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Noncovariance of field theories quantized on the light cone which are not scale invariant

C. R. Hagen
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
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It is shown from spectral representations involving the energy-momentum tensor that in general a field theory quantized on the light cone (as opposed to the case in which an equal-time-quantized theory is merely rewritten in terms of light-cone coordinates) does not possess a Lorentz-invariant vacuum state. The sole exception to this result occurs in cases for which one has scale invariance.

The possibility of using the light-cone formulation of a field theory to study high-energy limits has been recognized for some time. The attractiveness of this approach has been further enhanced by the recent realization that certain two-dimensional field theories become soluble in such coordinates. A well-known example of this is the so-called 't Hooft model.[1] Although it is known[2,3] that strictly canonical (i.e., quantized on the light cone) formulations of such theories[4] yield anomalies in the current divergence which contradict the equations of motion, one can in principle hope that there might be found a current definition which could restore gauge invariance.

In order to examine the feasibility of such a hope, the author has recently proposed that one examine the compatibility of current conservation with the condition that there exist a gauge-invariant vacuum state, i.e.,

$$Q\,|0\rangle = 0\,. \tag{1}$$

Using the Lehmann representation for the current correlation function, it was shown[5] that in fact (1) cannot be satisfied in light-cone coordinates for charges defined as

$$Q = \int dx^- dx_\perp j^+\,,$$

where $dx_\perp = dx_2 dx_3$ and light-cone coordinates are defined by

$$x^\pm = \frac{1}{\sqrt{2}}(x^1 \pm x^0)\,.$$

It is also easy to see that attempts to resolve this dilemma by introducing a smearing function $f(x^-, x_\perp)$ into the definition of Q cannot be successful for any nonvanishing f.

Since it may be possible to avoid the serious consequences of this theorem by using currents which are nonsinglets under the relevant gauge group, there is clearly considerable interest in ascertaining whether general considerations place any additional constraints upon light-cone field theories. It is the purpose of this paper to point out that there is no vacuum which is Lorentz invariant in such a theory unless it is simultaneously scale invariant. Concisely stated, the theorem states the following:

(a) If the energy-momentum tensor has the spectral form for the unordered product

$$\langle 0|T^{\mu\nu}(x)T^{\alpha}_{\alpha}(x')|0\rangle = (\partial^2 g^{\mu\nu} - \partial^\mu\partial^\nu) \times \int_0^\infty d\kappa^2 \sigma(\kappa^2)\Delta^{(+)}(x-x';\kappa^2)\,, \tag{2}$$

where

$$\Delta^{(+)}(x;\kappa^2) = 2\pi \int \frac{dk}{(2\pi)^4}\theta(k_+)\delta(\kappa^2 + k^2)e^{ikx}$$

and

(b) if the generators of Lorentz transformations are given by

$$J^{\mu\nu} = \int (x^\mu T^{\nu +} - x^\nu T^{\mu +})dx^- dx_\perp\,,$$

then there is no Lorentz-invariant vacuum which satisfies

$$J^{+-}|0\rangle = 0$$

unless

$$T^\mu_\mu = 0\,.$$

The proof is easily effected by taking $x^+ = 0$ and obtaining from (2)

$$\langle 0|[J^{+-}, T^{\alpha}_{\alpha}(x')]|0\rangle = \int dx^- dx_\perp x^- \partial_-^{\,2} \int_0^\infty d\kappa^2 \sigma(\kappa^2)\left[\frac{i}{4}\epsilon(x^- - x^{-\prime})\delta(x_\perp - x_\perp')\right], \tag{3}$$

where the term in square brackets is obtained by straightforward application of (2). From (3) there follows that

$$\langle 0|[J^{+-}, T^{\alpha}_{\alpha}(x')]|0\rangle = -\frac{i}{2}\int_0^\infty d\kappa^2\sigma(\kappa^2)\ ,$$

which, in view of the positivity of the weight function $\sigma(\kappa^2)$ (except in the scale-invariant case $T^\mu_\mu = 0$) establishes the claimed result. Since the theories of massless noninteracting scalar and spinor fields in two dimensions are known to be consistent as well as obviously scale invariant, no significant weakening of this condition on T^μ_μ is to be expected. It is to be emphasized that the convergence of the integral over κ^2 is not relevant to this conclusion as one could as easily project out intermediate states of fixed invariant mass, obtaining a similar contradiction of the Lorentz invariance of the vacuum.

In sum, the result obtained here displays in yet another way the significant differences which exist between ordinary field theories and those quantized on the light cone. It has been shown[6] that the mass spectrum of the 't Hooft model at least in the charged sectors does not coincide with that obtained in spacelike quantization. This work indicates that the very concept of a vacuum state (which in turn allows the definition of single-particle states) is generally absent in a canonical light-cone theory. Such theories must consequently require at the least a very different interpretation than that which is currently used or a significant departure from the usual canonical formalism.

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