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Nucleon decay in the nucleus

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We estimate the change in the baryon-number-violating nucleon lifetime inside a nucleus, compared to the free-space decay of the nucleon. For the decay modes $p\to e^+\pi^0$, $e^+\omega$ and $n\to e^+\pi^-$, we include the effects of pion or ω absorption and pion rescattering (including charge exchange). The rescattering effects are found to represent a correction of only a few percent or less to the total decay rate. The pion and ω absorption corrections depend sensitively on the nature of the nucleon-nucleon short-range correlations, varying from roughly 5 to 50 % depending on the hard-core radius. These modes lead to the emission of fast nucleons and a displaced bump in the positron spectrum.

I. INTRODUCTION

After the impressive phenomenological success[1] of the weak-electromagnetic unified gauge theory of $SU(2)\otimes U(1)$, ambitious attempts have been made to include the strong interactions in the program of unification.[2] Such grand unified theories have the far-reaching implication that the proton may be unstable, since quarks and leptons occur in the same multiplets. In these theories, the parameter $\sin^2\theta_W$, which is an undetermined parameter in the electroweak unified theories, is determined[3] to be 0.21 after higher perturbative effects are included. The value $\sin^2\theta_W=0.21$ is quite close to the experimental value $\sin^2\theta_W\simeq 0.23\pm 0.02$, obtained[1] by fitting a variety of weak-interaction data. Within the same calculation scheme, the proton lifetime is estimated to be $\sim 10^{31\pm 1}$ years.[3] The branching ratios for nucleon decay into a lepton plus various mesons can also be estimated. In several independent calculations,[4] the $p\to e^+\pi^0$ channel was found to be the most significant one due to the large phase space available, even though the $e^+\omega^0$ channel has a larger intrinsic decay amplitude. Thus several experiments[5] so far proposed to study the possible nucleon decays are designed to observe the electromagnetic radiations from the $\pi^0 e^+$ decay of the proton.

Since searches for proton decay will be carried out using materials other than pure hydrogen, it is important to study whether the decay rate and the final decay products are altered due to the presence of other nucleons in the nucleus. The effect we are interested in is a change in the lifetime due to *virtual* processes. In the familiar case of internal conversion, it is well known that virtual processes can substantially decrease the lifetime of nuclear electromagnetic transitions.[6] Hypernuclear decay has many features in common with our problem and has been considered at length by Dalitz and his collaborators.[7] There is some experimental evidence that the decay rates of Λ hypernuclei are increased over the free-Λ decay rate.[8] For proton decay, a different effect has recently been considered by Sparrow.[9] He shows that rescattering in the nucleus of *real* mesons resulting from nucleon decay can have rather important effects on the signal. We emphasize that the effect we are considering is quite different from this. Rescattering of real mesons cannot change the total decay rate nor can it change the associated positron spectrum, while the virtual processes that we calculate can change both of these quantities. An earlier estimate by Dover and Wang[10] suggests that the effect can be important.

The outline of the paper is as follows. In Sec. II, we discuss nucleon decay in the simplest nucleus, the deuteron. Besides the individual decays $p\to e^+\pi^0$, $e^+\omega^0$, and $n\to e^+\pi^-$, etc., there are the decays $pn\to e^+(n, \pi^- p, \pi^0 n, \ldots)$ via the virtual exchange of π^0, ω^0, etc. For definiteness in numerical calculations, we will use SU(5) couplings. We shall call the $pn\to e^+ n$ channel "absorption" and the rest we refer to as "rescattering." The rescattering part contains various Δ and N^* resonance contributions. Because $(e^+,\bar\nu)$ behaves like an isodoublet in SU(5), the hadrons in the final state must have $I=\frac{1}{2}$, thus the Δ does not contribute to the deuteron decay. For the deuteron wave function, we use the Hulthén form, and find that the effect from rescattering is rather small, in the range of a few percent compared to that of free decay. The total absorptive contribution from π^0 and ω^0 exchange shortens the nucleon lifetime by an amount that depends strongly on the hard-core radius r_c in the Hulthén wave function. For $r_c=0$, the absorptive effect shortens the lifetime by about 50%, while for $r_c=0.5$ fm, the effect is less than 10%. The signature of the $pn\to e^+ n$ channel is an e^+ of momentum $p_{e^+}\approx\frac{3}{4}m_N$, rather than $\frac{1}{2}m_N$ as in free decay, plus an energetic neutron in the final state.

In Sec. III, the nucleon decay in more complex nuclei is discussed. We utilize a simple independent-particle approximation (IPA), which treats each nucleon as moving independently in the average Hartree-Fock field provided by the rest of the nucleus. The harmonic-oscillator approximation is used for each single-particle orbital. In the case of ^{4}He, for which the formalism simplifies considerably, since the single-particle wave functions are all s states, our calculations give a result of $\delta\Gamma_{abs}/\Gamma^{free}_{^4He} \simeq 0.1$ from pion absorption, and a pion-rescattering effect of only a few percent.

In the Appendix, we give the derivation of the formalism used and discuss the effects for nuclei more complex than ^{4}He.

II. DECAY OF THE DEUTERON

In this section, we consider corrections to the deuteron lifetime due to meson rescattering and absorption. The processes we include are shown in Fig. 1. The free decay width Γ_D^{free} is given by the graphs of Figs. 1(a) and 1(b). Since in SU(5) the e^+ behaves essentially as an isospin-$\frac{1}{2}$ particle, the $ne^+\pi^-$ vertex contains a $\sqrt{2}$ relative to the $pe^+\pi^0$ vertex, and hence

$$\Gamma_D^{free} = \Gamma_p + \Gamma_n = 3\Gamma_p = 3\lambda^2 m_N/16\pi\,, \tag{2.1}$$

where m_N is the nucleon mass, and we have used a simple phenomenological form $\lambda\bar{u}\vec{\tau}\cdot\vec{\phi}u$ for the $N\to e^+\pi$ vertex. Here $\vec{\tau}$ is the pion isospin operator, u is a spinor for a spin-$\frac{1}{2}$ particle (N or e^+), and $\vec{\phi}$ is the pion field operator; in Eq. (2.1), but not generally, we neglect pion (μ_π) and electron masses relative to m_N. In this approximation, we also obtain Eq. (2.1) for a more general vertex for which $1\to(1+\alpha\gamma_5)/(1+\alpha)$.

The actual deuteron width Γ_D differs from Γ_D^{free} due to the meson rescattering processes of Figs. 1(c) and 1(d) and the meson absorption processes of Figs. 1(e) and 1(f). We can write

$$\Gamma_D = \Gamma_D^{free} + \delta\Gamma_{resc} + \delta\Gamma_{abs}\,. \tag{2.2}$$

The general method of deriving $\delta\Gamma_{resc}$ and $\delta\Gamma_{abs}$ for any nucleus is given in the Appendix. For the deuteron, the rescattering contribution $\delta\Gamma_{resc}$ can be expressed in the form

$$\frac{\delta\Gamma_{resc}}{\Gamma_D^{free}} = \frac{8}{\pi m_N}\int_0^{p_{max}} dp\, p_{\pi L}\sigma^{tot}_{I=1/2}(s)\{[\mathrm{Re}F(p)]^2 - [\mathrm{Im}F(p)]^2 + 2\,\mathrm{Re}F(p)\,\mathrm{Im}F(p)\alpha(p)\}\,, \tag{2.3}$$

where $p_{\pi L}$ is the pion laboratory momentum in the final state [see Eq. (2.8) below], and p is the positron momentum. Since the deuteron has isospin zero, only the π-nucleon total cross section in the isospin $I=\frac{1}{2}$ channel, $\sigma^{tot}_{I=1/2}(s)$, enters here. The ratio of the real to imaginary part of the πN amplitude is $\alpha(p)$. In Eq. (2.3), $F(p)$ is defined by

$$F(p) = \int_0^\infty dr\, e^{-iqr}\sin(pr)\psi_D(r)\,, \tag{2.4}$$

where $\psi_D(r)$ is the normalized deuteron relative wave function (the c.m. of the deuteron is assumed to be at rest), and $q=[(m_N-p)^2-\mu_\pi^2]^{1/2}\approx m_N-p$. The maximum e^+ momentum is determined from

$$2m_N = p_{max} + [p_{max}^2 + (m_N+\mu_\pi)^2]^{1/2}\,,$$

that is,

$$p_{max}/m_N = \tfrac{3}{4}\left(1-\frac{2\delta}{3}-\frac{\delta^2}{3}\right),$$

where $\delta = \mu_\pi/m_N$.

For the absorption correction due to the pion exchange graphs of Figs. 1(e) and 1(f), we have

$$\frac{\delta\Gamma_{abs}}{\Gamma_D^{free}} = \frac{3(g_{\pi^0NN})^2}{2m_N}|F(p)|^2\,, \tag{2.5}$$

where $p=\frac{3}{4}m_N$, $q=(m_N/4)(1-16\delta^2)^{1/2}$ in $F(p)$, for

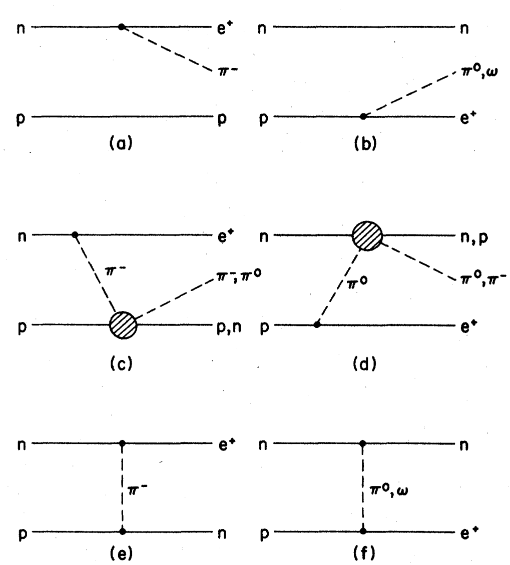

FIG. 1. Some of the processes involved in nucleon decay in the nucleus. Free decay, with a spectator nucleon, is represented in (a) and (b). The rescattering processes contributing to $\delta\Gamma_{resc}$ which we include here are shown in (c) and (d). The shaded blob corresponds to a meson-nucleon elastic or charge-exchange scattering amplitude. The absorptive processes leading to $\delta\Gamma_{abs}$ are depicted in (e) and (f).

a deuteron with zero c.m. momentum. The strong coupling constant is $(g_{\pi 0 NN})^2/4\pi \cong 13.4$.

We now discuss the evaluation of Eqs. (2.3) and (2.5). For the rescattering effect, we use a Hulthén wave function for the deuteron:

$$\psi_D(r) = C(e^{-\beta r} - e^{-\gamma r})/r\,, \tag{2.6}$$

where

$$C = [2\gamma\beta(\beta+\gamma)/4\pi]^{1/2}/(\gamma-\beta)\,,$$

$$\beta = (m_N E_B)^{1/2}/\hbar = 0.232 \text{ fm}^{-1}\,,$$

$$\gamma \approx 7\beta \approx 1.624 \text{ fm}^{-1}\,.$$

For this choice, $F(p)$ can be evaluated in closed form:

$$\mathrm{Re}F(p) = \frac{C}{2}\left\{\tan^{-1}\left[\frac{(\gamma-\beta)(p+q)}{(p+q)^2+\beta\gamma}\right] + \tan^{-1}\left[\frac{(\gamma-\beta)(p-q)}{(p-q)^2+\beta\gamma}\right]\right\},$$

$$\mathrm{Im}F(p) = \frac{C}{4}\ln\left\{\frac{[(p+q)^2+\gamma^2][(p-q)^2+\beta^2]}{[(p+q)^2+\beta^2][(p-q)^2+\gamma^2]}\right\}. \tag{2.7}$$

Note that the πN laboratory amplitude $\mathfrak{M}$, which appears in Figs. 1(c) and 1(d), in general has the pion off-shell. We replace $\mathfrak{M}$ by on-shell quantities by using the approximation $\mathrm{Im}\mathfrak{M}_{I=1/2} = -2m_N p_{\pi L}\sigma^{\rm tot}_{I=1/2}(\sqrt{s})$, where $\sqrt{s}$ is the invariant mass of the πN system in the on-shell final state; we take $\alpha(p)$ from observed $I=\frac{1}{2}$ real parts at energy $\sqrt{s}$. We ignore the presence of off-shell form factors and the angular dependence of $\mathfrak{M}$. Since the results we obtain for $\delta\Gamma_{\rm resc}$ represent rather small corrections in any case, it does not seem worthwhile to try to improve on this rough approximation. Note that if $q = p_{\pi L}$, the intermediate state pion is in fact on-shell; this occurs for $p = m_N/2$, well inside the domain of integration of the e^+ momentum p. Hence the off-shell excursions of $\mathfrak{M}$ are limited. (Care is required in integrating over this on-shell point to avoid double counting. This is discussed fully in Sec. III and the Appendix.) Explicitly, we have

$$s = 4m_N(m_N - p)\,,$$

$$\frac{p_{\pi L}}{m_N} = [\tfrac{1}{4}(3 - 4p/m_N - \delta^2)^2 - \delta^2]^{1/2}\,. \tag{2.8}$$

Note that in the limit of $p = p_{\max}$, $p_{\pi L} = 0$, so the phase space for the πN collision is correctly included. For $p = 0$, we find $p_{\pi L} \approx 1.4$ GeV/c, which is the maximum momentum sampled in the pion rescattering collision. We thus sweep over the $N^*(1470)$ resonance of the πN system; the $\Delta(1236)$ does not contribute for the deuteron, as mentioned before.

The values of $\sigma^{\rm tot}_{I=1/2}$ and $\alpha(p)$ were taken from Ref. (11). The resulting integrand which appears in Eq. (2.3) is displayed in Fig. 2. Note the large degree of cancellation; this occurs even for $\alpha(p) = 0$, since $[\mathrm{Re}F(p)]^2 - [\mathrm{Im}F(p)]^2$ changes sign near $p/m_N = \frac{1}{2}$. The fact that $\alpha(p)$ is large and positive (1.5–6.0) for $p_{\pi L} \leq 0.5$ GeV/c implies a *negative* value for $\delta\Gamma_{\rm resc}$ (leading to a slightly longer lifetime). We find

$$\frac{\delta\Gamma_{\rm resc}}{\Gamma_D^{\rm free}} \approx -0.02\,. \tag{2.9}$$

The calculation of pion absorption is somewhat more delicate, since it depends more crucially on the short-range properties of the deuteron wave function. If we use the Hulthén wave function of Eq. (2.6), we obtain a rather substantial effect:

$$\frac{\delta\Gamma_{\rm abs}}{\Gamma_D^{\rm free}} \approx 0.45\,. \tag{2.10}$$

However, Eq. (2.6) does not include the possibility of strong "hard-core" repulsion at short distances. To see that these short-range effects are important, we note that the dominant component of $F(p)$, that is, $\mathrm{Re}F(p)$, contains in its integrand the factor

$$\cos(qr)\sin(pr) \approx \cos\frac{m_N r}{4}\sin\frac{3m_N r}{4}\,,$$

which achieves its maximum value for $r \approx 2\pi/3m_N \approx 0.45$ fm. This maximum corresponds more or less to the conventional hard-core radius in the

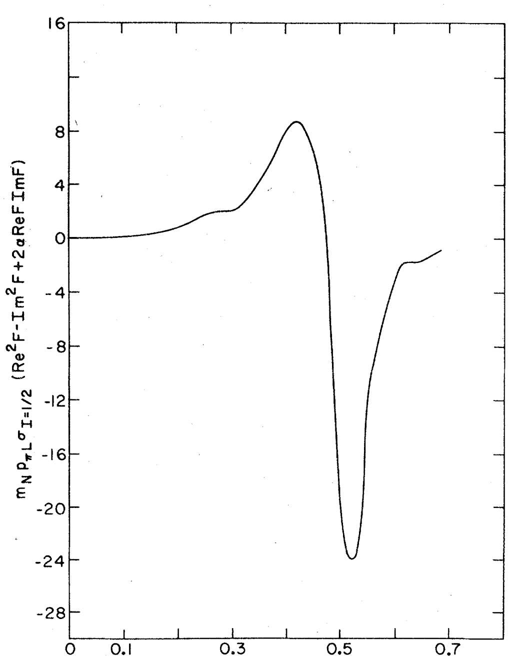

FIG. 2. Momentum dependence of the integrand of Eq. (2.3), illustrating the strong cancellations which occur in the rescattering correction $\delta\Gamma_{\rm resc}$. We have defined $F = F/C_0$, with $C_0{}^2 = \gamma\beta(\gamma+\beta)/2\pi(\gamma-\beta)^2$ and $x = p/m_N$.

NN potential, so our results will depend sensitively on the degree of wave-function suppression at short distances. The Hulthén wave function $\psi_D(r)$ remains finite as $r\to 0$, and so Eq. (2.10) represents an overestimate of $\delta\Gamma_{\rm abs}$. The effect of hard-core repulsion can be estimated by considering wave functions of the type[12]

$$\psi_D(r)=\begin{cases}0 \text{ for } r<r_c\,,\\ \dfrac{C}{r}e^{-\beta r}(1-e^{-\gamma(r-r_c)}) \text{ for } r\geq r_c\,,\end{cases} \tag{2.11}$$

where $\beta\approx(m_N E_B)^{1/2}\approx 0.232$ fm^{-1} ($E_B=2.23$ MeV is the deuteron binding energy) and γ and r_c have been adjusted to produce the observed rms radius and quadrupole moment for the deuteron. If we now vary the core radius r_c, we find typical values

$$\frac{\delta\Gamma_{\rm abs}}{\Gamma_D^{\rm free}}=\begin{cases}0.022 \text{ for } r_c=0.43 \text{ fm},\ \gamma=1.91 \text{ fm}^{-1}\,,\\ 0.052 \text{ for } r_c=0.5 \text{ fm},\ \gamma=2.37 \text{ fm}^{-1}\,.\end{cases} \tag{2.12}$$

Equation (2.12) implies a considerably smaller absorption effect than Eq. (2.10). Note that the change in $\delta\Gamma_{\rm abs}/\Gamma_D^{\rm free}$ is not monotonic with r_c.

In the preceding calculations, we have included only the pion degrees of freedom. There are other meson exchanges which interfere *coherently* with pion exchange in the absorption correction. The inclusion of these depends on a detailed model for the free-space branching ratios $p\to e^+\pi^0$, $e^+\rho^0$, $e^+\omega$, $e^+\eta$, etc., about which there is some disagreement.[4] These interferences can also occur in the rescattering correction, if the final state ($e^+\pi N$, say) is the same. However, in this case one can argue that the cross sections $\pi N\to\eta N$, ρN, ωN are much smaller in the allowed momentum region ($p_{\pi L}\leq 1.4$ GeV/c) than for the $\pi N\to\pi N$ process, and thus these effects would be unimportant.

We now estimate the absorptive contribution from the $p\to e^+\omega$ channel, which is estimated[4] to have a large intrinsic coupling strength. The ω contributions we have included appear in Figs. 1(b) and 1(f). We have omitted the ω in rescattering graphs such as 1(d). We take an effective Hamiltonian for the ω exchange as $H_v=ig_\omega\bar\psi\gamma_\mu\psi\omega^\mu$. The $f_\omega\sigma_{\mu\nu}$ tensor interaction term is neglected at the $N\to N\omega$ vertex due to the fact that f_ω/g_ω is small, and also dropped at the $p\to e^+\omega$ vertex, as suggested in the literature.[4] An explicit calculation shows that the interference between the ω and the π^0 exchanges vanishes. We also find that the interference between the ω and π^- exchanges gives very small contributions, only a few percent of the ω term by itself. Using the values $g_\omega^2/4\pi=10$ and $(\lambda_{\omega e^+p}/\lambda_{\pi^0e^+p})^2=3$, we find

$$\frac{\delta\Gamma_{\rm abs}}{\Gamma^{\rm free}}=\frac{3(g_{\pi^0NN})^2}{2m_N}\times[|F(p,q_\pi)|^2+1.4|F(p,q_\omega)|^2]\,, \tag{2.13}$$

where $q_{\omega,\pi}^2=(m_N-p)^2-\mu_{\omega,\pi}^2$, and $F(p,q)$ is defined in Eq. (2.4). Note that $q_\omega^2<0$, so that

$$F(p,q_\omega)=\int_0^\infty dr\,e^{-|q_\omega|r}\sin pr\,\psi_D(r)\,. \tag{2.14}$$

Using the Hulthén wave function of Eq. (2.6), we obtain

$$\frac{\delta\Gamma_{\rm abs}}{\Gamma_D^{\rm free}}\approx 0.52\,, \tag{2.15}$$

when both pion and ω exchange are included, to be compared with Eq. (2.10) for pions alone.

The rescattering effects from ω exchange are found to be very small, as for the pion. Equation (2.15) refers to $r_c=0$. If a hard core is included, the ω contribution to $\delta\Gamma_{\rm abs}$ is suppressed even more than the pion, by at least an additional order of magnitude. Similar considerations apply to η, η', or ρ exchange contributions to $\delta\Gamma_{\rm abs}$; for these, the intrinsic coupling constants at the nucleon decay vertex are expected to be smaller than for the ω, and they also suffer an order-of-magnitude suppression if a hard core is included.

The experimental signature for meson absorption in the deuteron would be a back-to-back e^+-neutron pair, each with momentum $p\approx\frac{3}{4}m_N\approx 700$ MeV/c. This would be a rather distinctive mode. In a more complex nucleus, as we show later, such a correlated e^+n pair can also result, but the momentum spectrum is broadened by the Fermi momentum associated with the c.m. motion of the interacting pair of nucleons.

III. NUCLEON DECAY IN COMPLEX NUCLEI

In this section, we present results for nucleon decay processes in complex nuclei. In general this is a complicated problem, since the pion resulting from the decay $N\to e^+\pi$ can enjoy multiple scattering in the nucleus, or suffer absorption on one, two, or more nucleons. We utilize a simple independent-particle approximation (IPA), which treats each nucleon as moving independently in the average Hartree-Fock field provided by the rest of the nucleus. The harmonic-oscillator approximation is used for each single-particle orbital. The antisymmetrized product wave function of a nucleon pair is decomposed into relative and c.m. parts. Unlike the deuteron case, where the c.m. of the pair is at rest, we integrate over the c.m.

momentum distribution here. The general formalism of the IPA applied to nucleon decay is outlined in the Appendix. For ^{4}He, the formalism simplifies considerably, since the single-particle wave functions are all s states without angular dependence. In this case, the pion-rescattering-correction analog to Eq. (2.3) for the deuteron becomes

$$\frac{\delta\Gamma_{\rm resc}}{\Gamma^{\rm free}_{^4{\rm He}}}=\frac{32\rho^2}{3\pi^2}\int_0^{1-\delta}x^2dx\int_0^{y(x)}dx'\int_{-1}^{\cos\theta_c(x,x')}d(\cos\theta)\exp[-2\rho^2(x^2+x'^2+2xx'\cos\theta)]$$
$$\times(\{[{\rm Re}\hat{F}(x,x')]^2-[{\rm Im}\hat{F}(x,x')]^2\}\hat{\sigma}+2\,{\rm Re}\hat{F}(x,x')\,{\rm Im}\hat{F}(x,x')\alpha\hat{\sigma})\,, \quad (3.1)$$

where $\delta=\mu_\pi/m_N$ and $\rho=bm_N$; b is the oscillator radius parameter. If[13] we adjust b to reproduce the rms charge radius $\langle r^2\rangle_{\rm ch}^{1/2}=1.674$ fm for ^{4}He, we obtain $b=(2\langle r^2\rangle_{\rm ch}/3)^{1/2}\approx1.367$ fm. In Eq. (3.1), we have defined dimensionless variables $x=p/m_N$ ($p=e^+$ momentum, as before), $x'=|\vec{K}/2-\vec{p}|/m_N=|\vec{K}'|/m_N$, $\vec{K}$ being the initial c.m. momentum of the nucleon pair, and $\cos\theta=\vec{K}'\cdot\vec{p}/|\vec{K}'||\vec{p}|$. The kinematical variables s, $p_{\pi L}$, and κ (c.m. momentum in the πN system) are obtained as

$$\frac{s}{4m_N^2}=1-x-x'^2-xx'\cos\theta\,,$$
$$\left(\frac{\kappa}{m_N}\right)^2=\frac{(s/m_N^2-1-\delta^2)^2-4\delta^2}{4s/m_N^2}\equiv\xi^2\,, \quad (3.2)$$
$$\left(\frac{2p_{\pi L}}{m_N}\right)^2=\left(\frac{s}{m_N^2}\right)^2-2(1+\delta^2)\frac{s}{m_N^2}+(1-\delta^2)^2$$
$$\equiv(2z)^2\,.$$

Note that s in Eq. (3.2) corresponds to an off-shell initial pair of nucleons (i.e., bound in the nucleus with binding energy $\ll m_N$) with total momentum $\vec{K}$ and energy $2M_N$, for which $s=(2m_N-p)^2-(\vec{K}-\vec{p})^2$. The requirement that the πN collision take place above threshold is $s/m_N^2\geq1+\delta$ (or $x=p_{\pi L}=0$); this provides the limits $y(x)$ and $\cos\theta_c(x,x')$ on the x' and $\cos\theta$ integrations in Eq. (3.1). In practice, these constraints are unimportant, since for large ρ the integrand is strongly focused near $x\approx x'\approx\frac{1}{2}$, $\cos\theta\approx-1$, where we have $s/m_N^2\approx2$.

The effective cross section $\hat{\sigma}$ which appears in Eq. (3.1) is defined by

$$\hat{\sigma}=m_Np_{\pi L}\sigma^{\rm tot}_{I=3/2}(s)\,. \quad (3.3)$$

As before, $\alpha={\rm Re}\mathfrak{M}(s)/{\rm Im}\mathfrak{M}(s)$ is the ratio of real to imaginary parts of the *forward* πN amplitude at energy $\sqrt{s}$. Note that $\hat{\sigma}$ and α depend on all three integration variables. Unlike the deuteron case, the (3,3) channel contributes for ^{4}He, and provides the dominant effect.

For α and $\sigma^{\rm tot}_{I=3/2}$, we use a parametrized form for the (3,3) resonance due to Carter *et al.*[14]:

$$\hat{\sigma}=\frac{2\pi z\Gamma_0^2}{\xi^2}\frac{1}{[(E_{\rm res}-E)^2+(\Gamma_0/2)^2]}\,,$$
$$\alpha=2(E_{\rm res}-E)/\Gamma_0\,, \quad (3.4)$$

where all momenta and energies are in units of m_N, and the resonance energy $E_{\rm res}$ and width Γ_0 are

$$E_{\rm res}=1.904\delta\,,$$
$$\Gamma_0=\frac{4\xi^3a_0^3\gamma^2\delta}{(E_{\rm res}+E)(1+\xi^2a_0^2)}\,, \quad (3.5)$$

with $E=(\xi^2+\delta^2)^{1/2}$, $a_0=0.6277/\delta$, $\gamma^2=0.1709$.

Finally, $\hat{F}(x,x')$ is the dimensionless version of the function F defined in the Appendix. It is analogous to the integral (2.4) for the deuteron, being an integral over the relative coordinates of the two nucleons, but it also takes into account the motion of their center of mass. In general,

$$F(\vec{p},\vec{K})=\frac{|\vec{p}-\vec{K}/2|}{4\pi}\int d^3r\frac{e^{-iqr}}{r}e^{-i(\vec{p}-\vec{K}/2)\cdot\vec{r}}\psi(\vec{r})\,, \quad (3.6)$$

where $\psi(\vec{r})$ is the relative wave function, which in the harmonic-oscillator approximation is

$$\psi(\vec{r})=\frac{2}{(\sqrt{2}b)^{3/2}}\frac{1}{\pi^{1/4}}\frac{1}{(4\pi)^{1/2}}e^{-r^2/4b^2}\,. \quad (3.7)$$

Then

$$\hat{F}(x,x')=\frac{\sqrt{4\pi}}{b}\frac{(\sqrt{2}b)^{3/2}\pi^{1/4}}{2}F(\vec{p},\vec{K})\,,$$
$$=\frac{|\vec{p}-\vec{K}/2|}{4\pi b}\int d^3r\frac{e^{-iqr}}{r}e^{-i(\vec{p}-\vec{K}/2)\cdot\vec{r}}e^{-r^2/4b^2}\,. \quad (3.8)$$

This is most conveniently evaluated using the form

$${\rm Re}\hat{F}(x,x')=\int_0^{\gamma_1}\exp(t^2-\gamma_1^2)dt+\int_0^{\gamma_2}\exp(t^2-\gamma_2^2)dt\,,$$
$${\rm Im}\hat{F}(x,x')=\frac{\sqrt{\pi}}{2}(e^{-\gamma_2^2}-e^{-\gamma_1^2})\,, \quad (3.9)$$

when $\gamma_{1,2}=\rho\{x'\pm[(1-x)^2-\delta^2]^{1/2}\}^{1/2}$.

The three-dimensional numerical integration gives for ^{4}He,

$$\frac{\delta\Gamma_{\rm resc}}{\Gamma^{\rm free}_{^4{\rm He}}}\approx-0.022\,, \quad (3.10)$$

a slight *lengthening* of the lifetime. It is amusing to note that the integral over the c.m. momentum $\vec{K}$ of the pair affects the total rate only very slightly (although it changes the shape of the positron spectrum). If we take $\vec{K}=0$ (as for the deuteron) the rescattering correction in ^{4}He is given by

$$\frac{\delta\Gamma_{\rm resc}}{\Gamma^{\rm free}_{^4{\rm He}}}=\frac{8}{3\pi^2(2\pi)^{1/2}\rho}\int_0^{x_0}dx\,\hat\sigma\{[{\rm Re}\hat F(x,x)]^2-[{\rm Im}\hat F(x,x)]^2+2\alpha\,{\rm Re}\hat F(x,x)\,{\rm Im}\hat F(x,x)\}, \quad (3.11)$$

where $x_0=\frac{3}{4}(1-2\delta/3-\delta^2/3)$. A numerical evaluation of Eq. (3.11) yields

$$\frac{\delta\Gamma_{\rm resc}}{\Gamma^{\rm free}_{^4{\rm He}}}\approx-0.019\,, \quad (3.12)$$

to be compared with Eq. (3.10).

We now turn to the absorption correction $\delta\Gamma_{\rm abs}$. The appropriate generalization of Eq. (2.5) is

$$\frac{\delta\Gamma_{\rm abs}}{\Gamma^{\rm free}_{^4{\rm He}}}=\frac{10(g_{\pi^0NN})^2\rho^2}{\pi^2}\int_0^1 dx\,x(1-x)\int_{\alpha_1(x)}^{\alpha_2(x)}\frac{dx'}{x'}\,|\hat F(x,x')|^2\exp[-2\rho^2(x^2-2x-x'^2+\tfrac{3}{2})]\,, \quad (3.13)$$

where $\alpha_1(x)=\max\{0,[(3+x^2-4x)^{1/2}-x]/2\}$, and $\alpha_2(x)=[(3+x^2-4x)^{1/2}+x]/2$. One can obtain an approximate analytic evaluation of Eq. (3.13) by noting that the exponential is maximal for $x'=x_0'=(x^2-2x+\frac{3}{2})^{1/2}$. However, $x_0'>\alpha_2(x)$ for $0\le x\le 1$, so the peak is always outside of the domain of the x' integration. For the dominant contribution from ${\rm Re}\hat F$, we can use (for large ρ)

$$\int_{\alpha_1(x)}^{\alpha_2(x)}\frac{dx'}{x'}\,|{\rm Re}\hat F(x,x')|^2\exp[-2\rho^2(x_0'^2-x'^2)]\approx\frac{|{\rm Re}\hat F(x,\alpha_2)|^2}{\alpha}\int_{\alpha_1(x)}^{\alpha_2(x)}\exp[-2\rho^2(x_0'^2-x'^2)]dx'$$
$$\approx\frac{|{\rm Re}\hat F(x,\alpha_2)|^2}{2\rho^2\alpha_2(x_0'+\alpha_2)}e^{-2\rho^2(x_0'^2-\alpha_2^2)}\,. \quad (3.14)$$

The remaining x integration can be done by noting that $x_0'^2-\alpha_2{}^2\approx\frac{16}{9}(x-\frac{3}{4})^2+\cdots$, and hence

$$\int_0^1\exp[-2\rho^2(x_0'^2-\alpha_2{}^2)]dx\approx3(\pi/2)^{1/2}/4\rho\,.$$

We finally arrive at

$$\frac{\delta\Gamma_{\rm abs}}{\Gamma^{\rm free}_{^4{\rm He}}}=\frac{45(g_{\pi^0NN})^2}{28\pi(2\pi)^{1/2}\rho^3}, \quad (3.15)$$

for large ρ. For $b=1.367$ fm, we obtain

$$\frac{\delta\Gamma_{\rm abs}}{\Gamma^{\rm free}_{^4{\rm He}}}\approx0.127$$

from Eq. (3.15), to be compared with the exact numerical evaluation of Eq. (3.13), which gives

$$\frac{\delta\Gamma_{\rm abs}}{\Gamma^{\rm free}_{^4{\rm He}}}\approx0.12\,. \quad (3.16)$$

Note that $\delta\Gamma\sim\rho^{-3}\sim(\text{volume})^{-1}$, as must be the case, since as b increases, one is putting only *four* particles into an increasingly larger volume.

By examination of the analytic form of the integrand in Eq. (3.1), in particular, the fact that it is proportional to ${\rm Im}(\mathfrak{M}F^2)$, one can also show that $\delta\Gamma_{\rm resc}/\Gamma^{\rm free}\propto1/\rho^3$ for large ρ. Individual terms in the expression vary as $1/\rho^2$ and the important cancellation of this behavior results from including the interference between the free decay and the forward elastic scattering. Were it not for this term we would obtain ${\rm Im}(\mathfrak{M}|F|^2)$ instead. This latter form is incorrect in principle, because it includes the effect of *real* mesons which are already accounted for in $\Gamma_{\rm free}$. The signal for real mesons is $1/\rho^2$ behavior; virtual mesons are of fixed range and so as the wave function becomes more extended the rate must fall as the density, proportional to $1/\rho^3$. Thus, this cancellation provides a check that we have not double counted real emission. We cannot give the analog of (3.15) in general, but in the approximation of (3.11) with $\hat\sigma$ taken to be constant, evaluated at $x=\frac{1}{2}$, we obtain for large ρ

$$\frac{\delta\Gamma_{\rm resc}}{\Gamma^{\rm free}_{^4{\rm He}}}\approx-\frac{2\hat\sigma(m_N/2)}{\pi^2(2\pi)^{1/2}\rho^3}\,. \quad (3.17)$$

Using the expression (3.4) for $\hat\sigma$ leads to

$$\frac{\delta\Gamma_{\rm resc}}{\Gamma^{\rm free}_{^4{\rm He}}}\approx-0.013\,, \quad (3.18)$$

in reasonable agreement with the result (3.10).

The results (3.10) and (3.16) refer only to the change in *total* decay rate. An even more interesting quantity is the change in the e^+ *momentum spectrum* arising from rescattering and absorption. First of all, the total free-space decay rate of ^{4}He is

$$\Gamma^{\rm free}_{^4{\rm He}}\approx2\Gamma_n+2\Gamma_p\approx3\lambda^2m_N/8\pi\,. \quad (3.19)$$

If we consider the nucleons in ^{4}He to have a distribution of momenta $\vec{K}$, and decaying into an e^+ of momentum $\vec{p}$ and a pion of momentum $\vec{K}-\vec{p}$, we can write

$$\Gamma^{\rm free}_{^4{\rm He}}=\frac{3\lambda^2}{(2\pi)^3}\frac{2b^3}{\sqrt{\pi}}\times\int d^3p\int d^3K\,e^{-K^2b^2}\delta(m_N{}^2-2m_Np+2pKz-\mu_\pi{}^2)\,, \quad (3.20)$$

where $z = \cos\theta_{\vec{K},\vec{p}}$. This can be cast into the form

$$\Gamma^{\rm free}_{^4{\rm He}} = \frac{3\lambda^2 \rho m_N}{2\pi^{3/2}} \int_0^\infty x\,dx \exp[-\rho^2(1-2x-\delta^2)^2/4x^2]\,. \quad (3.21)$$

We now define a spectrum function $S_{\rm free}(x)$ in terms of the integrand in Eq. (3.21):

$$S_{\rm free}(x) = \frac{4\rho}{\sqrt{\pi}} x \exp\left[-\rho^2\left(\frac{1-2x-\delta^2}{2x}\right)^2\right], \quad (3.22)$$

normalized such that $\int_0^\infty S_{\rm free}(x) \approx 1$. We can then compare $S_{\rm free}(x)$ with the modified spectrum $S(x)$ given by

$$S(x) = S_{\rm free}(x) + S_{\rm abs}(x) + S_{\rm resc}(x)\,, \quad (3.23)$$

where $S_{\rm abs}(x)$ and $S_{\rm resc}(x)$ are defined by

$$\frac{\delta\Gamma_{\rm abs}}{\Gamma^{\rm free}_{^4{\rm He}}} = \int dx\, S_{\rm abs}(x)\,, \qquad \frac{\delta\Gamma_{\rm resc}}{\Gamma^{\rm free}_{^4{\rm He}}} = \int dx\, S_{\rm resc}(x)\,. \quad (3.24)$$

The modifications $S_{\rm abs}$ and $S_{\rm resc}$ due to absorption and rescattering are shown separately in Fig. 3. The effect of rescattering is to deplete the positron spectrum for some momenta (most strongly for $x \approx 0.47$) and enhance it at higher momenta ($x \approx 0.57$). This shifts the main peak in the e^+ spectrum (at $x \approx \frac{1}{2}$) to slightly higher momentum and diminishes the peak height a bit, as shown in Fig. 4. The effect is rather insignificant, however. More interesting is the effect of absorption, which produces an additional peak near $x \approx \frac{3}{4}$, where it would occur if $\vec{K} = 0$ for the initial nucleon pair. The peak is broadened due to our inclusion of the average $\int d^3K$ over the c.m. momentum. As shown in Fig. 4, this pion absorption peak is still visible in the total spectrum, since it is kinematically quite distinguishable from the free-decay peak, as modified by rescattering. Unlike the deuteron case, in ^{4}He one can obtain correlated pairs of *charged* particles from the process $pp \to e^+p$, as well as fast neutrons from $pn \to e^+n$.

So far, the numerical calculations refer to ^{4}He. The formulas in the Appendix are applicable to larger nuclei, such as the interesting case of ^{16}O, but they are quite tedious to apply. Naively, the independent particle approximation we have used suggests that $\delta\Gamma$ goes as the number of pairs, i.e., $A(A-1)$. Clearly, this is oversimplified and will be much reduced by orthogonality of the wave functions involved. It is rather easy to see that, for large nuclei, the Fermi-gas model leads to $\delta\Gamma \propto V(k_F)^6 \propto A^2/V$, where V is the nuclear volume. Thus, $\delta\Gamma/\Gamma_{\rm free}$ is proportional to the density. This is a very plausible result and will lead to only a moderate increase in rate in heavier nuclei. Another model in which the pion rescatters coherently on the whole nucleus would give $\delta\Gamma_{\rm resc} \sim (A-1)^{2/3}$. The pion cross section on the

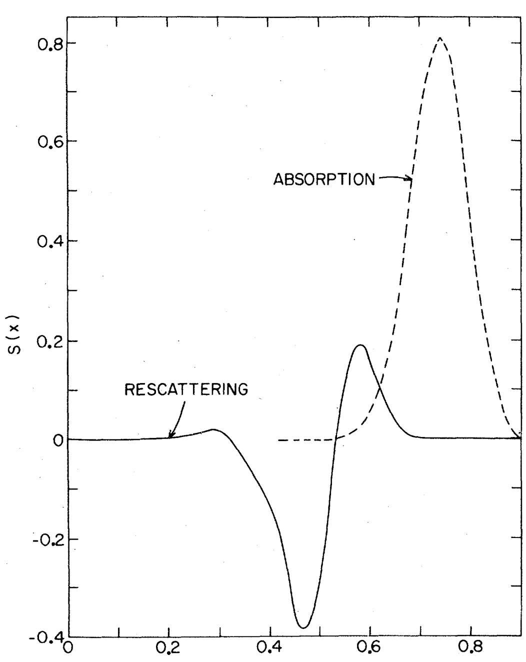

FIG. 3. Pion absorption and rescattering contributions to the positron spectrum $S(x)$. The dashed curve corresponds to $S_{\rm abs}(x)$, which is everywhere positive and peaks near $x = p/m_N \approx \frac{3}{4}$. The solid curve is the rescattering contribution $S_{\rm resc}(x)$, which alternates in sign, causing a shift of the main peak of the positron spectrum to higher momentum.

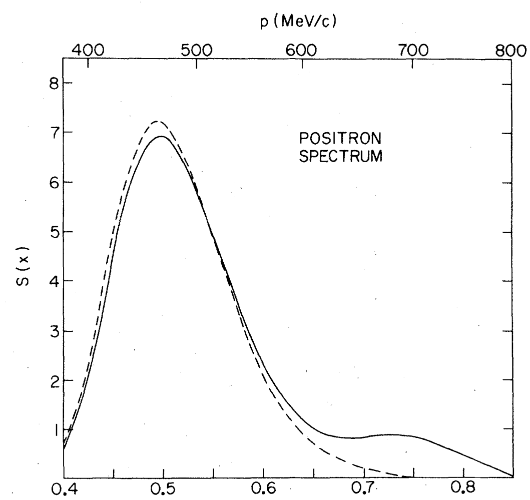

FIG. 4. The positron spectrum $S(x)$ of Eq. (3.23). The dashed line is the free spectrum $S_{\rm resc}(x)$ of Eq. (3.22). The solid curve includes the effects of pion rescattering and absorption, the latter producing the small peak at higher moments.

spectator nucleus is of the order of the geometrical limit $2\pi R^2$, where $R=\sqrt{5/3}\langle r^2\rangle_{A-1}{}^{1/2}$. Very roughly, the ratio of $\delta\Gamma_{\rm resc}$ in the IPA to that in the coherent model is

$$(4\times 2\pi R^2/4\times 3)\sigma_{\pi N}^{\rm tot}(s=2m_N{}^2)\approx 1$$

for ^{4}He. Thus, the qualitative size of the rescattering effect is about the same in these two (very different) models. We do not anticipate any dramatic enhancement of $\delta\Gamma_{\rm resc}$ due to multiparticle nuclear effects.

For $\delta\Gamma_{\rm abs}$, we have included only the one-nucleon absorption mechanism for the pion. This will give the sharpest experimental signature [a peak in $S(x)$] and is hence perhaps the most interesting absorption process. However, for *real* pions, the absorption on a correlated pair (for instance $\pi^- np\to nn$) dominates the one-nucleon mechanism. In light nuclei, one sees back-to-back neutron pairs from π^- absorption as a prominent mode.[15] In heavier nuclei, one gets even more complicated multiparticle emission processes. Here, the situation is quite different, since a *virtual* pion is produced *inside* the nucleus by nucleon decay. The balance of momentum transfer q and energy transfer ω, which tends to favor two-nucleon over one-nucleon absorption for a real pion, is altered for a virtual pion; for the latter, the one-nucleon mechanism is relatively more important.

ACKNOWLEDGMENT

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APPENDIX

We will describe here more completely the calculations outlined in Secs. II and III. The various approximations used will be introduced as we go along. We will restrict our attention to the π modes of decay partly because we expect these to show the largest nuclear effects because of the small π mass, partly because the other modes involve more problematical quantities (such as the ω-nucleon amplitude) and partly to keep the presentation as simple as possible. Other modes can be included by minor changes in the calculation described below. We will discuss the modes involving positron emission. The modes involving ν or μ emission can clearly be discussed in a similar fashion.

The effective Hamiltonian we will use is

$$\mathcal{H}(x)=\lambda_p\bar{e}(x)p(x)\phi^0(x)+\lambda_n\bar{e}(x)n(x)\phi^+(x)\,, \qquad \text{(A1)}$$

$p(x)$, $n(x)$, and $e(x)$ denote the proton, neutron, and *positron* fields while $\phi^i(x)$ denote the π fields. Ultimately we will choose the SU(5) values $\lambda_n=\sqrt{2}\lambda_p$ but for now we will leave them unrelated. (The coupling could have the form $a+ib\gamma_5$ but this will lead to the same result in the approximations we will make.)

Our nucleus is taken to be Z protons and N neutrons in independent-shell-model (spatial and spin) states $l_1\lambda_1\cdots l_Z\lambda_Z$ and $m_1\mu_1\cdots m_N\mu_N$. The transition to a state consisting of a positron of momentum $\vec{p}$ and helicity λ along with a nucleus in various states of disarray and possibly one or more mesons, which we denote collectively by α, is then determined by the matrix element

$$\begin{aligned}\langle\vec{p}\lambda\alpha|\mathcal{H}(x)|l_1\lambda_1\cdots l_Z\lambda_Z;m_1\mu_1\cdots m_N\mu_N\rangle \\ =\frac{e^{-i\vec{p}\cdot\vec{x}}}{[(2\pi)^3]^{1/2}}\left(\frac{m}{E_p}\right)^{1/2}\sum_r[\lambda_p\bar{u}_\lambda(p)\psi_{l_r\lambda_r}(x)(-1)^{r-1}\langle\alpha|\phi^0(x)|l_1\lambda_1\cdots\hat{l}_r\hat{\lambda}_r\cdots l_Z\lambda_Z;m_1\mu_1\cdots m_N\mu_N\rangle \\ +\lambda_n\bar{u}_\lambda(p)\psi_{m_r\mu_r}(x)(-1)^{Z+r-1}\langle\alpha|\phi^+(x)|l_1\lambda_1\cdots l_Z\lambda_Z;m_1\mu_1\cdots\hat{m}_r\hat{\mu}_r\cdots m_N\mu_N\rangle]\,. \qquad \text{(A2)}\end{aligned}$$

The caret indicates which single-particle state has been vacated by the decay; the factors $(-1)^{r-1}$ and $(-1)^{Z+r-1}$ come from commuting the corresponding operator to the left. (Evidently some ordering convention is understood, such as $E_{l_1}<E_{l_2}$, but that will not enter into our results.) m is the positron mass and E_p its energy, which we henceforth approximate by p, its momentum. $u_\lambda(p)$ is the standard fermion spinor for the positron and $\psi_{l\lambda}(x)$, $\psi_{m\mu}(x)$ are the Dirac spinors for the proton and neutron in the corresponding shell model state. Of course, the nonrelativistic approximation will be used for them so that

$$\psi_{l\lambda}(x)\approx\psi_l(x)\begin{pmatrix}\chi_\lambda\\0\end{pmatrix},$$

where χ_λ is the usual Pauli spinor for spin projection λ and $\psi_l(x)$ is just the shell-model spatial wave function.

The π fields $\phi(x)$ can be written in the form most appropriate to our boundary conditions.

$$\phi(x)=\phi^{\rm out}(x)+\int d^4y\,\Delta_{\rm adv}(x-y)j(y)\,, \qquad \text{(A3)}$$

where $\phi^{\rm out}(x)$ is a free field and

$\Delta_{adv}(x)=0$ for $x_0>0$.

The first term in (A3) gives the free decay of the nucleon into πe^+ while the second term leads to additional interaction with the remaining nucleons.

We will write the rate of decay of the nucleon as the sum of three terms,

$$\Gamma=\Gamma_{free}+\delta\Gamma_{abs}+\delta\Gamma_{resc}\ . \qquad (A4)$$

Γ_{free} will be the rate given by the first term of (A3) for an independent collection of Z protons and N nucleons and so is Z times the free-proton rate plus N times the free-neutron rate. Γ_{abs} is the contribution to the decay which results from the absorption of the (virtual) decay pion by another nucleon leading to a final state with no mesons. Γ_{resc} results from the rescattering of virtual or real pions from the residual nucleus. When the pion is real and the scattering is elastic, the final state is indistinguishable from the free decay and so those two processes can interfere. It is very important to take this into account to get a sensible physical result. As a result, $\delta\Gamma_{resc}$ can be positive or negative. It is convenient to write it as $\delta\Gamma_2+\delta\Gamma_{int}$, where $\delta\Gamma_2$ is a square and so positive while $\delta\Gamma_{int}$ represents the interference and may have either sign.

For completeness we repeat here the free rate:

$$\Gamma_{free}=\frac{m_N}{16\pi}\left(\frac{m_N{}^2-\mu_\pi{}^2}{m_N{}^2}\right)^2(\lambda_p{}^2Z+\lambda_n{}^2N)\ . \qquad (A5)$$

m_N denotes the nucleon mass and μ_π denotes the π mass.

Next we calculate Γ_{abs}. We assume this to be a two-nucleon process. Thus, although the second nucleon may scatter and lose energy to other nucleons after absorbing the π, we are assuming that this has a minor effect on the total rate (and the e^+ spectrum) though it surely will modify the nucleon spectrum. So the state $|\alpha\rangle$ will consist of a single free nucleon, proton, or neutron, with momentum $\vec{k}_f$ and helicity μ while the remaining nucleons remain in their initial state. Because of our nonrelativistic approximation for the initial nucleons the relevant matrix element is given by

$$\langle\vec{k}_f\mu\,|\,j^i(y)\,|\,l_r\lambda_r\rangle=ig\left(\frac{m_N}{E_f}\right)^{1/2}\frac{e^{-i\vec{k}_f\cdot\vec{y}}}{(2\pi)^{3/2}}\,e^{i(E_f-m_N)y_0}\bar{u}_\mu(k_f)\,\gamma_5\tau^i\psi_{l_r\lambda_r}(y)\ . \qquad (A6)$$

i is here the charge index of the π field and the nucleon charges are understood. The rate, summed on the outgoing positron spins, is then

$$\begin{aligned}\delta\Gamma_{abs}(2p)=g^2\int\frac{d^3p}{(2\pi)^3}\frac{m}{p}\int d^3x\int d^4y\int d^3x'\int d^4y'\int\frac{d^3k_f}{(2\pi)^3}\frac{m_N}{E_f}(2\pi)\delta(p+E_f-2m_N)\,e^{-i\vec{p}\cdot(\vec{x}-\vec{x}')}\\ \times\sum_{r,s}\lambda_p{}^2\Big[\psi^*_{l_r\lambda_r}(x')\frac{\not{p}}{2m}\psi_{l_r\lambda_r}(x)\psi^*_{l_s\lambda_s}(y')\,i\gamma_5u_\mu(k_f)\quad\bar{u}_\mu(k_f)\,i\gamma_5\psi_{l_s\lambda_s}(y)\\ -\psi^*_{l_s\lambda_s}(x')\frac{\not{p}}{2m}\psi_{l_r\lambda_r}(x)\psi^*_{l_r\lambda_r}(y')\,i\gamma_5u_\mu(k_f)\quad\bar{u}_\mu(k_f)\,i\gamma_5\psi_{l_s\lambda_s}(y)\Big]\\ \times e^{-i\vec{k}_f\cdot(\vec{y}-\vec{y}')}\Delta_{adv}(x-y)\Delta^*_{adv}(x'-y')\,e^{i(E_f-m_N)y_0}e^{-i(E_f-m_N)y_0'}\end{aligned} \qquad (A7)$$

for the case in which two protons are involved. The cases for two neutrons or one proton and one neutron, which are incoherent with this, are obtained by simple modification of this.

Because of our nonrelativistic approximation the first term in square brackets is

$$\sum_{\lambda_r\lambda_s}\left[\psi_{l_r}(x')\psi_{l_r}(x)\left(\frac{p}{2m}\right)\delta_{\lambda_r\lambda_r}\right]\left[\psi_{l_s}(y')\psi_{l_s}(y)\left(\frac{E_f-m_N}{2m_N}\right)\delta_{\lambda_s\lambda_s}\right]=\frac{p(E_f-m_N)}{mm_N}\psi^*_{l_r}(x')\psi_{l_r}(x)\psi^*_{l_s}(y')\psi_{l_s}(y)\ .$$

The second term is slightly different, namely,

$$\sum_{\lambda_r\lambda_s}\psi^*_{l_s}(x')\psi_{l_r}(x)\psi^*_{l_r}(y')\psi_{l_s}(x')\left(\frac{p}{2m}\right)\delta_{\lambda_r\lambda_s}\delta_{\lambda_r\lambda_s}\frac{E_f-m_N}{2m_N}=\frac{1}{2}\,\frac{p(E_f-m_N)}{mm_N}\,\psi^*_{l_s}(x')\psi_{l_r}(x)\psi^*_{l_r}(y')\psi_{l_s}(y)\ .$$

Note the important factor-of-$\frac{1}{2}$ difference. This is because the interference term occurs only when the π is absorbed by a nucleon in the same spin state as the decaying nucleon while it can also be absorbed in a nucleon in the other state. Evidently, we have assumed here in doing the spin sum that both spin states of any orbital state are populated. This is appropriate for the nuclei we are interested in and we will continue to assume so. (Similar results are obtained on summing over states of unpolarized nuclei.)

The time integrals may be done

$$\int dy_0 \Delta_{adv}(x-y)\, e^{i(E_f-m_N)y_0} = \frac{1}{4\pi}\,\frac{e^{-iq|\vec{x}-\vec{y}|}}{|\vec{x}-\vec{y}|}\,, \tag{A8}$$

where

$$q=[(E_f-m_N)^2-\mu_\pi^2]^{1/2}=[(p-m_N)^2-\mu_\pi^2]^{1/2}\,, \tag{A9}$$

is the momentum the π would have were it on shell. In fact, its momentum is $-\vec{p}$ added to the Fermi momentum of the decaying proton and the difference between these is crucial to the size of the effect.

Making these replacements in (A7) yields

$$\delta\Gamma_{abs}(2p)=g^2\frac{\lambda_p^2}{8\pi}\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3k_f}{(2\pi)^3}\left(\frac{E_f-m_N}{m_N}\right)\delta(p+E_f-2m_N)$$
$$\times\sum_{r,s}\left[\int d^3x\int d^3y\,\frac{e^{-iq|\vec{x}-\vec{y}|}}{|\vec{x}-\vec{y}|}\,e^{-i\vec{p}\cdot\vec{x}-i\vec{k}_f\cdot\vec{y}}\,\psi_{l_s}(y)\psi_{l_r}(x)\right]$$
$$\times\left\{\int d^3x'\int d^3y'\,\frac{e^{iq|\vec{x}'-\vec{y}'|}}{|\vec{x}'-\vec{y}'|}\,e^{i\vec{p}'\cdot\vec{x}'+i\vec{k}_f\cdot\vec{y}'}\,[2\psi^*_{l_s}(y')\psi^*_{l_r}(x')-\psi^*_{l_r}(y')\psi^*_{l_s}(x')]\right\}. \tag{A10}$$

There is no analogous contribution for two neutrons because the π^- emitted in the neutron decay cannot be absorbed by the other neutron. The neutron-proton contribution can easily be related to this by examining Figs. 1(e) and 1(f). (Remember that the π^- coupling is $\sqrt{2}$ times the $\pi^0 n$ coupling.)

$$\delta\Gamma_{abs}(np)=\frac{g^2}{8\pi}\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3k_f}{(2\pi)^3}\left(\frac{E_f-m_N}{m_N}\right)\delta(p+E_f-2m_N)$$
$$\times\sum_{r,s}\left[\int d^3x\int d^3y\,\frac{e^{-iq|\vec{x}-\vec{y}|}}{|\vec{x}-\vec{y}|}\,e^{-i\vec{p}\cdot\vec{x}-i\vec{k}_f\cdot\vec{y}}\,\psi_{l_s}(y)\psi_{m_r}(x)\right]$$
$$\times\left\{\int d^3x'\int d^3y'\,\frac{e^{iq|\vec{x}'-\vec{y}'|}}{|\vec{x}'-\vec{y}'|}\,e^{i\vec{p}\cdot\vec{x}'+i\vec{k}_f\cdot\vec{y}'}[4\lambda_n^2\psi^*_{l_s}(y')\psi^*_{m_r}(x')+\sqrt{2}\lambda_p\lambda_n\psi^*_{m_r}(y')\psi^*_{l_s}(x')]\right\}$$
$$+\left[\int d^3x\int d^3y\,\frac{e^{-iq|\vec{x}-\vec{y}|}}{|\vec{x}-\vec{y}|}\,e^{-i\vec{p}\cdot\vec{x}-i\vec{k}_f\cdot\vec{y}}\,\psi_{m_s}(y)\psi_{l_r}(x)\right]$$
$$\times\left\{\int d^3x'\int d^3y'\,\frac{e^{iq|\vec{x}'-\vec{y}'|}}{|\vec{x}'-\vec{y}'|}\,e^{i\vec{p}\cdot\vec{x}'+i\vec{k}_f\cdot\vec{y}'}[2\lambda_p^2\psi^*_{m_s}(y')\psi^*_{l_r}(x')+\sqrt{2}\lambda_p\lambda_n\psi^*_{l_r}(y')\psi^*_{m_s}(x')]\right\}. \tag{A11}$$

Turning now to Γ_2, instead of (A6) we are led to the matrix element $\langle R|j^i(y)|l_r\lambda_r\rangle$, where R denotes the coordinates of the nucleon and meson that result from the scattering of the decay π (real or virtual) on a nucleon in state $l_r\lambda_r$. Γ_2 will involve the sum over all of these coordinates constrained only by the δ function of energy conservation. In order to do the integrals it will be useful to Fourier analyze the initial state:

$$\langle R|j(\vec{y})|l_r\lambda_r\rangle=\int d^3k\,\tilde{\psi}_{l_r}(k)\langle R|j(\vec{y})|\vec{k}\lambda_r\rangle=\int d^3k\,e^{-i(\vec{Q}_R-\vec{k})\cdot\vec{y}}\tilde{\psi}_{l_r}(k)\langle R|j(0)|\vec{k}\lambda_r\rangle\,,$$

where $\tilde{\psi}_{l_r}(k)$ is just the Fourier transform of $\psi_{l_r}(r)$. The sum on final states leads then to integrals of the form

$$\int d^3K\int_R(2\pi)^4\delta^{(3)}(\vec{Q}_R+\vec{p}-\vec{K})\delta(E_R+p-2m_N)\langle R|j(0)|\vec{k}'\lambda_r\rangle^*\langle R|j(0)|\vec{k}\lambda_r\rangle$$
$$=-\frac{1}{2im_N}\int\frac{d^3K}{(2\pi)^3}[\mathfrak{M}_{\lambda_r\lambda_r}(\vec{K}-\vec{p}-\vec{k}',\vec{k}';\vec{K}-\vec{p}-\vec{k},\vec{k})-\mathfrak{M}_{\lambda_r\lambda_r}(\vec{K}-\vec{p}-\vec{k},\vec{k};\vec{K}-\vec{p}-\vec{k}',\vec{k}')^*] \tag{A12}$$

by unitarity. $\mathfrak{M}$ denotes the π-nucleon amplitude (any of $\pi^0p\to\pi^0p$, $\pi^0p\to\pi^+n$, $\pi^0n\to\pi^-p$, and so on as the case may be).

We make the following reasonable approximations: we neglect the dependence on the mass of the off-shell π leg; we neglect the relativistic transformations of the initial frame because of the small initial momentum so that $\langle R|j|k\lambda_r\rangle$ is interpreted as a laboratory-frame πN amplitude, but we do keep the energy variation. $\vec{K}$ obviously is the sum of the momenta of the two initial nucleons participating in the process. Finally, while we will retain the dependence on energy (or equivalently $\vec{K}$) we will neglect the variation with $\vec{k}$ and $\vec{k}'$ separately. This amounts to neglecting, in addition to the off-shell behavior already dropped, the angular variation of $\mathfrak{M}$. This is reasonable because the dominant contribution comes for rather-high-momentum π's so the angle does not change very rapidly as we integrate over the Fermi momentum. Altogether this leads to the replacement in (A12):

$$\mathfrak{M}_{\lambda_r\lambda_r}(\vec{K}-\vec{p}-\vec{k}', \vec{k}'; \vec{K}-\vec{p}-\vec{k}, \vec{k}) \to \mathfrak{M}_{\lambda_r\lambda_r}(\hat{s}),$$

where $\mathfrak{M}_{\lambda_r\lambda_r}(\hat{s})$ denotes the forward amplitude and

$$\hat{s} = (2m_N - p)^2 - (\vec{K}-\vec{p})^2 . \tag{A13}$$

The normalization of $\mathfrak{M}$ is

$$\mathrm{Im}\,\mathfrak{M}_{\lambda_r\lambda_r}(\hat{s}) = -[\hat{s}^2 - 2\hat{s}(m_N^2+\mu_\pi^2) + (m_N^2-\mu_\pi^2)^2]^{1/2}\sigma_{\mathrm{tot}}^{\lambda_r}(\hat{s}) . \tag{A14}$$

Using these results we obtain for the proton-proton contributions to $\delta\Gamma_2$ [the positron and decaying proton spin sums are done as in going from (A7) to (A10)].

$$\begin{aligned}\delta\Gamma_2(pp) = {}& \frac{\lambda_n^2}{8\pi}\int\frac{d^3p}{(2\pi)^3}\int d^3x\int d^3y\int d^3x'\int d^3y'\int d^3k\int d^3k'\int_R \delta(E_R+p-2m_N)\,e^{-i\vec{p}\cdot(\vec{x}-\vec{x}')}e^{-i(\vec{Q}_R-\vec{k})\cdot\vec{y}} \\ & \times e^{i(\vec{Q}_R-\vec{k}')\cdot\vec{y}'}\frac{e^{-iq|\vec{x}-\vec{y}|}}{|\vec{x}-\vec{y}|}\frac{e^{iq|\vec{x}'-\vec{y}'|}}{|\vec{x}'-\vec{y}'|} \\ & \times\sum_{\lambda_s, l_r, l_s}\langle R|j_0|k',\lambda_s\rangle\langle R|j_0|k,\lambda_s\rangle\,[\psi^*_{l_r}(x')\psi_{l_r}(x)\tilde{\psi}^*_{l_s}(k')\tilde{\psi}_{l_s}(k) - \tfrac{1}{2}\psi^*_{l_s}(x')\psi_{l_r}(x)\tilde{\psi}^*_{l_r}(k')\tilde{\psi}_{l_s}(k)] \\ = {}& -\frac{\lambda_n^2}{16\pi^2 m_N}\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3K}{(2\pi)^3}\sum_{\lambda_s, l_r, l_s}\mathrm{Im}\,\mathfrak{M}_{\lambda_s\lambda_s}(\hat{s})\int d^3x\int d^3y\int d^3x'\int d^3y'\,e^{-i\vec{p}\cdot(\vec{x}-\vec{x}')}e^{-i\vec{K}\cdot(\vec{y}-\vec{y}')} \\ & \times\frac{e^{-iq|\vec{x}-\vec{y}|}}{|\vec{x}-\vec{y}|}\frac{e^{iq|\vec{x}'-\vec{y}'|}}{|\vec{x}'-\vec{y}'|}[\psi^*_{l_r}(x')\psi_{l_r}(x)\psi^*_{l_s}(y')\psi_{l_s}(y) \\ & - \tfrac{1}{2}\psi^*_{l_s}(x')\psi_{l_r}(x)\psi^*_{l_r}(y')\psi_{l_s}(y)] .\end{aligned} \tag{A15}$$

The amplitude $\mathfrak{M}$ entering here is for $\pi^0p\to\pi^0p$. When we consider the other contributions to the $\delta\Gamma_2$ amplitude for $\pi^0n\to\pi^-p$, $\pi^0n\to\pi^0n$, etc., will enter. The whole thing is slightly messy but we write it out for convenience:

$$\begin{aligned}\delta\Gamma_2 = {}& -\frac{1}{16\pi^2 m_N}\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3K}{(2\pi)^3}\int d^3x\int d^3y\int d^3x'\int d^3y'\,e^{-i\vec{p}\cdot(\vec{x}-\vec{x}')}e^{-i\vec{K}\cdot(\vec{y}-\vec{y}')}\frac{e^{-iq|\vec{x}-\vec{y}|}}{|\vec{x}-\vec{y}|}\frac{e^{+iq|\vec{x}'-\vec{y}'|}}{|\vec{x}'-\vec{y}'|} \\ & \times\sum_{\substack{\lambda_s, l_r, l_s,\\ m_r, m_s}}\{\lambda_p^2\mathfrak{M}_{\lambda_s\lambda_s}(\pi^0p\to\pi^0p)[\psi^*_{l_r}(x')\psi_{l_r}(x)\psi^*_{l_s}(y')\psi_{l_s}(y) - \tfrac{1}{2}\psi^*_{l_s}(x')\psi_{l_r}(x)\psi^*_{l_r}(y')\psi_{l_s}(y)] \\ & + \lambda_n^2\mathfrak{M}_{\lambda_s\lambda_s}(\pi^-n\to\pi^-n)[\psi^*_{m_r}(x')\psi_{m_r}(x)\psi^*_{m_s}(y')\psi_{m_s}(y) - \tfrac{1}{2}\psi^*_{m_s}(x')\psi_{m_r}(x)\psi^*_{m_r}(y')\psi_{m_s}(y)] \\ & + \lambda_n^2\mathfrak{M}_{\lambda_s\lambda_s}(\pi^0p\to\pi^0n)\psi^*_{l_r}(x')\psi_{l_r}(x)\psi^*_{m_s}(y')\psi_{m_s}(y) + \lambda_n^2\mathfrak{M}_{\lambda_s\lambda_s}(\pi^-p\to\pi^-p)\psi^*_{m_s}(x')\psi_{m_s}(x)\psi^*_{l_r}(y')\psi_{l_r}(y) \\ & - \tfrac{1}{2}\lambda_p\lambda_n\mathfrak{M}_{\lambda_s\lambda_s}(\pi^-p\to\pi^0n)\psi^*_{l_r}(x')\psi_{m_s}(x)\psi^*_{m_s}(y')\psi_{l_r}(y) \\ & - \tfrac{1}{2}\lambda_p\lambda_n\mathfrak{M}_{\lambda_s\lambda_s}(\pi^0n\to\pi^-p)\psi^*_{m_s}(x')\psi_{l_r}(x)\psi^*_{l_r}(y')\psi_{m_s}(y)\} .\end{aligned} \tag{A16}$$

Finally, we calculate $\delta\Gamma_{\mathrm{int}}$. Again begin from the pp case. This is given by

$$\begin{aligned}\delta\Gamma_{\mathrm{int}}(pp) = {}& \frac{\lambda p^2}{2}\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3q}{[(2\pi)^3 2\omega_q]^{1/2}}\int d^3x\int d^3x'\int d^3y'\,e^{-i\vec{p}\cdot(\vec{x}-\vec{x}')}\delta(p+\omega_q-m_N) \\ & \times\sum_{r,s}\Bigg[\frac{e^{iq|\vec{x}'-\vec{y}'|}}{|\vec{x}'-\vec{y}'|}e^{-i\vec{q}\cdot\vec{x}}[\psi^*_{l_r}(x')\psi_{l_r}(x)\langle\vec{q}l_s\lambda_s|j^0(y')|l_s\lambda_s\rangle^* \\ & - \tfrac{1}{2}\psi^*_{l_s}(x')\psi_{l_r}(x)\langle\vec{q}l_s\lambda_s|j^0(y')|l_r\lambda_s\rangle^*]\Bigg] \\ & + \Bigg[\frac{e^{-iq|\vec{x}-\vec{y}'|}}{|\vec{x}-\vec{y}'|}e^{i\vec{q}\cdot\vec{x}'}(\psi^*_{l_r}(x')\psi_{l_r}(x)\langle\vec{q}l_s\lambda_s|j^0(y')|l_s\lambda_s\rangle \\ & - \tfrac{1}{2}\psi^*_{l_s}(x')\psi_{l_r}(x)\langle\vec{q}l_r\lambda_s|j^0(y')|l_s\lambda_s\rangle)\Bigg] ,\end{aligned} \tag{A17}$$

where $\vec{q}$ is the outgoing π^0 momentum. We will make the same approximation on the matrix element as before. Thus when we introduce the Fourier components we will approximate

$$\langle \vec{q}, k'\lambda_s | j^0(0) | k\lambda_s \rangle = \int d^3K \delta^{(3)}(\vec{k}' + \vec{q} + \vec{p} - \vec{K}) \langle \vec{q}, \vec{K} - \vec{q} - \vec{p}, \lambda_s | j^0(0) | k\lambda_s \rangle$$
$$\approx \int d^3K \delta^{(3)}(\vec{k}' + \vec{q} + \vec{p} - \vec{K}) \frac{(2\pi)^{3/2}}{(2\omega_q)^{1/2}} \frac{\mathfrak{M}_{\lambda_s\lambda_s}(\hat{s})}{(2\pi)^6 2m_N}.$$

We then have the integrals of the form

$$\int d^3q \int d^3k' \int d^3k \tilde{\psi}^*_{I_s}(k') \tilde{\psi}_{I_r}(k) \delta^{(3)}(\vec{k}' + \vec{q} + \vec{p} - \vec{K}) \delta(p + \omega_q - m_N) e^{i\vec{y}' \cdot (\vec{k} - \vec{K} + \vec{p})} e^{i\vec{q} \cdot \vec{x}}$$
$$= (2\pi)^{3/2} \psi_{I_r}(y') \int d^3q \, \tilde{\psi}^*_{I_s}(\vec{K} - \vec{p} - \vec{q}) \delta(p + \omega_q - m_N) e^{-i\vec{y}' \cdot (\vec{K} - \vec{p})} e^{i\vec{q} \cdot \vec{x}}$$
$$= \psi_{I_r}(y') e^{-i\vec{y}' \cdot (\vec{K} - \vec{p})} \int d^3y \int d^3q \, e^{i(\vec{K} - \vec{p} - \vec{q}) \cdot \vec{y}} \psi^*_{I_s}(y) \delta(p + \omega_q - m_N) e^{i\vec{q} \cdot \vec{x}}$$
$$= \int d^3y \, e^{i(\vec{K} - \vec{p}) \cdot (\vec{y} - \vec{y}')} \psi_{I_r}(y') \psi^*_{I_s}(y) 4\pi\omega_q \frac{\sin q |\vec{x} - \vec{y}|}{|\vec{x} - \vec{y}|}.$$

Assembling all of this into (A17) yields

$$\delta\Gamma_{\text{int}}(pp) = \frac{\lambda_p^2}{16\pi^2 m_N} \int \frac{d^3p}{(2\pi)^3} \int \frac{d^3K}{(2\pi)^3} \int d^3x \int d^3y \int d^3x' \int d^3y' e^{-i\vec{p} \cdot (\vec{x} - \vec{x}')}$$
$$\times \sum_{r,s} \left[\frac{e^{iq|\vec{x}' - \vec{y}'|}}{|\vec{x}' - \vec{y}'|} \frac{\sin q |\vec{x} - \vec{y}|}{|\vec{x} - \vec{y}|} \mathfrak{M}^*_{\lambda_s\lambda_s}(\hat{s}) [\psi^*_{I_r}(x') \psi_{I_r}(x) \psi^*_{I_s}(y') \psi_{I_s}(y) - \tfrac{1}{2} \psi^*_{I_s}(x') \psi_{I_r}(x) \psi^*_{I_r}(y') \psi_{I_s}(y)] \right.$$
$$\left. + \frac{e^{-iq|\vec{x} - \vec{y}|}}{|\vec{x} - \vec{y}|} \frac{\sin q |\vec{x}' - \vec{y}'|}{|\vec{x}' - \vec{y}'|} \mathfrak{M}_{\lambda_s\lambda_s}(\hat{s}) [\psi^*_{I_r}(x') \psi_{I_r}(x) \psi^*_{I_s}(y') \psi_{I_s}(y) - \tfrac{1}{2} \psi^*_{I_s}(x') \psi_{I_r}(x) \psi^*_{I_r}(y') \psi_{I_s}(y)] \right]. \tag{A18}$$

The inclusion of nn and np initial states obviously will combine in the same way in going from (A15) to (A16) and we will not write the complete result down.

Let us now consider explicitly ^{4}He with harmonic-oscillator wave functions, the same for proton and neutron. In addition, let us take $\lambda_n^2 = 2\lambda_p^2$. Since there are then only s states, this all collapses to a quite simple form,

$$\delta\Gamma_2 + \delta\Gamma_{\text{int}} = -\frac{\pi\lambda_p^2}{4m_N} \int \frac{d^3p}{(2\pi)^3} \int \frac{d^3K}{(2\pi)^3} \frac{16\pi^2}{|\vec{p} - \vec{K}/2|^2} \frac{b^3 e^{-b^2K^2/2}}{(2\pi)^{3/2}} \{[5 \, \text{Im}\mathfrak{M}_{1/2}(\hat{s}) + 4 \, \text{Im}\mathfrak{M}_{3/2}(\hat{s})][(\text{Re}F)^2 - (\text{Im}F)^2]$$
$$+ [5 \, \text{Re}\mathfrak{M}_{1/2}(\hat{s}) + 4 \, \text{Re}\mathfrak{M}_{3/2}(\hat{s})] 2 (\text{Re}F)(\text{Im}F)\}. \tag{A19}$$

Here $\mathfrak{M}_{1/2}$ and $\mathfrak{M}_{3/2}$ are the $I = \frac{1}{2}$, $\frac{3}{2}$ πN forward amplitudes, and the function F, depending on $\vec{p}$ and $\vec{K}$, is given by

$$F = \frac{|\vec{p} - \vec{K}/2|}{4\pi} \int d^3r \frac{e^{-iqr}}{r} e^{-i(\vec{p} - \vec{K}/2) \cdot \vec{r}} \times \frac{2}{(\sqrt{2}b)^{3/2}} \frac{1}{\pi^{1/4}} \frac{1}{\sqrt{4\pi}} e^{-r^2/4b^2}. \tag{A20}$$

For completeness

$$\psi(r) = \frac{2}{b^{3/2}} \frac{1}{\pi^{1/4}} \frac{1}{\sqrt{4\pi}} e^{-r^2/2b^2}.$$

Note that $(\text{Re}F)^2 - (\text{Im}F)^2$ appears in this expression, not $|F|^2$. This results from including the interference between elastic scattering and the free decay. It is very important physically and ensures that the free decay is not double counted in $\delta\Gamma_{\text{resc}}$. See Sec. III for a discussion of the physics of this point.

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