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Masses of the charmed and b-quark hadrons in a quark model

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Employing the nonrelativistic quark-model approach, the masses of charmed and b-quark hadrons are estimated.

I. INTRODUCTION

In the past few years, high-energy experiments have provided us with far-reaching developments. On one side, the discovery of the J/ψ particles[1] and of charmed mesons[2] and baryons[3] has confirmed the theoretical model of charm.[4] On the other side, the upsilon family, Υ, Υ' (Ref. 5) with masses 9.41 and 10.06 GeV discovered in the reaction

$$p + (\mathrm{Cu}, \mathrm{Pt}) \to \mu^+ + \mu^- + \text{anything}$$

could not be accommodated within the SU(4) framework, which compelled people to think beyond charm. The new particles are interpreted as a $b\bar{b}$ bound state of a new quark flavor (b) suggesting that a rich spectrum of new heavier particles might exist. Recently a bump at $M = 5.3$ GeV has been observed[6] in the channel $\psi K \pi$ at CERN (Wall experiment using a π beam of 150–175 GeV) which is expected to be a b-quark meson D_b, further confirming the existence of b-quark hadrons. The masses of the charmed[7] and b-quark[8] hadrons have been discussed by many authors. In this paper, we discuss their masses in a nonrelativistic quark-cluster model.[9] The notion of quark clusters was initially introduced by Gell-Mann and since then many authors[10] have discussed the properties of nonexotic hadrons, exotic hadrons, dibaryons, and multibaryon resonance considering the clustering of quarks. The basic idea behind it is that in some hadronic states, the constituent quarks (antiquark) tend to clump together into clusters, so it is better to consider the interaction among the constituent clusters rather than individual quarks.

We work in the quark-cluster model[9] where the potential[11] between the quark clusters has the qualitative features as suggested by quantum chromodynamics[12] (QCD). According to QCD, quarks come in three colors and interact via an octet of colored vector bosons. This interaction is relatively weak at short distances according to the property of asymptotic freedom but at large distances the interaction becomes strong and so the quark confinement is achieved. Within the potential picture,[11] the confining potential is often taken to depend linearly on the distance between quarks.[13] The justification for taking the clustering of quarks in hadrons comes from two principal sources. First, the experimental information on baryon resonances[14] appears to favor clustering of two quarks. Secondly, several theoretical models including the string model[15] and the bag model[16] also lead to clustering. Since the string and bag models are phenomenological manifestations of the QCD, clustering will also occur in that theory.

In Sec. II we discuss the general expression for the mass of a hadron and in Secs. II A and II B the masses of the mesons and baryons, respectively, are estimated. We see that the predicted masses are in good agreement with the recent available data.

II. MASS OF THE HADRON

The mass of a hadron can be written as the sum of the constituent cluster masses m_1 and m_2 plus an interaction energy E_{nL} which depends on the radial quantum number n and the orbital angular momentum L, i.e.,

$$m_{(h)} = m_1 + m_2 + E_{nL} \,. \tag{2.1}$$

We take the spin-dependent interaction also into consideration, i.e.,

$$V_{SS} \sim \vec{S}_1 \cdot \vec{S}_2 \; \nabla^2 V_c(r)/m_1 m_2 \,, \tag{2.2}$$

$$V_{SL} \sim \vec{S} \cdot \vec{L} \; (dV_c/dr)/\mu^2 r \,, \tag{2.3}$$

where $\vec{S}_1$ and $\vec{S}_2$ are the spins of the clusters, $\vec{L}$ is the orbital angular momentum between clusters, $\vec{S} = \vec{S}_1 + \vec{S}_2$, and V_c is the appropriate central potential. Omitting the mass dependence of the expectation values of $\nabla^2 V_c$ and $(1/r)\,(dV_c/dr)$ and explicitly taking into account the fact that the spin-spin interaction is short range (i.e., by including it only in states with $L = 0$), we have a simple expression for the mass of a hadron,

$$m(h) = m_1 + m_2 + E_{nL} + V_n \delta_{L0} \frac{m^2}{m_1 m_2} \vec{S}_1 \cdot \vec{S}_2 + V_{nL}\left(\frac{m}{2\mu}\right)^2 \vec{L} \cdot \vec{S} \,, \tag{2.4}$$

where V_n and V_{nL} are additional parameters, δ_{L0} is the Kronecker δ, μ is the reduced mass, and m is the scale factor which we shall take to the mass of the lightest quark, i.e., m_u. We use the particle notations of Singh *et al.*[17] and particle symbols denoting their masses.

A. Masses of the mesons

In the case of mesons, m_1 and m_2 are the masses of the constituent quark and antiquark. Considering the ground-state vector and pseudoscalar mesons, i.e., taking $n=1$ and $L=0$, the masses of the different mesons can be obtained in terms of the quark masses and parameters E_{10} and V_1, viz.,

$$\rho = 2m + E_{10} + \tfrac{1}{4}V_1 ,$$

$$D_c^* = m + m_c + E_{10} + \tfrac{1}{4}V_1\left(\frac{m}{m_c}\right) ,$$

$$\psi = 2m_c + E_{10} + \tfrac{1}{4}V_1\left(\frac{m}{m_c}\right)^2 , \qquad (2.5)$$

$$\pi = 2m + E_{10} - \tfrac{3}{4}V_1$$

$$D_c = m + m_c + E_{10} - \tfrac{3}{4}V_1\left(\frac{m}{m_c}\right)$$

and similarly for others. The value of the parameters can be calculated from the known masses of K^* and K mesons and come out to be

$$E_{10} = -53 \text{ MeV}, \quad V_1 = 603 \text{ MeV} . \qquad (2.6)$$

We take quark masses as

$$m_u = m_d = 335 \text{ MeV}, \quad m_s = 510 \text{ MeV},$$
$$m_c = 1650 \text{ MeV}, \quad m_b = 5000 \text{ MeV}. \qquad (2.7)$$

Using these values we calculate the masses of charmed and b-quark mesons (Table I) which are compared with experimental values. We see that the calculated masses of ordinary and charmed mesons are in reasonable agreement with experiment, and the predicted mass of D_b^* (5.29 GeV) is in excellent agreement with the observed value 5.3 GeV.[6]

B. Masses of the baryons

Baryons consist of three quarks. Because of the tendency of quarks to gather into clusters which are triplet and antitriplet of color SU(3), there is a possibility of a diquark and a quark in a baryon. Ida and Kobayashi,[10] and Lichtenberg and Tassie[10] have already suggested such structure of baryons.

TABLE I. Masses of the mesons in GeV.

Particles	Present analysis	Experimental values
ρ	0.768	0.77
K^*		0.89 [a]
ϕ	1.03	1.02
D_c^*	1.97	2.01
F_c^*	2.13	2.14
ψ	3.03	3.09
D_b^*	5.29	5.3
F_b^*	5.46	
G_b^*	6.59	
Υ	9.94	9.5
π	0.16	0.14
K		0.49 [a]
D_c	1.84	1.86
F_c	2.05	2.03
D_b	5.25	
F_b	5.44	
G_b	6.58	
$(D_b^* - D_b)$	40 MeV	

[a] Input.

So, for a baryon, one can take m_1 and m_2 as the masses of a quark and a diquark in Eq. (2.4). Masses of the different possible ground-state diquark clusters can be obtained from the following expressions:

$$C_{(q_1q_2)}^{S=1} = m_{q_1} + m_{q_2} + \hat{E}_{10} + \tfrac{1}{4}\hat{V}_1(m^2/m_{q_1}m_{q_2}) \qquad (2.8)$$

and

$$C_{(q_1q_2)}^{S=0} = m_{q_1} + m_{q_2} + \hat{E}_{10} - \tfrac{3}{4}\hat{V}_1(m^2/m_{q_1}m_{q_2}) , \qquad (2.9)$$

where C represents the cluster (diquark) mass, S is the spin, and q_1 and q_2 are the quarks constituting the diquark. $\hat{E}_{10}$ (different from E_{10}) is the ground-state energy appropriate to two quarks in an antitriplet state. Assuming linear superpositions of different clusters for a given baryon in a manner of Ref. 9 and using $SU(10) \supset SU(5) \otimes SU(2)_{\text{spin}}$-invariant wave functions[17] and Eqs. (2.4), we can then obtain the masses of baryons in terms of the masses of the quarks, diquarks, and the parameters E_{10} and V_1'. We obtain

$$\begin{aligned}
N &= m + \tfrac{1}{2}C^1_{(uu)} + \tfrac{1}{2}C^0_{(ud)} + E_{10} - \tfrac{1}{2}V_1'\left(\frac{m}{C^1_{(uu)}}\right),\\
\Lambda &= \tfrac{2}{3}m + \tfrac{1}{3}m_s + \tfrac{1}{3}C^0_{(ud)} + \tfrac{1}{2}C^1_{(us)} + \tfrac{1}{6}C^0_{(us)}\\
&\quad + E_{10} - \tfrac{1}{2}V_1'\left(\frac{m}{C'_{(us)}}\right),\\
\Sigma &= \tfrac{2}{3}m + \tfrac{1}{3}m_s + \tfrac{1}{3}C^1_{(uu)} + \tfrac{1}{6}C^1_{(us)} + \tfrac{1}{2}C^0_{(us)}\\
&\quad + E_{10} - \tfrac{1}{6}V_1'\left(\frac{m}{C^1_{(us)}} + \frac{2m^2}{m_s C^1_{(uu)}}\right),\\
\Xi &= \tfrac{1}{3}m + \tfrac{2}{3}m_s + \tfrac{1}{3}C^1_{(ss)} + \tfrac{1}{6}C^1_{(us)} + \tfrac{1}{2}C^0_{(us)}\\
&\quad + E_{10} - \tfrac{1}{6}V_1'\left(\frac{2m}{C^1_{(ss)}} + \frac{m^2}{m_s C^1_{(us)}}\right),\\
\Delta &= m + C^1_{(uu)} + E_{10} + \tfrac{1}{2}V_1'\left(\frac{m}{C^1_{(uu)}}\right),\\
\Sigma^* &= \tfrac{2}{3}m + \tfrac{1}{3}m_s + \tfrac{1}{3}C^1_{(uu)} + \tfrac{2}{3}C^1_{(us)} + E_{10}\\
&\quad + \tfrac{1}{6}V_1'\left(\frac{m^2}{m_s C^1_{(uu)}} + \frac{2m}{C^1_{(us)}}\right),\\
\Xi^* &= \tfrac{1}{3}m + \tfrac{2}{3}m_s + \tfrac{2}{3}C^1_{(us)} + \tfrac{1}{3}C^1_{(ss)} + E_{10}\\
&\quad + \tfrac{1}{6}V_1'\left(\frac{2m^2}{m_s C^1_{(us)}} + \frac{m}{C^1_{(ss)}}\right),\\
\Omega &= m_s + C^1_{(ss)} + E_{10} + \tfrac{1}{2}V_1'\left(\frac{m^2}{m_s C^1_{(ss)}}\right)
\end{aligned} \tag{2.10}$$

and similarly for the baryons containing c and b quarks. We take V_1' different from V_1 (for meson) because the spin-spin interaction has a very short range so that the diquark does not look like a point particle to the spectator quark. However E_{10} is taken to be the same because interaction giving this energy includes the confining interaction and therefore has a long range.

Using the values of the quark masses and of the parameters

$$\hat{V}_1 = 195 \text{ MeV}, \quad \hat{E}_{10} = 131 \text{ MeV}, \quad \text{and} \quad V_1' = 490 \text{ MeV}, \tag{2.11}$$

which can be obtained from the known masses of the ordinary baryons, we predict the mass spectrum for $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons given in Table II. It can be seen that the predicted masses for strange and known charmed baryons are in good agreement with the recently observed values.[3,18]

TABLE II. Masses of the $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons. Values in parentheses are experimental values.

$\frac{1}{2}^+$ particles	Masses in GeV	$\frac{3}{2}^+$ particles	Masses in GeV
N	0.94[a]	Δ	1.23[a]
Λ	1.128 (1.116)	Σ^*	1.37 (1.38)
Σ	1.162 (1.19)	Ξ^*	1.52 (1.53)
Ξ	1.333 (1.32)	Ω	1.67 (1.67)
Σ_c	2.43 (2.45)	Σ_c^*	2.46 (2.48)
Ξ_c	2.55	Ξ_c^*	2.62
Ω_c	2.73	Ω_c^*	2.78
Ξ_{cc}	3.68	Ξ_{cc}^*	3.73
Ω_{cc}	3.87	Ω_{cc}^*	3.90
Λ_c'	2.30 (2.29)	Ω_{ccc}^*	5.04
Ξ_c'	2.49		
Σ_b	5.73	Σ_b^*	5.78
Ξ_b	5.92	Ξ_b^*	5.95
Ω_b	6.10	Ω_b^*	6.12
Ξ_{cb}	7.06	Ξ_{cb}^*	7.07
Ω_{cb}	7.23	Ω_{cb}^*	7.25
Ω_{ccb}	8.38	Ω_{ccb}^*	8.38
Ξ_{bb}	10.40	Ξ_{bb}^*	10.42
Ω_{bb}	10.58	Ω_{bb}^*	10.59
Ω_{cbb}	11.72	Ω_{cbb}^*	11.73
Λ_b'	5.62	Ω_{bbb}^*	15.08
Ξ_b'	5.88		
Ξ_{cb}'	7.04		
Ω_{cb}'	7.23		

[a] Input.

III. CONCLUSION AND DISCUSSION

According to lowest-order QCD, quarks have a tendency to cluster into configurations which belong to $\underline{3}$ or $\underline{3}^*$ multiplets of color SU(3). Lichtenberg and Johnson[9] have given a simplified quark-cluster model by considering the approximation that triplet clusters act like quarks and antitriplet clusters act like antiquarks. Extending the same considerations we have discussed the masses of charmed and b-quark hadrons. It was shown that the masses of ordinary and charmed mesons are in reasonable agreement with the experiment. The predicted mass value 5.29 GeV for D_b^* is also in excellent agreement with 5.3 GeV, recently observed at CERN.[6] We predict the mass difference $(D_b^* - D_b) = 40$ MeV, whereas the predictions of Ono[19] and Eichten[19] are 34 and 50 MeV, respectively.

In the case of $J^P = \frac{1}{2}^+$ baryons our values Λ_c'(2.30 GeV) and Σ_c(2.43 GeV) are in reasonable agreement with the recently observed values Λ_c'(2.29 GeV) and Σ_c(2.45 GeV) at CERN.[18] The $J^P = \frac{3}{2}^+$

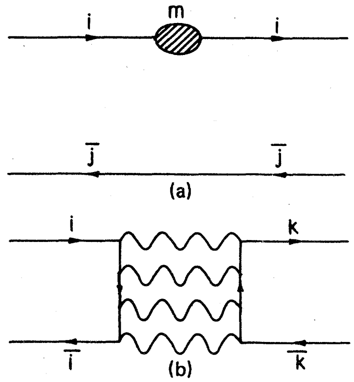

FIG. 1. (a) Meson-mass-matrix contribution due to quark masses. (b) Meson-mass contribution due to gluon annihilation.

baryon $\Sigma_c^*(2.46$ GeV) is also very close to the $\Sigma_c^*(2.48$ GeV) as observed in the photoproduction experiment[3] of Knapp *et al.*

Since, in strange and charm sectors, we obtain a remarkable agreement with the recent data except for the $c\bar{c}$ state, it is reasonable to assume that this model may work for hadrons which includes the b quark except for the $b\bar{b}$ state. A possible explanation for this disagreement in the case of $q\bar{q}$ states can be as follows.[20]

The gluons, which themselves carry energy (mass) and also give rise to diagonal and off-diagonal elements of the meson mass matrix due to transitions of the form $q_1q_1 \rightarrow$ gluons $\rightarrow q_2q_2$, are also contributing to the mass of the hadron. The effect of the gluon energy is usually assumed to be the same for all baryons with given L. For mesons $q_i\bar{q}_j$ with net flavor (i.e., $i \neq j$), the second diagram [Fig. 1(b)] makes no contribution to the mass matrix, for the gluons do not carry flavor. But for mesons with no net flavor ($i = j$) viz., $q\bar{q}(s\bar{s}, c\bar{c}, b\bar{b}\ldots)$, the second diagram [Fig. 1(b)] will also contribute. That may be the reason that such states do not fit well within the considered scheme as we have treated them at par with $q_i\bar{q}_j$ ($i \neq j$) states.

The validity of the model and the involved assumption of quark clustering for heavier hadrons will be further tested only in the future when more experimental information will be available.

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