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Predictions for hadronic transitions in the $b\bar{b}$ system

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Within the framework of the multipole expansion of quantum chromodynamics, we propose a model to compute the rates of hadronic transitions in the $b\bar{b}$ system and heavier $Q\bar{Q}$ states. Rates and estimated branching ratios of various transitions are given. The predicted result for $\Upsilon' \to \Upsilon\pi\pi$ agrees with recent measurements. We discuss the spin-flip $\pi\pi$ transitions $\Upsilon'' \to 1\,^1P_1\pi\pi$ and $2\,^3P_J \to 1\,^1S_0\pi\pi$ which can be used to reach the spin-singlet states $1\,^1P_1$ and $1\,^1S_0$.

I. INTRODUCTION

It is now generally accepted[1] that the Υ resonances are bound states of a heavy quark b and its antiquark $\bar{b}$ with a quark mass about 5 GeV. The four resonances discovered so far are presumably the spin-triplet states $1\,^3S_1$, $2\,^3S_1$, $3\,^3S_1$, and $4\,^4S_1$: Their spacings and the ratios of the leptonic widths agree with the expectation of potential models constructed to fit the charmonium system.

The $b\bar{b}$ system[2] has a much richer spectrum below the flavor threshold than the $c\bar{c}$ system. The various decay rates of these narrow states will provide valuable information on heavy-quark dynamics. Among all the Zweig-rule-forbidden decays hadronic transitions play an unusual role. They are dominant decay modes of states such as ψ' and Υ', yet it is not possible to compute the absolute rates from first principles. The radiated light hadrons carry only a small amount of energy, and the asymptotic freedom of quantum chromodynamics (QCD) is not of much use here. However, a general formalism for describing these transitions in QCD has been developed in terms of a multiple expansion of the gauge fields.[3-5] We have presented elsewhere[5] many predictions that follow simply from the symmetry properties of the transition amplitudes. Calculation of absolute rates would require the understanding of color confinement, e.g., how the gluons emitted by the heavy quarks convert into light hadrons. On the other hand, ratios of similar transitions in two families are largely determined by the relative sizes and quark masses of the two systems. For example, the naive estimate[2,3]

$$\frac{\Gamma(\Upsilon' \to \Upsilon\pi\pi)}{\Gamma(\psi' \to \psi\pi\pi)} \cong \frac{\langle r^2\rangle_{\Upsilon'}^2}{\langle r^2\rangle_{\psi'}^2} = \frac{1}{16} \qquad (1.1)$$

agrees very well with recent data from CESR (Refs. 6 and 7) and DORIS (Ref. 8). Therefore, we may hope to calculate the relative rates with some confidence.

The calculation of the hadronic transition rates not only tests the multipole-expansion formalism; it is also important in other respects. These rates, if large, will significantly reduce the branching ratios of photon transitions which are the main method to find the P states. There are also interesting transitions which will reach some otherwise inaccessible states (see discussion in Sec. IV).

In the present paper we will attempt to compute the absolute rates of the $\pi\pi$ and η transitions within the Υ family and for even heavier quarkonia. The measured rates of the transitions $\psi' \to \psi\pi\pi$ and $\psi' \to \psi\eta$ will be used to normalize the parameters in the models we will use.

To illustrate the basic ideas of our models let us consider a $\pi\pi$ transition between two spin-triplet states. Such a transition is dominated by two color-electric-dipole emissions. Its matrix element is given by[5]

$$M_{E1\text{-}E1} = i\frac{g^2}{6}\langle \Phi_f h|\vec{x}\cdot\vec{E}G(E_i)\vec{x}\cdot\vec{E}|\Phi_i\rangle\,, \qquad (1.2)$$

where $G(E_i)$ is the Green's function

$$G(E_i) = (E_i - H_8 - iD_0)^{-1} \qquad (1.3)$$

with E_i being the energy of the initial state, H_8 the Hamiltonian of the color-octet $Q\bar{Q}$ system, $\vec{E}$ the color-electric field, and

$$D_0 = \partial_0 - g\bar{A}_0\,. \qquad (1.4)$$

The matrix element (1.2) is represented in Fig. 1.

We encounter two fundamental difficulties when we attempt to calculate the matrix element (1.2):

1. The intermediate states between the dipole transitions are very complicated. Although they are overall color-singlet states, the $Q\bar{Q}$ system is in a color-octet state. The static potential for a color-singlet $Q\bar{Q}$ system extracted from the $c\bar{c}$ and $b\bar{b}$ system is not directly relevant to a color-octet $Q\bar{Q}$ pair.

2. The mechanism for the conversion of gluons into light hadrons is not understood.

Both problems are related to the question of

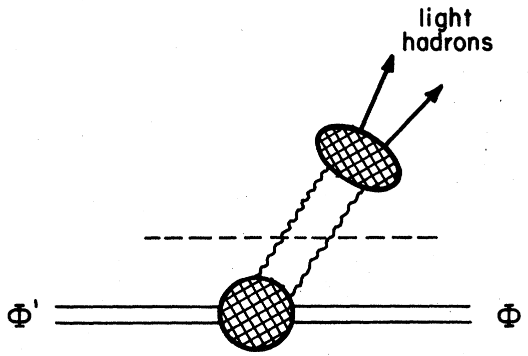

FIG. 1. Amplitude for a hadronic transition between heavy-quark states Φ′ and Φ.

"color confinement." Any suggestion to overcome the difficulty has to be simple-minded. Two simple options suggest themselves. One may, in the spirit of the parton model, consider the intermediate states as consisting of a free $Q\bar{Q}$ pair and a free gluon. At the other extreme, one may neglect the nonlocality of the resolvent and treat it as a local operator,

$$\frac{g^2}{6}\vec{x}\cdot\vec{E}G(E_i)\vec{x}\cdot\vec{E}\simeq c(\vec{x}\cdot\vec{E})^2 . \tag{1.5}$$

As we shall see in Sec. V, both treatments are not very satisfactory since neither takes into account the crucial property of the intermediate states that they are color-singlet hadrons with finite masses. We, therefore, propose a third alternative which we will mainly follow in this paper.

We observe that gluonic degrees of freedom play a crucial role in these states. The string vibrational states proposed by Tye and his collaborators[9,10] are also due to excitations of degrees of freedom other than quarks. If the string picture is correct in QCD, these new degrees of freedom must be due to gluons also. We propose to make the *ad hoc* identification of the intermediate states here with the vibrational states of the string. The resolvent $G(E_k)$ is then given by

$$G(E_i)=\sum_\sigma \frac{P_\sigma}{E_i-H_v^{(\sigma)}} , \tag{1.6}$$

where $H_v^{(\sigma)}$ is the effective $Q\bar{Q}$ Hamiltonian for vibrational states with a string excitation labeled by σ, and P_σ is the gluonic projection operator for this particular excitation. A prescription for constructing $H_v^{(\sigma)}$ from that of a color-singlet $Q\bar{Q}$ system has been given by Ref. 10. Now we have

$$M_{E1\text{-}E1}=i\frac{g^2}{6}\sum_\sigma\langle\Phi_f|x_k(E_i-H_v^{(\sigma)})^{-1}x_l|\Phi_i\rangle \times\langle h|E_kP_\sigma E_l|0\rangle . \tag{1.7}$$

Equation (1.7) has no predictive power since it involves an infinite number of unknowns. We will drastically simplify (1.7) by keeping only the contribution of the lowest excitation of the string. The matrix element now factorizes,

$$M_{E1\text{-}E1}=i\frac{g^2}{6}\langle\Phi_f|x_k(E_i-H_v)^{-1}x_l|\Phi_i\rangle \times\langle h|E_kPE_l|0\rangle , \tag{1.8}$$

where we have dropped the label σ and denoted by H_v and P the effective $Q\bar{Q}$ Hamiltonian and the projection operator for the sector of the lowest string excitation. For two-pion transitions we apply the ideas[5,11] of the soft-pion theorem to write

$$\frac{g^2}{6}\langle\pi_a(q_1)\pi_b(q_2)|E_kPE_l|0\rangle=\frac{\delta_{ab}}{[(2\omega_1)(2\omega_2)]^{1/2}}[a\delta_{kl}q_1^\mu q_{2\mu}+b(q_{1k}q_{2l}+q_{1l}q_{2k}-\tfrac{2}{3}\delta_{kl}\vec{q}_1\cdot\vec{q}_2)] . \tag{1.9}$$

This model is clearly an oversimplification: our ignorance of color confinement is summarized by an effective Hamiltonian H_v and two constants a and b. Once we have recognized the fundamental difficulty in computing the amplitude (1.2), the oversimplification becomes a virtue of the model. We hope that it incorporates the correct qualitative physics yet introduces a minimum number of arbitrary parameters. We must view in the same spirit the use of the string vibrational states. It does not imply that the success of the model necessarily depends on the existence of these states. It only means that we need a convenient representation of the hadronic states not found in the conventional quark model, and the vibrational states are the only ones in this category that we know.

In Sec. II spin-nonflip $\pi\pi$ transitions are calculated. The result for $\Upsilon'\to\Upsilon\pi\pi$ agrees very well with the recent measurements, and $\Upsilon''\to\Upsilon\pi\pi$ is predicted to have a width smaller than $\Gamma(\Upsilon'\to\Upsilon\pi\pi)$ in spite of a much larger phase space available to the $\pi\pi$ system. The η transitions are discussed in Sec. III. The rates are found to be rather small. The method used in Secs. II and III can only study transitions that are analogs of $\psi'\to\psi\pi\pi$ and $\psi'\to\psi\eta$. When applied to other transitions, it will introduce unknown parameters and no new prediction can be made. In Sec. IV we propose yet another method. The $\pi\pi$ system and the η are approximated by a two-gluon state. This method allows us to use the measured rates of $\psi' \to \psi\pi\pi$ and $\psi' \to \psi\eta$ to determine two coupling constants: g_E of (colored) $E1$ and g_M of (colored) $M1$ and $M2$ transitions. The rates for the (colored) $E1$-$M1$ induced transitions ${}^3S_1 \to {}^1P_1 + \pi\pi$ and ${}^3P_J \to {}^1S_0 + \pi\pi$ are then calculable. The cascade $\Upsilon'' \to 1\,{}^1P_1 + \pi\pi \to 1\,{}^1S_0 + \gamma + \pi\pi$ appears to be an exciting and promising way to reach *two* spin-singlet states. [The $E1$ transition $1\,{}^1P_1 \to 1\,{}^1S_0 + \gamma$ has a very large branching ratio of ~40%. See Sec. IV.] In Sec. V we discuss and summarize our results.

II. SPIN-NONFLIP $\pi\pi$ TRANSITIONS

These processes are induced by two electric-dipole transitions. The matrix element is given by (1.2). In our model, it is reduced to (1.8) and (1.9). We may write

$$(E_i - H_v)^{-1} = \sum_{kl} \frac{|v,kl\rangle\langle kl,v|}{E_i - E_{v,kl}}, \qquad (2.1)$$

where $|v,kl\rangle$ and $E_{v,kl}$ are vibrational eigenstates and corresponding eigenvalues in the sector of the lowest string excitation. An elementary but lengthy calculation gives the transition rate

$$\Gamma(\Phi_i \to \Phi_f \pi\pi) = \delta_{l_i l_f}\, \delta_{J_i J_f} (G|c_1|^2 - \tfrac{2}{3} H |c_2|^2) \left| \sum_l (2l+1) \begin{pmatrix} l_i & 1 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l & 1 & l_i \\ 0 & 0 & 0 \end{pmatrix} f^l_{if} \right|^2$$
$$+ (2l_i+1)(2l_f+1)(2J_f+1) \sum_k (2k+1)[1+(-1)^k] \begin{Bmatrix} s & l_f & J_f \\ k & J_i & l_i \end{Bmatrix}^2 H|c_2|^2$$
$$\times \left| \sum_l (2l+1) \begin{pmatrix} l_f & 1 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l & 1 & l_i \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} l_i & l & 1 \\ 1 & k & l_f \end{Bmatrix} f^l_{if} \right|^2, \qquad (2.2)$$

where l_i and J_i are the orbital and total angular momenta of the initial state Φ_i, respectively. The final state Φ_f has the corresponding quantum numbers l_f and J_f. Both Φ_i and Φ_f have the same spin s. We have defined

$$f^l_{if} = \sum_k \frac{1}{M_i - M_{kl}} \left[\int dr\, r^3 R_f(r) R^v_{kl}(r) \right] \left[\int dr'\, r'^3 R^v_{kl}(r') R_i(r) \right], \qquad (2.3)$$

where $R_i(r)$, $R_f(r)$, and $R^v_{kl}(r)$ are the radial wave functions of the initial, final, and intermediate vibrational states, respectively. A vibrational state is labeled by its orbital angular momentum l and the principal quantum number k. The initial state and the vibrational state have masses M_i and M_{kl}, respectively. The quantities G and H are the phase-space integrals

$$G = \frac{3}{4} \frac{M_f}{M_i} \pi^3 \int dM_{\pi\pi}^2 K \left(1 - \frac{4m_\pi^2}{M_{\pi\pi}^2}\right)^{1/2} (M_{\pi\pi}^2 - 2m_\pi^2)^2 \qquad (2.4)$$

$$H = \frac{1}{20} \frac{M_f}{M_i} \pi^3 \int dM_{\pi\pi}^2 K \left(1 - \frac{4m_\pi^2}{M_{\pi\pi}^2}\right)^{1/2} \left[(M_{\pi\pi}^2 - 4m_\pi^2)^2 \left(1 + \frac{2}{3} \frac{K^2}{M_{\pi\pi}^2}\right) + \frac{8K^4}{15 M_{\pi\pi}^4} (M_{\pi\pi}^4 + 2m_\pi^2 M_{\pi\pi}^2 + 6m_\pi^4) \right], \qquad (2.5)$$

with K given by

$$K = \frac{1}{2M_i} [(M_i + M_f)^2 - M_{\pi\pi}^2]^{1/2} [(M_i - M_f)^2 - M_{\pi\pi}^2]^{1/2}. \qquad (2.6)$$

We give here explicit formulas for a few transitions between spin triplets:

$$l_i = l_f = 0: \quad \Gamma = G|c_1|^2 |f^1_{if}|^2 \qquad (2.7)$$

$$l_i = 2,\ l_f = 0: \quad \Gamma = \tfrac{4}{15} H |c_2|^2 |f^1_{if}|^2 \qquad (2.8)$$

$$l_i = l_f = 1: \quad \Gamma(0 \to 0) = \tfrac{1}{9} G|c_1|^2 |f^0_{if} + 2f^2_{if}|^2, \quad \Gamma(0 \to 1) = \Gamma(1 \to 0) = 0,$$
$$\Gamma(0 \to 2) = 5\Gamma(2 \to 0) = \tfrac{10}{27} H|c_2|^2 |f^0_{if} + \tfrac{1}{5} f^2_{if}|^2, \quad \Gamma(1 \to 1) = \Gamma(0 \to 0) + \tfrac{1}{4}\Gamma(0 \to 2), \qquad (2.9)$$
$$\Gamma(1 \to 2) = \tfrac{5}{3}\Gamma(2 \to 1) = \tfrac{3}{4}\Gamma(0 \to 2), \quad \Gamma(2 \to 2) = \Gamma(0 \to 0) + \tfrac{7}{20}\Gamma(0 \to 2).$$

In (2.9) we have used the short-hand notation $\Gamma(J' \to J)$ for $\Gamma(\Phi_{J'} \to \Phi_J + \pi\pi)$.

To evaluate numerically the transition rates (2.7)–(2.9) in the Υ family we must specify H_v and determine the constants c_1 and c_2. We will discuss H_v later. Once H_v is given, the constant c_1 can be determined by the measured rate of $\psi' \to \psi\pi\pi$. The decay rates for $\Upsilon' \to \Upsilon\pi\pi$, $\Upsilon'' \to \Upsilon\pi\pi$, and $\Upsilon'' \to \Upsilon'\pi\pi$ are then predicted. However, the constant c_2 enters the decay rates for transitions such as $2\,^3P_{J'} \to 1\,^3P_J + \pi\pi$, $1\,^3D_J \to \Upsilon + \pi\pi$, etc. In an attempt to predict these rates we propose to fix c_2 by the following considerations. In the limit of massless pions the two processes

$$\Phi_i \to \Phi_f + \text{two real gluons},$$

$$\Phi_i \to \Phi_f + \pi\pi$$

have the same kinematics and there is a one-to-one correspondence between the two calculations of the integrated rates. We will assume that the ratio c_1/c_2 is correctly given by the two-gluon calculation. The result is [see Eq. (4.20)]

$$|c_2| = 3|c_1|\,. \tag{2.10}$$

We now turn to the consideration of H_v. Let us write it as

$$H_v = \frac{p^2}{m_Q} + V_v(r)\,.$$

The potential $V_v(r)$ for the vibrational states is related to the potential model we use for the color-singlet $Q\bar{Q}$ system. The prescription given by Buchmüller and Tye[10] is

$$V_v(r) = V(r) + \left[V_n(r) - \frac{1}{a^2}r\right] + \frac{A_v}{r}\,, \tag{2.11}$$

where $V(r)$ is the $Q\bar{Q}$ potential in the color-singlet sector and $(1/a^2)r$ is its linear piece. The potential $V_n(r)$ is defined implicitly by

$$V_n(r) = \frac{1}{a^2}r\left[1 + \frac{2\pi a^2 n}{(r-2d)^2 + 4d^2}\right]^{1/2}$$

$$\equiv \frac{1}{a^2}r[2 - \alpha_n^{\,2}(r)]^{-1/2}\,, \tag{2.12}$$

$$d = \frac{r^2\alpha_n(r)}{4a^2[2m_Q + (1/a^2)r\alpha_n(r)]}\,. \tag{2.13}$$

For our model we will set $n=1$ corresponding to the lowest string excitation. The constant A_v is adjusted to fit the location of the lowest vibrational state in the $c\bar{c}$ system. So far, there is no experimental confirmation of the existence of a vibrational state. According to Buchmüller and Tye,[10] the structure at $\sqrt{s}=3.96$ GeV or $\sqrt{s}=4.03$ GeV in $e^+e^- \to$ hadrons could be the candidate for the lowest $c\bar{c}$ vibrational state. In our calculation we will choose $M_v = 4.03$ GeV to be the ground state of the $c\bar{c}$ vibrational spectrum. The higher value for M_v is favored so that the vibrational states are farther away from the color-singlet $Q\bar{Q}$ states, and thereby the sensitivity of our results to their details is reduced.

In order to have some idea on the model dependence of our calculations we have considered three models defined below. The prescription (2.11) is natural for a potential $V(r)$ which has a linear piece at large distances and a *genuine* Coulomb potential at short distances. For models A and B this is true and we follow the prescription (2.11). However, for model C we simply set $A_v = 0$ and introduce an additive constant in $V_v(r)$ [Eq. (2.11)] so that M_v occurs at 4.03 GeV. Now, the three models are as follows.

Model A (Coulomb plus linear potential with parametrization given by Ref. 2):

$$V(r) = \frac{1}{a^2}r - \frac{\kappa}{r}\,, \tag{2.14}$$

$$a = 2.34\ \text{GeV}^{-1},$$
$$\kappa(c\bar{c}) = 0.52,\quad \kappa(b\bar{b}) = 0.48 \tag{2.15}$$
$$m_c = 1.84\ \text{GeV},\quad m_b = 5.17\ \text{GeV}\,.$$

Model B (Bhanot-Rudaz potential[12]):

$$V(r) = \begin{cases} -\dfrac{\kappa}{r}, & r \le e^{-1}\sqrt{\kappa}a \equiv e^{-1}r_0 \\[2mm] \dfrac{er_0}{a^2}\ln\dfrac{r}{r_0}, & e^{-1}r_0 \le r \le er_0 \\[2mm] \dfrac{1}{a^2}r, & r \ge er_0 \end{cases} \tag{2.16}$$

$$\kappa = 0.42\,,$$
$$a = 2.54\ \text{GeV}^{-1}\,, \tag{2.17}$$
$$m_c = 1.05\ \text{GeV},\quad m_b = 3.4\ \text{GeV}\,.$$

Model C (improved Richardson potential[13] as given by Buchmüller, Grunberg, and Tye[14]). For details of the potential the reader is referred to Ref. 14. It is characterized by a scale parameter $\Lambda_{\overline{MS}}$ which is chosen to be

$$\Lambda_{\overline{MS}} = 0.509\ \text{GeV}\,,$$
$$m_c = 1.48\ \text{GeV},\quad m_b = 4.88\ \text{GeV}\,. \tag{2.18}$$

For our calculations we have used the following experimental information[1,15,16]:

$$\Gamma_{\text{tot}}(\Upsilon) = 35^{+25(+9)}_{-10(-7)}\ \text{keV},$$
$$\Gamma_{ee}(\Upsilon) = 1.23 \pm 0.10(\pm 0.14)\ \text{keV}, \tag{2.19}$$
$$B_{\mu\mu}(\Upsilon) = [3.5 \pm 1.4(\pm 0.4)]\%\,,$$

$$\frac{\Gamma_{ee}(\Upsilon')}{\Gamma_{ee}(\Upsilon)} = 0.45 \pm 0.03 \pm 0.04\,,$$
$$\frac{\Gamma_{ee}(\Upsilon'')}{\Gamma_{ee}(\Upsilon)} = 0.32 \pm 0.03 \pm 0.03\,, \tag{2.20}$$

$$\Gamma_{\text{tot}}(\psi) = 215 \pm 40\ \text{keV}\,,$$
$$B(\psi' \to \psi\pi\pi) = (50 \pm 4)\%\,. \tag{2.21}$$

TABLE I. Values of parameters in different potential models.

Parameter	Model A	Model B	Model C
c_1^2	61.8×10^{-6}	12.0×10^{-6}	50.2×10^{-6}
c_2^2 (GeV^6)	1.93×10^{-6}	0.12×10^{-6}	0.46×10^{-6}
$g_E^2/4\pi$	0.54	0.24	0.49
$g_M^2/4\pi$	4.8	0.61	4.1

The quantities f_{if}^l are computed numerically. The constant c_1 is determined by (2.21) and given in Table I. The results for the transition rates are presented in Table II. We would like to make a few comments on these results:

1. Although the quark masses and the constant c_1 have quite different values in the three models, the results for the rates come out surprisingly consistent with one another. It indicates that the ratios of rates are indeed determined mostly by the scaling properties of heavy-quark systems.

2. The widths $\Gamma(\Upsilon' \to \Upsilon\pi\pi)$ and $\Gamma(\Upsilon'' \to \Upsilon'\pi\pi)$ are rather model independent. In particular the width $\Gamma(\Upsilon' \to \Upsilon\pi\pi) \simeq 6$–$7$ keV agrees with the earlier estimate (1.1). The transition $\Upsilon' \to \Upsilon\pi\pi$ has been observed experimentally. Three groups have measured the branching ratio:

$$B(\Upsilon' \to \Upsilon\pi^+\pi^-) = \begin{cases} (19\pm8)\%, & \text{LENA (Ref. 8)} \\ (20\pm7)\%, & \text{CUSB (Ref. 6)} \\ (19.1\pm3.1)\%, & \text{CLEO (Ref. 7)} \end{cases} \tag{2.22}$$

To compare our results with the data, we must convert our predictions into branching ratios. We have estimated other partial widths for Υ' (to be discussed later) and obtained the results

$$B(\Upsilon' \to \Upsilon\pi^+\pi^-) = \begin{cases} 18\%, & \text{model A} \\ 20\%, & \text{model B} \\ 16\%, & \text{model C} \end{cases} \tag{2.23}$$

The agreement between (2.22) and (2.23) is excellent. All three groups have also found that the mass distributions of the $\pi\pi$ system are consistent with the prediction of soft-pion theorem.[5,11]

3. The rate for $\Upsilon'' \to \Upsilon\pi\pi$ is surprisingly small. If we compare the phase-space integrals (2.4) for the two transitions $\Upsilon'' \to \Upsilon\pi\pi$ and $\Upsilon' \to \Upsilon\pi\pi$, their ratio is large,

$$\frac{G(\Upsilon'' \to \Upsilon\pi\pi)}{G(\Upsilon' \to \Upsilon\pi\pi)} \approx 33\,. \tag{2.24}$$

The matrix element for $\Upsilon'' \to \Upsilon\pi\pi$ is tremendously suppressed:

$$\left|\frac{f_{if}^1(\Upsilon'' \to \Upsilon\pi\pi)}{f_{if}^1(\Upsilon' \to \Upsilon\pi\pi)}\right|^2 \approx (2-4)\times10^{-3}\,. \tag{2.25}$$

The large suppression is due to two effects. First, there is a great deal of cancellation among different terms in the series for $f_{if}^1(\Upsilon'' \to \Upsilon\pi\pi)$. Second, many high vibrational levels contribute, so the mean distance from these levels to Υ'' is large. Because of the delicate cancellations, we cannot expect our results to be very reliable. This uncertainty is reflected in our results for

TABLE II. Hadronic transition rates of $b\bar{b}$ states. The total widths needed for computing the branching ratios given here are obtained by adding the hadronic transition rates in this table to the partial widths of the last column of Table III. Rates for other transitions $2\,^3P_{J'} \to 1\,^3P_J$ ($J' \neq J$) not listed in this table can be found by relations (2.9).

Transition	Model A Γ(keV) [B (%)]	Model B Γ(keV) [B (%)]	Model C Γ(keV) [B (%)]
$\Upsilon' \to \Upsilon\pi\pi$	7 [27]	7 [29]	6 [25]
$\Upsilon' \to \Upsilon\eta$	0.01 [0.04]	0.01 [0.04]	0.01 [0.04]
$\Upsilon'' \to \Upsilon\pi\pi$	0.3 [2]	0.5 [3]	0.9 [5]
$\Upsilon'' \to \Upsilon'\pi\pi$	0.6 [3]	0.5 [3]	0.4 [2]
$\Upsilon'' \to \Upsilon\eta$	0.003 [0.02]	0.003 [0.02]	0.005 [0.03]
$2\,^3P_0 \to 1\,^3P_0\,\pi\pi$	0.3 [0.05]	0.3 [0.05]	0.4 [0.06]
$2\,^3P_1 \to 1\,^3P_1\,\pi\pi$	0.4 [0.3]	0.3 [0.3]	0.4 [0.3]
$2\,^3P_2 \to 1\,^3P_2\,\pi\pi$	0.4 [0.2]	0.3 [0.2]	0.4 [0.2]
$2\,^3P_2 \to 1\,^3P_1\,\pi\pi$	0.02 [0.01]	0.03 [0.02]	0.01 [0.01]
$1\,^3D_1 \to \Upsilon\pi\pi$	24		

this transition given in Table II. They differ by almost a factor of 3 in the three models we have considered. Furthermore, the emitted pions are not very soft. The multipole expansion need not be valid nor is PCAC (partial conservation of axial-vector current) expected to work well. Therefore, we can only regard our results as an order-of-magnitude estimate. It is probably a reasonable guess that

$$\Gamma(\Upsilon'' \to \Upsilon\pi\pi) < \Gamma(\Upsilon' \to \Upsilon\pi\pi)\,. \qquad (2.26)$$

Additional evidence for our model will come from the confirmation of the mass distribution of the $\pi\pi$ system predicted by the soft-pion method.

4. The $\pi\pi$ transitions between $2\,^3P_{J'}$, and $1\,^3P_J$ have very small rates due to the small phase space available. Transitions between initial and final states with the same total angular momentum have substantially larger rates. Even so, their branching ratios are tiny since the P states are much broader.

5. We have calculated the $\pi\pi$ transition rates for heavy quarkonia as functions of the quark mass in model C. The results are shown in Fig. 2. The rate for $2\,^3S_1 \to 1\,^3S_1 + \pi\pi$ decreases very rapidly as the quark becomes heavier until it stabilizes for $m_Q \gtrsim 30$ GeV. The rapid decrease is due to the shrinking size of the $Q\bar{Q}$ system. When the quark is so heavy that the Coulomb potential dominates the binding, the phase space increases with m_Q. The two compensating factors stabilize the rate. In any case, the rates are all very small ($\lesssim$1 keV). So hadronic transitions will not be a major decay mode for a very heavy $Q\bar{Q}$ system.

In Table II we have also given the branching ratios of various transitions. These are obtained by estimating other partial widths according to standard methods. The three-gluon annihilation rates of Υ' and Υ'' are determined by

$$\frac{\Gamma(\Upsilon' \to 3g)}{\Upsilon_{ee}(\Upsilon')} = \frac{\Gamma(\Upsilon'' \to 3g)}{\Gamma_{ee}(\Upsilon'')} = \frac{\Gamma(\Upsilon \to 3g)}{\Gamma_{ee}(\Upsilon)}\,, \qquad (2.27)$$

$$\Gamma(\Upsilon \to 3g) = [1-(3+R)B(\Upsilon \to e^+e^-)]\Gamma_{tot}(\Upsilon)\,. \qquad (2.28)$$

The gluon-annihilation rates of the P states are estimated by the formulas given by Ref. 17. The $E1$ rates for various photon transitions are taken from Ref. 2. For convenience, these rates are listed in Table III. Incidentally, (2.28) leads to the value of $\alpha_s(\Upsilon)$:

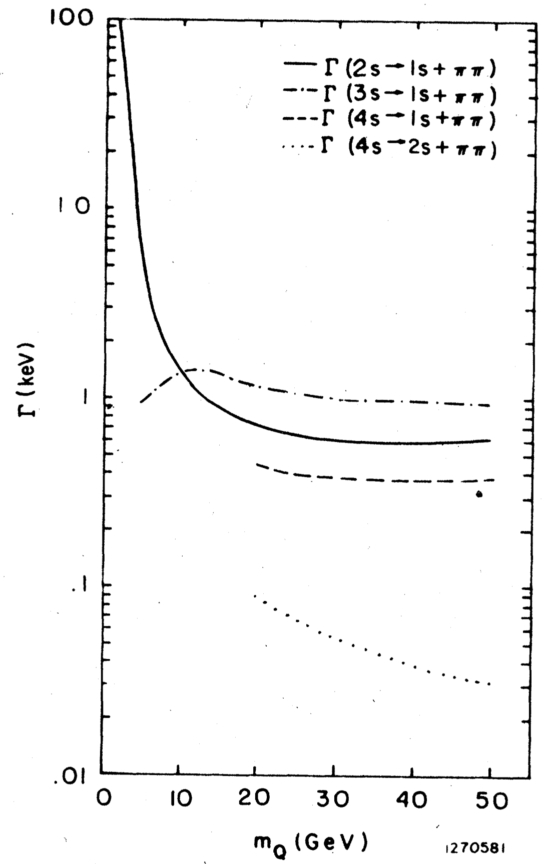

FIG. 2. Rates for $\pi\pi$ transitions as function of quark mass m_Q.

$$\alpha_s(\Upsilon) = 0.17\,. \qquad (2.29)$$

III. η TRANSITIONS

For simplicity we will consider only the transitions

$$n_i\,^3S_1 \to n_f\,^3S_1 + \eta$$

which cover the important processes $\psi' \to \psi\eta$, $\Upsilon' \to \Upsilon\eta$, and $\Upsilon'' \to \Upsilon\eta$. The leading multiples for the η transitions are $M1$-$M1$ and $E1$-$M2$. The matrix element is given by

$$M(\Phi_i \to \Phi_f + \eta) = M(M1\text{-}M1) + M(E1\text{-}M2)\,, \qquad (3.1)$$

$$M_1 \equiv M(M1\text{-}M1) = i\frac{g^2}{24m_Q^2}\langle\Phi_f\eta|(\vec{\sigma}-\vec{\sigma}')\cdot\vec{B}G(E_i)(\vec{\sigma}-\vec{\sigma}')\cdot\vec{B}|\Phi_i\rangle\,, \qquad (3.2)$$

$$M_2 \equiv M(E1\text{-}M2) = i\frac{g^2}{12m_Q}\langle\Phi_f\eta[\vec{x}\cdot\vec{E}G(E_i)\vec{S}\cdot(\vec{x}\cdot\vec{\nabla})\vec{B} + \vec{S}\cdot(\vec{x}\cdot\vec{\nabla})\vec{B}G(E_i)\vec{x}\cdot\vec{E}]|\Phi_i\rangle\,, \qquad (3.3)$$

TABLE III. Theoretical estimates of partial widths of $b\bar{b}$ states (keV).

State	Gluon annihilation[a]	$(3+R_{\rm had})\Gamma_{ee}$[b]	Photon transition[c]	Partial sum
$\Upsilon(9.46)$	27	8	0	35
$\Upsilon'(10.02)$	12	3.7	2.5	18.2
$\Upsilon''(10.36)$	8.2	2.6	5.6	16.4
$1\,^3P_2(9.92)$	760		30	790
$1\,^3P_1(9.92)$	60		30	90
$1\,^3P_0(9.92)$	200		30	230
$1\,^1P_1(9.92)$	40		30	70
$2\,^3P_2(10.27)$	600		33	633
$2\,^3P_1(10.27)$	80		33	113
$2\,^3P_0(10.27)$	160		33	194

[a] The gluon-annihilation widths given here replace those in Ref. 2. The Γ_{3g} for Υ, Υ', and Υ'' are obtained by (2.27) and (2.28) and are model-independent. Those for the P states are obtained by using the formulas of Ref. 17 and the wave functions of Ref. 2 and $\alpha_s=0.17$.

[b] A value of $R_{\rm had}=3.7$ is used.

[c] These are taken from Ref. 2 with corrected photon energies by using the experimental values of the Υ masses and the improved $b\bar{b}$ spectrum given in the note added in proof of Ref. 2.

where $\vec\sigma$ and $\vec\sigma'$ are the Pauli matrices for Q and $\bar{Q}$, respectively, and $\vec{S}$ is the total spin of the $Q\bar{Q}$ system. In our model (3.2) and (3.3) reduce to

$$M_1=i\frac{g^2}{24m_Q{}^2}\Phi_f\,|\,(\sigma-\sigma')_k(E_i-H_v)^{-1}(\sigma-\sigma')_l\,|\,\Phi_i\rangle \times\langle\eta\,|\,B_kPB_l\,|\,0\rangle\,, \tag{3.4}$$

$$M_2=i\frac{g^2}{12m_Q}\frac{1}{3}\langle\Phi_f\,|\,\sum_k x_k(E_i-H_v)^{-1}x_kS_l\,|\,\Phi_i\rangle \times\langle\eta\,|\,\vec{E}\cdot P\vec\nabla B_l+\vec\nabla B_lP\cdot\vec{E}\,|\,0\rangle\,, \tag{3.5}$$

where we have made use of the fact that both Φ_i and Φ_f are $l=0$ states. Each of M_1 and M_2 has a reduced matrix element. But the measured rate of $\psi'\to\psi\eta$ is the only available experimental information. Our model must be simplified further. We observe that

$$\langle\eta\,|\,B_kB_l\,|\,0\rangle=\sum_n\langle\eta\,|\,B_kP_nB_l\,|\,0\rangle\,. \tag{3.6}$$

It is, therefore, not totally unreasonable to assume that each term in the sum has the same symmetry in indices k and l as $\langle\eta\,|\,B_kB_l\,|\,0\rangle$. We will make the assumption. Then

$$\langle\eta\,|\,B_kPB_l\,|\,0\rangle=0 \tag{3.7}$$

since the only available pseudotensor is antisymmetric in k and l. Let us define

$$\frac{g^2}{12}\langle\eta\,|\,(\vec{E}\cdot\vec\nabla)PB_k+\vec\nabla B_kP\cdot\vec{E}\,|\,0\rangle=i\frac{c_3}{\sqrt{2\omega}}q_k\,, \tag{3.8}$$

where $\vec{q}$ is the momentum of η. The matrix element (3.1) now becomes

$$M(\Phi_i\to\Phi_f+\eta)=M_2 =-\frac{c}{\sqrt{2\omega}}\vec\epsilon_f^{\,*}\times\vec\epsilon_i\cdot\vec{q}\cdot\frac{1}{3m_Q}f_{if}^1\,, \tag{3.9}$$

where $\vec\epsilon_i$ and $\vec\epsilon_f$ are the polarization vectors of the initial and final $Q\bar{Q}$ states, respectively, and f_{if}^1 is the amplitude defined in (2.3) with $l=1$. The decay rate calculated from (3.9) is

$$\Gamma(\Phi_i\to\Phi_f+\eta)=\frac{8\pi^2}{27}\frac{M_fc_3{}^2}{M_im_Q{}^2}|f_{if}^1|^2q^3\,. \tag{3.10}$$

In our model, a $\pi\pi$ transition and an η transition between two spin-triplet s states are related. From (2.7) and (3.10) we find

$$\frac{\Gamma(\Phi_i\to\Phi_f\pi\pi)/G}{\Gamma(\Phi_i\to\Phi_f\eta)/(q^3M_f/M_i)}=\frac{27}{8\pi^2}\left(\frac{c_1}{c_3}\right)^2m_Q{}^2\,, \tag{3.11}$$

where G and q are the phase-space integral (2.4) and the η momentum, respectively. For example, Eq. (3.11) gives

$$\frac{\Gamma(\Upsilon''\to\Upsilon\pi\pi)/G}{\Gamma(\Upsilon''\to\Upsilon\eta)/(q^3M_{\Upsilon''}/M_\Upsilon)} =\left(\frac{m_b}{m_c}\right)^2\frac{\Gamma(\psi'\to\psi\pi\pi)/G}{\Gamma(\psi'\to\psi\eta)/(q^3M_{\psi'}/M_\psi)}\,. \tag{3.12}$$

When the data (2.21) and[18]

$$B(\psi'\to\psi\eta)=(2.18\pm0.14\pm0.35)\% \tag{3.13}$$

are used, (3.12) predicts

$$\frac{\Gamma(\Upsilon'' \to \Upsilon\pi\pi)}{\Gamma(\Upsilon'' \to \Upsilon\eta)} = 19\left(\frac{m_b}{m_c}\right)^2 \simeq 150\,, \tag{3.14}$$

where the quark masses of $m_b = 5.17$ GeV and $m_c = 1.84$ GeV are used. As we have emphasized earlier, we cannot reliably calculate the rate for $\Upsilon'' \to \Upsilon\pi\pi$. The same is true of the rate for $\Upsilon'' \to \Upsilon\eta$. We hope that the prediction for the ratio (3.14) is more reliable. These predictions (3.12) and (3.14) follow from (3.7); they provide a test of this assumption.

The constant c_3 is fixed by (2.21) and (3.31) and shown in Table I. The rates computed from (3.10) are given in Table II. We notice that the rates are rather small. So far, no η transition in the Υ family has been detected experimentally. We have also applied (3.10) to heavier $Q\bar{Q}$ systems. The results are so small that they will not be presented here.

IV. TWO-GLUON TRANSITIONS

So far we have considered only those hadronic transitions in the Υ family which are the analogs of $\psi' \to \psi\pi\pi$ and $\psi' \to \psi\eta$ in the charmonium system. However, the $b\bar{b}$ system has a much richer spectrum and many other interesting transitions are possible. A particularly exciting example is the spin-flip parity-changing $\pi\pi$ transition

$$\Upsilon(3\,{}^1S_3) \to \Upsilon(1\,{}^1P_1) + \pi\pi \tag{4.1}$$

and its subsequent $E1$ transition

$$\Upsilon(1\,{}^1P_1) \to \Upsilon(1\,{}^1S_0) + \gamma\,. \tag{4.2}$$

This cascade offers the possibility of detecting *two* spin-singlet ($b\bar{b}$) states $1\,{}^1P_1$ and $1\,{}^1S_0$. Another alternative to reach $1\,{}^1S_0$ is

$$\Upsilon(3\,{}^1S_3) \to \Upsilon(2\,{}^3P_J) + \gamma \searrow \Upsilon(1\,{}^1S_0) + \pi\pi\,. \tag{4.3}$$

The rates for the photon transitions in (4.2) and (4.3) can be calculated by a standard $E1$ dipole formula. The $\pi\pi$ transitions are more difficult to estimate. The model described in earlier sections cannot predict the rates for (4.1) and (4.3). The $\pi\pi$ cascades are induced by $E1$-$M1$ transitions, and the matrix element $\langle\pi\pi|E_k P B_l|0\rangle$ needed involves unknown parameters.

In a bold attempt to provide some guide for the experimentalists, we will resort to a different approach. The basic idea is to make as many predictions as possible from the two pieces of experimental information presently available,

$$\Gamma(\psi' \to \psi\pi\pi) = 107 \text{ keV}\,, \tag{4.4}$$

$$\Gamma(\psi' \to \psi\eta) = 4.7 \text{ keV}\,. \tag{4.5}$$

The objective can be accomplished if we approximate the hadronic transition rates by gluon emission rates,[19] namely,

$$\Gamma(\Phi_i \to \Phi_f\pi\pi) \cong \Gamma(\Phi_i \to \Phi_f gg)\,, \tag{4.6}$$

$$\Gamma(\Phi_i \to \Phi_f\eta) \cong \Gamma(\Phi_i \to \Phi_f (gg)_{0^-})\,. \tag{4.7}$$

In (4.7) the two gluons are projected into a $J^P = 0^-$ state to simulate the η meson. If one calculates the rates in the conventional perturbation theory, there is only one coupling constant to be determined. We will allow for the possibility that a heavy quark may have an anomalous color-magnetic moment. The first three terms in the multipole expansion are

$$H_I = -\tfrac{1}{4}g_E(\lambda_a - \bar{\lambda}_a)\vec{r}\cdot\vec{E}_a - \frac{g_M}{4m_Q}\tfrac{1}{2}(\lambda_a - \bar{\lambda}_a)[(\vec{\sigma} - \vec{\sigma}')\cdot\vec{B}_a + \tfrac{1}{2}(\vec{\sigma} + \vec{\sigma}')\cdot(\vec{r}\cdot\vec{D})\vec{B}_a]\,, \tag{4.8}$$

where λ_a, $\vec{\sigma}$ and $\bar{\lambda}_a, \vec{\sigma}'$ are the color matrices and Pauli spin matrices for a quark and an antiquark, respectively. In this model we have

$$M[\Phi_i(s=1) \to \Phi_f(s=1)g_1g_2] = M(E1\text{-}E1) = i\frac{g_E^2}{6}\langle\Phi_f|x_k(E_i - H_v)^{-1}x_l|\Phi_i\rangle\langle g_1g_2|E_kE_l|0\rangle\,, \tag{4.9}$$

$$\begin{aligned} M[\Phi_i(s=1) \to \Phi_f(s=1)(g_1g_2)_{0^-}] &= M(M1\text{-}M1) + M(E1\text{-}M2) \\ &= iP\left[\frac{g_M^2}{24m_Q^2}\right]\langle\Phi_f|(\sigma - \sigma')_k(E_i - H_v)^{-1}(\sigma - \sigma')_l|\Phi_f\rangle\langle g_1g_2|B_kB_l|0\rangle \\ &\quad + \frac{g_Eg_M}{12m_Q}\langle\Phi_f|x_k(E_i - H_v)^{-1}(\sigma + \sigma')_l x_j + (\sigma + \sigma')_l x_j(E_i - H_v)^{-1}x_k|\Phi_i\rangle \\ &\quad \times\langle g_1g_2|E_k\nabla_j B_l|0\rangle\,, \end{aligned} \tag{4.10}$$

$$M[\Phi_i(s=1)\to\Phi_f(s=0)g_1g_2]=M(E1-M2)$$
$$=-i\frac{g_Eg_M}{12m_Q}\langle\Phi_f|(\sigma-\sigma')_k(E_i-H_v)^{-1}x_l+x_l(E_i-H_v)^{-1}(\sigma-\sigma')_k|\Phi_i\rangle$$
$$\times\langle g_1g_2|B_kE_l|0\rangle\,, \tag{4.11}$$

where s denotes the spin eigenvalue and P is the projection operator into the 0^- state

$$P=\frac{(\hat{k}_1-\hat{k}_2)\vec{\epsilon}_1^{\,*}\times\vec{\epsilon}_2^{\,*}}{\sqrt{2}(1-\hat{k}_1\cdot\hat{k}_2)} \tag{4.12}$$

where $\hat{k}_1$, $\hat{k}_2$, $\vec{\epsilon}_1$, and $\vec{\epsilon}_2$ are the unit vectors along the gluon momenta $\vec{k}_1$ and $\vec{k}_2$, and their polarization vectors, respectively. A straightforward calculation gives the rates

$$\Gamma[\Phi_i(s)\to\Phi_f(s)+\pi\pi]\cong\Gamma[\Phi_i(s)\to\Phi_f(s)gg]=\left(\frac{g_E{}^2}{2}\right)^2\frac{1}{9\pi^3}\frac{(M_i-M_f)^7}{140}(2J_f+1)(2l_i+1)(2l_f+1)$$
$$\times\sum_k(2k+1)[1+(-1)^k]\begin{Bmatrix}s & l_f & J_f\\ k & J_i & l_i\end{Bmatrix}^2$$
$$\times\left|\sum_l(2l+1)\begin{pmatrix}l_f & 1 & l\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}l & 1 & l_i\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}l_i & l & 1\\ 1 & k & l_f\end{Bmatrix}f_{if}^l\right|^2, \tag{4.13}$$

$$\Gamma[\Phi_i(s=1)\to\Phi_f(s=1)\eta]\cong\Gamma[\Phi_i(s=1)\to\Phi_f(s=1)(gg)_{0^-}]=\left(\frac{g_M{}^2}{3m_Q{}^2}\right)^2\frac{1}{12\pi^3}\frac{(M_i-M_f)^7}{140}|h_{if}|^2$$
$$+\left(\frac{g_Eg_M}{3m_Q}\right)^2\frac{1}{12\pi^3}\frac{(M_i-M_f)^9}{6804}|h'_{if}|^2\,, \tag{4.14}$$

$$\Gamma[\Phi_i(s=1)\to\Phi_f(s=0)\pi\pi]\cong\Gamma[\Phi_i(s=1)\to\Phi_f(s=0)gg]=\left(\frac{g_Eg_M}{6m_Q}\right)^2\frac{(M_i-M_f)^7}{315\pi^3}(2l_f+1)\begin{pmatrix}l_f & 1 & l_i\\ 0 & 0 & 0\end{pmatrix}^2|g_{if}|^2\,. \tag{4.15}$$

Equation (4.14) only applies to transitions in which both Φ_i and Φ_f have zero orbital angular momentum. The amplitude f_{if}^l is defined by (2.3) and the others are given by

$$g_{if}=\sum_k\frac{1}{E_i-E_{kl_f}}\int dr\,r^2R_f(r)R_{kl_f}^v(r)\int dr'r'^3R_{kl_f}^v(r')R_i(r')$$
$$+\sum_k\frac{1}{E_i-E_{kl_i}}\int dr\,r^3R_f(r)R_{kl_i}^v(r)\int dr'r'^2R_{kl_i}^v(r')R_i(r')\,, \tag{4.16}$$

$$h_{if}=\sum_{k(l=0)}\frac{1}{E_i-E_{kl}}\int dr\,r^2R_f(r)R_{kl}^v(r)\int dr'r'^2R_{kl}^v(r')R_k(r')\,, \tag{4.17}$$

$$h'_{if}=\sum_{k(l=1)}\frac{1}{E_i-E_{kl}}\int dr\,r^3R_f(r)R_{kl}^v(r)\int dr'r'^3R_{kl}^v(r')R_i(r')\,. \tag{4.18}$$

We should mention the shortcomings of the model before the numerical results are discussed. The phase space for the light hadrons in these transitions is generally not large enough to neglect the masses, while the gluons are treated as massless. Furthermore, the η meson is poorly approximated by a two-gluon system since the latter does not have a definite mass. Despite these complaints, it is instructive to find out its consequences simply because this is the only way to estimate the transition rates for new processes such as (4.1) and (4.3).

The coupling constants g_E and g_M are determined by (4.4) and (4.5); their values are given in Table I. The rates of several transitions calculated in potential models A, B, and C are given in Table IV.

It is gratifying to note that the rates of spin-nonflip $\pi\pi$ transitions and the η transitions in Tables II and IV are very similar with two exceptions. The rates for $\Upsilon''\to\Upsilon'\pi\pi$ and $\Upsilon''\to\Upsilon\eta$ are much bigger in the two-gluon model. This is understand-

TABLE IV. Two-gluon transition rates of $b\bar{b}$ states (in keV). The process in parentheses is the one to be represented by the two-gluon transition.

Transition	Model A	Model B	Model C
$\Upsilon' \to \Upsilon gg$ ($\Upsilon' \to \Upsilon\pi\pi$)	6	7	5
$\Upsilon'' \to \Upsilon gg$) ($\Upsilon'' \to \Upsilon\pi\pi$)	0.2	0.3	0.6
$\Upsilon'' \to \Upsilon' gg$ ($\Upsilon'' \to \Upsilon'\pi\pi$)	4	5	2
$\Upsilon' \to \Upsilon(gg)_{0^-}$ ($\Upsilon' \to \Upsilon\eta$)	0.03	0.03	
$\Upsilon'' \to \Upsilon(gg)_{0^-}$ ($\Upsilon'' \to \Upsilon\eta$)	0.4	1.2	
$\Upsilon'' \to 1\,^1P_1 gg$ ($\Upsilon'' \to 1\,^1P_1\pi\pi$)	0.2	0.2	0.1
$2\,^3P_J \to 1\,^1S_0 gg$ ($2\,^3P_J \to 1\,^1S_0\pi\pi$)	0.6	3	2

able. In $\Upsilon'' \to \Upsilon'\pi\pi$ the actual phase space is so small that it is very poor to approximate the pions by massless gluons. In $\Upsilon'' \to \Upsilon\eta$ the phase space is rather large. The two-gluon representation of the η is also poor and gives a very strong energy dependence [see Eq. (4.14)]. From the examples of $\Upsilon' \to \Upsilon\pi\pi$ and $\Upsilon'' \to \Upsilon\pi\pi$ we may conclude that when the phase space available to the two pions is comparable to or larger than that of $\psi' \to \psi\pi\pi$ the two-gluon model is reasonable for a $\pi\pi$ transition.

Therefore, the spin-flip parity-changing $\pi\pi$ transitions should have a better chance of success in the model. The transition $\Upsilon'' \to 1\,^1P_1 + \pi\pi$ has a respectable, although small, rate of ~0.2 keV corresponding to a branching ratio of order 1%. The subsequent photon transition $1\,^1P_1 \to 1\,^1S_0 + \gamma$ has a very large branching ratio[20] (~40%; see Table III) due to the fact that an axial-vector meson cannot annihilate into two gluons. Once the spin-singlet $1P$ is found, the spin-singlet $1S$ should be easy to detect. Alternatively, it is also possible to reach the $1\,^1S_0$ by $\Upsilon'' \to 2\,^3P_J + \gamma$, $2\,^3P_J \to 1\,^1S_0 + \pi\pi$. The best chance is to cascade through the narrow axial-vector state $2\,^3P_1$.

Finally, we verify the claim in Sec. II that the two-gluon model leads to the relation (2.10). To see this, we set $m_\pi = 0$ and $M_i/M_f = 1$ in (2.4) and (2.5). We then have

$$G = \frac{4\pi^3}{35}(M_i - M_f)^7 ,\qquad H = \frac{2\pi^3}{105}(M_i - M_f)^7 . \tag{4.19}$$

When (4.19) is substituted into (2.2) we find the formulas (2.2) and (4.13) agree provided

$$c_2{}^2 = 9c_1{}^2 = \frac{1}{6}\left(\frac{g_E{}^2}{6\pi^3}\right)^2 \tag{4.20}$$

which establishes (2.10).

V. SUMMARY AND DISCUSSIONS

In this paper we have proposed a method (with two variations) to calculate the rates of hadronic transitions within a $Q\bar{Q}$ system in the framework of multipole expansion of QCD. Emphasized in the Introduction and again here is the difficulty of carrying out such a program because of our ignorance of color confinement. Therefore, a useful model should contain as few free parameters as possible. Our model is constructed to meet this requirement. The measured rates of $\psi' \to \psi\pi\pi$ and $\psi' \to \psi\eta$ are used to determine the parameters in the model. The goal is to predict the rates for as many transitions as possible in the Υ family. The details are presented in Secs. II–IV. In summary the most important conclusions are as follows.

1. The predicted rate for $\Upsilon' \to \Upsilon\pi\pi$ is in excellent agreement with the recent measurements. The transition $\Upsilon'' \to \Upsilon'\pi\pi$ should have a small rate ($\lesssim 1$ keV) due to its small Q value. The rate for $\Upsilon'' \to \Upsilon\pi\pi$ is also expected to be small [$< \Gamma(\Upsilon' \to \Upsilon\pi\pi)$], although large cancellations in the amplitude make it difficult to predict this rate reliably. The two simple options mentioned in Sec. I would have led to very different results: (a) If $G(E_i)$ is approximated by the Green's function for free particles as in the parton model, then we get

$$\Gamma(\Upsilon' \to \Upsilon\pi\pi) = 4\text{ keV},\quad \Gamma(\Upsilon'' \to \Upsilon\pi\pi) = 0.1\text{ keV},\quad \Gamma(\Upsilon'' \to \Upsilon'\pi\pi) = 6\text{ keV}. \tag{5.1}$$

(b) If $G(E_i)$ is approximated by a constant by neglecting its nonlocality, then we obtain

$$\Gamma(\Upsilon' \to \Upsilon\pi\pi) = 20\text{ keV},\quad \Gamma(\Upsilon'' \to \Upsilon\pi\pi) = 35\text{ keV},\quad \Gamma(\Upsilon \to \Upsilon'\pi\pi) = 0.5\text{ keV}. \tag{5.2}$$

Not only do the two sets of predictions (5.1) and (5.2) differ markedly from our results in Table II, they do not agree between themselves. Clearly our model gives the best result for $\Upsilon' \to \Upsilon\pi\pi$. Further testing of our model will come from future measurements on the other transitions. One should also study and compare the mass distributions of the $\pi\pi$ system in the three transitions $\Upsilon' \to \Upsilon\pi\pi$, $\Upsilon'' \to \Upsilon\pi\pi$, and $\Upsilon'' \to \Upsilon'\pi\pi$.

2. The treatment given in Secs. II and III and that in Sec. IV usually give consistent results except for the transition $\Upsilon'' \to \Upsilon\eta$. (We are not concerned with the discrepancy in another transition $\Upsilon'' \to \Upsilon'\pi\pi$ which is simply due to the pion's mass effect.) The predictions for this η transition by the two methods differ by almost two orders of magnitude. The small rate $\Gamma(\Upsilon'' \to \Upsilon\eta)$ predicted in Sec. III is related to the small rate for $\Upsilon'' \to \Upsilon\pi\pi$. This relation is a consequence of the assumption (3.7). On the other hand, the large rate predicted in Sec. IV is due to the poor representation of η by a two-gluon system. Perhaps the truth lies somewhere between the two extremes. We will need experimental information to help us understand the η transition better.

3. The $b\bar{b}$ system has a richer spectrum than the $c\bar{c}$ system. Consequently, new types of hadronic transitions are possible for the first time. In Sec. IV we have considered two spin-flip $\pi\pi$ transitions: $\Upsilon'' \to 1\,{}^1P_1 + \pi\pi$ and $2\,{}^3P_J \to 1\,{}^1S_0 + \pi\pi$ These processes offer new alternatives to reach the spin-singlet states $1\,{}^1P_1$ and $1\,{}^1S_0$. The theoretically interesting state $1\,{}^1P_1$ is almost impossible to detect by any other means.

4. The rates for the transitions $2\,{}^3P_{J'} \to 1\,{}^3P_J + \pi\pi$ in the $b\bar{b}$ system are disappointingly small. It will not be easy to test the relations among different transitions predicted by the Wigner-Eckart theorem.[5]

5. As the mass of a heavy quark increases, the rates for hadronic transitions decrease rapidly. Hadronic transitions will cease to be dominant decay modes of a very heavy $Q\bar{Q}$ bound state.

Overall, we are encouraged by the agreement of our prediction and the data for $\Upsilon' \to \Upsilon\pi\pi$. This is by no means a trivial result. First of all, the validity of the multipole expansion for the $c\bar{c}$ system is marginal at best. Yet we have to rely on the rate of $\psi' \to \psi\pi\pi$ as input to our calculations. Furthermore, to arrive at a branching ratio, so our result can be compared with the data, we must estimate other partial widths theoretically. These estimates involve theoretical as well as experimental uncertainties. Therefore, the agreement between the model and experiment indicates that our results will provide at least a guide as to what to expect from these transitions.

Note added in proof. Recently, both experimental groups (CLEO and CUSB) at the Cornell Electron Storage Ring have observed the transition $\Upsilon'' \to \Upsilon\pi\pi$. The branching ratio has been measured to be [for details, see talks by D. Schamberger and A. Silverman, in Proceedings of the 1981 International Symposium on Lepton and Photon Physics at High Energies, Bonn, (unpublished)]

$$\frac{\Gamma(\Upsilon'' \to \Upsilon\pi^+\pi^-)}{\Gamma(\Upsilon'' \to \text{all})} = \begin{cases} (3.9 \pm 1.9)\% & \text{(CLEO)} \\ (9.7 \pm 4.3)\% & \text{(CUSB)} \\ (4.8 \pm 1.7)\% & \text{(CLEO and} \\ & \text{CUSB combined).} \end{cases}$$

This is to be compared with our predictions of 1.3%, 1.8%, and 3.4% for models A, B, and C, respectively. Our predictions agree with the CLEO result and the world average, and they compare favorably with the CUSB measurement. Most importantly, the data confirms the theoretical expectation that the rate of this transition should be smaller than that of $\Upsilon' \to \Upsilon\pi\pi$. It therefore lends further support to the general ideas of the multipole expansion as the mechanism for hadronic transitions.

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