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Wave-function mixing of flavor-degenerate baryons

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There are two baryons at the center of an octet weight diagram of $SU(3)_{\text{flavor}}$ both of which contain the same quarks. We call these baryons "flavor degenerate." We discuss the symmetry of the wave functions of these baryons in terms of a quark-ordering principle and a mixing angle. We explore the consequences of the mixing on predictions of the masses and magnetic moments of flavor-degenerate baryons, including baryons containing heavy quarks. We obtain a simple approximate expression for the mixing angles in terms of the quark masses. The mixing angles turn out to be rather small, but still change some of the calculated magnetic moments substantially. The effect of mixing on the baryon masses is much smaller.

I. INTRODUCTION

In the past few years, a number of quark-model calculations of masses[1-7] and magnetic moments[8-14] of "heavy" baryons (i.e., baryons containing heavy quarks, such as the c, b, and t quarks) have appeared. These calculations have generally used quark wave functions constructed in analogy with wave functions previously used for the light u, d, and s quarks.

For baryons containing two or three identical quarks, the Pauli principle requires definite spin wave functions and the procedures used previously for light quarks are directly applicable to heavy quarks as well. However, this is not necessarily the case with spin-$\frac{1}{2}$ baryons containing three quarks of different flavors. We refer to such baryons as "flavor degenerate" because they correspond to the twofold-degenerate states in the middle of the baryon-octet weight diagram in $SU(3)_{\text{flavor}}$. For the flavor-degenerate baryons the Pauli principle does not apply directly, and there is an ambiguity in how the three different quarks are to be ordered in the spin function of the baryon.

We illustrate this ambiguity by considering the usual magnetic-moment formulas in the quark model. For spin-$\frac{1}{2}$ baryons with a pair of identical quarks, the standard quark-model magnetic-moment prediction is[15-17]

$$\mu = \tfrac{1}{3}(2\mu_1 + 2\mu_2 - \mu_3)\,, \tag{1}$$

with the Pauli principle requiring that the pair of identical quarks be taken as the first two.[18] For a pair of flavor-degenerate baryons, one moment is given by Eq. (1) and the other by[15-17]

$$\mu' = \mu_3\,, \tag{2}$$

but the pure Pauli principle puts no restriction on the quark ordering.

For the light-baryon octet, the Pauli principle can be extended using isospin to the ud pair, and this removes the ordering ambiguity. The Σ^0 has moment μ and the Λ has moment μ', each with the s quark third. The fact that isospin is not an exact symmetry leads to some small mixing between the Σ^0 and Λ, which we discuss later.

For heavy flavor-degenerate baryons, such as the Ξ_c and Ξ_c',[19] composed of the u, s, and c quarks for instance, isospin does not resolve the ambiguity in the quark ordering. The usual procedure in Refs. 8–14 has been to pick the two lightest quarks (u and s) as the first two quarks. Then use of Eqs. (1) and (2) leads to

$$\mu(\Xi_c^+) = \tfrac{1}{3}(2\mu_u + 2\mu_s - u_c) = 0.70 \tag{3}$$

and

$$\mu(\Xi_c^{+\prime}) = \mu_c = 0.39\,. \tag{4}$$

The numerical values for all magnetic moments are in units of nuclear magnetons and come from the input values

$$\mu_u = -2\mu_d = 1.86\,,\quad \mu_s = -0.61\,,\quad \mu_c = 0.39\,, \tag{5}$$

which assume Dirac moments for quarks and use the measured proton and Λ moments,[20] and a mass for the c quark of $m_c \simeq 1.6$ GeV, as suggested by ψ-meson spectroscopy.

Without a principle such as isospin to guide us, other choices could be made for the quark ordering. For instance, picking the u quark as the third quark results in

$$\mu(\Xi_c^+) = \tfrac{1}{3}(2\mu_c + 2\mu_s - \mu_u) = -0.77 \tag{6}$$

and

$$\mu(\Xi_c^{+\prime}) = \mu_u = 1.86\,. \tag{7}$$

These are completely different values from those of Eqs. (3) and (4), showing the nature of the ambiguity. Remarkably, the sign of $\mu(\Xi_c^+)$ in Eq. (6) is opposite that given in Eq. (3).

Similar, but not quite as striking, ambiguities can occur in heavy-baryon mass calculations.[2-6] (In Ref. 1, the mass matrix is diagonalized in such a way that the ordering ambiguity never arises. In Ref. 7, a specific model is used to mix the flavor-degenerate states, removing the ambiguity in the same way as is done in this paper.)

The quark-ordering ambiguities discussed here also appear in group-theoretic treatments of heavy baryons,[21,22] but there take the form of which SU(2) subgroup is used to distinguish heavy baryons. (In the case of baryons containing only light quarks the subgroups are isospin, U spin, and V spin.[23])

Some calculations of magnetic moments have assumed the moment operator proportional to the charge[13,14,21] rather than the charge-to-mass ratio, even though the former assumption does not work so well for light-hyperon moments. The difference becomes crucial for baryons containing heavy quarks. Here we shall assume that the quark moments are proportional to their charge-to-mass ratios.

Our main purpose in this paper is to discuss the ambiguities and mixing that occur for flavor-degenerate baryons and to suggest a more general principle (arising from the color-magnetic interaction) than isospin for resolving the ambiguities within the quark model. The color-magnetic interaction also leads to predictable mixing between such baryons that somewhat modifies their predicted magnetic moments and produces a small (generally negligible) shift in their masses.

In Sec. II of this paper we review the mixing formalism for flavor-degenerate baryons and in Sec. III, the dynamics of mass mixing. In Sec. IV we use the color-magnetic spin interaction to resolve the ambiguity in quark ordering. Applied to flavor-degenerate baryons, the color-magnetic interaction predicts their mixing and consequent shifts in their predicted magnetic moments and masses. In Sec. V we include electromagnetic mixing for the Σ^0-Λ pair and discuss its effects on the Λ magnetic moment. In Sec. VI we discuss our results.

II. MIXING FORMALISM

All our considerations are for the static quark model with neglect of relativistic, orbital, and exchange effects. We further assume an internal degree of freedom (such as color) in which the three quarks are antisymmetrized.[24] Then the use of Fermi statistics and the absence of relative orbital angular momentum for any two spin-$\frac{1}{2}$ quarks of the same flavor requires them to be in the symmetric spin-1 state. This requirement forces any spin-$\frac{1}{2}$ baryon with two identically flavored quarks and one odd quark to have the unique wave function[18]

$$\psi = q_1 q_2 q_3 \chi\,, \tag{8}$$

having the spin function

$$\chi = (2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)/\sqrt{6} \tag{9}$$

with the pair of identical quarks taken as the first two. We do *not* further symmetrize with respect to quark flavor, treating different quarks (even u and d) as distinguishable particles. This makes for simpler calculations (with the same results) and facilitates the extension to heavy-quark baryons without ud pairs. Since there is no fully symmetric spin-$\frac{1}{2}$ function for three spin-$\frac{1}{2}$ quarks, the above considerations show there is no baryon of spin $\frac{1}{2}$ with three identical quarks (without orbital excitation).

For the standard spin-$\frac{1}{2}$ baryon octet, formed of combinations of the u, d, and s quarks, the above procedure works for the six baryons with a pair of identically flavored quarks. It does not work for baryons such as the Σ^0 and Λ with three distinct quarks, uds. For the Σ^0 and Λ, one possibility is to use the isospin formalism. Because the Σ^0 is in an isospin triplet with the Σ^+ and Σ^-, it is natural to give the Σ^0 the spin function χ with the u and d quarks chosen as the first two, so that

$$\Sigma^0 = uds\chi\,. \tag{10}$$

The Λ can then be chosen as

$$\Lambda = uds\chi'\,, \tag{11}$$

with the spin function

$$\chi' = (\uparrow\downarrow - \downarrow\uparrow)\uparrow/\sqrt{2} \tag{12}$$

orthogonal to χ. The spin function χ' couples the first two quarks in the antisymmetric spin-0 state and thus cannot be used if the first two quarks are identical.

Having thus formed the spin-$\frac{1}{2}$ baryon octet, we now consider what would happen if we did not have isospin to guide us for the Σ^0 state. Then there would be three possible states, depending on which quark was chosen as the third quark. We can generate the other two Σ^0 states by the quark flavor (but not spin) interchanges $q_1 \rightleftharpoons q_3$, leading to

$$\Sigma_U^0 = sdu\chi \tag{13}$$

and $q_2 \rightleftarrows q_3$, leading to

$$\Sigma_V^0 = us\,d\chi\,. \tag{14}$$

As the subscripts U and V suggest, these are just the familiar central states of the U- and V-spin triplets.[23] These three Σ^0 states are neither orthogonal nor linearly independent, satisfying the relation

$$\Sigma_I^0 + \Sigma_U^0 + \Sigma_V^0 = 0\,. \tag{15}$$

There are also Λ_U and Λ_V states chosen orthogonal to the corresponding Σ^0 states. The three different Λ^0 states can all be written in terms of the Σ^0 states so that

$$\Lambda_I = (\Sigma_V^0 - \Sigma_U^0)/\sqrt{3} \tag{16}$$

with the other two Λ's given by the interchanges $(I \rightleftarrows U)$ and $(I \rightleftarrows V)$. Thus, even though we now have six ambiguous states, only two of them are independent, corresponding to the twofold degeneracy in the middle of the baryon-octet weight diagram in SU(3).

This ambiguity in choosing the central states can lead to ambiguous predictions, further compounded by the fact that the two physical states could be linear combinations of the mathematical states discussed above. We treat this situation by using a mixing formalism, writing the physical baryon states B and B' as normalized linear combinations of any one of the Σ^0-like states and its orthogonal Λ-like partner:

$$B = \cos\theta\, q_1 q_2 q_3 \chi - \sin\theta\, q_1 q_2 q_3 \chi' \tag{17}$$

and

$$B' = \sin\theta\, q_1 q_2 q_3 \chi + \cos\theta\, q_1 q_2 q_3 \chi'\,. \tag{18}$$

That the mixing representation of Eqs. (17) and (18) can replace the I, U, V representation of Eqs. (10)–(16) can be seen by the observation that $\theta = 60^\circ$ corresponds (up to a sign) to the quark interchange $q_1 \rightleftarrows q_3$, $\theta = -60^\circ$ to $q_2 \rightleftarrows q_3$, and $\theta = 90^\circ$ to $B \rightleftarrows B'$.

With mixing, the magnetic moments of flavor-degenerate baryons B and B' are given by[25]

$$\mu(B) = \mu_B \cos^2\theta - \mu_{BB'} \sin 2\theta + \mu_B' \sin^2\theta \tag{19}$$

and

$$\mu(B') = \mu_B \sin^2\theta + \mu_{BB'} \sin 2\theta + \mu_B' \cos^2\theta\,, \tag{20}$$

where μ_B and μ_B' are the unmixed moments given by Eqs. (1) and (2). The unmixed transition moment $\mu_{BB'}$ is given in the quark model by[17,18]

$$\mu_{BB'} = (\mu_2 - \mu_1)/\sqrt{3}\,. \tag{21}$$

For small mixing angles, Eqs. (19) and (20) can be approximated by

$$\mu(B) \simeq \mu_B - 2\theta\mu_{BB'} \tag{22}$$

and

$$\mu(B') \simeq \mu_B' + 2\theta_{BB'}\,. \tag{23}$$

The mixing also affects the transition moment between the two flavor-degenerate baryons. The physical transition moment is given by

$$\begin{aligned}\mu(B,B') &= \mu_{BB'} \cos 2\theta - \tfrac{1}{2}(\mu_B' - \mu_B)\sin 2\theta \\ &\simeq \mu_{BB'} - \theta(\mu_B' - \mu_B)\,.\end{aligned} \tag{24}$$

The physical transition moment between a flavor-degenerate baryon and a nondegenerate excited baryon B^* with the same quark content (e.g., Σ^0 and Σ^{0*}) is given by

$$\begin{aligned}\mu(B^*,B) &= \mu_{B^*B} \cos\theta - \mu_{B^*B'} \sin\theta \\ &\simeq \mu_{B^*B} - \theta\mu_{B^*B'}\end{aligned} \tag{25}$$

or

$$\begin{aligned}\mu(B^*,B') &= \mu_{B^*B'} \cos\theta + \mu_{B^*B} \sin\theta \\ &\simeq \mu_{B^*B'} + \theta\mu_{B^*B}\,,\end{aligned} \tag{26}$$

where μ_{B^*B} and $\mu_{B^*B'}$ are the unmixed transition moments. For s-wave spin-$\frac{3}{2}$ baryons, these are given by[17,18]

$$\mu_{B^*B} = \sqrt{2}\,(\mu_1 + \mu_2 - 2\mu_3)/3 \tag{27}$$

and

$$\mu_{B^*B'} = (\tfrac{2}{3})^{1/2}(\mu_1 - \mu_2)\,. \tag{28}$$

III. MIXING DYNAMICS

The mixing angle θ is determined as that angle for which the linear combinations of Eqs. (17) and (18) diagonalize the 2×2 part of the mass operator

$$\mathfrak{M} = \begin{bmatrix} M & \epsilon \\ \epsilon & M' \end{bmatrix} \tag{29}$$

connecting the two unmixed states B and B'. This leads to

$$\tan 2\theta = -2\epsilon/\Delta\,, \tag{30}$$

where $\Delta = M - M'$. There is also a shift δM ($\delta M'$) in the B (B') mass given by

$$\begin{aligned}\delta M = -\delta M' &= -\epsilon \tan\theta \\ &= [(\tfrac{1}{2}\Delta)^2 + \epsilon^2]^{1/2} - \tfrac{1}{2}\Delta\,.\end{aligned} \tag{31}$$

The mass shifts in Eq. (31) are equal and opposite so that the combination $(M_B + M_{B'})$ remains invariant under mixing (as expected for the trace of a matrix). This means that the prediction for the mass sum of a pair of flavor-degenerate baryons does not depend on mixing and is unambiguous with respect to quark ordering.[2] Inspection of Eqs. (19) and (20) shows that this is also true for the sum of the magnetic moments of a flavor-degenerate pair.

Since the baryon B' has the spin state χ', the interchange of the first two quarks ($q_1 \rightleftarrows q_2$) in its wave function results in $B' \rightleftarrows -B'$. On the other hand, B with the spin function χ is unchanged. This means that the off-diagonal mass matrix element will change sign under the interchange ($q_1 \rightleftarrows q_2$). However, Eq. (30) shows that θ also would change sign so that the mass shifts of Eqs. (31) which depend on $\epsilon \tan\theta$, are not affected. As far as physical mass predictions are concerned, the relative ordering of the first two quarks is irrelevant.

The sign of $\mu_{BB'}$ as given by Eq. (21) does change sign under the interchange ($q_1 \rightleftarrows q_2$) and, from Eqs. (24) and (26), so will physical transition moments to B'. The relative signs of transition moments are observable in interference between decays to the same state. The relative signs will not change if the same quark ordering is used consistently for each state of the same quarks. Then there will be no physical dependence on the relative order of the first two quarks. When the first two quarks are the u and d (e.g., for the Λ), it is convenient to choose the order ud to correspond with the Condon-Shortley phase convention when comparison is made with the isospin formalism.

The mass matrix of Eq. (29) can only be determined by a dynamical assumption. We consider a model[26,17] where the mass operator $\mathfrak{M}$ is given by a sum of one-body operators m_i and spin-dependent two-body operators D_{ij}^S that include projection operators onto spin state S ($S = 1, 0$):

$$\mathfrak{M} = \sum_i m_i + \sum_{i<j} D_{ij}^S . \tag{32}$$

The D_{ij}^S are independent of which baryon the quarks are in and where they are in the baryon if we make the SU(6)-like assumption that all the spin-$\frac{1}{2}$ baryons have the same symmetric spatial wave function. This assumption has generally been made in Refs. 1–18, and long experience with the quark model has shown it to be a relatively good assumption as far as baryon mass differences are concerned.

With this mass operator, baryon masses are easily calculated as the diagonal elements of $\mathfrak{M}$. The mass M of a baryon with quark ordering $q_1q_2q_3$ and spin function χ is[26,17]

$$M = m_1 + m_2 + m_3 + D_{12}^1 + \tfrac{1}{4}(D_{13}^1 + D_{23}^1) + \tfrac{3}{4}(D_{13}^0 + D_{23}^0) , \tag{33}$$

while the mass M' of a baryon with the same quark ordering and spin function χ' is

$$M' = m_1 + m_2 + m_3 + D_{12}^0 + \tfrac{3}{4}(D_{13}^1 + D_{23}^1) + \tfrac{1}{4}(D_{13}^0 + D_{23}^0) . \tag{34}$$

The off-diagonal elements ϵ of the mass operator are given by[2]

$$\epsilon = \langle q_1q_2q_3\chi \,|\mathfrak{M}|\, q_1q_2q_3\chi'\rangle = \frac{\sqrt{3}}{4}\,(D_{13}^0 - D_{13}^1 + D_{23}^1 - D_{23}^0) . \tag{35}$$

The off-diagonal elements exist only for flavor-degenerate baryons where all three quarks are different (otherwise there is no χ'). The mass difference between a pair of flavor-degenerate baryons is given from Eqs. (33) and (34) by

$$\Delta = M - M' = D_{12}^1 - D_{12}^0 + \tfrac{1}{2}(D_{13}^0 - D_{13}^1 + D_{23}^0 - D_{23}^1) . \tag{36}$$

Combining Eqs. (30), (35), and (36) we have for the mixing angle for quark order 1, 2, 3

$$\tan 2\theta_{123} = -\frac{2\epsilon}{\Delta} = \frac{\sqrt{3}\,(D_{13}^1 - D_{13}^0 + D_{23}^0 - D_{23}^1)}{2(D_{12}^1 - D_{12}^0) + D_{13}^0 - D_{13}^1 + D_{23}^0 - D_{23}^1} . \tag{37}$$

We note that any one of the six possible quark orderings can be used in the original mixing equations (17) and (18), and while θ depends on this ordering, the final diagonalized mass matrix and the physical baryons B and B' do not. In this respect, there is no longer an ambiguity in physical results for different quark orderings. As a matter of calculational simplicity and for physical insight, however, it is natural to choose a quark ordering that makes the mixing angle θ smallest. Furthermore, since we cannot calculate θ exactly, it is better to have an error in a small quantity than in a large one. We therefore take as our criterion for choosing the quark order for flavor-degenerate baryons that order which minimizes θ.

For the uds quark combination leading to the physical Σ^0 and Λ baryons, inspection of Eq. (37) shows that the equality of u-s and d-s forces (charge symmetry) makes $\theta = 0$ if the u and d quarks are chosen as the first two, so that this is the obvious choice. With violations of charge symmetry known to be small, letting the u and d quarks be the first two is still the natural choice in calculating Σ^0-Λ mixing effects. For the heavy baryons with two nucleon-type quarks, charge symmetry also suggests that they be chosen as the first two. However, for heavy baryons with only one (or no) nucleon quark, charge symmetry cannot be used as a guide and a new principle is required.

IV. COLOR-SPIN MIXING

For heavy flavor-degenerate baryons, two questions confront us: Which quark order is best, and what is the mixing angle θ for a given order?

From Eq. (37) for θ, we notice that the numerator and denominator each depends on the difference in interaction energies between the two spin states and thus, only on the spin-dependent part of the quark-quark interaction energies. Considerations from quantum chromodynamics (QCD) lead to a quark-quark potential with a spin-dependent electro-color-magnetic interaction, which, for s waves, has the form[27,28]

$$D_{ij}^{S}=\frac{\lambda_{ij}\vec{s}_i\cdot\vec{s}_j}{m_i m_j}. \qquad (38)$$

The interaction strength λ_{ij} is given from QCD and QED by

$$\lambda_{ij}=\frac{8\pi}{3}\left(\tfrac{2}{3}\alpha_s - Q_i Q_j \alpha\right)\langle\delta(\vec{r}_i-\vec{r}_j)\rangle, \qquad (39)$$

where Q_i and Q_j are the quark charges in units of the proton charge, α_s is the strong-interaction (quark-gluon) coupling strength analogous to the electromagnetic fine-structure constant α $(=\frac{1}{137})$, and $\langle\delta(r_i-r_j)\rangle$ is the expectation value of the δ function. With the assumption of SU(6) symmetry of the spatial wave functions and the neglect of electromagnetic interactions ($\alpha \ll \alpha_s$), the only flavor dependence in Eq. (38) is through the quark masses in the denominator. Then the difference between quark-quark interaction energies that only differ in total spin is

$$D_{ij}^{1}-D_{ij}^{0}=\frac{\lambda}{m_i m_j}, \qquad (40)$$

where $\lambda_{ij}\simeq\lambda$, a constant. Then Eq. (37) for θ can be rewritten as

$$\tan 2\theta=\frac{\sqrt{3}\,(m_2-m_1)}{2m_3-m_1-m_2}, \qquad (41)$$

which is the key equation for answering our two questions.

For any quark order, Eq. (41) gives the mixing angle explicitly in terms of the quark masses. The best choice of quark orderings is to pick m_1 and m_2 to give the smallest difference m_2-m_1 since that makes θ, as given by Eq. (41), smallest. Since successive quark-mass differences seem to increase with increasing quark mass (for the u, d, s, c, and b quarks), this leads to the simple rule to order the quarks in flavor-degenerate baryons in increasing order of their masses to achieve the least mixing. This corresponds to what has been done intuitively in most calculations.[1-14]

There is still the question of what values to use for the quark masses in Eq. (41), but the mixing angles are all small and not too sensitive to the exact values of these masses. For an average nucleon-quark mass m we take

$$m\simeq m_u\simeq m_d\simeq 335\ \text{MeV}, \qquad (42)$$

as suggested from the nucleon magnetic moments in the simplest quark model.[18]

With SU(6)-symmetric wave functions, the mass difference m_s-m_u is given in this model by[28,29]

$$m_s-m=M_\Lambda-M_p\simeq 180\ \text{MeV}, \qquad (43)$$

leading to a strange-quark mass

$$m_s=515\ \text{MeV}. \qquad (44)$$

The c and b quark masses are suggested by heavy-quarkonium spectroscopy to be about

$$m_c\simeq 1.6\ \text{GeV and } m_b\simeq 5.0\ \text{GeV}. \qquad (45)$$

With the above masses, we can use Eq. (41) to estimate the mixing angles due to the color-magnetic interaction. These are listed in Table I for each flavor-degenerate baryon pair, along with the effect of the mixing on the magnetic-moment predictions. The mixing angle θ_{uds} also includes an electromagnetic contribution, and is discussed further in the next section. The mixing angles θ_{udc} and θ_{udb} are not listed because they are negligible, as are all induced mass shifts. (We shall estimate θ_{udc} and θ_{udb} in the next section.) Because the mixing angles are negligible, the moments of spin-$\frac{1}{2}$ heavy baryons containing two nucleon quarks are given by the standard quark model of Eqs. (1) and (2). The moments of heavy spin-$\frac{1}{2}$ baryons containing two quarks of the same flavor are given by Eq. (1), and the moments of spin-$\frac{3}{2}$ baryons are given by

$$\mu(B^*)=\mu_1+\mu_2+\mu_3. \qquad (46)$$

We do not list in Table I the moments of these unmixed baryons.[30]

The color-magnetic interaction is smaller for heavy quarks than for light ones. For this reason, some of the heavy s-wave spin-$\frac{3}{2}$ baryons (B^*) may be stable under strong decay and have dominant electromagnetic decay into the corresponding B or B' baryon. The physical transition moments for these decays are not listed, but can be readily calculated from Eqs. (25)–(28) using the appropriate mixing angle.

V. Σ^0-Λ MIXING

Electromagnetic and color-magnetic SU(3)-breaking interactions both contribute to Σ^0-Λ mixing, but the Σ^0-Λ mixing can be found in terms of other hyperon masses without any assumption about the form of the interaction.

The off-diagonal matrix element ϵ connecting the Σ^0 and Λ is given from Eq. (35) by

$$\epsilon=\frac{\sqrt{3}}{4}\left(D_{us}^{0}-D_{us}^{1}+D_{ds}^{1}-D_{ds}^{0}\right). \qquad (47)$$

It can be seen from Eq. (32) that this same combination of interaction energies occurs in certain linear combinations of baryon masses, so that we

TABLE I. Color-spin mixing for flavor-degenerate baryon magnetic moments (using $\mu_u = -2\mu_d = 1.86$, $\mu_s = -0.61$, $\mu_c = 0.39$, and $\mu_b = -0.06$, all in nuclear magnetons). Values of mixed and unmixed moments are in nuclear magnetons.

Quark content	Mixing angle (rad)	Baryon pair	Unmixed moment	Mixed moment
		Σ^0	0.82	0.86
uds	0.011[a]	Λ	−0.61	−0.65
		(Σ^0, Λ)	−1.61	−1.59
		Ξ_c^+	0.70	0.89
usc	0.066	$\Xi_c'^+$	0.39	0.20
		$(\Xi_c^+, \Xi_c'^+)$	−1.43	−1.40
		Ξ_c^0	−1.16	−1.18
dsc	0.066	$\Xi_c'^0$	0.39	0.41
		$(\Xi_c^0, \Xi_c'^0)$	0.18	0.08
		Ξ_b^0	0.85	0.90
usb	0.017	$\Xi_b'^0$	−0.06	−0.11
		$(\Xi_b^0, \Xi_b'^0)$	−1.43	−1.41
		Ξ_b^-	−1.01	−1.02
dsb	0.017	$\Xi_b'^-$	−0.06	−0.05
		$(\Xi_b^-, \Xi_b'^-)$	0.18	0.16
		Ξ_{cb}^+	1.52	1.71
ucb	0.13	$\Xi_{cb}'^+$	−0.06	−0.25
		$(\Xi_{cb}^+, \Xi_{cb}'^+)$	−0.85	−0.62
		Ξ_{cb}^0	−0.34	−0.53
dcb	0.13	$\Xi_{cb}'^0$	−0.06	0.13
		$(\Xi_{cb}^0, \Xi_{cb}'^0)$	0.76	0.70
		Ω_{cb}^0	−0.13	−0.27
scb	0.12	$\Omega_{cb}'^0$	−0.06	0.08
		$(\Omega_{cb}^0, \Omega_{cb}'^0)$	0.58	0.56

[a]Including electromagnetic mixing angle.

can write

$$\epsilon = (M_{\Xi^{*-}} - M_{\Xi^{*0}} - M_{\Xi^-} + M_{\Xi^0})/2\sqrt{3}$$
$$= -0.92 \pm 0.29 \text{ MeV} \tag{48}$$

or

$$\epsilon = (M_{\Sigma^{*-}} - M_{\Sigma^{*+}} - M_{\Sigma^-} + M_{\Sigma^+})/2\sqrt{3}$$
$$= -0.81 \pm 0.20 \text{ MeV}. \tag{49}$$

These two independent results agree well within the experimental errors and we use their statistical average

$$\epsilon = -0.84 \pm 0.17 \text{ MeV} \tag{50}$$

as our estimate for ϵ. Then θ is given by $-\epsilon/\Delta$, where $\Delta = M - M'$. Because the mixing is so small, it is a good approximation to replace $M - M'$ by the difference between the physical Σ^0 and Λ masses ($\Delta \simeq M_{\Sigma^0} - M_\Lambda = 77$ MeV). Then we get

$$\theta = -\epsilon/\Delta = 0.011 \pm 0.002 \tag{51}$$

for the Σ^0-Λ mixing angle.

Some time ago other authors[31,32] derived an expression for θ, using only electromagnetic breaking of SU(3), obtaining

$$\theta = (M_p - M_n + M_{\Sigma^0} - M_{\Sigma^+})/(\sqrt{3}\Delta) = 0.014. \tag{52}$$

At first glance, it is surprising that the result of Eq. (51) is so close to the purely electromagnetic result, especially since the color-magnetic mixing due to the d-u mass difference is believed to be[33] considerably larger than the mass-corrected electromagnetic contribution to θ.

We can understand this by considering the quark-model derivation of Eq. (52). From Eq. (32), we can write

$$M_n - M_p + M_{\Sigma^+} - M_{\Sigma^0} = D_{dd}^1 - D_{ud}^1 + \tfrac{1}{4}(D_{us}^1 - D_{ds}^1)$$
$$+ \tfrac{3}{4}(D_{us}^0 - D_{ds}^0). \tag{53}$$

If it is assumed that the only SU(3)-symmetry breaking in Eq. (53) arises from the electro-color-magnetic interaction of Eqs. (38) and (39), then Eq. (53) can be reduced to

$$M_n - M_p + M_{\Sigma^+} - M_{\Sigma^0} = (2 + m_s/m)(D_{ds}^1 - D_{us}^1). \tag{54}$$

With the same assumption, Eq. (47) for ϵ can be written

$$\epsilon = \sqrt{3}(D_{ds}^1 - D_{us}^1). \tag{55}$$

Comparison of Eqs. (54) and (55) shows

$$\epsilon = \frac{\sqrt{3}(M_n - M_p + M_{\Sigma^+} - M_{\Sigma^0})}{(2 + m_s/m)} = -0.88 \text{ MeV} \tag{56}$$

or

$$\theta = \frac{-\epsilon}{\Delta} = \frac{\sqrt{3}(M_p - M_n + M_{\Sigma^0} - M_{\Sigma^+})}{\Delta(2 + m_s/m)} = 0.011. \tag{57}$$

With the further SU(3) assumption that $m_s = m$, this reduces to Eq. (52).

We note that the reduction of Eq. (53) to Eq. (54) follows for either the electromagnetic or the color-magnetic spin-spin interaction because the strange quark has the same electric charge as the d quark (U-spin invariance, except for the m_s/m mass ratio). It is for this reason that the Σ^0-Λ mixing angle is the same whichever interaction is used to derive Eq. (56). The near agreement of Eq. (56) for ϵ with Eqs. (48) and (49) indicates that any SU(3)-breaking contributions other than those given by the spin-spin interaction are not large. Isgur,[33] using procedures similar to ours in a specific quark model, has also estimated $\theta_{\Sigma^0\text{-}\Lambda} \simeq 0.01$ from a combination of different individual contributions.

The Σ^0-Λ mixing can be extended to isospin mixing in heavy baryons with quark combinations *udh*, with *h* being any heavy quark, by simply scaling the Σ^0-Λ results with appropriate mass factors. From Eq. (41), we see that for *udh* baryons the mixing angle scales as

$$\theta_{udh} \propto 1/(m_h - m)\,. \tag{58}$$

Then the mixing angle θ_{udh} for heavy quark *h* will be related to θ_{uds} by

$$\theta_{udh} = [(m_s - m)/(m_h - m)]\theta_{uds}\,. \tag{59}$$

For charmed baryons, this gives

$$\theta_{udc} = 0.0015\,, \tag{60}$$

while for bottom baryons,

$$\theta_{udb} = 0.0004\,, \tag{61}$$

both of which are probably negligible.

The effect of Σ^0-Λ mixing on the Λ magnetic moment has been considered by Isgur and Karl[34] using Eq. (52). Here we discuss the effect of the spin-spin mixing of Eqs. (47)–(51) on $\mu(\Lambda)$.

With the Σ^0-Λ mixing angle $\theta = 0.011$, the quark-model prediction for the Λ magnetic moment is shifted to

$$\mu(\Lambda) = \mu_s + 2\theta\mu_{\Sigma^0\Lambda} = \mu_s - 0.035 \pm 0.006\,. \tag{62}$$

The Λ moment has been accurately measured to be $\mu(\Lambda) = -0.613 \pm 0.005$, and this value has been used in many calculations as a measure for μ_s (and consequently m_s). We see from Eq. (62) that a more consistent measure for models such as this with SU(6)-symmetric wave functions would be

$$\mu_s = -0.58 \pm 0.01\,. \tag{63}$$

This can be compared with the Dirac quark moment (for a strange-quark mass of 515 MeV) derived from the equation

$$\mu_s = -\tfrac{1}{3}(M_p/m_s) = -0.61\,. \tag{64}$$

This is reasonable agreement, but not as good as using the uncorrected moment $\mu_s = -0.61$ instead of Eq. (63). We emphasize, however, that the appropriate comparison is between Eqs. (63) and (64), because the Σ^0-Λ mixing is inevitable in the model.

Another way to use Eq. (62) is to use the strange-quark Dirac magnetic moment of -0.61 to predict $\mu(\Lambda) = -0.65$, again not in as good agreement with experiment as the uncorrected result.

VI. SUMMARY

The ambiguity in quark ordering that arises for flavor-degenerate baryons can be resolved if the spin-spin interaction between quarks has the form $\lambda\vec{s}_i \cdot \vec{s}_j/(m_i m_j)$. This form was originally proposed on phenomenological grounds by Zel'dovich and Sakharov[28] and is expected to follow from one-gluon exchange in QCD.[27] The best quark ordering for flavor-degenerate baryons is that which has the two quarks closest together in mass as the first two. Their relative order then does not matter as long as it is kept the same for all flavor-degenerate baryons with the same three quarks. For the known quarks (u, d, s, c, b) the quark mass spectrum is such that this rule can be followed by always ordering the quarks in flavor-degenerate baryons from lightest to heaviest.

The mixing angle relating the physical flavor-degenerate baryons to their corresponding mathematical pair is given simply (for baryons containing at most one nucleon quark) in terms of the quark masses by Eq. (41). Using this equation, the quark ordering described above leads to the smallest mixing angles and thus mitigates any uncertainties involved in the derivation. We have listed the mixing angles of flavor-degenerate baryons containing at most one nucleon quark in Table I, as well as the Σ^0-Λ angle.

For flavor-degenerate baryons containing two nucleon quarks, electromagnetic interactions must also be taken into account. We obtain the mixing angles for such baryons containing a heavy quark in terms of the Σ^0-Λ mixing angle of Eqs. (51) or (57) and the scaling principle of Eq. (59). The values of θ for these baryons are given by Eqs. (60) and (61).

The largest mixing angle found is $\theta = 0.13$ rad. The largest mixing angle possible for any possible new quark masses would be $\theta = 15^\circ = 0.26$ rad. This would be for the case of equal quark-mass spacing, which is not expected even for new quarks.

With the mixing angles all being so small, the mass shifts from the simple unmixed masses of flavor-degenerate baryons, as given by Eq. (31) are generally negligible, being at most 2% of the B-B' mass difference. This is because the mass shifts are second order in the mixing angles. This means that, if the quark order is optimized as above, mixing is not necessary in practice to calculate baryon masses. Magnetic-moment shifts are first order in the mixing angle, and mixing does affect them, especially the magnetic moment of the Λ-like flavor-degenerate baryon of each pair. The effect of mixing on flavor-degenerate moments is shown in Table I.

The mixing formalism applied to the Σ^0-Λ flavor-degenerate pair leads to a change in the strange-quark moment as determined by the accurate Λ moment measurement[20] from -0.61 to -0.58. This has the effect of worsening agreement with the Dirac moment expected for a strange-quark mass of 515 MeV as predicted from the Λ-p mass difference,[29] and the proton moment. The Σ^0-Λ mixing angle calculated including color-magnetic mixing turns out to be almost the same as that calculated years ago using SU(3) with purely electromagnetic breaking. This apparent coincidence is explained by the fact that the U-spin breaking in Eq. (54) due to $m_s \neq m_d$ is seen to be small.

Our main conclusions are that mixing can be important for the magnetic moments of flavor-degenerate baryons, but can usually be neglected for their masses if the two quarks closest in mass are symmetrized (or antisymmetrized) as the first two quarks in our ordering. The mixing angles can be uniquely determined from the quark masses if the mixing interaction is due to the mass-dependent spin-spin interaction suggested by one-gluon exchange in QCD.

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