Comments

Comments are short papers which comment on papers of other authors previously published in the **Physical Review**. *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Inclusive semileptonic decays of explicitly flavored heavy particles

J. L. Cortes*
Laboratoire de Physique Théorique et Hautes Energies, Paris, France†
(Received 15 December 1980)

The inclusive semileptonic decays of flavored heavy particles are investigated in detail. The inclusive decay rate is in agreement with the standard quark-parton-model estimation.

The experimental large D^+-D^0 lifetime difference[1] has been generally interpreted among theorists as a signal that the nonleptonic D decays cannot be understood with a simple perturbative picture such as naive quark models.[2] Even in the more simple case of inclusive semileptonic decays, the validity of the quark-parton model has been questioned[3] on the basis of an incompatibility of the estimate for the inclusive semileptonic decay rate with exclusive modes. The crucial point of these analyses is the fact that the computed decay rate turns out to be strongly reduced with respect to the simple estimate by Gaillard, Lee, and Rosner.[2]

We will discuss in this paper the inclusive semileptonic decays in order to test the validity of the simple perturbative picture in this case. Our results are in agreement with the naive quark-parton-model estimate and a consistent picture of semileptonic processes is obtained.

The inclusive semileptonic decay rate for $P\to Xl\bar{\nu}$ (X means anything) is given by

$$\Gamma_{\rm SL}=\frac{G_F^{\,2}}{24\pi^3}\int dq^2 d\nu(\nu^2-q^2)^{1/2}[3q^2W_1+(\nu^2-q^2)W_2]\,, \tag{1}$$

where q and P denote the lepton pair and decaying particle (with mass M) momenta, respectively. $\nu=P\cdot q/M$ and the structure functions W_1 and W_2 are defined, on the analogy of deep-inelastic scattering, from the hadronic tensor through

$$\begin{aligned}W_{\mu\nu}(P,q)&\equiv\int d^4x\, e^{iq\cdot x}\langle P|[J_\mu(x),J_\nu^\dagger(0)]|P\rangle\\&=-g_{\mu\nu}MW_1(q^2,\nu)+P_\mu P_\nu\frac{W_2(q^2,\nu)}{M}+\cdots\,,\end{aligned} \tag{2}$$

where the dots stand for terms proportional to q_μ or q_ν which can be ignored for our purposes (semileptonic decay rate) because their contributions are of the order of $m_l^{\,2}/M^2$.

Now one can use the following argument to justify the use of the quark-parton model: Due to the high mass of the decayed particle, the momentum q_μ will be large ($\sim M$) and the integral over x in Eq. (2) will be dominated by short distances ($\sim 1/M$) leading to the standard quark-parton picture (see Fig. 1).

We define in the rest frame of the decaying particle the variable ξ,

$$\xi=\frac{k^0+|\vec{k}|}{M}\,, \tag{3}$$

limited by[4]

$$0\leqslant\xi\leqslant 1\,. \tag{4}$$

ξ has a simple traditional parton interpretation, giving just the momentum fraction of the parton in an infinite-momentum frame. The condition $|\vec{k}|\geqslant 0$ and (4) lead to

$$k^2\leqslant M^2\xi^2\,. \tag{5}$$

The parton model allows us to relate the hadronic matrix element in Eq. (2) to a calculable quark matrix element[5]

$$\langle P|[J_\mu,J_\nu]|P\rangle=\frac{1}{4\pi}\int d\xi\, dk^2 d\Omega_k\frac{f(\xi,k^2)}{\frac{1}{2}(\xi+k^2/\xi M^2)}\langle k|[J_\mu,J_\nu]|k\rangle\,, \tag{6}$$

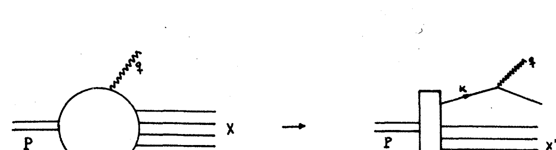

FIG. 1. Inclusive semileptonic decays in the quark-parton picture.

where the factor $\frac{1}{2}(\xi + k^2/\xi M^2)$ is the ratio of hadron and quark fluxes k^0/m, and $f(\xi, k^2)$ is the quark distribution function normalized by

$$\int_0^1 d\xi \int_{-\infty}^{M^2\xi^2} dk^2 f(\xi, k^2) = 1 \,. \tag{7}$$

The structure function W_2 can be obtained from the $k_\mu k_\nu$ term in the partonic tensor $\langle k|[J_\mu, J_\nu]|k\rangle$ while W_1 receives contributions from the terms proportional to $g_{\mu\nu}$ and $k_\mu k_\nu$. If one multiplies the parton tensor by $g_{\mu\nu} - q_\mu q_\nu/q^2$ and $\hat{P}_\mu \hat{P}_\nu$ $[\hat{P} = P - (M\nu/q^2)q]$, one obtains $W_{1,2}$ in terms of integrals of $k \cdot P$ or $k \cdot q$ that can be reexpressed in terms of ξ and k^2. The result for the relevant combination in the decay rate is

$$(\nu^2 - q^2)^{1/2}[3q^2 W_1 + (\nu^2 - q^2)W_2] = \int d\xi\, dk^2 \frac{f(\xi, k^2)}{(M^2\xi^2 - k^4/M^2\xi^2)} (k^2 - q^2)(2q^2 + k^2) \,, \tag{8}$$

where the on-mass-shell constraint $\delta((k-q)^2)$ has been used in Eq. (6) to integrate over the angular dependence of $\vec{k}$ which gives

$$0 \le q_- \le \frac{k^2}{\xi M} \le q_+ \le \xi M \,, \tag{9}$$

where $q_\pm \equiv q_0 \pm q_3 = \nu \pm (\nu^2 - q^2)^{1/2}$. The final result for the inclusive semileptonic decay rate (1) can be written, after integration over q^2 and ν, as

$$\Gamma_{\rm SL} = \frac{G_F^2}{192\pi^3} \int_0^1 d\xi \int_0^{\xi^2 M^2} dk^2\, k^6 \frac{f(\xi, k^2)}{\frac{1}{2}(\xi M + k^2/\xi M)} \,, \tag{10}$$

where one recognizes the free-quark decay rate $(G_F^2/192\pi^3)k^5$ and the Lorentz factor $\sqrt{k^2}/k^0$ due to the fact that this quark is not at rest in the rest frame of the heavy particle.

To go on, we have to specify the quark distribution function reflecting the long-distance interaction responsible for quark confinement. Since the quantum-chromodynamics (QCD) bound-state problem has not been solved one is forced to make a variety of reasonable guesses.

If one puts the heavy quark on the mass shell

$$f(\xi, k^2) = \delta(k^2 - m^2) g(\xi) \,, \tag{11}$$

the semileptonic decay rate becomes

$$\Gamma_{\rm SL} = \Gamma_{\rm SL0} \frac{m}{M} \int_{m/M}^1 d\xi \frac{g(\xi)}{\frac{1}{2}(\xi + m^2/\xi M^2)} \,, \tag{12}$$

where

$$\Gamma_{\rm SL0} = \frac{G_F^2 m^5}{192\pi^3} \tag{13}$$

corresponds to the free heavy-quark decay. Using the normalization (7), one obtains *independent* of the function g the following inequalities:

$$1 - \frac{(1 - m/M)^2}{1 + m^2/M^2} \le \frac{\Gamma_{\rm SL}}{\Gamma_{\rm SL0}} \le 1 \,. \tag{14}$$

Therefore one reobtains the simple estimate by Gaillard, Lee, and Rosner[2] $\Gamma_{\rm SL} \simeq \Gamma_{\rm SL0}$ for any reasonable ratio of heavy-quark and heavy-particle masses.

If the $\delta(k^2 - m^2)$, coming from the pole in the quark propagator in QCD perturbation theory, is replaced for confined quarks by[6] $\gamma^2/[(k^2 - m^2)^2 + \gamma^4]$, where the width γ reflects the range over which the strong interactions have a large coupling strength (which can be expected to be of the order of 500 MeV), then the simple quark-model estimate $\Gamma_{\rm SL0}$ is once more obtained for $m^2 \gg \gamma^2$ and the naive result seems already reliable for charmed particles.[7]

This is in contrast with previous analyses[3] based on the scaling hypothesis for the structure functions.[8] Their result can be obtained from (10) by taking $f(\xi, k^2) = \delta(k^2 - \xi^2 M^2) g(\xi)$. The semileptonic decay rate

$$\Gamma_{\rm SL} = \frac{G_F^2 M^5}{192\pi^3} \int_0^1 d\xi\, \xi^5 g(\xi) \tag{15}$$

is now strongly reduced when compared with the naive estimate: Due to the ξ^5 factor, $\int_0^1 \xi^5 g(\xi) \ll 1$ for any $g(\xi)$ except when g is concentrated around $\xi \sim 1$. This case is in fact realized in the parton model: The condition $k^2 \sim m^2$ implies in this case that the structure function $g(\xi)$ is constrained to only large values of $\xi(\sim m/M)$, hence $\Gamma_{\rm SL} \sim \Gamma_{\rm SL0}$. If one compares the above inclusive estimate with the exclusive mode $0^-(M) \to 0^- l\nu$, which can be estimated in terms of the semileptonic form factor f_+,

$$\Gamma(0^-(M) \to 0^- l\nu) = \Gamma_{\rm SL0} \tfrac{1}{4} |f_+|^2 \left(\frac{M}{m}\right)^5 \,, \tag{16}$$

the possible conflict[9] between inclusive and exclusive widths disappears and the experimental result[10] $\Gamma(0^-(M) \to 0^- l\nu) \sim \frac{1}{2}\Gamma_{\rm SL}$ can now be easily understood.

In summary we show that the quark-parton model provides a consistent picture of semileptonic decays of heavy particles. Nonleptonic decays in contrast are much more complicated, the inclusive width being now[9]

$$\Gamma_{\rm NL} = \int d^4x \langle P|[H_w(x), H_w(0)]|P\rangle \,, \tag{17}$$

where $H_w(x)$ is the nonleptonic weak Hamiltonian. It is to be presumed there will be short- as well as long-distance effects in this case and the quark-parton picture, based on short-distance dominance, will be modified. This can be the reason for the large difference in the lifetimes and semileptonic branching ratios of D^0 and D^+.[11]

I am indebted to X. Y. Pham for several useful discussions, to Michel Gourdin for reading the manuscript, and the Laboratoire de Physique Théorique et Hautes Energies for their hospitality. This work was partially supported by Comisión Asesora de Investigación Científica y Técnica, Spain, and partially supported by the Joliot-Curie Foundation.

---

*On leave of absence from Zaragoza University, Spain.

†Laboratoire Associé au CNRS. Postal Address: Université Pierre et Marie Curie, Tour 16 - ler étage, 4, place Jussieu, 75230 Paris Cedex 05, France.

[1]N. Ushida *et al.*, Phys. Rev. Lett. 45, 1053 (1980).

[2]M. K. Gaillard, B. W. Lee, and J. L. Rosner, Rev. Mod. Phys. 47, 277 (1975); J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B100, 313 (1975).

[3]X. Y. Pham and R. P. Navabi, Phys. Rev. D 18, 220 (1978); Y. Tosa and S. Okubo, *ibid.* 22, 168 (1980).

[4]The upper limit comes from the condition $E_{x'} \geq |\vec{P}_{x'}|$ and the lower limit from $|\vec{k}| \geq 0$ for $k^2 \leq 0$ and from $M \geq \sqrt{k^2} + M_{x'}$ for $k^2 \geq 0$.

[5]See, for example, R. Barbieri, J. Ellis, M. K. Gaillard, and G. G. Ross, Nucl. Phys. B117, 50 (1976).

[6]A. De Rújula and F. Martin, Phys. Rev. D 22, 1787 (1980).

[7]If one applies the above argument to K decays one should expect in this case $m_s \sim \gamma$ and therefore a semileptonic decay rate lower than the naive estimate, in agreement with experiment. Anyway, the application of the quark parton model up to the kaon mass cannot be taken seriously.

[8]Pham and Navabi take $x = q^2/2M\nu$ as the scaling variable while Tosa and Okubo use a modified variable which takes into account the large mass of the parent hadron.

[9]Y. Tosa and S. Okubo, Phys. Rev. D 22, 168 (1980).

[10]W. Bacino *et al.*, Phys. Rev. Lett. 43, 1073 (1979).

[11]M. Bander, D. Silverman, and S. Soni, Phys. Rev. Lett. 44, 7 (1980); H. Fritzsch and P. Minkowski, Phys. Lett. 90B, 455 (1980); W. Bernreuther, O. Nachtmann, and B. Stech, Z. Phys. C 4, 257 (1980); I. Bigi, *ibid.* 5, 313 (1980); M. Suzuki, Nucl. Phys. B177, 413 (1981).