of itself, and which will become extremely relevant if the upcoming proton-decay experiments yield a negative result.

Finally to conclude we remark that if we enlarge our chiral gauge theory by putting members of different families into common fundamentals [for instance an $SU(16)_L \times SU(16)_R$ based on $u_i, d_i, s_i, c_i, \nu_e, e, \mu, \nu_\mu$ treated as one big family), there will still only be two residual global symmetries, since any chiral theory can possess at most two extra U(1) invariances. Consequently every possible kind of symmetry (i.e., within families or between families) will ultimately be broken in the vacuum leaving only Q, B_V, and $L_V - N_R$ as residual symmetries, thus explaining the division of the world into baryons and leptons, i.e., into at most two species which have their own global quantum numbers.

Note added

(a) We would like to make an additional comment regarding our solution to the axion problem given in Sec. IV. Strictly speaking our analysis only shows that we can rotate away the angle θ either in the unbroken theory or in the broken theory at the grand unification mass scale. Since the triangle anomaly is mass independent, the anomaly itself is not affected by the spontaneous breakdown mechanism which produces all the mass scales of our model. Consequently we do not anticipate that θ would undergo renormalization and acquire a nonzero value in the low-energy region of the theory. Nonetheless, without a detailed treatment of the renormalization properties of instantons which would accompany their quantization, we cannot rule out the possibility that θ is renormalized. However, it would appear to us that even if such a renormalization effect exists it is likely to be very small (exponentially small perhaps as it is a radiative correction to a tunneling probability). But without a detailed analysis we must, for the moment, make what we regard as a mild assumption, namely that θ is not renormalized at all, or if it is that such a renormalization is very small.

(b) It is of interest to ask what we would need to do to our model in order to actually get proton decay. The simplest possibility is to gauge the additional fermion-number generator $F_V = L_V + 3B_V$ so that the full local symmetry is now $SU(8)_L \times SU(8)_R \times U(1)_{L+R}$. In this case the baryon number B_V is completely local. Since there is no known massless gauge boson coupled to B_V experimentally, the local gauge boson coupled to B_V must acquire a mass by the Higgs mechanism. Hence B_V must be spontaneously broken, with the pattern of expectation values for the Higgs fields being modified so as to give one extra Goldstone boson compared to the previous situation in which only $SU(8)_L \times SU(8)_R$ is local. Thus with the additional gauging of $U(1)_{L+R}$ we produce a model in which baryon number is a good symmetry at the level of the Lagrangian and is necessarily broken in the vacuum. Provided the axial $U(1)_{L-R}$ symmetry still remains global, we can continue to use the 't Hooft mechanism for the axial fermion number $F_A = L_A + 3B_A$ so as to remove the axion from the physical spectrum as before.

With the above remarks in mind, we note that our $SU(8)_L \times SU(8)_R$ theory has a natural embedding in SU(16) if we put all of the fermions of the $(8,1) \oplus (1,8)$ representation into the fundamental $\underline{16}$ of SU(16). A purely left-handed $\underline{16}$ consists of left-handed fermions and the charge conjugates of right-handed fermions. Thus in SU(16) we must put particles and antiparticles into the same irreducible representation, and so we anticipate proton decay after spontaneous breakdown. Explicitly we find that this SU(16) contains a local $SU(8)_L \times SU(8)_R \times U(1)_{L+R}$ subgroup so that proton decay simply follows our remarks above. More interestingly, we note that this SU(16) does not contain $U(1)_{L-R}$ as a subgroup. Hence in models built on $SU(16) \times U(1)_{L-R}$, where SU(16) is local and $U(1)_{L-R}$ is global, there will be proton decay but again the axion will be removed by the 't Hooft mechanism.

(c) After we finished our work we became aware that the proton-decay aspects of maximal gauging theories such as SU(16) and its subgroups had also been discussed recently by Pati, Salam, and Strathdee. See J.C. Pati and A. Salam, International Centre for Theoretical Physics, Trieste, Report No. IC/80/72 (unpublished); J.C. Pati, A. Salam, and J. Strathdee, Nucl. Phys. B185, 445 (1981).

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APPENDIX: SIMPLE METHOD FOR CALCULATING ANOMALY FACTORS

In this appendix we show how to calculate the anomaly factor associated with an arbitrary irreducible representation of an SU(n) group. This will then enable us to determine what choices of fermions are needed in order to make our $SU(8)_L \times SU(8)_R$ theory anomaly-free. The problem of calculating the arbitrary anomaly factor has in fact already been solved in the literature,[22] and our interest here is only is presenting a very simple alternative method of calculating which will complement the published works.

To define notation we introduce SU(n) generators F^α ($\alpha = 1, \ldots, n$), with an m-dimensional representation $\psi_a^{(m)}$ ($a = 1, \ldots, m$) satisfying

$$[F^\alpha, \psi_a^{(m)}] = iT^\alpha_{ab}\psi_b^{(m)}. \tag{A1}$$

For the fundamental representation

$$T^\alpha = \lambda^\alpha/2\,, \tag{A2}$$

where

$$[\lambda^\alpha, \lambda^\beta] = 2if_{\alpha\beta\gamma}\lambda^\gamma \tag{A3}$$

and

$$\{\lambda^\alpha, \lambda^\beta\} = 2d_{\alpha\beta\gamma}\lambda^\gamma + \frac{4}{n}\delta_{\alpha\beta}I\,. \tag{A4}$$

The anomaly in the gauge couplings of gauge bosons A_μ^α, A_μ^β, and A_μ^γ due to a closed $\psi_a^{(m)}$ loop is proportional to $\eta A^{\alpha\beta\gamma}(n,m)$, where $\eta = \pm 1$ for left- or right-handed fermions, respectively, and

$$A^{\alpha\beta\gamma}(n,m) = \mathrm{Tr}\,T^\alpha\{T^\beta, T^\gamma\}. \tag{A5}$$

We note that $A^{\alpha\beta\gamma}(n,m)$ is a group invariant and is symmetric under the interchange of its indices, and is thus proportional to $d_{\alpha\beta\gamma}$, i.e.,

$$A^{\alpha\beta\gamma}(n,m) = \tfrac{1}{2}d_{\alpha\beta\gamma}\mathcal{A}(n,m)\,. \tag{A6}$$

The coefficients $\mathcal{A}(n,m)$ thus contain the explicit group-theoretical dependence of the anomaly after all kinematic factors associated with Feynman diagrams etc. have been removed, and have been normalized so that the fundamental representation satisfies

$$\mathcal{A}(n,n) = 1 \tag{A7}$$

with its conjugate satisfying

$$\mathcal{A}(n,n^*) = -1\,. \tag{A8}$$

Having determined the group-theoretical factor for the anomaly associated with the fundamental representation we can now generate the anomaly factor for an arbitrary representation by taking products of fundamentals. In order to extract out the irreducible representations of SU(n) contained in the product n^k of k fundamentals it is convenient to use Young diagrams. We now proceed by construction and begin with $k=2$. For $k=2$ there are two irreducible representations in the decomposition of $n \times n$, the totally symmetric representation of dimension $n(n+1)/2$ and the totally antisymmetric representation of dimension $n(n-1)/2$. We shall calculate the trace in Eq. (A5) in the $n \times n$ space itself so that T^α is a four-index tensor $(i,j,k,l=1,\ldots,n)$

$$T^\alpha = [\lambda^\alpha_{ik} \otimes (I)_{jl} + (I)_{ik} \otimes \lambda^\alpha_{jl}]/2 . \tag{A9}$$

To project out the representations we introduce projector operators for the symmetric and antisymmetric combinations

$$P = \tfrac{1}{2}(\delta_{ik}\delta_{jl} \pm \delta_{il}\delta_{jk}) \tag{A10}$$

and evaluate

$$\mathrm{Tr} P T^\alpha \{T^\beta, T^\gamma\} \tag{A11}$$

to obtain

$$\begin{aligned} \alpha(n, n(n+1)/2) &= n+4 , \\ \alpha(n, n(n-1)/2) &= n-4 . \end{aligned} \tag{A12}$$

As a quick check on these relations we note that in the SU(3) decomposition of $\underline{3} \times \underline{3} = \underline{6} + \underline{3}^*$ the $\underline{3}^*$ has $\alpha = -1$ as it should. In the SU(4) decomposition $\underline{4} \times \underline{4} = \underline{10} + \underline{6}$ the $\underline{6}$ is anomaly free which is to be expected since it is self-conjugate. Also in the SU(5) decomposition of $\underline{5} \times \underline{5} = \underline{15} + \underline{10}$ the $\underline{10}$ has $\alpha = +1$ and hence cancels the $\underline{5}^*$, thus confirming the original cancellation noted by Georgi and Glashow in Ref. 5.

The same procedure can now be applied to larger values of k. However, simplification follows by noting that because of the projector summation used previously we will find that in the irreducible decomposition of n^k each anomaly factor will be a polynomial of degree $k-1$ in n, with the sum of the anomaly factors for all the relevant irreducible representations being kn^{k-1}. Further, for the arbitrary product of two irreducible representations m_1 and m_2 we can express T^α in the m_1, m_2 product space analogously to our construction of Eq. (A9) to obtain

$$m_1 \alpha(n, m_2) + m_2 \alpha(n, m_1) = \sum_j \alpha(n, \mu_j) \tag{A13}$$

summed over all irreducible representations μ_j contained in

$$m_1 \otimes m_2 = \sum \mu_j . \tag{A14}$$

Also, we have found a closed form expression for the anomaly factors associated with the completely symmetric and completely antisymmetric irreducible representations contained in n^k, which are given respectively as

$$\begin{aligned} \alpha(n, {}^{n+k-1}C_k) &= \frac{(n+2k)(n+k)!}{(k-1)!\,(n+2)!} \\ \alpha(n, {}^{n}C_k) &= \frac{(n-2k)(n-3)!}{(k-1)!\,(n-k-1)!} , \quad k<n \end{aligned} \tag{A15}$$

$$\alpha(n, {}^{n}C_k) = 0 , \quad k \geq n \tag{A16}$$

(we give the derivation of these relations below, but note now that the relation for $k \geq n$ follows since it involves multiplication by a singlet). Finally the anomaly factor of the conjugate representation satisfies

$$\alpha(n, m^*) = -\alpha(n, m) . \tag{A17}$$

Using the above information it is then possible to determine all the $k>2$ anomaly factors without needing to construct explicit projector operators or evaluate traces at all.

We illustrate the procedure for $k=3$. For $k=3$ the fully antisymmetric representation has an anomaly factor which is quadratic in n. For $n=3$ the representation is a singlet, and is hence anomaly free. For $n=4$ a column of three boxes is equivalent to $\underline{4}^*$ for which $\alpha = -1$, and for $n=5$ a column of three boxes is equivalent to $(\underline{5}^* \times \underline{5}^*)$ antisymmetric for which $\alpha = -1$ according to Eqs. (A12) and (A17) above. Solving then gives

$$\alpha(n, {}^{n}C_3) = (n-3)(n-6)/2 . \tag{A18}$$

(This procedure shows that the arbitrary totally antisymmetric anomaly factor can also be found algebraically.) As a check on Eq. (A18) we note that for $n=6$ α vanishes, which is to be expected since the $\underline{20}$ of SU(6) is self-conjugate.

To calculate the other anomaly factors for $k=3$ we now use Young diagrams. The multiplication of the ${}^{n}C_2$ representation (a column of two boxes) by a fundamental contains the ${}^{n}C_3$ representation discussed above and a mixed symmetry representation of dimension $(n+1)n(n-1)/3$ (see e.g., Table I). The sum of their anomalies is given by Eq. (A13) as

$$(n-4)n + \tfrac{1}{2}n(n-1) \times 1 . \tag{A19}$$

From Eq. (A18) we thus obtain

$$\alpha(n, 2^{n+1}C_3) = (n-3)(n+3) . \tag{A20}$$

As a check on this relation we note that for SU(3) this mixed representation is the $\underline{8}$ which is self-adjoint and hence anomaly free. Finally we also note that the multiplication of the ${}^{n+1}C_2$ representation (a row of two boxes) by a fundamental contains this same mixed representation and the fully symmetric ${}^{n+2}C_3$ representation. The sum of their anomalies is obtained similarly and is given as

$$(n+4)n + \tfrac{1}{2}n(n+1) \times 1 \tag{A21}$$

so that

$$\alpha(n, {}^{n+2}C_3) = (n+3)(n+6)/2\,, \tag{A22}$$

which exhausts all the irreducible representations in n^3.

By now we have obtained enough totally symmetric and totally antisymmetric representations to be able to generalize their anomaly factors to any k to obtain Eqs. (A15) and (A16). For the arbitrary n^k we can proceed constructively using $n^{k-1},\ldots,n$ and we present the results for $k=4$ in Table I, where we have collected together all the anomaly factors that we have explicitly calculated in this paper. Repeated application of our technique (which can also be used to determine the dimensionality of an arbitrary Young diagram) then enables us to determine any $\alpha(n,m)$ as desired.

The most interesting aspect of our results is that the factors given in Table I are all reasonably small integers. Moreover, the overall sign of each anomaly can be changed by choosing the conjugate representation or by taking right- or left-handed fermions. Thus essentially any SU(n) group can be made anomaly-free by a judicious choice of the low-lying representations given in Table I. Thus it would appear to us that anomalies present no serious formal consistency problems for the renormalizability of non-Abelian gauge theories. Moreover, even though the presence of the required extra representations of fermions may well involve many new fermions, in grand unified theories only a few of them will be observable color singlets.

We can now use our results to make our $SU(8)_L \times SU(8)_R$ model anomaly-free. From Eqs. (A7) and (A8) we see that the anomalies of each $(8,1) \oplus (1,8)$ representation can be canceled by a mirror $(8^*,1)\oplus(1,8^*)$ representation. Alternatively we note from Eqs. (A12) and (A17) that $\alpha(8,28^*) = -4$, so that we can cancel the anomalies of a complete set of four families of $(8,1)\oplus(1,8)$ representations by a single $(28^*,1)\oplus(1,28^*)$ representation. Both procedures are acceptable, and either will serve to make our model formally anomaly-free as discussed in Sec. II.

Having now developed our method we note that it can also be applied to an arbitrary $\mathrm{Tr}\,T^\alpha T^\beta \ldots T^\gamma$

TABLE I. Anomaly factors and dimensions of the low-lying representations together with the Young diagram that characterizes each representation.

Young diagram	Dimension m	$\alpha(n,m)$
$\ydiagram{1}$	n	1
$\ydiagram{2}$	${}^{n+1}C_2$	$n+4$
$\ydiagram{1,1}$	${}^{n}C_2$	$n-4$
$\ydiagram{3}$	${}^{n+2}C_3$	$(n+3)(n+6)/2$
$\ydiagram{2,1}$	$2\,{}^{n+1}C_3$	$(n-3)(n+3)$
$\ydiagram{1,1,1}$	${}^{n}C_3$	$(n-3)(n-6)/2$
$\ydiagram{4}$	${}^{n+3}C_4$	$(n+3)(n+4)(n+8)/3!$
$\ydiagram{3,1}$	$3\,{}^{n+2}C_4$	$(n+4)(n^2+n-8)/2$
$\ydiagram{2,2}$	$(n-1)n^2(n+1)/12$	$(n-4)n(n+4)/3$
$\ydiagram{2,1,1}$	$3\,{}^{n+1}C_4$	$(n-4)(n^2-n-8)/2$
$\ydiagram{1,1,1,1}$	${}^{n}C_4$	$(n-3)(n-4)(n-8)/3!$

and not just to the three index $A^{\alpha\beta\gamma}(n,m)$ of Eq. (A5). Particularly, in this paper we are also interested in the anomalies of the extra global $U(1)_L$ and $U(1)_R$ symmetries of the full $U(8)_L \times U(8)_R$. Thus we calculate

$$A^{\alpha\beta}(n,m) = \mathrm{Tr}\, T^\alpha T^\beta \tag{A23}$$

with the anomaly factor for the coupling of a U(1) gauge boson to two SU(8) gauge bosons then being given by $Y(n,m)A^{\alpha\beta}(n,m)$, where $Y(n,m)$ is the U(1) quantum number of the representation (n,m). Noting that we can write $A^{\alpha\beta}(n,m)$ as

$$A^{\alpha\beta}(n,m) = \beta(n,m)\delta_{\alpha\beta}/2\,, \tag{A24}$$

we thus only need to calculate the factor $\beta(n,m)$. For the fundamental this is done directly from Eq. (A4) to give

$$\beta(n,n) = 1\,. \tag{A25}$$

By introducing projectors as in the derivation of Eq. (A12) we also find that

$$\begin{aligned}\beta(n, n(n+1)/2) &= n+2\,,\\ \beta(n, n(n-1)/2) &= n-2\,.\end{aligned} \tag{A26}$$

Now as discussed in Sec. IV it is our wish in our model that $U(1)_L$ and $U(1)_R$ not be anomaly-free so that axial baryon number does possess an anomaly. Thus for our $SU(8)_L \times SU(8)_R$ anomaly cancellation mechanisms we must additionally now require that the mirror fermions not satisfy $Y(8,8^*) = -Y(8,8)$ [so that the $(8^*,1)\oplus(1,8^*)$ fermions are mirror fermions for the $SU(8)_L \times SU(8)_R$ currents only and not mirror fermions for the extra $U(1)_L \times U(1)_R$ currents as well]. Or alternatively noting that $\beta(8,28) = 6$ we must require that the $(28^*,1)\oplus(1,28^*)$ representation not satisfy $6Y(8,28^*) = -4Y(8,8)$, a choice we are free to make. Thus we have no difficulty in keeping $SU(8)_L \times SU(8)_R$ anomaly-free with anomalies still being present in the extra $U(1)_L \times U(1)_R$ symmetries, as required for our work.

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