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Effect of curvature-squared terms on cosmology

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We consider the effect on Friedmann cosmology of adding all possible algebraic terms quadratic in the curvature tensor to the usual Einstein-Hilbert action for gravitation. Assuming that the vacuum energy density (cosmological constant) is not extraordinarily large in any phase of the evolution of the universe, we find that the cosmological solutions of this more general theory are indistinguishable from standard Friedmann models all the way from the initial singularity to the present.

In quantum gravity it has become fashionable recently to consider Lagrangians constructed from curvature-squared terms in addition to the usual term linear in the scalar curvature R. The coupling coefficients appearing in these Lagrangians can be chosen so that the resulting gravitational theories possess several desirable features: renormalizability,[1] asymptotic freedom,[2] and a Euclidean functional integral convergent on the metric conformal factor.[3] It is consequently of interest to compare classical solutions of these more general theories to the usual solutions. In this paper we consider Friedmann cosmologies. These are of particular interest because of the importance of the early universe in grand unified theories of elementary particles,[4] and, of course, only near a singularity (such as that at $t \simeq 0$) should we expect to see deviations from Einstein's theory. This is because a typical quadratic action density, R^2, is comparable to the Einstein-Hilbert action density $(16\pi G)^{-1}R$ only in a region of extreme curvature where $R \gtrsim (16\pi G)^{-1}$ (taking $c=\hbar=1$).

The most general algebraic gravitational action S_G which is at most quadratic in the curvature tensor can conveniently be written as[5,6]

$$S_G = -\int d^4x\sqrt{-g}\,[\tfrac{1}{2}\alpha R^2 + \tfrac{1}{2}\beta C_{\mu\nu\sigma\tau}C^{\mu\nu\sigma\tau} + (16\pi G)^{-1}R], \tag{1}$$

where $C_{\mu\nu\sigma\tau}$ is the Weyl conformal curvature tensor[7] and α, β are dimensionless constants. We have not included an explicit cosmological constant term in (1) since such a term may instead be incorporated into the matter action.

Isotropic, homogeneous cosmological models are based on the Robertson-Walker metric[8]:

$$ds^2 = dt^2 - \Omega^2(t)(1-kr^2)^{-1}dr^2 - \Omega^2(t)\,r^2d\theta^2 - \Omega^2(t)\,r^2\sin^2\theta\, d\phi^2, \tag{2}$$

where t is the cosmic time, Ω is the (dimensionless) cosmic scale factor, normalized to unity at the present, and $k^{-1/2}$ is the present radius of curvature of three-space. If we introduce "conformal time" $\tau = \int dt\,\Omega^{-1}(t)$, Eq. (2) becomes

$$ds^2 = \Omega^2(\tau)[d\tau^2 - (1-kr^2)^{-1}dr^2 - r^2d\theta^2 - r^2\sin^2\theta\, d\phi^2]. \tag{3}$$

In these coordinates, the entire dynamics of the gravitational field is carried in the metric conformal factor Ω.

If a metric $g_{\mu\nu}$ is a conformal rescaling of another metric $\bar{g}_{\mu\nu}$, $g_{\mu\nu} = \Omega^2\bar{g}_{\mu\nu}$, then the associated curvature scalars $R(g)$ and $R(\bar{g})$ are related by $R(g) = \Omega^{-2}R(\bar{g}) - 6\,\Omega^{-3}\bar{\Box}\Omega$, where $\bar{\Box}$ is the covariant d'Alembertian operator formed from $\bar{g}_{\mu\nu}$. The Weyl tensor scales under a conformal transformation, $C_{\mu\nu\sigma\tau}(g) = \Omega^2 C_{\mu\nu\sigma\tau}(\bar{g})$. Applying these relations to the Robertson-Walker metric (3),

$$g_{\mu\nu} = \Omega^2(\tau)\bar{g}_{\mu\nu}, \tag{4a}$$

$$\bar{g}_{\mu\nu} = \mathrm{diag}[1, -(1-kr^2)^{-1}, -r^2, -r^2\sin^2\theta], \tag{4b}$$

$$\sqrt{-g} = \Omega^4 r^2 \sin\theta (1-kr^2)^{-1/2}, \tag{4c}$$

$$R(\bar{g}) = -6k, \tag{4d}$$

$$R(g) = -6(k\Omega^{-2} + \Omega^{-3}\ddot{\Omega}), \tag{4e}$$

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$$C_{\mu\nu\sigma\tau}(\bar{g})=0\,, \tag{4f}$$

where a dot means $d/d\tau$. It is a remarkable fact that relations (4c), (4e), and (4f) can be inserted directly into the action (1); the equations of motion for Ω are still the same, but the action simplifies greatly:

$$S_G(\Omega)=-V_3\int d\tau\,\{18\alpha[k^2+2k\Omega^{-1}\ddot{\Omega}+(\Omega^{-1}\ddot{\Omega})^2] - 3(8\pi G)^{-1}[k\Omega^2-(\dot{\Omega})^2]\}\,, \tag{5}$$

where we have integrated by parts and discarded a boundary term to transform $\Omega\ddot{\Omega}$ to $-(\dot{\Omega})^2$, and where V_3 is the present volume of three-space:

$$V_3\equiv\int dr\,d\theta\,d\phi\,r^2(1-kr^2)^{-1/2}\sin\theta=\begin{cases}2\pi^2k^{-3/2} & (k>0)\,,\\ \infty & (k\leq 0)\,.\end{cases} \tag{6}$$

In fact, splitting $g_{\mu\nu}$ into Ω and $\bar{g}_{\mu\nu}$ (where $\det\bar{g}_{\mu\nu}$ is a specified function) always preserves the equations of motion and has equivalent action for any spacetime dependence of Ω. If $\bar{g}_{\mu\nu}$ is fixed *a priori*, the theory is called scalar gravity [see Eq. (8) of Ref. 3 for the general formula].

Since all dependence on β has disappeared from (5) by virtue of the conformal flatness (4f) of the Robertson-Walker metric, we conclude that *any isotropic, homogeneous cosmology which satisfies the field equations arising from (1) or (5) is independent of* β. (More generally, any geometry for which $\bar{C}_{\mu\nu\sigma\tau}\bar{C}^{\mu\nu\sigma\tau}$ vanishes will not depend on β.)

We add to S_G the matter action S_M describing the material content of the universe. The energy-momentum tensor $T^{\mu\nu}$ of matter, defined by

$$\frac{\delta S_M}{\delta g_{\mu\nu}}=-\tfrac{1}{2}\sqrt{-g}\,T^{\mu\nu}\,, \tag{7}$$

acts as the source of the gravitational field and is covariantly conserved ,

$$T^{\mu\nu}{}_{;\nu}=0\,. \tag{8}$$

For the Robertson-Walker metric only the conformal factor Ω is dynamical, and (7) simplifies to

$$\frac{\delta S_M}{\delta\Omega}=-\Omega^3\sqrt{-\bar{g}}\,T\,, \tag{9}$$

where $T\equiv g_{\mu\nu}T^{\mu\nu}$ is the trace of the energy-momentum tensor.

For the purposes of cosmology, the matter content of the universe is assumed to be a perfect fluid of density ρ and pressure P, for which

$$T^\mu_\nu=\mathrm{diag}(\rho,-P,-P,-P)\,. \tag{10}$$

The energy conservation equation (8) becomes

$$\frac{d}{d\Omega}(\rho\Omega^3)+3P\Omega^2=0\,. \tag{11}$$

To supplement (10) and (11) we must give the equation of state relating pressure and density:

$$P=P(\rho)\,. \tag{12}$$

There are three principal fluids of interest in cosmology: one, a cold, pressureless dust appropriate to the matter-dominated era, two, radiation, consisting of massless or ultrarelativistic massive particles, believed to characterize the early universe, and, three, the "vacuum fluid" of constant density and pressure. This third case has traditionally been interpreted as describing an empty universe with a nonvanishing cosmological constant. More recently, it has been suggested[9] that if spontaneously broken gauge theories undergo a symmetry-restoring phase transition in the early universe, a large vacuum energy density should appear, possibly dominating the evolution of the universe for a brief period.[10]

Substituting the equation of state (12) for each fluid in the conservation equation (11), we may solve for the density and pressure as functions of Ω, and thereby determine via (10) the trace $T(\Omega)$ of T^μ_ν in each of the three cases:

$$P_M=0\quad\text{(matter)}\,, \tag{13a}$$

$$P_R=\rho_R/3\quad\text{(radiation)}\,, \tag{13b}$$

$$P_V=-\rho_V\quad\text{(vacuum)}\,, \tag{13c}$$

$$\rho_M(\Omega)=\rho_{0M}\Omega^{-3}\quad\text{(matter)}\,, \tag{14a}$$

$$\rho_R(\Omega)=\rho_{0R}\Omega^{-4}\quad\text{(radiation)}\,, \tag{14b}$$

$$\rho_V(\Omega)=\rho_{0V}\quad\text{(vacuum)}\,, \tag{14c}$$

$$T_M(\Omega)=\rho_{0M}\Omega^{-3}\quad\text{(matter)}\,, \tag{15a}$$

$$T_R(\Omega)=0\quad\text{(radiation)}\,, \tag{15b}$$

$$T_V(\Omega)=4\rho_{0V}\quad\text{(vacuum)}\,, \tag{15c}$$

where ρ_0 is the density when $\Omega=1$. In all three cases T is independent of time derivatives of Ω, so (9) may be integrated directly:

$$S_M(\Omega)=-V_3\int d\tau\int_0^\Omega d\omega\,\omega^3T(\omega)\,, \tag{16}$$

where ω is an integration variable. In particular

$$S_M=-V_3\times\begin{cases}\rho_{0M}\int d\tau\,\Omega\quad\text{(matter)}\,, & (17a)\\ 0\quad\text{(radiation)}\,, & (17b)\\ \rho_{0V}\int d\tau\,\Omega^4\quad\text{(vacuum)}\,. & (17c)\end{cases}$$

Equation (17b) seems to suggest that the radiation content of the universe is irrelevant to its evolution. This apparent paradox is resolved below.

Combining (5) with (16) we arrive at the total action $S_T(\Omega)$:

$$\begin{aligned}S_T(\Omega) &\equiv S_G(\Omega)+S_M(\Omega)\\ &=-V_3\int d\tau\left\{18\alpha[k^2+2k\Omega^{-1}\ddot{\Omega}+(\Omega^{-1}\ddot{\Omega})^2]-3(8\pi G)^{-1}[k\Omega^2-(\dot{\Omega})^2]+\int_0^{\Omega}d\omega\,\omega^3T(\omega)\right\}\\ &\equiv -V_3\int d\tau\,L(\Omega,\dot{\Omega},\ddot{\Omega}).\end{aligned} \tag{18}$$

Evidently, L is a one-dimensional, higher-derivative Lagrangian. Varying Ω in (18) yields a fourth-order equation of motion. However, because L has no explicit time dependence, by analogy to ordinary mechanics, there exists a constant of the motion (energy), $V_3\tilde{\rho}$, which depends on derivatives of Ω no higher than third. The constant density $\tilde{\rho}$ is easily found to be

$$\begin{aligned}\tilde{\rho} &= \left[\frac{\partial L}{\partial\dot{\Omega}}-\left(\frac{\partial L}{\partial\ddot{\Omega}}\right)^{\cdot}\right]\dot{\Omega}+\frac{\partial L}{\partial\ddot{\Omega}}\ddot{\Omega}-L\\ &=18\alpha[-k^2+(\Omega^{-1}\ddot{\Omega})^2+2k(\Omega^{-1}\dot{\Omega})^2-2(\Omega^{-2}\ddot{\Omega})^{\cdot}\,\dot{\Omega}]\\ &\quad+3(8\pi G)^{-1}(k\Omega^2+\dot{\Omega}^2)-\int_0^{\Omega}d\omega\,\omega^3T(\omega).\end{aligned} \tag{19}$$

Note that the kinetic contribution to the energy $V_3\tilde{\rho}$ is not positive semidefinite, which is characteristic of higher-derivative actions. From (15),

$$\int_0^{\Omega}d\omega\,\omega^3T(\omega)=\begin{cases}\rho_{0M}\Omega & \text{(matter)},\\ 0 & \text{(radiation)},\\ \rho_{0V}\Omega^4 & \text{(vacuum)}.\end{cases} \tag{20}$$

Setting $\alpha=0$ and using (20), Eq. (19) can be written

$$\Omega^{-4}(k\Omega^2+\dot{\Omega}^2)=\frac{8\pi G}{3}\times\begin{cases}\rho_{0M}\Omega^{-3}+\tilde{\rho}\Omega^{-4} & \text{(matter)},\\ \tilde{\rho}\Omega^{-4} & \text{(radiation)},\\ \rho_{0V}+\tilde{\rho}\Omega^{-4} & \text{(vacuum)}.\end{cases} \tag{21}$$

This is just the first-order Friedmann cosmology equation of standard general relativity, the 00 component of the Einstein field equations, which requires us to identify the constant of the motion $\tilde{\rho}$ with the radiation density constant ρ_{0R}. Radiation thereby contributes to the equation of motion for Ω.

When $\alpha\neq 0$, we seek solutions of the generalized Friedmann Eq. (19) which can be rewritten as

$$\Omega^{-4}(\dot{\Omega}^2+k\Omega^2)+48\pi G\alpha\Omega^{-4}[-k^2+(\Omega^{-1}\ddot{\Omega})^2+2k(\Omega^{-1}\dot{\Omega})^2-2(\Omega^{-2}\ddot{\Omega})^{\cdot}\,\dot{\Omega}]=\frac{8\pi G}{3}\rho, \tag{22}$$

where ρ is the total density of matter, radiation, and vacuum energy. For a universe predominantly filled[11] by radiation and/or vacuum energy, $\rho=\rho_{0R}\Omega^{-4}+\rho_{0V}$, where ρ_{0R} and ρ_{0V} are, respectively, the radiation and vacuum densities when $\Omega=1$ and where the solution Ω of Eq. (22) is a function of τ, k, ρ_{0R}, ρ_{0V}, and α; i.e., $\Omega=\Omega(\tau,k,\rho_{0R},\rho_{0V},\alpha)$. To proceed, we set $\alpha=0$ in (22) and replace ρ_{0R} by a different constant $\hat{\rho}_{0R}$, which results in the ordinary Friedmann equation

$$\Omega_F^{-4}(\dot{\Omega}_F^{\,2}+k\Omega_F^{\,2})=\frac{8\pi G}{3}(\hat{\rho}_{0R}\Omega_F^{-4}+\rho_{0V}) \tag{23}$$

with solution $\Omega_F\equiv\Omega(\tau,k,\hat{\rho}_{0R},\rho_{0V},0)$. By substituting Ω_F into the full equation (22), and using the fact that Ω_F satisfies (23), we verify the remarkable result that Ω_F is also an exact solution of the generalized Friedmann equation provided that

$$\hat{\rho}_{0R}=\rho_{0R}[1-8(8\pi G)^2\alpha\rho_{0V}]^{-1}. \tag{24}$$

That is,

$$\begin{aligned}\Omega(\tau,k,\rho_{0R},\rho_{0V},\alpha)&=\Omega_F\\ &=\Omega(\tau,k,\rho_{0R}[1-8(8\pi G)^2\alpha\rho_{0V}]^{-1},\rho_{0V},0).\end{aligned} \tag{25}$$

Therefore, *any isotropic, homogeneous cosmology which satisfies the standard Friedmann equation for a radiation and/or vacuum energy-dominated universe is also an exact solution of the generalized Friedmann equation (22) arising from the action (1) after rescaling the radiation density in accordance with (24).*[12] Note that if $\rho_{0V}=0$ (pure radiation) or $\rho_{0R}=0$ (pure vacuum energy), no rescaling is required and the Friedmann solutions directly satisfy (22). These "pure" solutions are independent of α.

It is easy to see that the rescaling of the radiation energy density implied by (24) must hold in spacetimes much more general than the Friedmann cosmologies. The field equations arising from the action (1) with $\beta=0$ for an arbitrary spacetime filled with radiation and/or vacuum energy are

$$T_{R\mu\nu}+\rho_{0V}g_{\mu\nu}=(8\pi G)^{-1}G_{\mu\nu}+\alpha(2R_{;\mu;\nu}-2g_{\mu\nu}\Box R+2RR_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R^2)\,, \qquad (26)$$

where $G_{\mu\nu}\equiv R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R$ is the Einstein tensor. If $\alpha=0$, the trace of (26) is

$$R=-32\pi G\rho_{0V}\,, \qquad (27)$$

since $T_R=0$. Because R is constant, the full field equations (26) with $\alpha\neq 0$ reduce to

$$T_{R\mu\nu}+\rho_{0V}g_{\mu\nu}=(8\pi G)^{-1}G_{\mu\nu}+\alpha(2RR_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R^2)=(8\pi G)^{-1}(1+16\pi G\alpha R)G_{\mu\nu}+\tfrac{1}{2}\alpha g_{\mu\nu}R^2\,. \qquad (28)$$

Rearrangement of (28) and division by $1+16\pi G\alpha R$ yields

$$(8\pi G)^{-1}G_{\mu\nu}=(1+16\pi G\alpha R)^{-1}T_{R\mu\nu}+(1+16\pi G\alpha R)^{-1}(\rho_{0V}-\tfrac{1}{2}\alpha R^2)\,. \qquad (29a)$$

Substituting (27) into (29), we have finally

$$(8\pi G)^{-1}G_{\mu\nu}=[1-8(8\pi G)^2\alpha\rho_{0V}]^{-1}T_{R\mu\nu}+\rho_{0V}g_{\mu\nu} \qquad (29b)$$

which is just the ordinary Einstein equation with a rescaled energy-momentum tensor $\hat{T}_{R\mu\nu}$ defined by the analog of Eq. (24):

$$\hat{T}_{R\mu\nu}=T_{R\mu\nu}[1-8(8\pi G)^2\alpha\rho_{0V}]^{-1}\,. \qquad (30)$$

To establish limits on α (since β is irrelevant to cosmology), we consider the nonrelativistic, static, weak-field limit of the field equations arising from the action (3) when $\beta=0$. In this limit, the gravitational potential $V(r)$ due to a point mass M at $r=0$ takes the form[13]

$$V(r)=\frac{-GM}{r}-\frac{GM}{3r}\exp[-(48\pi G\alpha)^{-1/2}r]\,. \qquad (31)$$

$V(r)$ has a well-behaved Newtonian limit containing a Yukawa term $e^{-\mu r}/r$ if $\alpha\geq 0$. Taking $\alpha<0$ yields the oscillatory term $e^{i|\mu|r}/r$, most likely indicating the unacceptable presence of tachyons in the underlying quantum theory. In order that the Yukawa contribution to $V(r)$ not spoil agreement with Newtonian gravitation for distances $\gtrsim 1$ cm, we have the limits

$$0\leq\alpha\lesssim 10^{64}\,. \qquad (32)$$

By (24), the effective radiation density $\hat{\rho}_{0R}$ becomes infinite and changes sign when $\rho_{0V}=[8(8\pi G)^2\alpha]^{-1}\approx 10^{98}\alpha^{-1}$ g cm^{-3}, or, using (32), when $\rho_{0V}\gtrsim 10^{34}$ g cm^{-3}. This vacuum energy density is at least 10^{13} times larger than the densities considered in Refs. 9 and 10. Moreover, if $\alpha\approx 1$, as is plausible by analogy to gauge theories, ρ_{0V} would have to reach the Planck density (10^{94} g cm^{-3}) to significantly rescale ρ_{0R}. Barring such extremely large vacuum densities, we conclude from (24) that $\hat{\rho}_{0R}\approx\rho_{0R}$ and thus from (25) that Ω is indistinguishable from the usual Friedmann solution. In particular, for $\rho_{0V}\ll 10^{98}\alpha^{-1}$ g cm^{-3} the presence of $\frac{1}{2}\alpha R^2+\frac{1}{2}\beta C_{\mu\nu\sigma\tau}C^{\mu\nu\sigma\tau}$ in the action (1) cannot (classically) halt the collapse of a contracting homogeneous, isotropic universe to a singularity.[14]

The recent history of the universe ($\Omega\gtrsim 10^{-4}$) appears to have been matter dominated, $\rho=\rho_{0M}\Omega^{-3}$, in which case (22) can be written

$$\Omega^{-4}(\dot{\Omega}^2+k\Omega^2)=\frac{8\pi G}{3}\Omega^{-3}\{\rho_{0M}-18\alpha\Omega^{-1}[-k^2+(\Omega^{-1}\ddot{\Omega})^2+2k(\Omega^{-1}\dot{\Omega})^2-2(\Omega^{-2}\ddot{\Omega})^{\cdot}\dot{\Omega}]\}\,. \qquad (33)$$

While we have not discovered an exact solution of (33), the second term on the right can be viewed as a correction to the matter density constant ρ_{0M}. At the present epoch, $\Omega=1$, $\dot{\Omega}=H\approx 10^{-28}$ cm^{-1} $\approx 10^{-61}$, where H is the Hubble parameter, equal to 10^{-61} in Planck units where $G=c=\hbar=1$. In these units we can also estimate $\ddot{\Omega}\lesssim H^2\approx 10^{-122}$, $\Omega\lesssim H^3\approx 10^{-183}$, $k\lesssim H^2\approx 10^{-122}$. Since $\alpha\lesssim 10^{64}$, we find the correction to ρ_{0M} is

$$18\alpha\Omega^{-1}[-k^2+(\Omega^{-1}\ddot{\Omega})^2+2k(\Omega^{-1}\dot{\Omega})^2-2(\Omega^{-2}\ddot{\Omega})^{\cdot}\dot{\Omega}]\lesssim 72\alpha H^4\approx 10^{-178}\,. \qquad (34)$$

Because $\rho_{0M}\approx 10^{-29}$ g cm$^{-3}\approx 10^{-123}$, the correction (34) is completely negligible and at the present the ordinary Friedmann equation is applicable. To see that this conclusion holds throughout the matter-dominated era, we resort to an approximate solution. For Ω small, we can ignore k, simplifying the ordinary Friedmann equation to

$$\Omega_F^{-4}\dot{\Omega}_F^{\,2}=\frac{8\pi G}{3}\rho_{0M}\Omega_F^{-3}\,, \qquad (35)$$

with solution $\Omega_F(\tau)=2\pi G\rho_{0M}\tau^2/3$. We therefore attempt a solution of the full equation (33) (with $k=0$) by

$$\Omega(\tau)\approx\frac{2\pi G\rho_{0M}\tau^2}{3}+\epsilon(\tau)\,, \qquad (36)$$

where $\epsilon\ll 1$. Inserting (36) into (33), setting $k=0$, and keeping only those terms which are both first order ϵ and dominant as $\tau\to\infty$, we find

$$\Omega(\tau) \approx \frac{2\pi G\rho_{0M}\tau^2}{3} + \frac{648\alpha}{5\rho_{0M}\tau^4}. \quad (37)$$

The correction to the ordinary Friedmann solution is significant only for times $\tau \lesssim (648\alpha/5\rho_{0M})^{1/4} \approx 10^{47} \approx 10^4$ sec, which is long before matter dominance begins. *A solution of the generalized Friedmann equation (33) appropriate to a matter-dominated universe is approximated within negligible error by a matter-dominated solution of the ordinary Friedmann equation.*

For $\Omega \ll 1$, the universe should be radiation dominated. It may nevertheless contain a small amount of nonrelativistic matter (e.g., heavy monopoles[15]). We now verify that such a matter contamination cannot qualitatively alter our previous conclusions. We have previously established that for pure radiation, $\Omega = \Omega_F$, where Ω_F is a solution of the ordinary Friedmann equation (with $k = 0$ since $\Omega \ll 1$):

$$\Omega_F^{-4}\dot{\Omega}_F^{\,2} = \frac{8\pi G\rho_{0R}}{3}\,\Omega_F^{-4}. \quad (38)$$

The solution of (38) is $\Omega_F = (8\pi G\rho_{0R}/3)^{1/2}\tau$, so for the full theory with some matter present, we write the solution as

$$\Omega(\tau) \approx \left(\frac{8\pi G\rho_{0R}}{3}\right)^{1/2}\tau + \epsilon(\tau), \quad (39)$$

where again $\epsilon \ll 1$. Substituting (39) into the generalized Friedmann equation (22), with $k = 0$ and $\rho = \rho_{0R}\Omega^{-4} + \rho_{0M}\Omega^{-3}$, and keeping only terms first order in ϵ and dominant as $\tau \to 0$, we find

$$\Omega(\tau) \approx \left(\frac{8\pi G\rho_{0R}}{3}\right)^{1/2}\tau + \frac{\pi G\rho_{0M}\,\rho_{0R}}{810\alpha}\tau^6. \quad (40)$$

The correction term is rapidly vanishing as $\tau \to 0$, so that at early enough times the matter contamination is indeed inconsequential.

We conclude that, barring fantastically large vacuum energy densities, the evolution of an isotropic, homogeneous universe for which the gravitational action is (1), cannot be distinguished from that of the standard Friedmann models. Including higher and higher powers of the curvature tensor and its derivatives in the gravitational action (view the Lagrangian as a scalar formed from a power series in $R_{\mu\nu\sigma\tau}$ and its derivatives) necessitates the introduction of higher and higher powers of dimensional constants. We have seen that the effect on Friedmann cosmology of the second-order terms is negligible; we expect the effects due to third- and higher-order terms to be smaller still. None of these higher-derivative theories can then substantially deviate from the standard cosmology. Since the Friedmann solutions also arise in Newtonian gravitation,[16] which is the nonrelativistic limit of Einstein's theory, this demonstrates the universality of the Friedmann models across a wide class of possible gravitational theories.[17]

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[1]K. S. Stelle, Phys. Rev. D 16, 953 (1977).

[2]E. Tomboulis, Phys. Lett. 97B, 77 (1980).

[3]K. I. Macrae, Phys. Lett. 103B, 103 (1980). Quadratic terms are also natural from the viewpoint of gauge theories. See L. N. Chang, K. I. Macrae, and F. Mansouri, Phys. Lett. 57B, 59 (1975); Phys. Rev. D 13, 235 (1976); K. I. Macrae, *ibid.* 18, 3737 (1978); 18, 3761 (1978); 18, 3777 (1978), and the references therein.

[4]J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Report No. TH. 2858-CERN, 1980 (unpublished).

[5]Our curvature conventions are

$$R^{\mu}{}_{\nu\sigma\tau} \equiv \partial_\sigma \Gamma^{\mu}{}_{\tau\nu} - \partial_\tau \Gamma^{\mu}{}_{\sigma\nu} + \Gamma^{\mu}{}_{\sigma\lambda}\Gamma^{\lambda}{}_{\tau\nu} - \Gamma^{\mu}{}_{\tau\lambda}\Gamma^{\lambda}{}_{\sigma\nu},$$
$$R_{\nu\tau} \equiv R^{\mu}{}_{\nu\mu\tau}.$$

[6]Only two of a possible three curvature-squared terms are required in S_G because of the Gauss-Bonnet theorem in four dimensions. Terms such as $(\Box R)^2$ are quadratic but not "algebraic" since extra derivatives are included.

[7]S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972), p. 145.

[8]Reference 7, p. 412.

[9]S. A. Bludman and M. A. Ruderman, Phys. Rev. Lett. 38, 255 (1977).

[10]E. W. Kolb and S. Wolfram, Astrophys. J. 239, 428 (1980).

[11]We say that a universe is "dominated" by a particular fluid if that fluid contributes by far the largest portion to the total contents of the universe. In that event, we take the density ρ appearing in the evolution equation for Ω to be the density of the dominant fluid alone, and solve for Ω in this "pure fluid" case. The implicit assumption is that this solution is a good approximation to the true situation when all fluids are present; that is, we assume that Ω is stable against small perturbations in the contents of the universe. Below we explicitly verify the validity of this assumption for the case of a radiation-filled universe contaminated by a small amount of nonrelativistic matter.

[12]Since (22) is a third-order ordinary differential equation, we expect three distinct solutions. We consider

(25) to be the physical solution because it goes over smoothly to the standard Friedmann solution as $\alpha \to 0$. Presumably, the other two solutions of (22) do not share this property.

[13]K. S. Stelle, Gen. Relativ. Gravit. 9, 353 (1978).

[14]In making these estimates, we have assumed that G has its present value 2.61×10^{-66} cm^2. If gauge symmetries are broken by the existence of scalar Higgs fields ϕ_i, then in curved spacetime we expect the normal Higgs scalar Lagrangian to be augmented by the nonminimal term $\sum_i \eta_i \phi_i^2 \sqrt{-g} R$, where the η_i are dimensionless constants [see, e.g., L. S. Brown and J. C. Collins, Ann. Phys. (N.Y.) 130, 215 (1980) and T. S. Bunch, *ibid.* 131, 118 (1981)]. Then, the effective gravitational constant $\hat{G}$ in any constant Higgs phase would be given by $\hat{G}^{-1} = G_0^{-1} + 16\pi \sum_i \eta_i \phi_{i0}^2$, where ϕ_{i0} is the vacuum expectation value of ϕ_i and G_0 is whatever gravitational constant exists in the completely symmetric phase. As the symmetries are restored in the various phases, $\hat{G}$ changes. There can also be changes in $\hat{G}$ due to quantum loop effects, and finally, there can be alterations in both α and β. See the forthcoming paper of K. Macrae and R. Riegert.

[15]J. Ellis, T. K. Gaisser, and G. Steigman, Nucl. Phys. B177, 427 (1981).

[16]Reference 7, p. 474.

[17]One can obtain non-Friedmann cosmologies by introducing torsion into the theory. See A. V. Minkevich, Phys. Lett. 80A, 232 (1980).