PHYSICAL REVIEW D VOLUME 24, NUMBER 9 1 NOVEMBER 1981

New source of charge-symmetry violation in the nucleon-nucleon system

A. W. Thomas
TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3

P. Bickerstaff
School of Physics, University of Melbourne, Parkville, Victoria, Australia 3052

A. Gersten
*Physics Department, University of British Columbia, Vancouver, British Coloumbia, Canada V6T 1W5
and Ben-Gurion University, Beer-Sheva, Israel*
(Received 6 July 1981)

Within the framework of the cloudy bag model we show that the small difference of up- and down-quark masses leads to a small increase in the $\pi^0 nn$ coupling constant relative to $\pi^0 pp$. There may be many experimental consequences of this observation but, as an example, $|a_{nn}|-|a_{pp}| \cong 0.3$ fm because of this effect.

In the past two of three years there has been great interest in the so-called chiral bag models.[1–8] Essentially the idea is that whereas chiral $SU(2)\times SU(2)$ is one of the best symmetries of the strong interaction,[9] most phenomenological models of quark confinement violate it rather badly. The MIT bag model is a very useful case study, because the whole source of chiral-symmetry violation is localized on the bag surface. By introducing a pion field (at the first, crude stage elementary, and no longer describable in a bag model) into the theory, it is possible to compensate for the surface term so that the $SU(2)\times SU(2)$ symmetry is restored—at least in the absence of quark and pion mass terms.

The cloudy bag model (CBM) is one theory of this kind which has proven rather successful in a number of applications.[3–5] It is a basic assumption of the CBM that pionic corrections can be calculated as a relatively small perturbation on the original MIT model. Thus it should make sense to expand the full Lagrangian density in powers of $\vec{\phi}$ (the pion field). This leads to expressions for the pion coupling to bare nucleons, Δ's, and so on. Once the truncation in powers of $\vec{\phi}$ has been made the theory is completely renormalizable, and the renormalizations are, moreover, finite and small. We refer to Refs. 10, 11, and 5 for discussion of the rigorous bounds on the probability of finding n pions in the physical nucleon ($\langle n\rangle < 0.9$), and for calculations of nucleon electromagnetic properties. In the latter case there is a significant improvement over the original MIT model,[12,13] for both the magnetic moments and the neutron charge radius.

At the same time as these developments involving the substructure of the nucleon have been going on, there has been a great deal of activity, both experimental and theoretical, on the question of the violation of charge symmetry in the nucleon-nucleon system. The classical system in which this has been studied a great deal is the $^1S_0$ scattering length for nn compared with pp (Coulomb corrected). Currently the best experimental values, -17.1 ± 0.2 fm (Ref. 14) and -18.6 ± 0.6 fm (Ref. 15), respectively, indicate a small violation of charge symmetry. However, there is considerable discussion of the interpretation of the errors quoted.

A recent experiment at LAMPF failed to see a charge-symmetry-violating forward-backward asymmetry in the reaction $np \to d\pi^0$ at the level of 0.5%.[16] Complex experiments under way at Indiana and TRIUMF hope to see a small difference in the position of the zero in P and A in np elastic scattering, which should provide the most sensitive test to date.

On the theoretical side, apart from electromagnetic effects which are relatively straightforward, most calculations of charge-symmetry violation are based on boson-exchange models of the nucleon-nucleon force—e.g., ρ-ω and π-η mixing. An excellent summary of the present experimental and theoretical situation is to be found in the proceedings of the workshop held at TRIUMF earlier this year[17]; see also Ref. 18.

In the absence of a theory which connects the nucleon-nucleon force with the substructure of the nucleon, there has never been an estimate of the direct effects on NN scattering of the violation of $SU(2)\times SU(2)$ at the quark level. In particular, in order to explain the mass splittings within multiplets such as the nucleon, Σ, Ξ, Δ, and so on, one expects a splitting of the u- and d-quark masses.[19] Within the framework of the bag model we recently obtained a value of $(m_d - m_u)$ of 4–5 MeV.[20] Of course it is common to calculate the effects of n-p mass differences (an indirect inclusion of effects of $m_u \neq m_d$).

 ©1981 The American Physical Society

What we shall now show is that the π^0 coupling constant to the neutron is about 0.4% bigger than that to the proton as a consequence of these small quark mass differences.

In the original formulation of the CBM, in the case where $SU(2) \times SU(2)$ is still an exact symmetry, the Lagrangian density has the form[4]

$$\mathcal{L}_{CBM}(x) = (i\bar{q}_0 \not{\partial} q_0 - B)\theta_v - \tfrac{1}{2}\bar{q}_0 e^{i\vec{\tau}\cdot\vec{\phi}\gamma_5/f} q_0 \Delta_s + \tfrac{1}{2}(D_\mu \phi)^2 \ . \quad (1)$$

Here q_0 is the two-isospin-component (u and d) quark field, B is the phenomenological energy density associated with making the bag, and $\vec{\phi}$ is the pion field. The bag volume is defined by θ_v, and Δ_s is a surface δ function [$\theta_v = \theta(R-r)$ and $\Delta_s = \delta(r-R)$ in the static, spherical case]. The parameter f is readily identified as the pion decay constant (93 MeV). Finally, $D_\mu \phi$ is the appropriate covariant derivative, so that the whole Lagrangian density is invariant under the chiral tranformation

$$q_0 \to q_0 + \frac{i}{2}\vec{\tau}\cdot\vec{\epsilon}\gamma_5 q_0, \quad \bar{q}_0 \to \bar{q}_0 + \frac{i}{2}\bar{q}_0 \vec{\tau}\cdot\vec{\epsilon}\gamma_5 \ , \quad (2)$$

$$\vec{\phi} \to \vec{\phi} - \vec{\epsilon} f + f\left[1 - \left(\frac{\phi}{f}\right)\cot\left(\frac{\phi}{f}\right)\right]\hat{\phi}\times(\vec{\epsilon}\times\hat{\phi}) \ ; \quad (3)$$

that is

$$D_\mu \vec{\phi} = \partial_\mu \vec{\phi} - \left[1 - j_0\left(\frac{\phi}{f}\right)\right]\hat{\phi}\times(\partial_\mu\vec{\phi}\times\hat{\phi}) \ . \quad (4)$$

Very recently it has been discovered[21] that by making a transformation on the quark fields one obtains a Lagrangian density in which the current-algebra constraints are somewhat easier to see. In fact, defining

$$q = S q_0, \quad \bar{q} = \bar{q}_0 S \ , \quad (5)$$

with

$$S = e^{i\vec{\tau}\cdot\vec{\phi}\gamma_5/2f} \ , \quad (6)$$

the Lagrangian density becomes

$$\mathcal{L}'(x) = (i q \not{D} q - B)\theta_v - \tfrac{1}{2}\bar{q}q\Delta_s + \tfrac{1}{2}(D_\mu\phi)^2 + \frac{1}{2f}\bar{q}\gamma^\mu\gamma_5\vec{\tau}q\cdot(D_\mu\vec{\phi})\theta_v \ , \quad (7)$$

where the covariant derivative on the (new) quark fields is

$$\not{D} q = \not{\partial} q - i\left[\frac{\cos(\phi/f) - 1}{2}\right]\vec{\tau}\cdot(\hat{\phi}\times\not{\partial}\hat{\phi})q \ . \quad (8)$$

A full discussion of Eq. (7) can be found elsewhere,[21,22] but some remarks are necessary here. First, the covariant derivative on the quark fields [Eq. (8)] leads to the Weinberg-Tomozawa relation for low-energy pion scattering from any hadron describable in the bag model. Second, the pion coupling to the bag is a derivative coupling—once again in agreement with current algebra. (For the present time we avoid a discussion of exactly how derivative coupling to the quarks translates into coupling to the hadrons.)

Of course it is true that the original quark fields q_0 are not equivalent to the new q. What the transformation has in fact done is to change from the simple representation of $SU(2) \times SU(2)$ shown in Eq. (2), to a new, nonlinear representation in which chiral transformations mix quark states with different numbers of pions. (In this representation $\bar{q}q$ is, in fact, chiral invariant.) The new representation is preferable in many ways; for example, the Lagrangian density (7) decomposes much more readily as $\mathcal{L}_{MIT} + \mathcal{L}_\pi + \mathcal{L}_{interaction}$ (as used in earlier work). Moreover, the key elements which make Eq. (7) chiral invariant, namely, $\not{\partial} q \to \not{D} q$ and derivative coupling, can be used for *any* model of quark confinement—they are not specific to the bag model.

In order to obtain an $NN\pi$ vertex from Eq. (7), we follow the earlier procedure[4] of defining a P space (of three-quark baryons) and a Q space (the rest), and expanding to lowest order in ϕ. The appropriate $NN\pi$ interaction vertex is then simply the matrix element of the derivative-coupling term in Eq. (7) between MIT-bag nucleons. Clearly the $NN\pi$ coupling constant (at $k=0$) is $(1/2f)$ times the expectation value of $\gamma^\mu\gamma_5\vec{\tau}$ in the nucleon bag. That is simply $(g_A^{bag}/2f)$, where g_A^{bag} is the MIT-bag-model value of the axial-vector coupling constant. Since it was shown in Refs. 5 and 11 that in the truncated theory (higher powers of ϕ dropped) renormalization of coupling constants is small ($\sim 10\%$ for $NN\pi$), we shall not consider this further. (As we have pointed out explicitly in Ref. 5 the large surface correction to g_A found by Jaffe[2] and others is not present in the CBM, because the pion field is not excluded from the bag volume.)

Of course the enormous improvement in the prediction of g_A (1.09 for massless quarks) in comparison with nonrelativistic quark models ($\frac{5}{3}$) was one of the great successes of the MIT bag model. (With the recoil correction of Donoghue and Johnson[23] the bag value becomes 1.22 for massless quarks.) The reason for the suppression of g_A is the presence of small Dirac components in the quark wave functions. Obvioulsy, as the quark mass gets larger this suppression is less effective and g_A goes up.

The result promised earlier, namely, $f_{\pi^0 nn} > f_{\pi^0 pp}$, is now trivially obtained. The d quark is 5 MeV more massive than the u, and hence, for neutral-current coupling,

$$\frac{g_A^n}{g_A^p}=1+\frac{3}{5}\delta \ , \qquad (9)$$

where the ratio of the appropriate spatial matrix elements for single up and down quarks is $(1-\delta)$. Using the equations of Donoghue *et al.*,[13] $(m_d-m_u)=5$ MeV (Ref. 20) implies $\delta=6.4\times10^{-3}$, and hence g_A^n/g_A^p is 0.4% greater than one.

At the present level of accuracy of neutral-current experiments it is not possible to imagine seeing such a small effect. For the future one may hope. We prefer to examine the effects on pion production through the relationship $\sqrt{4\pi}\,f_{NN\pi}/m_\pi=g_A^{\rm bag}/2f$, noted above. This leads to a prediction that the $nn\pi^0$ coupling constant is 0.4% greater than that for $pp\pi^0$. In order to estimate the significance of this charge-symmetry violation, we have calculated the implied difference in nn and pp scattering lengths. For the Bryan-Gersten potential[24] (model D) this leads to $|a_{nn}|-|a_{pp}^{\rm no\ Coul}|=+0.3$ fm, which is in the same direction as the experiment, although a little small. (Recall, however, that the errors may be somewhat low.) Of course, we must add the caution that this is only one possible source of charge-symmetry violation in the low-energy N-N system.[17,25]

In summary, we stress that the source of charge-symmetry violation shown here is new. It is a long-range effect, which should survive in even the most recent models of N-N scattering which take into account the large size of the nucleon bag.[26] (A pessimist might doubt whether ρ-ω and π-η mixing will survive, since they are of short range and the bags overlap at 1.5–2.0 fm.) The effect of this violation should be seen at an appropriate level in many systems. In particular we think of different widths for Δ decay to π^0, of forward-backward asymmetry in $np\rightarrow d\pi^0$ (which may be enhanced in polarization measurements), and so on.

[1]A. Chodos and C. B. Thorn, Phys. Rev. D 12, 2733 (1975).
[2]R. L. Jaffe, lectures at the 1979 Erice School "Ettore Majorana" (unpublished).
[3]G. A. Miller, A. W. Thomas, and S. Théberge, Phys. Lett. 91B, 192 (1980); Comments Nucl. Part. Phys. 10, 101 (1981).
[4]S. Théberge, A. W. Thomas, and G. A. Miller, Phys. Rev. D 22, 2838 (1981); 23, 2106 (E) (1981).
[5]A. W. Thomas, S. Théberge, and G. A. Miller, Phys. Rev. D 24, 216 (1981).
[6]M. V. Barnhill III, Phys. Rev. D 20, 723 (1979); M. V. Barnhill III and A. Halprin, *ibid.* 21, 1916 (1980).
[7]G. E. Brown and M. Rho, Phys. Lett. 82B, 177 (1979).
[8]C. DeTar, Phys. Rev. D 24, 752 (1981); 24, 762 (1981).
[9]H. Pagels, Phys. Rep. C 16, 221 (1975).
[10]L. R. Dodd, A. W. Thomas, and R. F. Alvarex-Estrada, Phys. Rev. D 24, 1961 (1981).
[11]S. Théberge, G. A. Miller, and S. W. Thomas, Can. J. Phys. (to be published).
[12]A. Chodos *et al.*, Phys. Rev. D 9, 3471 (1974); K. Johnson, Acta Phys. Pol. B 6, 865 (1975).
[13]J. Donoghue, E. Golowich, and B. Holstein, Phys. Rev. D 12, 2875 (1975).
[14]H. Guratzsch *et al.*, Nucl. Phys. A342, 239 (1980).
[15]B. Gabioud *et al.*, Phys. Rev. Lett. 42, 1508 (1979).
[16]C. L. Hollas *et al.*, in Ref. 17.
[17]*Proceedings of the Charge-Symmetry Breaking Workshop*, edited by N. E. Davison, J. P. Svenne, and W. T. H. van Oers (TRIUMF, Vancouver, 1981).
[18]E. M. Henley and G. A. Miller, in *Mesons in Nuclei*, edited by M. Rho and D. Wilkinson (North-Holland, Amsterdam, 1979), p. 405.
[19]S. Weinberg, in *A Festschrift for I. I. Rabi*, edited by L. Motz (New York Academy of Sciences, New York, 1977); P. Langacker and H. Pagels, Phys. Rev. D 19, 2070 (1979).
[20]P. Bickerstaff and A. W. Thomas, University of Melbourne report, 1981 (unpublished).
[21]A. W. Thomas, J. Phys. G (to be published).
[22]A. W. Thomas, lectures at the Vacation School, University of Melbourne, 1981, Report No. UM-P-81/29 (unpublished).
[23]J. Donoghue and K. Johnson, Phys. Rev. D 21, 1975 (1980).
[24]R. A. Bryan and A. Gersten, Phys. Rev. D 6, 341 (1972); 7, 2802 (E) (1973).
[25]P. Langacker and D. A. Sparrow, Phys. Rev. Lett. 43, 1559 (1979).
[26]C. DeTar, Phys. Rev. D 22, 1451 (1979).