Pseudoscalar glueball, the axial-vector anomaly, and the mixing problem for pseudoscalar mesons

C. Rosenzweig, A. Salomone, and J. Schechter
Physics Department, Syracuse University, Syracuse, New York, 13210
(Received 13 July 1981)

If the $G(1440)$ observed in $\psi \rightarrow \gamma G$ is a pseudoscalar glueball its relationship with other pseudoscalar mesons must be understood. We present a simple, unified picture of these mesons in which there must be mixing between glue matter and quark matter. Our model, an extension of an effective Lagrangian which solved the U(1) problem by incorporating the axial-vector anomaly, dictates a relationship between η' and G. We are readily able to explain why the quark-matter meson η' is at least as prominent as the glueball G in the gluon-dominated reaction $\psi \rightarrow \gamma X$.

At long last a viable candidate for the much-sought-after glueball has emerged. The ψ meson is observed to decay via photon emission into a state, the $G(1440)$.[1] Mounting speculation[2-6] asserts that this state is in fact a glueball with pseudoscalar quantum numbers. For the purposes of this paper we accept this speculation and consider how such a meson fits in with the generally accepted picture of the 0^- meson spectrum.

It is interesting that the first discovered glueball should appear in the pseudoscalar sector since this has been one of the most mysterious parts of the meson spectrum. Constituent models, while notably successful for the vast majority of hadron states, have been less successful when it comes to describing the Goldstone nature of the pseudoscalar multiplet. There is a related problem with single-octet mixing. In constituent models ideal mixing is natural if not inevitable. The rather nonideal mixing between the η and η' is a problem.

There exists a complimentary picture of low-energy quantum chromodynamics (QCD) given by an effective chiral Lagrangian.[7] In this picture the underlying chiral symmetry is made manifest and the current-algebra results are reproduced in a simple way. Recently, following an initial suggestion of Witten,[8] an effective Lagrangian has been constructed in which the traditional difficulty of such schemes—the "U(1) problem" (crudely: why is the η' meson so much more massive than the π and K mesons?)—has been overcome.[9-11] In this model the anomalous conservation law of the axial-vector baryon "matter" current J_μ^5

$$\partial_\mu J_\mu^5 = \partial_\mu K_\mu = \frac{3g^2}{16\pi^2} F_{\mu\nu}\tilde{F}_{\mu\nu} \ , \qquad (1)$$

holds automatically. K_μ represents a well-known combination of the *fundamental* Yang-Mills "glue" fields. An interesting feature of this model is that the glue-field combination $\partial_\mu K_\mu$ is dominated by (or "dual to") the η' matter field. The model predicts that while the η' behaves and mixes as an SU(3)-singlet quark-antiquark combination it has a special gluonic side to its character which accounts for its relatively large mass. From the latter point of view it appears that the η' is in fact *the* low-energy 0^- glueball. At first glance this would appear to be in contradiction with the presumption that the new $G(1440)$ is *the* low-energy 0^- glueball. However we shall show here that there is no contradiction. The $G(1440)$ may be simply incorporated into the effective Lagrangian. What is more interesting is that one very naturally accounts for the somewhat peculiar pattern of η, η', and $G(1440)$ production in the cascade decays of the ψ.

The existence of a pseudoscalar glueball also presents a challenge to constituent models. To see what the problem might be we review the motivation for predicting physical glue states in QCD. Local color-singlet operators built from products of quark fields, e.g., $\bar{q}_i\gamma_5 q_j$, produce hadrons when applied to the vacuum. Similarly we expect local color-singlet operators built of products of glue fields to create glueballs when applied to the vacuum. A pseudoscalar glueball would arise from the operator $F_{\mu\nu}\tilde{F}_{\mu\nu}$. This operator is a total divergence [see Eq. (1)] so its role as a generator of a physical state may be suspect.

Constituent models do not know about instantons, axial-vector baryon currents, or the U(1) problem. They readily predict a pseudoscalar glueball. The best of them even correctly predict the mass[4] but unless we have some model which incorporates the unique properties of $F_{\mu\nu}\tilde{F}^{\mu\nu}$ and is compatible with the constituent models we must proceed with caution. We construct such a model by recalling the original effective Lagrangian:

$$\mathcal{L} = -\tfrac{1}{2}\,\mathrm{Tr}(\partial_\mu M \partial_\mu M^\dagger) - V_0(M, M^\dagger) + \tfrac{1}{2}c_1(\partial_\mu K_\mu)^2 + \frac{i}{12}\partial_\mu K_\mu(\ln\det M - \ln\det M^\dagger) + \mathrm{Tr}[A(M+M^\dagger)] \ . \quad (2)$$

M is (we assume three light flavors) a 3×3 matrix describing scalar and pseudoscalar mesons. In the simplest approximation one may include just the pseudoscalars by making the nonlinear replacement

$$M = \frac{F_\pi}{2}\exp\left(\frac{2i\phi}{F_\pi}\right) \ , \quad (3)$$

where $F_\pi \simeq m(\pi)$ is the pion decay constant and ϕ the pseudoscalar nonet. In (2) V_0 is a chiral-$\mathrm{U}(3)\times\mathrm{U}(3)$-invariant potential, A is proportional to the quark mass matrix, and the parameter c_1 describes the new physics associated with the η' mass. The easiest phenomenological way to treat the glue quantity $\partial_\mu K_\mu$ in (2) is to replace it by a 0^- field $\tilde{G}_1$. The Lagrange equation of motion for this field causes it to become proportional to the quantity $(\ln\det M - \ln\det M^\dagger)$ which from (3) can be seen to be essentially the η' field.[9] (This is only approximately true in the linear model.) In order to extend (2) to include the $G(1440)$ we notice that if a physical pseudoscalar glueball exists it should certainly also be associated with the quantity $\partial_\mu K_\mu$ and therefore

$$\partial_\mu K_\mu = \tilde{G}_1 + \tilde{G}_2 \ . \quad (4)$$

We can think of this equation as stating that the operator $\partial_\mu K_\mu$ is to be saturated by two distinct glueball fields. One of these $\tilde{G}_1$ will be dual to quark matter as before while the other $\tilde{G}_2$ could correspond to the physical glueball sate. It might be natural to think of $\tilde{G}_2$ as a radial excitation of the lower state $\tilde{G}_1$, but this is not necessary. In the full Lagrangian we must include as before a wrong-sign "mass" term for $\tilde{G}_1$ and *both* "kinetic" and "mass" terms with arbitrary coefficients for $\tilde{G}_2$. Thus the extended Lagrangian, which still automatically satisfies the axial-vector anomaly equation (1) (in the $A=0$ limit), is[12]

$$\mathcal{L} = -\tfrac{1}{2}\,\mathrm{Tr}(\partial_\mu M \partial_\mu M^\dagger) - V_0(M, M^\dagger) + \tfrac{1}{2}c_1\tilde{G}_1^2 - \tfrac{1}{2}c_2\tilde{G}_2^2 - \tfrac{1}{2}c_3(\partial_\mu \tilde{G}_2)^2 + \frac{i}{12}(\tilde{G}_1+\tilde{G}_2)(\ln\det M - \ln\det M^\dagger) + \mathrm{Tr}[A(M+M^\dagger)] \ . \quad (5)$$

In terms of the quantities in (5) the correctly normalized bare η_0' and bare G_0 fields are given by

$$\eta_0' = \sqrt{3}c_1 F_\pi \tilde{G}_1 \ , \quad (6a)$$

$$G_0 = \sqrt{c_3}\tilde{G}_2 \ . \quad (6b)$$

The equation of motion for $\tilde{G}_1$ was used in obtaining (6a). The matrix of squared masses which follows from (5) and (6) in the η_0, η_0', G_0 basis is readily found to be

$$\begin{pmatrix} \frac{4K-\pi}{3} & \frac{-2\sqrt{2}}{3}(K-\pi) & 0 \\ \frac{-2\sqrt{2}}{3}(K-\pi) & \frac{(2K+\pi)}{3}+\alpha & \beta \\ 0 & \beta & \gamma \end{pmatrix} \ , \quad (7)$$

where the particle symbol represents the (mass)^2. The quantities α, β, and γ are given by

$$\alpha = (3c_1F_\pi^2)^{-1}, \quad \beta = (3c_3F_\pi^2)^{-1/2}, \quad \gamma = c_2/c_3 \ .$$

Furthermore, the important equation (4) can be rewritten as

$$\partial_\mu K_\mu = \sqrt{3}F_\pi(\alpha\eta_0' + \beta G_0) \ . \quad (8)$$

The mass matrix (7) is precisely that which can be written down in a large variety of models. It might appear as if we have accomplished nothing more than the accommodation of a 0^- glue field in a chiral Lagrangian. But this ignores the nontrivial information contained in (8) which determines the contribution of the η_0' and G_0 to $\partial_\mu K_\mu$ completely in terms of the parameters appearing in the mass matrix. Since $\partial_\mu K_\mu$ is the most reasonable candidate for the effective-glue-field combination produced by two gluons in a 0^- state we expect some practical consequences. To the extent that we expect the glueball state G_0 to contribute to $\partial_\mu K_\mu$ we must also expect mixing between η_0' and G_0. The surprising feature is that the smaller the η_0'-G_0 mixing the greater the dominance of η_0' in the glue-field composite $\partial_\mu K_\mu$.

The most direct way to test the model is to study its implications for the pseudoscalar-mixing problem. The "physical" η, η', and $G(1440)$ fields are obtained as those combinations which diagonalize the matrix (7). The processes which we analyze purely in terms of mixing are $X\to\gamma\gamma$, $\pi^- p\to Xn$, $\psi\to\gamma X$ where X is one of η, η', G. The first two processes have long been used as tests of mixing models[13] but the last may need some comment. In perturbative QCD the decay $\psi\to\gamma+\text{hadrons}$ occurs via $\psi\to\gamma+2$ gluons $\to\gamma+\text{hadrons}$.[14,15] The calculated[6,16] branching ratio for the first step of this process ($\psi\to\gamma gg$) for the two gluons in a 0^- state with mass less than 1.6 GeV is about 1%, in good agreement with the measured ratios for $\psi\to\gamma(\eta+\eta'+G)$. Hence there is little room for other contributions, such as a $c\bar{c}$ admixture in η', to the decay mechanism.

It is an attractive hypothesis to saturate the second step (2 gluons in $0^-\to$ hadrons) by our $\partial_\mu K_\mu$.[17] Equation (8) informs us that the hadrons are the bare η_0' and G_0 produced in proportion α/β. To find the relative strengths for $\psi\to\gamma(\eta,\eta',G)$ we need only know (apart from phase-space considerations) the admixture of η_0 and G_0 in the physical particles. Table I lists the relevant experimental information and our predictions. The η' and G masses ($m_G = 1440$) were used together with the ratio $\Gamma(\psi \to \gamma\eta')/\Gamma(\psi \to \gamma G)$ to determine[18] the unknown parameters α, β, γ which appear in our Lagrangian and the mixing matrix. The top row in Table I represents the currently accepted experimental values, or ranges of values.[19] The next three rows are the predictions of our model for various values of $\Gamma(\psi \to \gamma\eta')/\Gamma(\psi \to \gamma G)$ used as input. The overall consistency between the model and experiment is very encouraging. We readily explain the large number of η's produced in $\psi \to \gamma X$ and the dominance in this decay mode of η'. The perturbative QCD analysis favors formation of glue matter over quark matter from the two emitted gluons accompanying the photon. Yet the data prefer[19] at least as much η' production as G production. The glue-matter duality relation clearly explains this phenomena. Conversely the observation of G in $\psi \to \gamma X$ forces, via (8), β to be nonzero and inevitably, from (7), leads to a non-negligible matter component in G. This in turn provides for a substantial partial width into $\gamma\gamma$ contrary to some expectations.[3]

Success is achieved in our predictions for $\eta' \to \gamma\gamma$ and the relative production of η and η' in high-energy $\pi^- p$ reactions. Less successful are our values for the η mass and the rate for $\eta \to \gamma\gamma$. This, however, may not be a problem (see below).

One difficulty we have to face is the hadronic production of G. No evidence for $\pi^- p \to Gn$ exists. Suppression of G production relative to η' production is predicted but it is not clear whether our model provides sufficient suppression. A key datum to decide this would be the branching ratio for $G \to \eta\pi\pi$, since the most stringent experimental limit on G production is sensitive only to this decay mode. At most the branching ratio is $\frac{1}{2}$ and could conceivably be as small as $\frac{1}{5}-\frac{1}{10}$. The smaller value would allow our model to coexist with present experiments. If we accept the Crystal Ball data $[\Gamma(\psi \to \gamma\eta') \lesssim 1.9 \times \Gamma(\psi \to \gamma G)$ and $\Gamma(\psi \to \gamma\eta') \simeq 5.7\Gamma(\psi \to \gamma\eta)]$ our model is not merely in qualitative agreement, but in good quantitative agreement with experiment. For this case $\alpha = 0.77$, $\beta = 0.33$, $\gamma = 2.0$, all in GeV^2. The resulting mixing is

$$\begin{aligned} \eta &= 0.94\eta_0 + 0.33\eta_0' - 0.06G_0 \ , \\ \eta' &= -0.33\eta_0 + 0.90\eta_0' - 0.28G_0 \ , \\ G &= -0.03\eta_0 + 0.28\eta_0' + 0.96G_0 \ . \end{aligned} \tag{9}$$

The present model appears even more attractive if we compare it with one which might have been a reasonable first guess. Such a model would require the domination of the two-gluon 0^- channel in $\psi \to \gamma gg$ by a single glueball which would be the "bare" $G(1440)$. η and η' production would arise by mixing. A mass matrix identical to (7) would appear but with somewhat different interpretation. α would still be necessary to solve the U(1) problem but would be considered a purely phenomenological term or a generalized annihilation term. β and γ would still represent η_0'-G_0 mixing and the bare G_0 mass, respectively. In the language of Eq. (8) G_0 dominance of the two-gluon state would read $\partial_\mu K_\mu \propto G_0$. The very last row in our table presents the results. The predictions for $\psi \to \gamma\eta$, $\pi^- p \to \eta' n$, and $\pi^- p \to Gn$ seem rather bad, and become even worse if we allow a bigger share of $\psi \to \gamma X$ for η'. These failures can be traced to the need for very large η'-G mixing in order to provide the necessary prominence of η' in ψ decay. Glue-matter duality naturally avoids this pitfall.

We have presented our model in the most simple form possible in order to emphasize how a basic feature of QCD physics, the anomalous conservation law (1), is capable of properly accounting for important experimental features. Embellishments are possible by considering further SU(3)-symmetry breaking,[7,20] higher-order $1/N_c$ corrections,[21] and additional mixing with radial excitations and charm states.[22]

TABLE I. Consequences of our mixing model. The first row summarizes the experimental results. The next three rows are predictions of our model with the first column used as input. The final row is for an alternative model described in the text.

$\frac{\Gamma(\psi\to\eta'\gamma)}{\Gamma(\psi\to G\gamma)}$	$\frac{\Gamma(\eta\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}$	$\frac{\Gamma(\eta'\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}$	$\frac{\Gamma(G\to\gamma\gamma)}{\Gamma(\eta'\to\gamma\gamma)}$	$\frac{\Gamma(\psi\to\eta'\gamma)}{\Gamma(\psi\to\eta\gamma)}$	$\frac{\pi^-p\to\eta n}{\pi^-p\to\eta' n}$	$\frac{\pi^-p\to Gn}{\pi^-p\to\eta' n}$	m_η^2 (GeV^2)
1.9–0.6	40 ± 10	660 ± 280		5.7–1.8	1.9 ± 0.25	$\ll 1$	0.3
1.9	78	590	0.4	5.4	2.2	0.15	0.250
1	75	550	0.75	5	2.3	0.28	0.253
0.67	72	510	1.2	4.7	2.4	0.45	0.257
0.67	67	435	2.2	22	2.7	0.82	0.263

We have no doubt that such embellishments would improve the accuracy of our experimental predictions, but at the price of obscuring the crucial role of the anomaly.

These corrections might easily improve our predictions for the η. We briefly mention one of these. Since $\partial_\mu K_\mu$ is proportional to the square of the strong coupling constant, Eq. (8) suggests that the numerical values of α, β, and γ might "run" hand-in-hand with the coupling constant. Their effective values would be different for η mixing than for η' and G. Such effects have been considered in previous topological and QCD studies of the η, η' system.[23] The variation of α_s would also enhance the low-energy part of $\psi \rightarrow \gamma X$ and favor η production. With a 40% change in α_s between 0.3 and 2 GeV, subsequent changes in α, β, γ would cause only small changes in our predictions for G and η', but would substantially improve the predictions for η.

The simplest possible model which reflects the important role of the axial-vector anomaly has been shown to provide an excellent qualitative description of the expanded pseudoscalar spectrum and its observed mixing pattern. The solution of the U(1) problem via the anomaly naurally leads to the preeminence of η' rather than G in ψ radiative decay.

[1]D. Scharre *et al.*, Phys. Lett. 97B, 329 (1980).

[2]M. Chanowitz, Phys. Rev. Lett. 46, 981 (1981).

[3]K. Ishikawa, Phys. Rev. Lett. 46, 978 (1981).

[4]J. Donoghue, K. Johnson, and B. A. Li, Phys. Lett. 99B, 416 (1981).

[5]C. E. Carlson *et al.*, Phys. Lett. 98B, 110 (1981).

[6]R. Lacaze and H. Navelet, CEN-Saclay Report No. Dph-T/81-2 (unpublished).

[7]J. Schechter and Y. Ueda, Phys. Rev. D 3, 2874 (1971), and references therein.

[8]E. Witten, Nucl. Phys. B160, 57 (1980).

[9]C. Rosenzweig, J. Schechter, and G. Trahern, Phys. Rev. D 21, 3388 (1980).

[10]P. DiVecchia and G. Veneziano, Nucl. Phys. B171, 253 (1980); P. Nath and R. Arnowitt, Phys. Rev. D 23, 473 (1981); E. Witten, Ann. Phys. 128, 363 (1980).

[11]For related approaches explicitly saturating the Ward identities see G. Veneziano, Nucl. Phys. B159, 212 (1979); H. Goldberg, Phys. Rev. Lett. 44, 363 (1980); K. Milton, W. Palmer, and S. Pinsky, Phys. Rev. D 22, 1647 (1980).

[12]Our basic equations (7) and (8) are unchanged even if we replace the glue terms in (5) by the more general form

$$\frac{1}{2}c_1\tilde{G}_1{}^2-\frac{1}{2}c_2\tilde{G}_2{}^2-\frac{1}{2}c_3(\partial_\mu\tilde{G}_2)^2+c_4\tilde{G}_1\tilde{G}_2 +\frac{i}{12}(\tilde{G}_1+\lambda\tilde{G}_2)(\ln\det M-\ln\det M^\dagger)$$

and the ansatz (4) by

$$\partial_\mu K_\mu=\tilde{G}_1+\lambda\tilde{G}_2\ .$$

α remains the same as before but β and γ now become $\beta=(3c_3F_\pi{}^2)^{-1/2}(\lambda-c_4/c_1)$ and $\gamma=c_3{}^{-1}(c_2+c_4{}^2/c_1)$.

[13]For a review see S. Okubo, Phys. Rev. D 16, 2236 (1978).

[14]T. Applequist *et al.*, Phys. Rev. Lett. 34, 365 (1974).

[15]M. Chanowitz, Phys. Rev. D 12, 918 (1975).

[16]A. Billoire *et al.*, Phys. Lett. 80B, 381 (1979).

[17]K. Kawarabayashi and N. Ohta, Nucl. Phys. B175, 477 (1980). For related approaches see V. A. Novikov *et al.*, *ibid.* 165, 55 (1980); W. A. Bardeen and V. I. Zakharov, Phys. Lett. 91B, 111 (1980); M. B. Voloshin and V. I. Zakharov, Phys. Rev. Lett. 45, 688 (1980).

[18]It turned out to be much more convenient to solve the mixing problem and determine α, β, γ by using the $(\text{mass})^2$ of the η as input and allowing this mass to vary over the range 0.250 to 0.270 GeV^2. We presented the calculation with $\Gamma(\psi\rightarrow\gamma\eta')/\Gamma(\psi\rightarrow\gamma G)$ as an input since the mixing is very sensitive to $m_\eta{}^2$ which required at least three significant digits. This extreme sensitivity to m_η is just a reflection of the same phenomenon in the 2×2 η-η' mixing case where it is, strictly speaking, impossible to solve the mixing problem when m_η is chosen to be its experimental value. On the other hand when $m_{\eta'}$ is chosen at its physical value one predicts m_η and the mixing angle fairly well. Additional corrections as mentioned in the text can overcome this problem.

[19]The data on radiative ψ decays are nicely summarized in D. L. Scharre, Report No. SLAC-PUB-2519, 1980 (unpublished). The $\gamma\gamma$ decay rates were taken from Particle Data Group, Rev. Mod. Phys. 52, S1 (1980). Data on $\pi^- p$ are from N. Stanton *et al.*, Phys. Rev. Lett. 42, 346 (1979); W. D. Apel, Phys. Lett. 83B, 131 (1979).

[20]R. Arnowitt and P. Nath, Northeastern Univ. Reports Nos. NUB 2468, 1980 and NUB 2469, 1980 (unpublished).

[21]P. DiVecchia *et al.*, Nucl. Phys. B181, 318 (1981).

[22]F. D. Gault and A. B. Rimmer, Z. Phys. C 8, 353 (1981).

[23]C. Rosenzweig, Phys. Rev. D 13, 3080 (1976); T. Inami *et al.*, Phys. Lett. 61B, 60 (1976); H. Fritzsch and J. D. Jackson, *ibid.* 66B, 365 (1977).