Scalar-mediated proton decay

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We analyze the difference in experimental signature between vector-mediated and scalar-mediated $\Delta S = 0$ proton decay. Relatively model-independent results are $\Gamma(\omega e^+):\Gamma(\rho^0 e^+):\Gamma(\eta e^+)::1.0:0.42:0.0$ for pure vector mediation and $1.22\Gamma(\rho^0 e^+) < \Gamma(\omega e^+) + 1.3\Gamma(\eta e^+)$ if scalar mediation contributes as well. Finally, a detailed study is carried out of proton decay mediated by Higgs scalars belonging to a single 5-plet of SU(5).

I. INTRODUCTION

It is possible to enumerate the operators of lowest dimension which can induce proton decay.[1] For $\Delta S = 0$ modes, these can be written as

$$O_1^{(\sigma)} = \epsilon_{ijk}\bar{d}_{iR}^c u_{jR}(\bar{u}_{kL}^c l_L - \bar{d}_{kL}^c \nu_L), \quad (1a)$$

$$O_2^{(\sigma)} = \epsilon_{ijk}\bar{d}_{iL}^c u_{jL}\bar{u}_{kR}^c l_R, \quad (1b)$$

$$O_3^{(\sigma)} = \epsilon_{ijk}\bar{d}_{iL}^c u_{jL}(\bar{u}_{kL}^c l_L - \bar{d}_{kL}^c \nu_L), \quad (1c)$$

$$O_4^{(\sigma)} = \epsilon_{ijk}\bar{d}_{iR}^c u_{jR}\bar{u}_{kR}^c l_R, \quad (1d)$$

where $l_L \equiv \frac{1}{2}(1+\gamma_5)l$, $\bar{u}_{kR}^c \equiv \bar{u}_k^c(1-\gamma_5)/2$, etc., and the superscript σ is a lepton generation index. Correspondingly, the Hamiltonian for $\Delta S = 0$ proton decay has the form

$$H^{\Delta S=0} = \sum_{i,\sigma} a_i^{(\sigma)} O_i^{(\sigma)}. \quad (2)$$

The $\{a_i^{(\sigma)}\}$ are model-dependent constants. For example, if only vector gauge bosons contribute significantly to proton decay, then $a_3^{(\sigma)} = a_4^{(\sigma)} = 0$.[1] This possibility has been extensively addressed in the literature.[2] Even with the restriction to vector-boson-induced decay, there remains dependence of $a_1^{(\sigma)}, a_2^{(\sigma)}$ on the nature of the symmetry group at the unification mass. This affects decay rates for individual modes.[3] Obviously, by allowing as well for scalar exchange, matters become even more complicated, and seemingly less amenable to analysis.

Yet given the significance of this subject to physics, it is surely important to confront the matter of distinguishing between scalar and vector exchange in a relatively model-independent manner. We carry out such an analysis in Sec. II. Our method can be viewed as an extension of the bag-model calculation presented in Ref. 3, with the emphasis on branching ratios. Some discussion relating the bag-model method to other approaches is also contained in Sec. II.

One reason for the almost total concentration on vector-boson-mediated processes in proton-decay calculations thus far[4] is the lack of a compellingly attractive model involving scalar exchange. Choosing among competing scalar models, each with its own arbitrary parameters, is not an appealing situation. However, there *is* a simplest case, in which the scalars belong to a Higgs 5-plet in SU(5). Despite the phenomenological shortcomings of such a model,[2] we feel it a useful exercise on pedagogic grounds alone to analyze it carefully. Besides, a more meaningful lower bound on the scalar mass is achievable if matrix elements can be reliably estimated. This is done in Sec. III.

Throughout this paper, we employ resonances to characterize multimeson final states. For example, an $I=1$ pion pair becomes a ρ meson whereas an $I=0$ pion three-body state is represented by an ω meson. The problem of describing an $I=J=0$ two-pion state is addressed in an Appendix, which follows a summary of our results and a discussion thereof in Sec. IV.

II. SCALAR VS VECTOR EXCHANGE

In our opinion it is nearly impossible to present a *strictly* model-independent analysis of the scalar-vs-vector issue. The present state of hadron dynamics is too primitive. Thus theorists can honestly compute with equally reasonable but distinct models, and yet reach rather different conclusions. A case in point is the difference between bag and harmonic-oscillator estimates of $\pi \bar{l}$ modes in nucleon decay.[5] As explained in Ref. 6, either by varying the form factors used in the bag-model approach or by modifying the procedure by which oscillator parameters are chosen, the discrepancy between the competing models can be minimized. Unfortunately, how the parameters in either model are chosen is evidently a matter of taste.

With this caveat in mind, we employ the following approach. We consider only $\Delta S = 0$ processes, and make no assumptions regarding the $\{a_i^{(\sigma)}\}$ of Eq. (2). It is in this sense that the analysis of this section is model independent. Modes with $\Delta S \neq 0$ are not considered because to do so would introduce new parameters analogous to the $\{a_i^{(\sigma)}\}$. Our predictive power would not be improved. We emphasize branching ratios rather than antilepton polarization in our analysis. However, a discussion of the latter appears in the Conclusion. Also, the reader should consult Ref. 1 for an informative treatment of polarization.

Rather than give an indepth account of the bag calculation we sketch its main features and refer the reader to Ref. 3 for further details. It is straightforward to compute two-body matrix elements for all the operators of Eq. (1).[7] We exhibit the relevant matrix elements in tabular form:

	O_1, O_2	O_3, O_4
$\rho^0 e^+$	$I+\frac{23}{3}J$	$-5I-\frac{11}{3}J$
ωe^+	$3I+7J$	$-3I-7J$
$\pi^0 e^+$	$3I+7J$	$-3I-7J$
ηe^+	$\sqrt{3}(I-3J)$	$-\sqrt{3}(3I-J)$
$\rho^+\bar{\nu}_e$	$-\sqrt{2}(I+\frac{23}{3}J)$	$\sqrt{2}(5I+\frac{11}{3}J)$
$\pi^+\bar{\nu}_e$	$-\sqrt{2}(3I+7J)$	$\sqrt{2}(3I+7J)$

(3)

The wave-function overlap integrals I, J are given by

$$I=\int_0^1 u^2 du\, j_0{}^3(pu), \tag{5}$$

$$J=\frac{\omega-mR}{\omega+mR}\int_0^1 u^2 du\, j_0(pu) j_1{}^2(pu), \tag{6}$$

where j_0, j_1 are spherical Bessel function, m is the nonstrange-quark mass, R is the bag radius, p is the confined-quark wave number, and $\omega=(p^2+m^2R^2)^{1/2}$. In this paper we employ $R^{-1}=200$ MeV, $m=33$ MeV, and compute $p=2.1176$.

For each mode the matrix elements are equal for $O_{1,2}$ and separately $O_{3,4}$. This is a consequence of the S-wave approximation in the bag-model computations of Refs. 3 and 5. To take recoil corrections into account, we multiply the matrix elements of Eqs. (4) and (5) by the form factor $(1+k^2/\kappa^2)^{-n}$ with $\kappa=1$ GeV and k^2 the bag momentum transfer.[8] We take $n=1, 2$ in this paper to display the sensitivity of our findings to the model dependence mentioned at the beginning of this section. Finally, we relate the bag amplitudes to appropriate plane-wave amplitudes,[3,7] square, and integrate over phase space. Using isospin relations, it is trivial to obtain neutron decay amplitudes from their proton counterparts.

Table I contains relative branching ratios for nucleon decay in the case of pure vector-boson mediation. Obviously there are two results insensitive to recoil corrections: (i) $\Gamma(\rho^0 e^+)/\Gamma(\omega e^+) = 0.42$ and (ii) $\Gamma(\eta e^+)/\Gamma(\omega e^+) \simeq 0$. The latter result arises because of fierce cancellations which occur in the ηe^+ amplitude. In our opinion, the pion modes are as unreliable as "model-independent" tests of vector-mediated nucleon decay. They are simply too dependent on one's choice of quark dynamics.

TABLE I. Relative branching ratios for vector-boson-mediated nucleon decay. Numerical entries are given for both monopole and dipole form factors.

Proton	Neutron	Dipole	Monopole
ωe^+	$\omega\bar{\nu}_e$	1.0	1.0
$\rho^0 e^+$	$\rho^0\bar{\nu}_e$	0.42	0.42
$\pi^0 e^+$	$\pi^0\bar{\nu}_e$	0.29	0.72
ηe^+	$\eta\bar{\nu}_e$	0.0	0.0
$\rho^+\bar{\nu}_e$	$\rho^- e^+$	1.0	0.59
$\pi^+\bar{\nu}_e$	$\pi^- e^+$	0.69	1.0

If scalar bosons contribute to nucleon decay with or without vector exchange, we must allow for $a_{3,4}^{(\sigma)} \neq 0$. Just to get insight as to the impact of the $a_{3,4}^{(1)}$ terms, let us temporarily take $a_{1,2}^{(1)}=0$. We then obtain the proton-decay branching ratios $\rho^0 e^+$: ωe^+: $\pi^0 e^+$: ηe^+:: 1.0:0.64:0.18:0.40 (1.0:0.63:0.46:0.50) for dipole (monopole) form factors, respectively. Evidently the operators O_3, O_4 tend to produce the $\rho^0 e^+, \eta e^+$ final states more copiously than do O_1, O_2. We might expect to find a remnant of this pattern for the general case $a_{1,2,3,4}^{(\sigma)} \neq 0$. Let us now return to our model-independent analysis.

If we treat the $\{a_i^{(\sigma)}\}$ as complete unknowns, the most easily derivable relation involves the ωe^+, $\pi^0 e^+$ modes, for which $\Gamma(\omega e^+)/\Gamma(\pi^0 e^+)=1.38$ (monopole), 3.48 (dipole). This follows from Eq. (3) times form-factor and phase-space contributions. It is unfortunately model dependent. A superior approach is to employ just the ωe^+, $\rho^0 e^+$, ηe^+ modes. Within this limitation, the most useful relation derivable is the inequality

$$1.22\Gamma(\rho^0 e^+) < \Gamma(\omega e^+) + 1.19\Gamma(\eta e^+) \tag{7a}$$

for a monopole form factor, or

$$1.23\Gamma(\rho^0 e^+) < \Gamma(\omega e^+) + 1.49\Gamma(\eta e^+) \tag{7b}$$

for the dipole form factor. We discuss this result in Sec. IV. However, we point out here that it appears to be relatively insensitive to the choice of form factor. The corresponding neutron result is not phenomenologically useful because antineutrino emission is involved, and one cannot experimentally determine the antineutrino flavor.

III. AN SU(5) MODEL OF HIGGS-BOSON-MEDIATED PROTON DECAY

In this section we analyze proton decay arising from Higgs scalars contained within a *single* 5-plet of SU(5). Let us first establish some notation. Noting that operators for $\Delta S=+1$ proton decay can be obtained from those with $\Delta S=0$ by replacing d quarks with s quarks, we define[9]

$$\begin{aligned}
O_1'^{(\sigma)} &= \epsilon_{ijk}\bar{s}^c_{iR}u_{jR}(\bar{u}^c_{kL}l_L-\bar{d}^c_{kL}\nu_L)\,,\\
O_2'^{(\sigma)} &= \epsilon_{ijk}\bar{d}^c_{iR}u_{jR}\bar{s}^c_{kL}\nu_L\,,\\
O_3'^{(\sigma)} &= \epsilon_{ijk}\bar{s}^c_{iL}u_{jL}\bar{u}^c_{kR}l_R\,,\\
O_4'^{(\sigma)} &= \epsilon_{ijk}[\bar{s}^c_{iL}u_{jL}(\bar{u}^c_{kL}l_L-\bar{d}^c_{kL}\nu_L)-\bar{d}^c_{iL}u_{jL}\bar{s}^c_{kL}\nu_L]\,,\\
O_5'^{(\sigma)} &= \epsilon_{ijk}[\bar{s}^c_{iL}u_{jL}(\bar{u}^c_{kL}l_L-\bar{d}^c_{kL}\nu_L)+2\bar{d}^c_{iL}u_{jL}\bar{s}^c_{kL}\nu_L]\,,\\
O_6'^{(\sigma)} &= \epsilon_{ijk}\bar{s}^c_{iR}u_{jR}\bar{u}^c_{kR}l_R\,.
\end{aligned} \tag{8}$$

The operators $O_4'^{(\sigma)}, O_5'^{(\sigma)}$, thus defined, do not mix under renormalization. The $\Delta S=1$ Hamiltonian has the general form

$$H^{\Delta S=1}=\sum_{i,\sigma}a_i'^{(\sigma)}O_i'^{(\sigma)}. \tag{9}$$

It is straightforward in SU(5) to determine the Higgs-boson–fermion interaction.[2,10] For simplicity, consider first the case of *zero mixing*. We obtain (see Fig. 1)

$$\frac{g_2}{\sqrt{2}M_w}H_k[m_d(\epsilon_{ijk}\bar{u}^c_{iR}d_{jR}+\bar{u}_{kR}e^c_R-\bar{d}_{kR}\nu^c_{eR}) - m_u(\epsilon_{ijk}\bar{u}^c_{iL}d_{jL}+\bar{u}_{kL}e^c_L)-m_s\bar{s}_{kR}\nu^c_{\mu R}]+\text{H.c.} \tag{10}$$

In Eq. (10), the SU(2) gauge coupling constant g_2 and the fermion masses m_w, m_d, m_s are all determined at the unification mass. Incidentally, at this scale the electron and down-quark mass parameters are equal, $m_e=m_d$. For definiteness, we adopt the convention of displaying the quark masses rather than their lepton counterparts.

Particle mixing can be taken into account as follows. Observe that the Higgs particles interact with fermions via two distinct vertices, in which the SU(5) fermionic representations couple as 10×10 and $5^*\times10$, respectively. The most economical parametrization is to mix the $T_3=-\frac{1}{2}$ fermions (e,μ,τ and d,s,b) in the former vertex and inverse-mix the $T_3=+\frac{1}{2}$ quarks (u,c,t) in the latter. See Ref. 2 for details. With this procedure it is clear that ν_τ emission in proton decay does *not* occur in our model. For, notice in Eq. (10) that neutrinos appear only in the $5^*\times10$ vertex and couple there only to $T_3=-\frac{1}{2}$ quarks. Our conclusion regarding ν_τ emission follows immediately. Moreover, our analysis implies that only mixing between the first two generations need be considered. This is indeed fortunate because it renders our model essentially free of mixing-angle ambiguities. In our numerical work, we employ the valid approximations (see Ref. 11) $\cos\theta_1\simeq1$ and $\sin\theta_1\cos\theta_3\simeq\sin\theta_1$.

In its short-distance version, the proton-decay Hamiltonian in our model is given by

$$\begin{aligned}
H=\frac{1}{2}\left(\frac{g_2}{M_WM_S}\right)^2\Big(&m_um_d(O_3^{(1)}+O_4^{(1)})-m_d^2O_1^{(1)}-m_u^2O_2^{(1)}+m_sm_dO_2'^{(2)}-\tfrac{1}{3}m_sm_u(O_5'^{(2)}-O_4'^{(2)})\\
&+\sin\theta_C\{m_um_d(O_4^{(2)}+\tfrac{2}{3}O_4'^{(1)}+\tfrac{1}{3}O_5'^{(1)})-m_u^2(O_2^{(2)}+O_3'^{(1)})-m_sm_d[O_1'^{(1)}+O_1^{(2)}(l)]+m_sm_u[O_3^{(2)}(l)+O_6'^{(1)}]\}\\
&+\sin^2\theta_C\{m_um_s[\tfrac{2}{3}O_4'^{(2)}(l)+\tfrac{1}{3}O_5'^{(2)}(l)+O_6'^{(2)}]-m_u^2O_3'^{(2)}-m_s^2O_1'^{(2)}(l)\}\Big)\,,
\end{aligned} \tag{11}$$

where the notation $O_i^{(\sigma)}(l)$ means to take just the lepton part of that operator and omit the neutrino part. This Hamiltonian can induce proton decay as depicted in Fig. 2.

To compute transition amplitudes with Eq. (11), we must be able to both determine the mass parameters therein and also to compute the operator matrix elements at some energy scale μ. There are several ways to accomplish this. The method we employ here is to fix the mass parameters by extrapolating their low energy values up to the unification scale of Eq. (11) with renormalization-group equations.[12] Thereafter in the calculation, they are simply fixed numbers. The enhancement factors associated with finite renormalizations of the operators in Eqs. (11) have been comprehensively tabulated in Ref. 13. An approach alternative to ours is to interpret the mass parameters of Eq. (11) as parts of propagators and renormalize the large collection of operators in Eq. (11) all at once.

One reason we have chosen to estimate the fermion masses at the unification scale is to explicitly acknowledge a phenomenological shortcoming of this model—the light-fermion mass ratios are not correctly reproduced. This implies a genuine am-

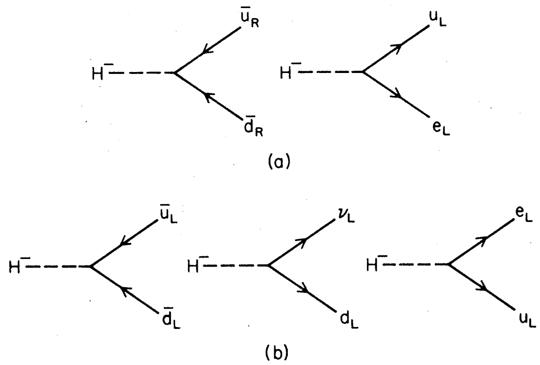

FIG. 1. Examples of trilinear Higgs-boson–fermion vertices in SU(5). The subscripts L,R pertain to the chirality of the corresponding fermions.

biguity as to which mass values to employ, m_d or m_e, m_μ or m_s. We have considered each possibility—first quark masses, then lepton masses. The values (at unification scale) employed are (in units of GeV) $m_s=0.04$, $m_d=0.0022$, $m_u=0.0010$, $m_\mu=0.074$, and $m_e=3.6\times10^{-4}$. Our qualitative results are fortunately independent of the fermion-mass ambiguity. Finally the operator enhancement factors used here are $O_1(3.4)$, O_2 (3.6), O_3 (6.8), O_4 (3.0), $O_1'(3.4)$, O_2' (3.6), O_3' (3.6), O_4' (6.8), O_5' (2.9), and O_6' (3.0).

Substantial simplification obtains upon inserting numerical fermion-mass values into Eq. (11). We find (including the enhancement factors)

$$H=\left(\frac{g_2}{M_W M_S}\right)^2(3.14\times10^{-4})\times[O_2'^{(2)}-0.91\,O_1'^{(2)}(l)+0.29\,O_4'^{(2)}-0.21\,O_1'^{(1)}+0.21\,O_1^{(2)}(l)+0.20\,O_3^{(2)}(l)+\cdots] \tag{12a}$$

for m_s, m_d, or

$$H=\left(\frac{g_2}{M_W M_S}\right)^2(0.98\times10^{-3})\times[-O_1'^{(2)}(l)+0.17\,O_4'^{(2)}+0.12\,O_3^{(2)}(l)-0.07\,O_5'^{(2)}+0.05\,O_6'^{(1)}+\cdots] \tag{12b}$$

for m_μ, m_e. The leading terms, $O_1'^{(2)}(l)$ and $O_2'^{(2)}$, in Eqs. (12a) and (12b) give rise to μ^+ emission in proton decay.

The lack of agreement between the Hamiltonian operators of (12a) and (12b) arises from the inadequacy of this simple model to correctly reproduce the mass ratios of light fermions. Nonetheless, it is instructive to consider proton decay in somewhat more detail. For definiteness we employ Eq. (12b) to compute assorted decay widths, which are presented in Table II. The importance of the $p\to K^0\mu^+$ mode for this case is evident. We can use the information in Table II to obtain a bound on M_S as follows. Taking the present proton lifetime to be $\tau_p>10^{30}$ yr, we see that 2.1×10^{-62} GeV $>\Gamma_{\rm tot}(p)>\Gamma(p\to K^0\mu^+)$. We take $\Gamma(p\to K^0\mu^+)=(g_2/M_W M_S)^4\eta\times10^{-9}$ where $\eta\sim1$ represents the theoretical uncertainties embodied by the scatter of $K^0\mu^+$ decay rates in Table II. Since g_2 is evaluated at

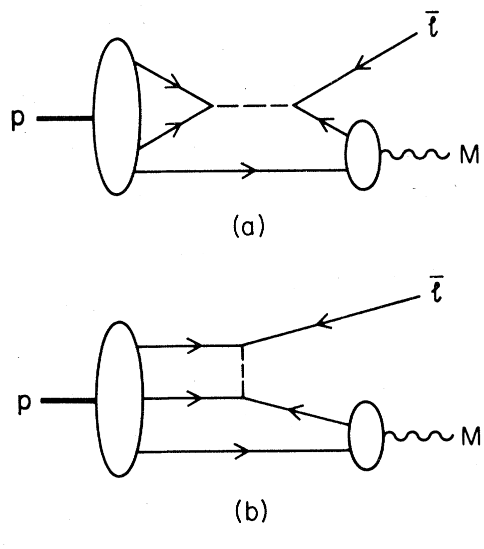

FIG. 2. Scalar-mediated decay of a proton (p) into a meson (M) and a antilepton ($\bar{l}$). The internal dashed line represents a color-bearing scalar particle.

TABLE II. Selected decay rates. Cases (a) and (b) refer to dipole and monopole form factors, respectively. All entries are computed using Eq. (12b). Each rate is to be multiplied by $(g_2/M_W M_S)^4$, where M_W and M_S are to be expressed in GeV.

Mode	Rate (10^{-13} GeV)
	(a)
$\omega\mu^+$	1.6
K^0e^+	2.4×10^{-2}
$K^+\bar{\nu}_\mu$	4.5
$K^0\mu^+$	2.0×10^1
	(b)
$\omega\mu^+$	1.7
K^0e^+	3.4×10^{-2}
$K^+\bar{\nu}_\mu$	6.4
$K^0\mu^+$	2.9×10^1

the unification mass (GUM), we have $g_2 = (4\pi\alpha_{\rm GUM})^{1/2} = 0.55$. Upon inserting the value $M_W = 81.6$ GeV into the decay rate inequality we obtain the bound

$$M_S > 2.1\eta^{1/4}\delta \times 10^{10}\ {\rm GeV}, \qquad (13)$$

where δ is a number greater than but near unity which takes account of the variation of vector- and scalar-boson mass with energy scale.

IV. CONCLUSION

Suppose the proton is found to decay and that sufficient data can be accumulated to allow determination of individual branching modes. Then within the dynamical framework of the bag model we can state with reasonable confidence that experimental signals for pure vector-boson mediation of the decay amplitude are $\Gamma(\rho^0 e^+) = 0.42\Gamma(\omega e^+)$, $\Gamma(\eta e^+) \simeq 0$, and probably $\Gamma(\omega e^+) > \Gamma(\pi^0 e^+)$.[14] Specific examples of these relations appear in Ref. 3 where branching rates for the unification groups SU(5), SO(10), and $SU(2)_L \times SU(2)_R \times U(1)$ are computed. The bag model is a model of relativistic quarks. Its results generally differ from those of nonrelativistic SU(6). In particular this is the case for the ηe^+ mode, the smallness of whose branching ratio follows from dynamic, not group-theoretic, considerations. Analogous relations for neutron decay [e.g., $\Gamma(\rho^0\bar{\nu}_e) = 0.42\Gamma(\omega\bar{\nu}_e)$] are not as useful because they involve antineutrino emission. It is not practical to expect the antineutrino flavor to be measured.

If vector-mediated decay amplitudes are contaminated or even dominated by scalar-exchange processes, the above relations must be replaced by the weaker statement $1.2\Gamma(\rho^0 e^+) < \Gamma(\omega e^+) + \epsilon\Gamma(\eta e^+)$ where $1.2 < \epsilon < 1.5$ is a mildly model-dependent number. The weakening of predictive power is a simple consequence of vector-mediation operators being a subset of scalar-mediation operators. As pointed out in Sec. II, the operators $O_3^{(\sigma)}, O_4^{(\sigma)}$, which are introduced by dropping the assumption of vector mediation, tend to be effective at ρ^0 and η production. Hence, on a nonrigorous level one might look for deviations of the vector-mediation relations via enhancement of the $\rho^0 e^+$, ηe^+ modes.

The author is quite aware of the usefulness that model-independent statements regarding $\pi\bar{l}$ would have to experimentalists engaged in proton-decay experiments. It is anticipated that claims of model independence involving pion modes will appear in the literature. We can only reemphasize our comments of Sec. II that every calculation of nucleon decay is based on some set of dynamical assumptions and that pion-emission amplitudes have shown themselves to be sensitive to such assumptions. However, matters are not entirely bleak, for if and when pion modes are observed in proton decay, the measured branching ratios will clarify the nature of structure corrections, and hence allow a resolution of the vector vs scalar question via predictions like those contained in Table I.

Measurement of the $\bar{l}$ polarization is also useful in probing the nature of the agent which induces proton decay. Weinberg has pointed out that for pure vector exchange the $\bar{l}$ polarization is independent of final states separately for $\Delta S = 0$ and $\Delta S = 1$ transitions.[1] Scalar contributions can be expected to disturb this universality property, e.g., the e^+ polarization in the $\rho^0 e^+$, ωe^+ modes are no longer expected to be equal. However, provided that the static-bag relation $\langle e^+\pi^0 | O_1 | p\rangle = \langle e^+\pi^0 | O_3 | p\rangle$ holds as well for *moving* bag states, the equality of e^+ polarizations in $p \to \pi^0 e^+$, ωe^+ is maintained even with scalar exchange present. Also, in view of the bag-model result $\langle e^+\eta | O_1 | p\rangle \cong 0$, measurement of the e^+ polarization in $p \to \eta e^+$ would yield information on the relative magnitudes of $a_3^{(1)}$ and $a_4^{(1)}$, viz.,

$$P = (|a_3^{(1)}|^2 - |a_4^{(1)}|^2)/(|a_3^{(1)}|^2 + |a_4^{(1)}|^2).$$

The model of Sec. III amusingly serves to remind us that relations between ω, ρ, η, and π modes might be useless if each rate is too small to be measured. Of course, the spectacular nature of the decay pattern $\Gamma_{tot}(p) \simeq \Gamma(p \to KX)$ would be strong evidence of the type of mass dependence present in Higgs-boson couplings. Moreover in the specific model Eq. (12b), the μ^+ in the dominant two-body $K^0\mu^+$ mode is strongly left-handed polarized. It would be hard to misinterpret such singular data. Finally, we note in passing that our bound on the Higgs-boson mass is comparable to earlier estimates based on Higgs-boson-mediated proton decay.[2]

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APPENDIX

The problem of trying to estimate the decay rate of a proton into a positron plus an S-wave pion pair is a nettlesome one. It would be simplest to characterize the pion pair in terms of some $I = J$

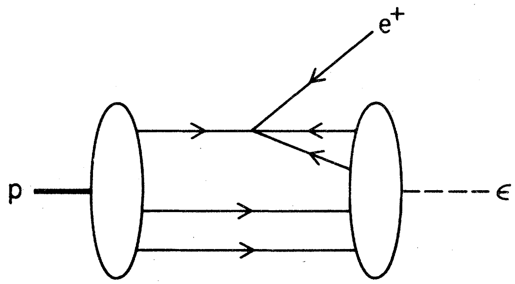

FIG. 3. The decay $p\rightarrow \epsilon e^+$. The meson ϵ is taken to be a $q^2\overline{q}^2$ composite.

$=0$ resonant state (called ϵ here). In this appendix we take seriously the suggestion of Ref. 15 that ϵ is a scalar, isoscalar $q^2\overline{q}^2$ configuration of mass around 690 MeV, and show that such a state cannot contribute to proton decay.

The dynamics that produces the low mass of $\epsilon(q^2\overline{q}^2)$ dictates its symmetry structure. Each of the diquarks in $\epsilon(q^2\overline{q}^2)$ is color and spin symmetric and flavor antisymmetric (see Ref. 16 for the explicit wave function). However, as shown in Fig. 3 the ud diquark is propagated undisturbed from the initial proton state. Such a diquark is necessarily color antisymmetric, and thus the amplitude for $p\rightarrow e^+\epsilon(q^2\overline{q}^2)$ vanishes.

In Ref. 3, a P-wave $q\overline{q}$ pair of total energy 700 MeV is used to characterize an $I=J=0$ resonance. The associated branching ratio in proton decay is found to be insignificant. This is consistent with the results of a totally different approach employing S-matrix techniques.[17] Evidently, protons prefer not to decay into S-wave pion pairs.

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[1]S. Weinberg, Phys. Rev. Lett. 43, 1566 (1979); F. Wilczek and A. Zee, *ibid*. 43, 1571 (1979).

[2]These are compiled in P. Langacker, Phys. Rep. (to be published). In addition, two papers which arrived as this manuscript was being typed are H. Arisue, Kyoto report, 1981 (unpublished) and W. B. Rolnick, Phys. Rev. D 24, 2464 (1981).

[3]For a study of this dependence, see E. Golowich, Phys. Rev. D 22, 1148 (1980).

[4]However, see M. Machacek, Nucl. Phys. B159, 37 (1979).

[5]For a harmonic-oscillator calculation see, e.g., G. Kane and G. Karl, Phys. Rev. D 22, 2808 (1980). For bag-model calculations see Ref. 3 or J. Donoghue, Phys. Lett. 92B, 99 (1980).

[6]J. Donoghue and G. Karl, Phys. Rev. D 24, 230 (1981).

[7]Since we are interested only in *relative* branching ratios, we suppress common numerical factors such as bag normalization. For 1^- meson emission, the *zero-helicity* amplitude is given.

[8]See Table II of Ref. 3. Also a discussion of alternative procedures within the bag-model framework is given in Sec. III of Ref. 3.

[9]L. Abbott and M. Wise, Phys. Rev. D 22, 2208 (1980).

[10]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).

[11]We employ the convention given in Particle Data Group, Rev. Mod. Phys. 52, S41 (1980).

[12]E. g., see A. Buras, J. Ellis, M. Gaillard, and D. Nanopoulos, Nucl. Phys. B135, 66 (1978); D. Nanopoulos and D. Ross, *ibid*. B157, 273 (1979).

[13]L. Abbott and M. Wise, Phys. Rev. D 22, 2208 (1980).

[14]We feel that $\omega\mu^+$, $\rho^0\mu^+$ modes would be difficult to detect (e. g., phase-space suppression) and hence do not consider them here.

[15]R. L. Jaffe, Phys. Rev. D 15, 267 (1977).

[16]E. Golowich, Phys. Rev. D 23, 2610 (1981).

[17]M. B. Wise, R. Blankenbecler, and L. F. Abbott, Phys. Rev. D 23, 1591 (1981).