Propagation of a neutrino in a homogeneous magnetic field

Gerry McKeon
Department of Applied Mathematics, University of Western Ontario, London, Canada N6A 5B9
(Received 26 June 1981)

A neutrino, although it has no electric charge, can interact with a magnetic field by virtue of self-energy processes in which the virtual particles are charged. In this note, we examine the one-loop correction to the neutrino propagator in the presence of a homogeneous magnetic field in the context of the standard Weinberg-Salam model for leptons, using a technique introduced by Sapirstein.

I. INTRODUCTION

The Weinberg-Salam model of leptons[1] involves the vertices ν-e^--W^+ and ν-e^--ϕ^+. The neutrino (ν) does not couple directly to the electric field, but the electron (e^-), the intermediate vector boson (W^+), and the Higgs boson (ϕ^+) do carry an electric charge. Consequently the virtual processes of Fig. (1) lead to an effective interaction between the neutrino and an external magnetic field, on account of the electric charge carried by the virtual particles. Evaluation of the neutrino propagator in this situation is complicated by the fact that both of the virtual particles carry an electric charge. A method for handling this problem has been developed by Sapirstein[2] in determining the magnetic moment of a quark in a constant colored magnetic field.

We will ignore the effect of the external magnetic field on the spontaneous symmetry breaking.[3]

II. THE WEINBERG-SALAM MODEL IN AN EXTERNAL MAGNETIC FIELD

The Lagrangian for the standard Weinberg-Salam model of leptons[4] has the following parts relevant to our discussion:

$$\mathcal{L}_{\text{gauge}} = -\tfrac{1}{4}F^i_{\mu\nu}F^{i\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu}, \tag{1a}$$

$$\mathcal{L}_{\text{leptons}} = \bar{R}i\gamma^\mu(\partial_\mu + ig'B_\mu)R + \bar{L}i\gamma^\mu\left(\partial_\mu + \frac{i}{2}g'B_\mu - ig\frac{\tau^i}{2}A^i_\mu\right)L, \tag{1b}$$

$$\mathcal{L}_{\text{scalars}} = \left(\partial_\mu\phi^\dagger + \frac{ig'}{2}B_\mu\phi^\dagger + ig'\frac{\tau^i}{2}A^i_\mu\phi^\dagger\right) \times \left(\partial^\mu\phi - \frac{ig'}{2}B^\mu\phi - ig\frac{\tau^i}{2}A^{i\mu}\phi\right) - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2, \tag{1c}$$

$$\mathcal{L}_{\text{inter}} = -G_e(\bar{R}\phi^\dagger L + \bar{L}\phi R), \tag{1d}$$

where

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\epsilon^{ijk}A^j_\mu A^k_\nu, \tag{2a}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{2b}$$

$$L = \tfrac{1}{2}(1-\gamma_5)\begin{pmatrix}\nu\\ e^-\end{pmatrix}, \tag{2c}$$

$$R = \tfrac{1}{2}(1+\gamma_5)e^-, \tag{2d}$$

$$\phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}. \tag{2e}$$

If $\mu^2 < 0$ and $\lambda > 0$, spontaneous symmetry breakdown occurs without destabilizing the system. The field ϕ, in the tree approximation (which we will not try to improve upon), develops a vacuum expectation value

$$\langle\phi\rangle_0 = \begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix} \tag{3}$$

upon a suitable orientation of the internal symmetry space. The vector fields A^i_μ, B_μ develop a mass, except for a component

$$A_\mu = (gB_\mu + g'A^3_\mu)/(g^2+g'^2)^{1/2} \tag{4}$$

which is identified with the photon.

We now assume the existence of an external electromagnetic field

$$F_{\mu\nu} = \partial_\mu\alpha_\nu - \partial_\nu\alpha_\mu, \qquad e\alpha_\mu = gA^3_\mu = g'B_\mu, \tag{5}$$

where $e = gg'/(g^2+g'^2)^{1/2}$ is the electric charge.

It is now possible to identify those parts of the Lagrangian of Eq. (1) that contribute to processes

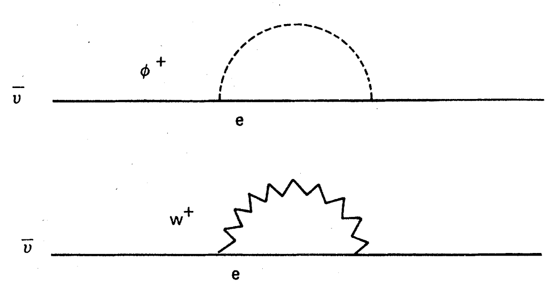

FIG. 1. Diagrams containing charged virtual particles.

involving charged virtual particles. If we set

$$\phi = \langle\phi\rangle_0 + f\,,$$

$$W^{\pm}_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \mp iA^2_\mu)\,,$$

then

$$\mathcal{L}_\phi = (\partial_\mu + ie\alpha_\mu)f^{+*}(\partial^\mu - ie\alpha^\mu)f^+ + \frac{v^2g^2}{4}W^+_\mu W^{-\mu} + \frac{ivg}{2}W^-_\mu(\partial^\mu - ie\alpha^\mu)f^+ - \frac{ivg}{2}W^+_\mu(\partial^\mu + ie\alpha^\mu)f^{+*}\,, \tag{6a}$$

$$\mathcal{L}_\psi = \bar{e}\left(i\gamma^\mu(\partial_\mu + ie\alpha_\mu) - \frac{G_e v}{\sqrt{2}}\right)e\,, \tag{6b}$$

$$\begin{aligned}\mathcal{L}_w = {}& \tfrac{1}{2}[W^-_\nu(\partial_\mu - ie\alpha_\mu)(\partial^\mu - ie\alpha^\mu)W^{+\nu} \\ &- W^-_\mu(\partial^\mu - ie\alpha^\mu)(\partial_\nu - ie\alpha_\nu)W^{+\nu} - 2ieW^-_\mu F^{\mu\nu}W^+_\nu] \\ &+ \tfrac{1}{2}[W^+_\nu(\partial_\mu + ie\alpha_\mu)(\partial^\mu + ie\alpha^\mu)W^{-\nu} \\ &- W^+_\mu(\partial^\mu + ie\alpha^\mu)(\partial_\nu + ie\alpha_\nu)W^{-\nu} + 2ieW^+_\mu F^{\mu\nu}W^-_\nu]\,,\end{aligned} \tag{6c}$$

lead to the appropriate propagators for the charged particles, provided we use an appropriate gauge condition such as

$$\mathcal{L}_g = -\left((\partial_\lambda - ie\alpha_\lambda)W^{+\lambda} - \frac{ivg}{2}f^+\right) \times \left((\partial_\sigma + ie\alpha_\sigma)W^{-\sigma} + \frac{ivg}{2}f^{+*}\right). \tag{7}$$

Vertices arise from the interaction terms

$$\mathcal{L}_I = -G_e\left[\bar{e}f^{+*}\left(\frac{1-\gamma_5}{2}\right)\nu + \bar{\nu}f^+\left(\frac{1+\gamma_5}{2}\right)e\right] + \sqrt{2}g\left[\bar{\nu}W^+\cdot\gamma\left(\frac{1-\gamma_5}{2}\right)e + \bar{e}W^-\cdot\gamma\left(\frac{1-\gamma_5}{2}\right)\nu\right]. \tag{8}$$

III. CALCULATION OF THE NEUTRINO PROPAGATOR

It is now possible to determine the lowest-order corrections to the neutrino propagator induced by an external electromagnetic field.

From Eqs. (6) and (7), the appropriate propagators are

$$S(x',x) = \langle e(x')\bar{e}(x)\rangle = \left\langle x'\left|\frac{1}{\gamma\cdot\Sigma - m}\right|x\right\rangle = \left\langle x'\left|\frac{1}{\Sigma^2 - m^2 - M}(\gamma\cdot\Sigma + m)\right|x\right\rangle, \tag{9a}$$

$$D(x',x) = \langle f^+(x')f^{+*}(x)\rangle = \left\langle x'\left|\frac{1}{\pi^2 - \mu^2}\right|x\right\rangle, \tag{9b}$$

and

$$D_{\mu\nu}(x'x) = \langle W^+_\mu(x')W^-_\nu(x)\rangle = \left\langle x'\left|\frac{-1}{g^{\mu\nu}(\pi^2 - \mu^2) + 2ieF^{\mu\nu}}\right|x\right\rangle, \tag{9c}$$

where $m = G_e v/\sqrt{2}$, $\mu^2 = v^2g^2/4$, $\pi = i\partial + e\alpha$, $\Sigma = i\partial - e\alpha$, and $M = (e/2)\sigma_{\alpha\beta}F^{\alpha\beta}$.

The expression corresponding to the diagrams of Fig. (1) is

$$I = \int dx' \int dx\left[K_1\bar{\nu}(x')\left(\frac{1+\gamma_5}{2}\right)S(x',x)D(x',x)\left(\frac{1-\gamma_5}{2}\right)\nu(x) + K_2\bar{\nu}(x')\left(\frac{1+\gamma_5}{2}\right)\gamma^\mu S(x',x)D_{\mu\nu}(x',x)\gamma^\nu\left(\frac{1-\gamma_5}{2}\right)\nu(x)\right], \tag{10}$$

where $K_1 = (-G_e)^2$ and $K_2 = (\sqrt{2}g)^2$. If we use the identity

$$\frac{1}{a} = -i\int_0^\infty ds\, e^{isa}\,,$$

then Eq. (10) becomes, upon dropping the term linear in m,

$$\begin{aligned}I = \int dx\,dx'\bar{\nu}(x')\Bigg(&K_1\left\langle x'\left|-i\int_0^\infty ds\, e^{-ism^2}\gamma\cdot\Sigma e^{is\Sigma^2}\right|x\right\rangle e^{-isM}\left\langle x'\left|-i\int_0^\infty dt\, e^{-it\mu^2}e^{it\pi^2}\right|x\right\rangle \\ &- K_2\gamma_\mu\left\langle x'\left|-i\int_0^\infty ds\, e^{-ism^2}\gamma\cdot\Sigma e^{is\Sigma^2}\right|x\right\rangle e^{-itM}\left\langle x'\left|-i\int_0^\infty dt\, e^{-2etF_{\mu\nu}}e^{-it\mu^2}e^{it\pi^2}\right|x\right\rangle\gamma_\nu\Bigg)\left(\frac{1-\gamma_5}{2}\right)\nu(x)\,.\end{aligned} \tag{11}$$

Sapirstein[2] has indicated how to evaluate the matrix elements for the case in which $F_{12} = B$. We arrive at the results

$$\begin{aligned}D(x'x) = -i\int_0^\infty \frac{dt}{t^2}e^{-it\mu^2}\frac{-i}{16\pi^2}\frac{eBt}{\sin(eBt)}\exp\left(\frac{i}{4t}[(x'^3 - x^3)^2 - (x'^0 - x^0)^2]\right) \\ \times\exp\left(\frac{ieB}{4\tan(eBt)}[(x'^2 - x^2)^2 + (x'^1 - x^1)^2]\right)\exp\left(\frac{ieB}{2}(x^1 + x'^1)(x^2 - x'^2)\right),\end{aligned} \tag{12a}$$

$$S(x',x)=\frac{-1}{16\pi^2}\int_0^\infty \frac{ds}{s^2}e^{-ism^2}\frac{eBs}{\sin(eBs)}\left(\frac{-1}{2s}\right)$$
$$\times\left([\gamma^0(x'^0-x^0)-\gamma^3(x'^3-x^3)]+\frac{eBs}{\sin(eBs)}e^{isM}[-\gamma^1(x'^1-x^1)-\gamma^2(x'^2-x^2)]\right)e^{-isM}$$
$$\times\exp\left(\frac{i}{4s}[(x'^3-x^3)^2-(x'^0-x^0)^2]\right)\exp\left(\frac{ieB}{4\tan(eBt)}[(x'^2-x^2)^2+(x'^1-x^1)^2]\right)$$
$$\times\exp\left(\frac{-ieB}{2}(x^1+x'^1)(x^2-x'^2)\right). \qquad (12b)$$

[Note the slight difference between the result of Eq. (12b) and Eq. (2.23) of Ref. 2.]

We are interested in fields whose strength is weak (i.e., $eB\ll\mu^2$) and hence we will use the approximations

$$\sin(eBS)\simeq\tan(eBS)\simeq eBS.$$

Equations (11) and (12) thus combine to give

$$I=\int dx\,dx'\bar\nu(x')\left(\frac{1}{16\pi^2}\right)^2$$
$$\times\int_0^\infty\frac{ds}{s^2}\int_0^\infty\frac{dt}{t^2}\left(\frac{-1}{2s}\right)\exp\left(\frac{i}{4w}[(x'^1-x^1)^2+(x'^2-x^2)^2+(x'^3-x^3)^2-(x'^0-x^0)^2]\right)$$
$$\times e^{-it\mu^2}(K_1\{[\gamma^0(x'^0-x^0)-\gamma^3(x'^3-x^3)]+e^{isM}[-\gamma^1(x'^1-x^1)-\gamma^2(x'^2-x^2)]\}e^{-isM}$$
$$-K_2\gamma_\mu\{[\gamma^0(x'^0-x^0)-\gamma^3(x'^3-x^3)]+e^{isM}[-\gamma^1(x'^1-x^1)-\gamma^2(x'^2-x^2)]\})e^{-isM}\gamma_\nu e^{-2etF_{\mu\nu}}\left(\frac{1-\gamma_5}{2}\right)\nu(x), \qquad (13)$$

where

$$1/w=1/s+1/t.$$

Again expanding to first order in eB,

$$e^{isM}\simeq1+ieBs\sigma_3,$$
$$e^{-2etF_{\mu\nu}}\simeq g^{\mu\nu}-2etF^{\mu\nu},$$

we see that Eq. (13) has a term linear in B:

$$\delta I=\int dx\,dx'\bar\nu(x')\left(\frac{1}{16\pi^2}\right)^2\int_0^\infty\frac{ds}{s^2}\int_0^\infty\frac{dt}{t^2}\left(\frac{-1}{2s}\right)e^{-(i/4w)(x'-x)^2}e^{-it\mu^2}$$
$$\times\{(K_1-2K_2)(-ieBs\sigma_3)[\gamma^0(x'^0-x^0)-\gamma^3(x'^3-x^3)]$$
$$+K_2(4ieBt\sigma_3)[\gamma^0(x'^0-x^0)-\gamma^3(x'^3-x^3)]$$
$$+(2eBs)(K_1+2K_2)[\gamma^2(x'^1-x^1)-\gamma^1(x'^2-x^2)]\}\left(\frac{1-\gamma_5}{2}\right)\nu(x). \qquad (14)$$

Equation (12) can be inverted to show that

$$x^\mu e^{-(i/4w)(x'-x)^2}=(16\pi^2 i)(2w^3)\langle x'|(i\partial_\mu)e^{-iw(i\partial)^2}|x\rangle. \qquad (15)$$

Consequently, Eq. (14) becomes

$$\delta I=-\int dx\int dx'\bar\nu(x')\left(\frac{-i}{16\pi^2}\right)\int_0^\infty\frac{ds}{s^2}\int_0^\infty\frac{dt}{t^2}\frac{w^3}{s}e^{-it\mu^2}$$
$$\times\{[(-ieBs\sigma_3)(K_1-2K_2)+(4ieBt\sigma_3)K_2]\langle x'|[\gamma^0(i\partial^0)-\gamma^3(i\partial^3)]e^{iw(i\partial)^2}|x\rangle$$
$$+(2eBs)(K_1+2K_2)\langle x'|[\gamma^2(i\partial^1)-\gamma^1(i\partial^2)]e^{iw(i\partial)^2}|x\rangle\}\left(\frac{1-\gamma_5}{2}\right)\nu(x). \qquad (16)$$

The integrals over x and x' in Eq. (16) can now be evaluated upon introducing a complete set of states $|k\rangle$ such that $\langle x|k\rangle = e^{ik\cdot x}/(2\pi)^2$. Our expression for δI then becomes

$$\delta I = \bar{\nu}(k)\left(\frac{-i}{16\pi^2}\right)\int_0^\infty \frac{ds}{s^2}\int_0^\infty \frac{dt}{t^2}\frac{w^3}{s}e^{-it\mu^2}e^{iwk^2}\{[(-ieBs\sigma_3)(K_1-2K_2)+(4ieBt\sigma_3)K_2](\gamma^0k^0-\gamma^3k^3)$$
$$+(2eBs)(K_1+2K_2)(\gamma^2k^1-\gamma^1k^2)\}\left(\frac{1-\gamma_5}{2}\right)\nu(k)\,.$$

The integrals over s and t can be evaluated upon setting $is=\lambda\nu$ and $it=\lambda(1-\nu)$. If $k^2\simeq 0$, we finally arrive at the expression

$$\delta I = \frac{ieB}{16\pi^2}\bar{\nu}(k)[\tfrac{1}{2}(K_1-6K_2)(\gamma^0k^0-\gamma^3k^3)\sigma_3+i(K_1+2K_2)(\gamma^1k^2-\gamma^2k^1)]\left(\frac{1-\gamma_5}{2}\right)\nu(k)\,. \tag{17}$$

Equation (17) gives the correction to the neutrino propagator to lowest order in B.

The equation of motion of a neutrino in an external magnetic field will effectively be altered from the free field equation of motion $(\gamma\cdot k-m)\nu=0$ as a result of this effective interaction. The phenomenological consequences are currently being examined.

[1]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, *in Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8)*, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968).
[2]J. Sapirstein, Phys Rev. D 20, 3246 (1979).
[3]A. Salam and J. Strathdee, Nucl. Phys. B90, 203 (1975).
[4]E. S. Abers and B. W. Lee, Phys. Rep. 9C, 1 (1973).